ME 2016 - Computing Techniques
Fall 2018

Homework 2
Due Thursday, September 20 (at the end of class – hardcopy only)

Taylor series and truncation errors
Please BOX your final results

Problem 1: Textbook problem 4.5 **(20 points)**

4.5 Use zero- through third-order Taylor series expansions to predict $f(3)$ for

$$f(x) = 25x^3 - 6x^2 + 7x - 88$$

using a base point at $x = 1$. Compute the true percent relative error ε_t for each approximation.

Problem 2: **(20 points)**
Use the forward, backward and centered difference approximations derived in class to estimate the first derivative of the function considered in Problem 1 above. Evaluate the derivative at x = 2 using a step size of h = 0.2. Compare your results with the true value of the derivative by calculating the true percent relative error and comment on your results.

Problem 3: **(20 points)**
In this problem, you will derive an expression for the *centered* finite divided difference approximation of the *second* derivative:

$$f''(x_i) \cong \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$$

1) Write the 4th-order Taylor series expansions for $f(x_{i+1})$ and $f(x_{i-1})$ and manipulate them to obtain the above approximate expression for $f''(x_i)$ (*Hint:* I showed you a similar calculation in class when I derived the centered difference approximation for the first derivative).
2) What is the order of the error for this approximation? Justify your answer by using the results of the calculation that you performed in part 1.

Problem 4: **(40 points)**

The so-called MacLaurin series expansion for $\sin x$ is given by:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

1) Prove that this expansion is actually a special case of the Taylor series studied in class. *Hint:* write down the 5th order Taylor series for $\sin x$ (use equation (1) in my notes) and make a wise choice for the value of a.

2) Use the MacLaurin series for $\sin x$ to prove that the limit of $\frac{\sin x}{x}$ is 1 when $x \to 0$.

3) Write a MATLAB script named HW2_q3 to plot $\sin x$ for $-\pi \le x \le \pi$ (using increments of 0.1 for x); on the same plot, include the MacLaurin expansions for $\sin x$ using 1, 2, 3 and 4 terms over the same range. Use different line colors and include a legend to differentiate your plots.
4) Write a second Matlab script named HW2_q4 to compute sin(π) using its MacLaurin expansion with up to 10 terms, and generate a plot that displays the remainder R_n as a function of the number of terms.

What conclusions can you draw from your results to questions 3) and 4)?

Include a print-out of your scripts and of your plots along with a handwritten hard copy of your homework, but ***do not upload the scripts*** to T-square.

As usual, please make sure that your scripts are formatted according to the instructions and that your plots have descriptive titles and appropriate labels.