Reducing Dimensionality of Monthly Changes in US Treasury Securities Yields

– Shanshan Shangguan

Introduction

This analysis aims to reduce the dimensionality of a dataset containing monthly changes in US Treasury securities yields using principal component analysis (PCA). Specifically, I use data from the FRED online macroeconomic database, which includes six different series: GS1, GS2, GS3, GS5, GS7, and GS10.

Data Retrieval and Exploration

To begin, I used the FRED API to query and retrieve six series of monthly changes in US Treasury Securities yields by setting the "units" parameter in the URL to "chg". Figure 1 below shows the plot of these six series. All series follow a similar pattern but with varying magnitudes of variation. To test the correlation between the six series, I calculated the correlation matrix and plotted it as a heat map (Figure 2). The heat map shows significant correlations between each security and its neighboring securities.

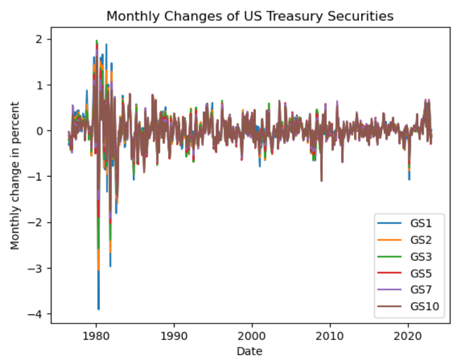


Figure 1

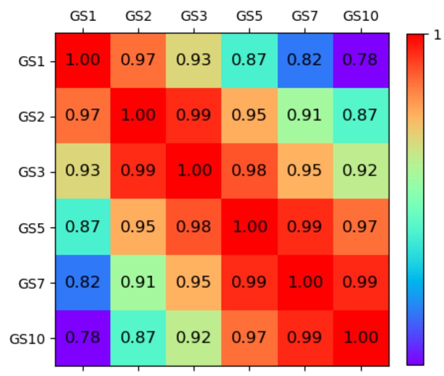


Figure 2

Data Preparation

Before applying PCA, we need to prepare the data by applying appropriate transformations. For time series data, it is recommended to calculate the differences between consecutive observations. This helps to eliminate trends or other patterns that might exist in the data, so that we can focus on the changes or fluctuations that are of interest. As the monthly changes of yields through FRED API are retrieved, this step is already fulfilled. Additionally, the data was scaled using the MinMaxScaler function from the scikit-learn library, ensuring that each series was on a comparable scale. This is essential for techniques like PCA that rely on measures of distance or similarity between data points. The data was scaled to [-1, 1] to preserve the signs of values.

Data Modeling

After preprocessing the data, I applied PCA to reduce its dimensionality. PCA is a common technique used in finance to identify the key drivers of variation and reveal the underlying structure of complex datasets. Since the dataset I obtained is dense, meaning it has few zero values, PCA was an appropriate choice for dimensionality reduction.

To decide how many principal components to retain, I plotted the cumulative explained variance against the number of components included (Figure 3). I included two principal components for my model because the cumulative variance barely increased after adding the first two. Also, the two components explained over 99% of the data variance. The two selected factors can capture almost all variations of the six securities' monthly changes. The plot of the transformed data (see Appendix) shows similar patterns as the original data, but it is important to note that the plots are not exactly the same because the transformed data is a linear combination of the original data after scaling.

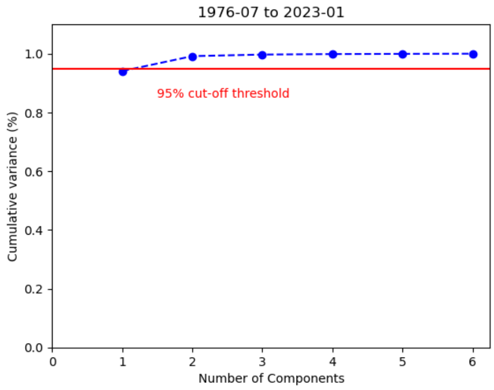


Figure 3

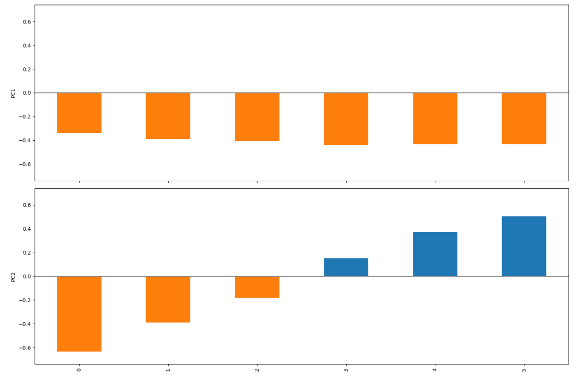


Figure 4

Interpretations

To interpret the reduced dimensions, I looked at the loadings of the original variables on each principal component (Figure 4). The loadings indicate the contribution of each original variable to the principal component, and can help us understand the underlying factors that drive changes in the data. The loadings of the two components are [-0.34 -0.39 -0.41 -0.44 -0.43 -0.43] and [-0.63 -0.39 -0.18 0.15 0.37 0.50], respectively. By using these two vectors, we can calculate the transformed data.

The first principal component, which explains over 94% of the total variance, all six series had relatively high loadings. This suggests that the first component represents a common trend or factor that is driving changes in all of the series and has significant effects on monthly changes of the six securities. I interpreted this as a general trend in US Treasury Securities yields and/or an indicator of overall market performance. The second component, which explains 5% of the total variance, represents a factor driving changes in the short-term and long-term Securities yields in opposite directions. The securities with longer maturities have higher loadings. I interpreted this as a flattening or steepening of the yield curve, with a smaller difference between short-term and long-term yields indicating a flatter curve.

Test Stability of the Model

To test the stability of the model, I split the dataset into five subsets with three years of overlapping periods. Then I applied PCA, with the number of components equal to two, separately to each subset. This approach is similar to the rolling window analysis commonly used for time series data. I observed that the loadings of the two principal components remained consistent across the subsets, and they agree with the loadings obtained using the entire dataset (see Appendix for statistics and graphs). Although the sign of the second factor's loading changes from one subset to another, the magnitudes of the loadings remain similar across subsets; this suggests that the overall importance of that series to the principal component has remained stable over time. I also tested the model by changing the number of subsets to 7 and 10, the loadings are consistent. Based on these results, I concluded that the PCA model was stable over time.

Conclusion and Next Steps

In conclusion, I applied PCA to reduce the dimensionality of a dataset of monthly changes in six US Treasury Securities yields. The analysis revealed that the first two principal components captured the majority of the variation in the data. I tested the stability of the result by applying PCA to subsets of the data and comparing the resulting principal components. Next steps for this analysis would include further exploring the relationship between the identified principal components and other economic factors to gain a deeper understanding of the underlying patterns in the data. Additionally, it would be worthwhile to explore other dimensionality reduction techniques, such as Independent Component Analysis (ICA) and Autoencoders, and compare their results. Overall, I believe PCA is useful and appropriate in this scenario, and further exploration can help identify key trends and inform investment decisions.

Appendix

Please be advised that this Notebook has been designed to offer a high level of automation, allowing for easy modification of data and variables. I have taken care to provide annotations detailing my thought process, as well as docstrings for key functions to facilitate your understanding and use of the code.

Import libraries and set constants

In [1]:

```
import json
import pandas as pd
import requests
import numpy as np
import matplotlib.pyplot as plt
from sklearn.preprocessing import MinMaxScaler
from sklearn.decomposition import PCA
```

In [2]:

```
# FRED_API_Key is a Python file I created, it includes my API key as a variable called "a
# Please replace the api_key with your own FRED API key when running the following code
import FRED_API_Key
api_key = FRED_API_Key.api_key

SERIES_ID_LIST = [1, 2, 3, 5, 7, 10] # the numbers after "GS" in the series ID
```

Data Retrieval

In [3]:

```
def get_6_series(api_key, units="lin"):
    """
    Retrieves the six U.S. Treasury securities data series using the FRED API.

    Parameters:
    - api_key (str): a string representing the FRED API key
    - units (str): a string representing the unit of observation of the data, default is

    Returns:
    - data (pd.DataFrame): a pandas DataFrame containing the retrieved data series and da
    """
    series_list = []
    for i in SERIES_ID_LIST:
        series_list.append(get_dataframe("GS{i}".format(i = i), api_key, units))

    res = series_list[0]
    for series in series_list[1:]:
        res = res.merge(series, on='date')

    return res


# Helper functions:
def get_response(series_id, api_key, units):
    url = "https://api.stlouisfed.org/fred/series/observations?" + \
          "series_id={series_id}&api_key={api_key}&units={units}&file_type=json"
    url_formatted = url.format(series_id = series_id, api_key = api_key, units = units)

    response = requests.get(url_formatted)
    if response.status_code != 200:
        raise Exception("Bad response from API, status code = {}".format(response.status_
    return response

def get_dataframe(series_id, api_key, units):
    response = get_response(series_id, api_key, units)
    data = pd.DataFrame(response.json()['observations'])[['date', 'value']].iloc[1:]\
```

```
            .assign(date = lambda cols: pd.to_datetime(cols['date']))\
            .assign(value = lambda cols: cols['value'].astype(float))\
            .rename(columns = {'value': series_id})

    return data
```

In [4]:

```
# The function get_6_series can retrieve the six series data from FRED,
# the unit is 'chg', meaning we are asking for the monthly changes of the six series
data = get_6_series(api_key, 'chg')
data
```

Out[4]:

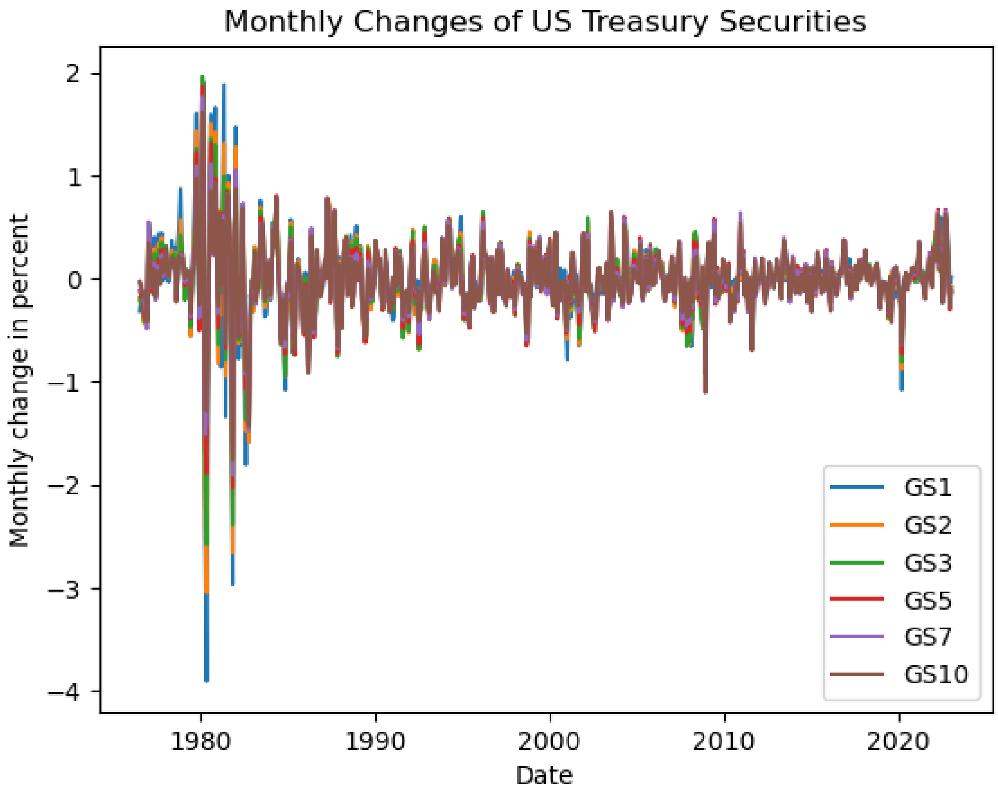
	date	GS1	GS2	GS3	GS5	GS7	GS10
0	1976-07-01	-0.32	-0.21	-0.19	-0.12	-0.05	-0.03
1	1976-08-01	-0.20	-0.22	-0.26	-0.18	-0.12	-0.06
2	1976-09-01	-0.16	-0.21	-0.20	-0.18	-0.17	-0.18
3	1976-10-01	-0.34	-0.44	-0.42	-0.38	-0.25	-0.18
4	1976-11-01	-0.21	-0.17	-0.15	-0.23	-0.30	-0.12
...	...	...	...	...	...	...	...
554	2022-09-01	0.61	0.61	0.65	0.67	0.66	0.62
555	2022-10-01	0.54	0.52	0.50	0.48	0.45	0.46
556	2022-11-01	0.30	0.12	-0.04	-0.12	-0.10	-0.09
557	2022-12-01	-0.05	-0.21	-0.29	-0.30	-0.27	-0.27
558	2023-01-01	0.01	-0.08	-0.14	-0.12	-0.13	-0.09

559 rows × 7 columns

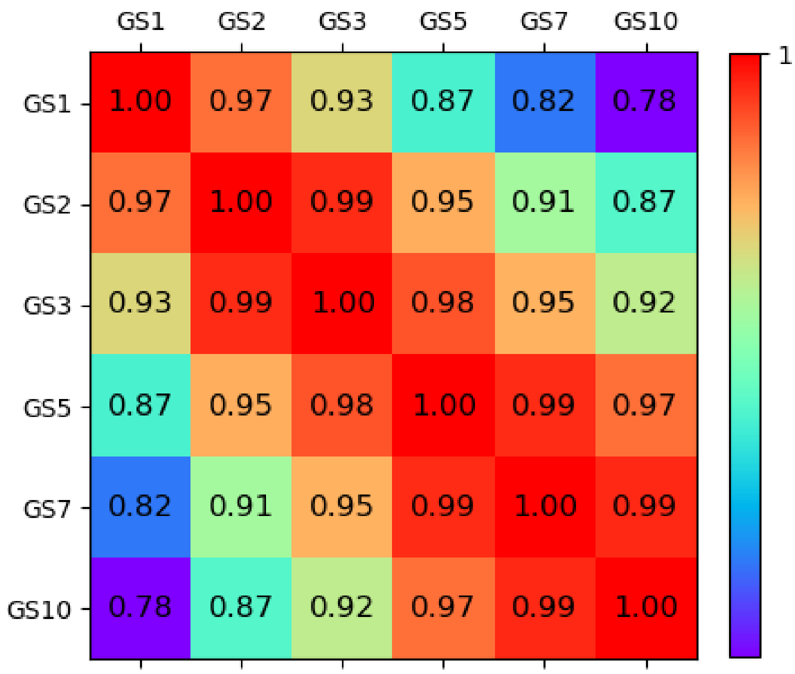
Data Exploration

In [5]:

```
fig, ax = plt.subplots()

# plot each series on the same graph
for col in data.columns[1:]:
    ax.plot(data['date'], data[col], label=col)

# set axis labels and legend
ax.set_xlabel('Date')
ax.set_ylabel('Monthly change in percent')
plt.title('Monthly Changes of US Treasury Securities')
ax.legend()
plt.show()
```



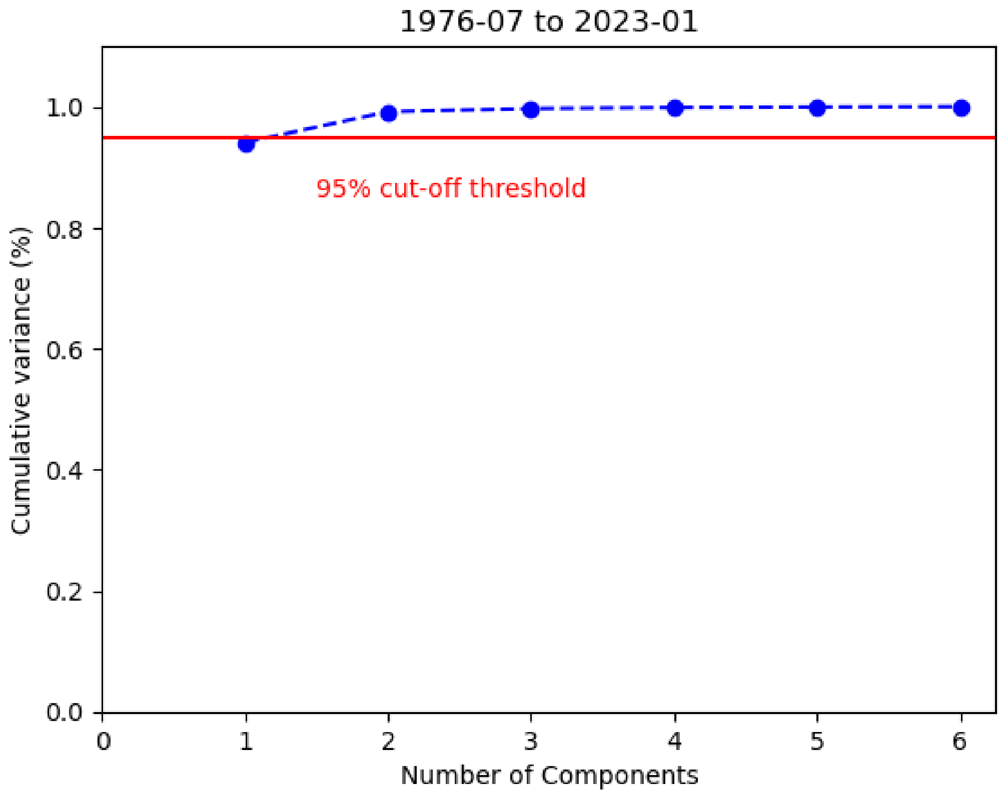
In [6]:
```
# Correlation matrix of the monthly changes of the securities yields

# np.corrcoef method views each row of the data as a variable, each column as an observat
# We need to take the transpose of data
corr_mat = np.corrcoef(data[data.columns[1:]].T)
img = plt.matshow(corr_mat, cmap=plt.cm.rainbow)
plt.colorbar(img, ticks = [-1, 0, 1], fraction=0.045)

# Set tick labels
plt.xticks(range(len(SERIES_ID_LIST)), ["GS" + str(x) for x in SERIES_ID_LIST])
plt.yticks(range(len(SERIES_ID_LIST)), ["GS" + str(x) for x in SERIES_ID_LIST])

# Add correlation values to plot
for x in range(corr_mat.shape[0]):
    for y in range(corr_mat.shape[1]):
        plt.text(x, y, "%0.2f" % corr_mat[x,y], size=12, color='black', ha="center", va="

plt.show()
```



PCA on the entire dataset

In [7]:

```
# Then we need to choose how many components to include in the PCA model

def select_num_pc(data):
    scaler = MinMaxScaler(feature_range=(-1, 1))
    data_pca = scaler.fit_transform(data[data.columns[1:]])
    pca = PCA().fit(data_pca)

    xi = np.arange(1, 7, step=1)
    y = np.cumsum(pca.explained_variance_ratio_)

    plt.ylim(0.0,1.1)
    plt.plot(xi, y, marker='o', linestyle='--', color='b')

    plt.xlabel('Number of Components')
    plt.xticks(np.arange(0, 7, step=1))
    plt.ylabel('Cumulative variance (%)')
    plt.title(data.iloc[0]['date'].strftime('%Y-%m') + ' to ' + \
              data.iloc[-1]['date'].strftime('%Y-%m'))

    plt.axhline(y=0.95, color='r', linestyle='-')
    plt.text(1.5, 0.85, '95% cut-off threshold', color = 'red', fontsize=10)

    ax.grid(axis='x')


select_num_pc(data)
```

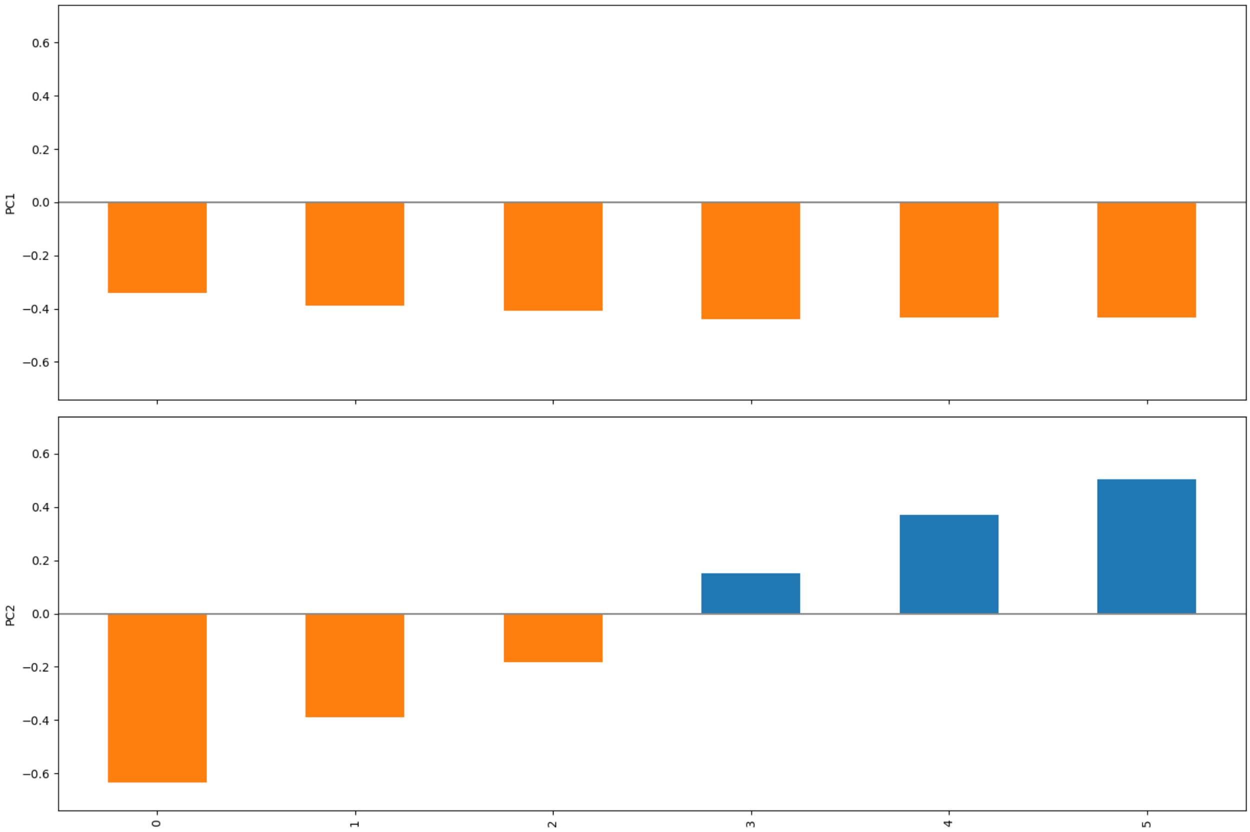


In [8]:

```
# n_components is the number of components we want to include for our model

def plot_loadings(data, n_components):
    """Plot the loadings of principal components from a given DataFrame.

    Parameters:
    - data (pd.DataFrame): The input data, which should include a 'date' column and
      columns for the securities to be analyzed.
    - n_components (int): The number of principal components to use for the analysis.

    Returns:
    - None

    This function applies PCA to the input data and plots the loadings of the first
    `n_components` principal components. The loadings show the correlations between
    the original securities and the principal components.

    The function first scales the data to the range [-1, 1] using MinMaxScaler. It then
    applies PCA using the specified number of components and calculates the explained
    variance ratio and singular values. Finally, it plots the loadings of each
    principal component using a bar chart.
    """
    scaler = MinMaxScaler(feature_range=(-1, 1))
    data_pca = scaler.fit_transform(data[data.columns[1:]])

    pca = PCA(n_components=n_components)
    opt_model = pca.fit(data_pca)

    print("Date Range: ", data.iloc[0]['date'].strftime('%Y-%m') + ' to ' + \
           data.iloc[-1]['date'].strftime('%Y-%m'))
    print("Singular Values: ", opt_model.singular_values_)
    print("Variance Explained by Each PC: ", opt_model.explained_variance_ratio_)
    print("Total Variance Explained: ", np.sum(opt_model.explained_variance_ratio_))

    loadings = pd.DataFrame(pca.components_, columns=data_pca.dtype.names)
    print("Loadings: ", '\n', loadings)

    maxPC = 1.01 * np.max(np.max(np.abs(loadings), axis=0), axis=0)
    f, axes = plt.subplots(n_components, 1, figsize=(15, 10), sharex=True)
    for i, ax in enumerate(axes):
        pc_loadings = loadings.loc[i, :]
        colors = ['C0' if l > 0 else 'C1' for l in pc_loadings]
```

```python
        ax.axhline(color='#888888')
        pc_loadings.plot.bar(ax=ax, color=colors)
        ax.set_ylabel(f'PC{i+1}')
        ax.set_ylim(-maxPC - 0.1, maxPC + 0.1)
    plt.tight_layout()
    plt.show()


plot_loadings(data, 2)
```

```
Date Range:  1976-07 to 2023-01
Singular Values:  [9.39243945 2.18025805]
Variance Explained by Each PC:  [0.94102199 0.05070593]
Total Variance Explained:  0.9917279249899535
Loadings:
          0         1         2         3         4         5
0 -0.341039 -0.388491 -0.407591 -0.439411 -0.431493 -0.432861
1 -0.634561 -0.389532 -0.184457  0.150289  0.369107  0.502739
```



In [ ]:

Now we can transform the data into the two principal components

In [9]:

```python
scaler = MinMaxScaler(feature_range=(-1, 1))
data_pca = scaler.fit_transform(data[data.columns[1:]])
pca = PCA(n_components=2)
opt_model = pca.fit(data_pca)
data_transformed = pd.DataFrame(opt_model.transform(data_pca)).rename({0: "PC1", 1: "PC2"

# data_trans_date is the dataframe of transformed data with the date column
data_trans_date = pd.DataFrame(data.date).join(data_transformed)
data_trans_date
```

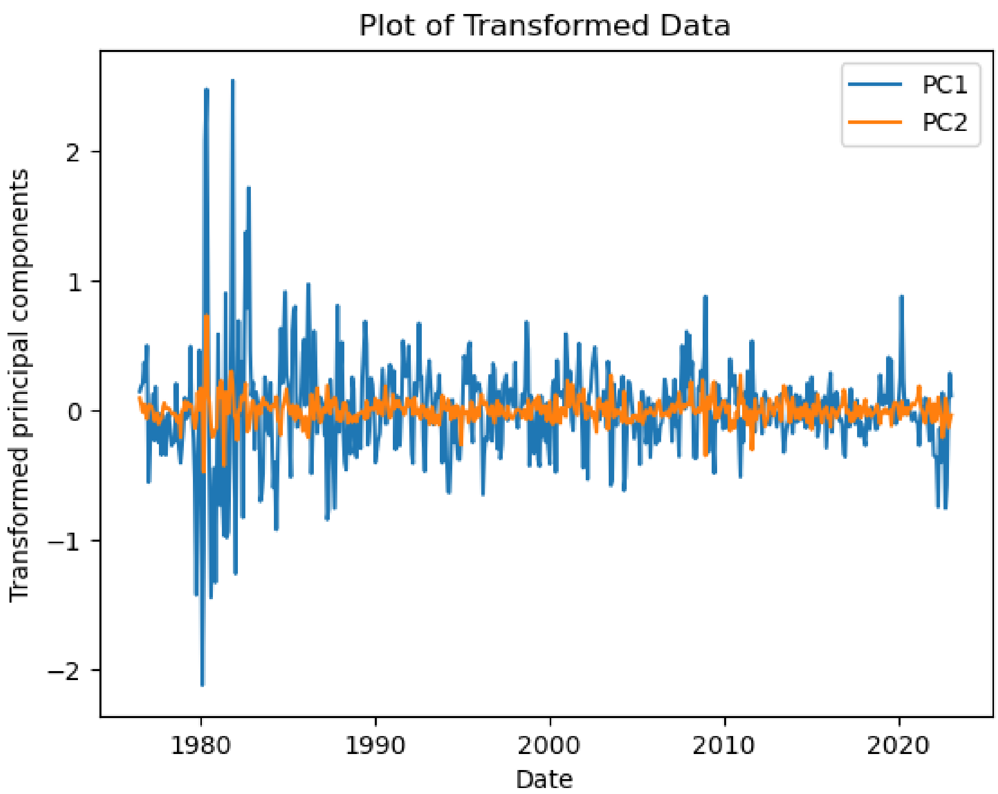
Out[9]:

	date	PC1	PC2
0	1976-07-01	0.143469	0.092325
1	1976-08-01	0.181236	0.045701
2	1976-09-01	0.206788	-0.015341
3	1976-10-01	0.367373	0.046363
4	1976-11-01	0.223878	-0.026863
...	...	...	...
554	2022-09-01	-0.757139	0.090176
555	2022-10-01	-0.574510	0.027103
556	2022-11-01	0.019330	-0.134983
557	2022-12-01	0.283779	-0.088295
558	2023-01-01	0.109652	-0.038194

559 rows × 3 columns

In [10]:

```
# plot of the transformed data
fig, ax = plt.subplots()

ax.plot(data_trans_date['date'], data_trans_date["PC1"], label="PC1")
ax.plot(data_trans_date['date'], data_trans_date["PC2"], label="PC2")

# set axis labels and legend
ax.set_xlabel('Date')
ax.set_ylabel('Transformed principal components')
plt.title('Plot of Transformed Data')
ax.legend()
plt.show()
```



In [ ]:

Test the stability of the model

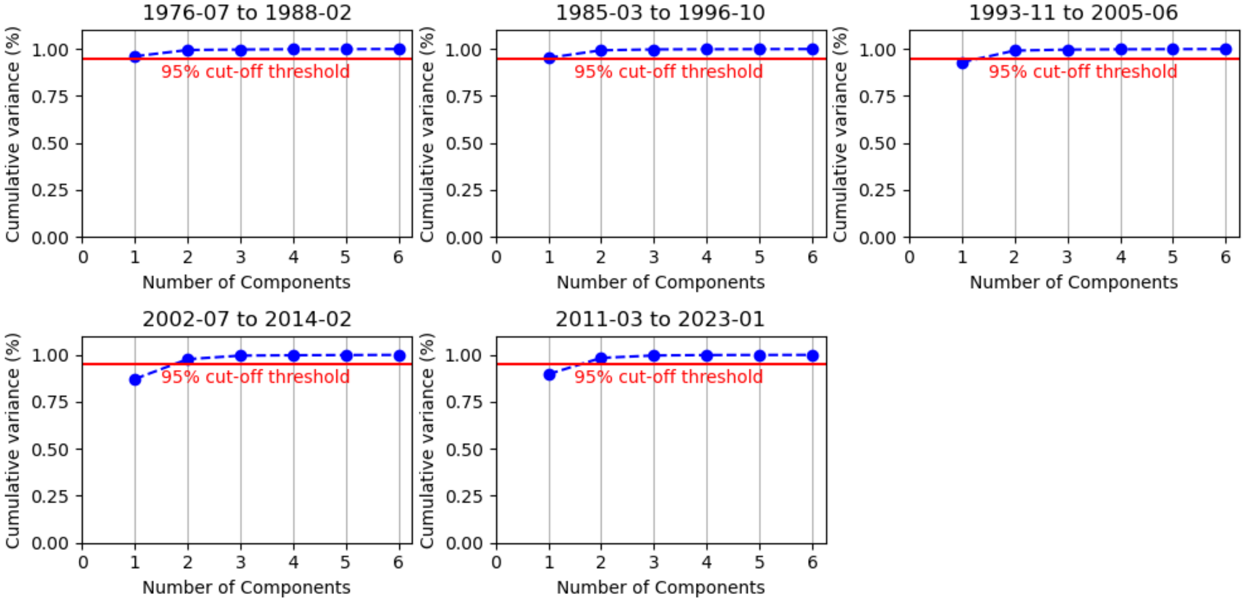
In [11]:
```
def split_data(data, num_subsets, overlap):
    """
    Split the given data into a specified number of subsets with overlaps.

    Parameters:
    - data (pandas.DataFrame): The input data to be split.
    - num_subsets (int): The number of subsets to create.
    - overlap (int): The number of overlapping observations(months) between each subset.

    Returns:
    - List of pandas.DataFrame: A list containing the split subsets of the input data.
    """
    subset_len = (data.shape[0] + 1 - overlap) // num_subsets
    data_list = []
    for i in range(0, num_subsets):
        if i == 0:
            subset = data.iloc[:subset_len + overlap]
        elif i == num_subsets - 1:
            subset = data.iloc[i * subset_len:]
        else:
            subset = data.iloc[i * subset_len : (i + 1) * subset_len + overlap]
        data_list.append(subset)
    return data_list

# The following line splits the data into 5 subsets with 36 months of overlap
subset_list = split_data(data, 5, 36)
```

In [12]:
```
for i in subset_list:
    print(i.shape) # shape (number of rows, number of columns) of each subset
```

```
(140, 7)
(140, 7)
(140, 7)
(140, 7)
(143, 7)
```

After we obtain the 5 subsets, we can plot the cumulative variance explained versus number of principle components chosen for each subset. The title of the each plot is the date range of the subset. For all subsets, the number of components chosen should be two based on the graphs below.

In [13]:
```
%matplotlib inline
fig = plt.figure(figsize=(10, len(subset_list)))
for i in range(0, len(subset_list)):
    ax = fig.add_subplot((len(subset_list) + 2) // 3, 3, i+1)
    subset = subset_list[i]
    # call the previous function select_num_pc
    select_num_pc(subset)

plt.tight_layout()
plt.show()
```

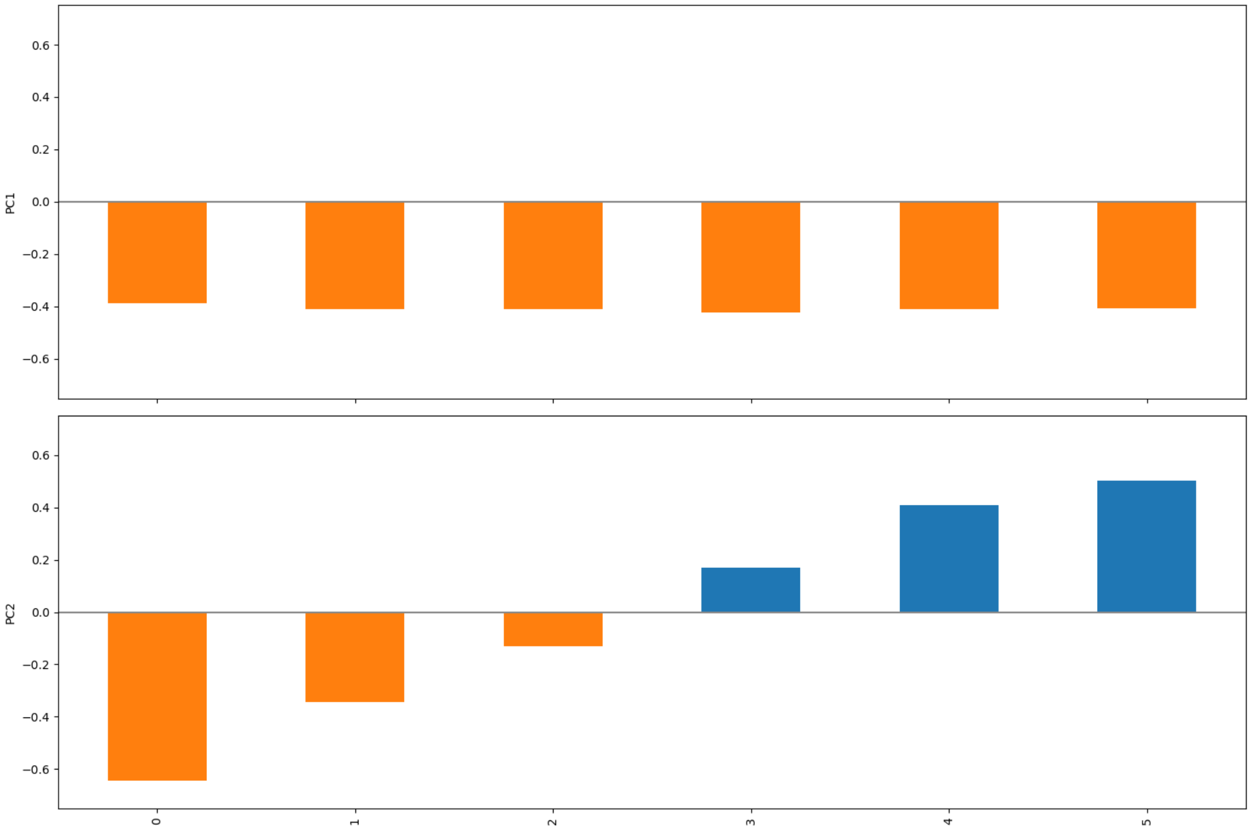


In [ ]:

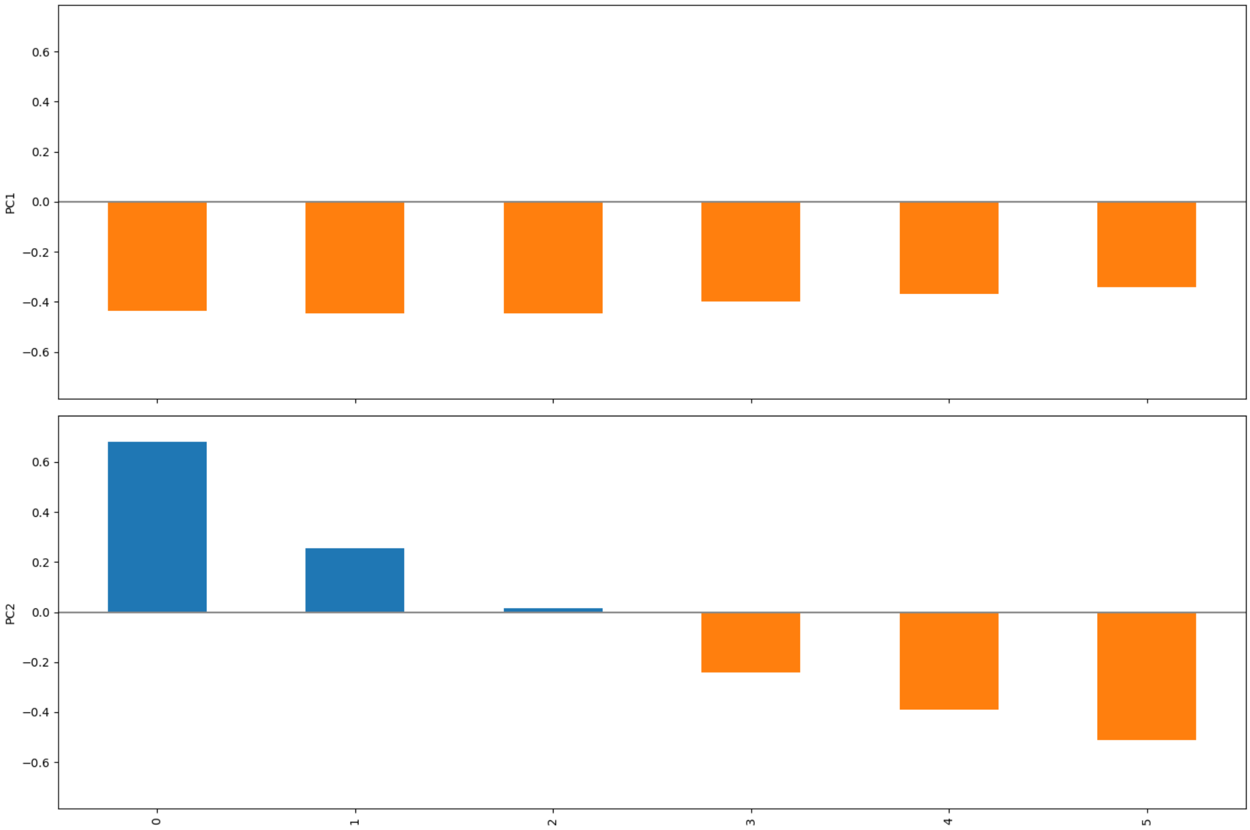
Note: although the loadings of the second PC have opposite signs, their magnitudes agree with each other. We can conclude that the PCA model is consistent.

In [14]:
```
for subset in subset_list:
    plot_loadings(subset, 2)
```

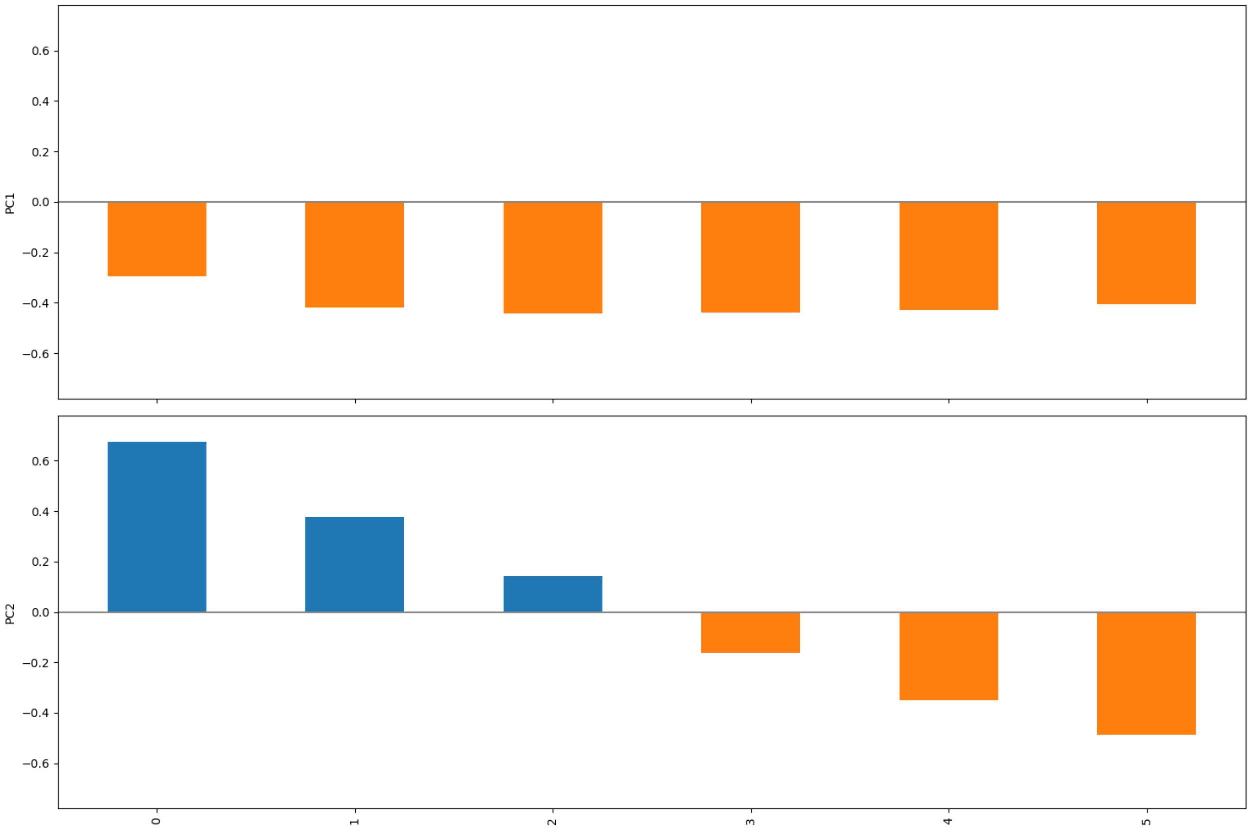
```
Date Range:  1976-07 to 1988-02
Singular Values:  [7.80718811 1.45774034]
Variance Explained by Each PC:  [0.96138852 0.03351737]
Total Variance Explained:  0.9949058941525208
Loadings:
          0         1         2         3         4         5
0 -0.387684 -0.409683 -0.410761 -0.423121 -0.409441 -0.407999
1 -0.645290 -0.342917 -0.131513  0.170779  0.409423  0.501917
```



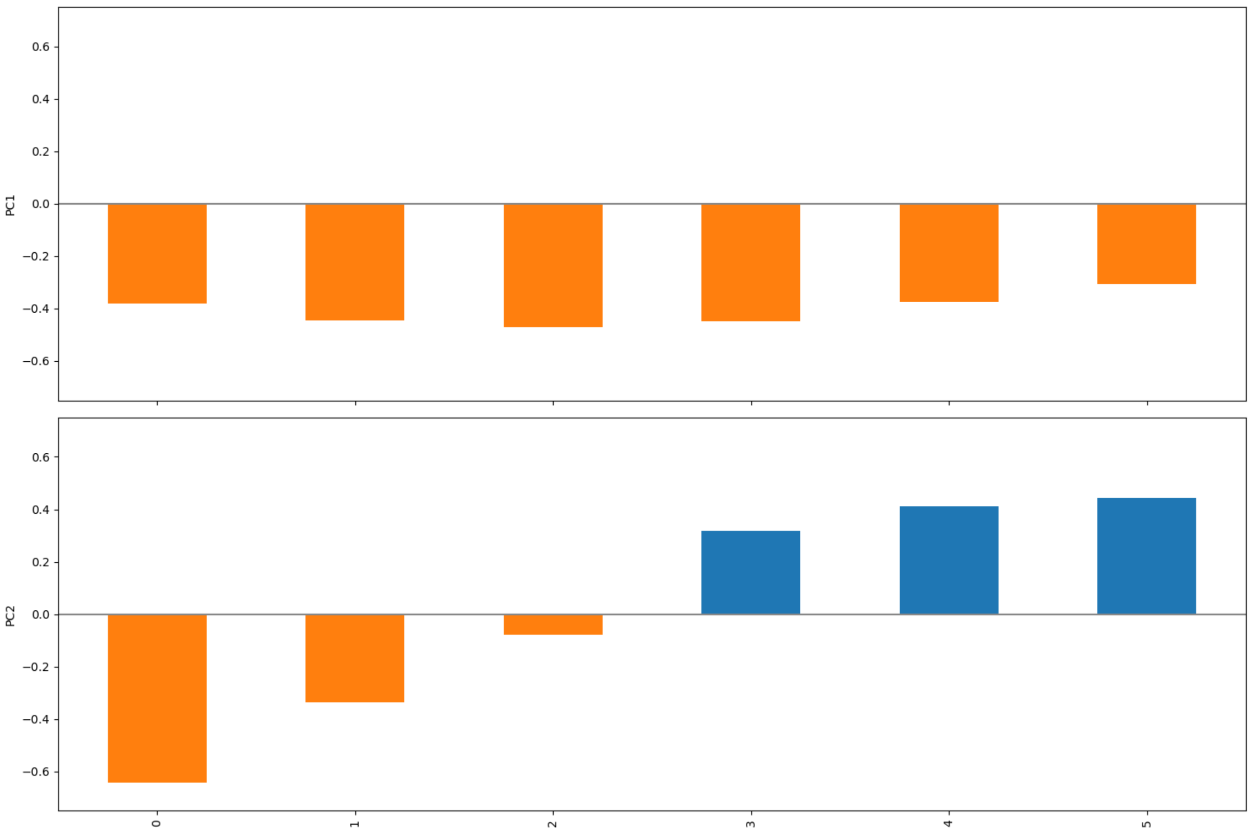
```
Date Range:  1985-03 to 1996-10
Singular Values:  [11.05095002  2.26714921]
Variance Explained by Each PC:  [0.95332303 0.04012371]
Total Variance Explained:  0.9934467403973184
Loadings:
          0         1         2         3         4         5
0 -0.434501 -0.447168 -0.447179 -0.397278 -0.368436 -0.343083
1  0.679816  0.253955  0.015392 -0.242194 -0.391735 -0.510887
```



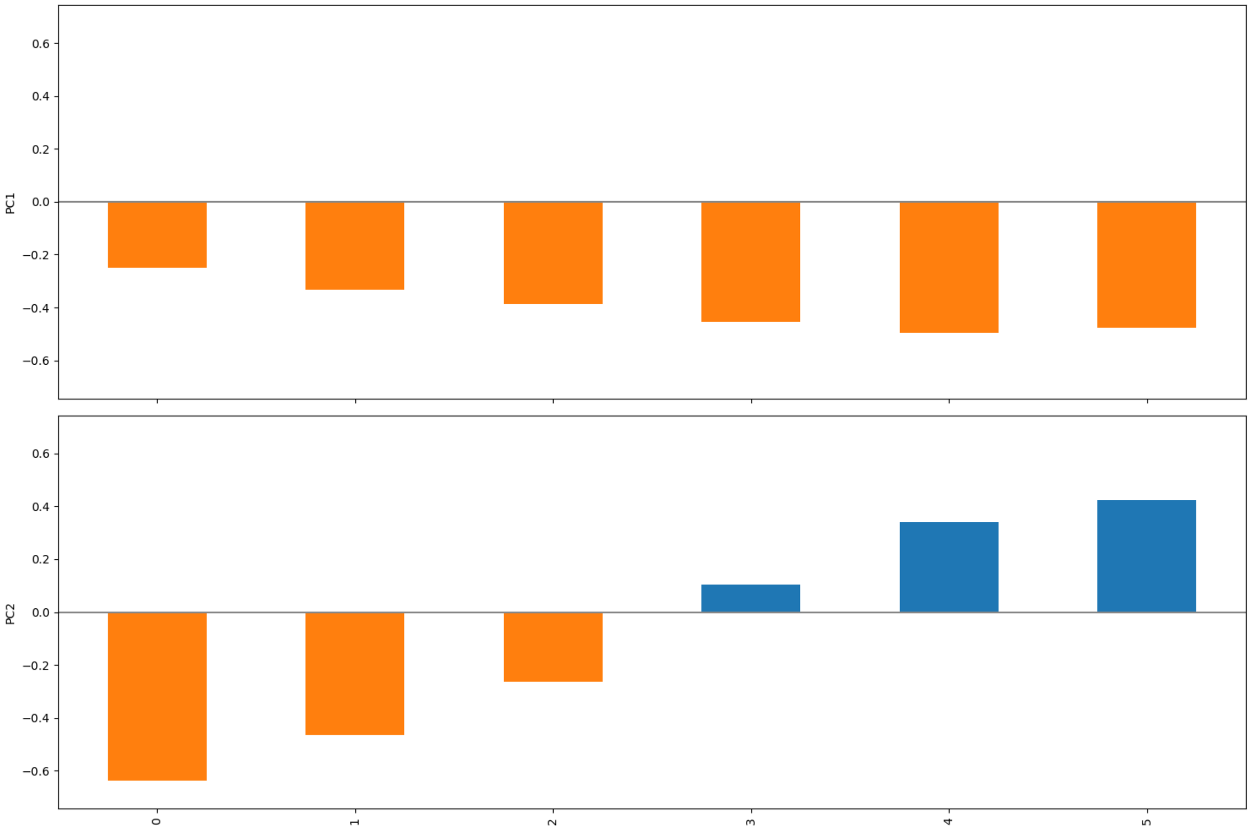

```
Date Range:  1993-11 to 2005-06
Singular Values:  [11.11463911  2.87494653]
Variance Explained by Each PC:  [0.9295511  0.06219308]
Total Variance Explained:  0.991744184514962
Loadings:
          0         1         2         3         4         5
0 -0.293906 -0.418465 -0.441882 -0.440641 -0.428105 -0.407195
1  0.673255  0.377332  0.142649 -0.162936 -0.348520 -0.485783
```




```
Date Range:  2002-07 to 2014-02
Singular Values:  [8.89686874 3.09724097]
Variance Explained by Each PC:  [0.87156935 0.10562764]
Total Variance Explained:  0.9771969924475125
Loadings:
           0        1         2         3         4         5
0 -0.380053 -0.44669 -0.470779 -0.447731 -0.375213 -0.305200
1 -0.644716 -0.33549 -0.077736  0.317119  0.411867  0.442205
```



```
Date Range:  2011-03 to 2023-01
Singular Values:  [6.98261986 2.16796788]
Variance Explained by Each PC:  [0.8973518  0.08650309]
Total Variance Explained:  0.983854888905102
Loadings:
          0         1         2         3         4         5
0 -0.249820 -0.333474 -0.387356 -0.453024 -0.494408 -0.476099
1 -0.638121 -0.465467 -0.264471  0.104058  0.339821  0.424134
```



In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]: