Week 4: 27/10/2021 – Wednesday

1. Outline of meeting

The meeting was held on Zoom. The first half of the meeting was dedicated to bug-fixing the code from the previous week and correct any misunderstandings from the topic. The second half of the meeting was dedicated to the Mattis Bardeen Theory. A discussion was had regarding the topic and a careful elaboration to the best of my understanding was given. An outline of the Mattis Bardeen Theory will be elaborated in section 3 of this diary. Essentially, Mattis Bardeen theory is derived from the fundamental principles of superconductivity and takes the band gap into consideration. From this, the task that arose from this is to create the plots of L_{int} vs T using Mattis Bardeen Approximations to find the L_{int} from the band gap energy. Then, the resistive part of the impedance can be found, and a plot of R vs T can be made. The outline of the theory and task specification is given below.

2. Specification of Tasks

i) Plot out L_{int} over a temperature range of 0.05 – 4 K from Mattis Bardeen Approximations using equation 3.33 for a superconducting film with the following properties:

- Normal state conductivity: $\sigma_n = 6.0 \times 10^7$
- Thickness: $t = 20 \times 10^{-9}\, m$
- Critical temperature: $T_c = 1.5\, K$
- Frequency: $f = 500 \times 10^6\, Hz$

ii) Use equation 3.38 to calculate R as a function of temperature

3. Outline of Theory and Methodology for Task

Mattis Bardeen Theory: *"The London Equations derived in previous weeks hold well but they are not derived from any fundamental principles of superconductivity and does not take into account the idea of a band gap. Another assumption of the London Model is that it assumes that the electrons in the superconducting state are just simply electrons which do not scatter and will all be accelerated independently if an electric field is applied."*

Source: Lumped Element Kinetic Inductance Detector – Dr. Simon Doyle Thesis

Building from Mattis Bardeen Theory, the full effects of the band gap and non-local treatment of Cooper pairs leads to the Mattis Bardeen Integrals:

$$\frac{\sigma_1}{\sigma_n} = \frac{2}{\hbar\omega}\int_{\Delta}^{\infty}[f(E) - f(E+\hbar\omega]g(E)dE + \frac{1}{\hbar\omega}\int_{\Delta-\hbar\omega}^{\Delta}[1 - f(E+\hbar\omega)]g(E)dE$$

$$\frac{\sigma_2}{\sigma_n} = \frac{1}{\hbar\omega}\int_{\Delta-\hbar\omega,-\Delta}^{\Delta}\frac{[1-2f(E+\hbar\omega)][E^2+\Delta^2+\hbar\omega E]}{[\Delta^2 - E^2]^{\frac{1}{2}}[(E+\hbar\omega)^2 - \Delta^2]^{\frac{1}{2}}}$$

To simplify, the integrals can be approximated when in the limits $k_B T << \Delta(0)$ and $\hbar\omega << \Delta(0)$ to the Mattis Bardeen Approximations:

$$\frac{\sigma_1}{\sigma_n} = \frac{2\Delta(\mathrm{T})}{\hbar\omega}\exp(\left(-\frac{\Delta(0)}{\mathrm{k_B T}}\right)\mathrm{K_0}(\frac{\hbar\omega}{2\mathrm{k_B T}})[2\sinh(\frac{\hbar\omega}{2\mathrm{k_B T}})]$$

$$\frac{\sigma_2}{\sigma_n} = \frac{\pi\Delta(\mathrm{T})}{\hbar\omega}[1 - 2\exp(\left(-\frac{\Delta(0)}{\mathrm{k_B T}}\right)\exp\left(\frac{-\hbar\omega}{2\mathrm{k_B T}}\right)\mathrm{I_0}(\frac{\hbar\omega}{2\mathrm{k_B T}})]$$

Where Δ is the band gap energy, I_0 and k_0 are modified Bessel functions of the first and second kind respectively. $\Delta(\mathrm{T})$ in this case was approximated to $\Delta(0)$ since $T \cong 0$.

Following this, the London Penetration depth can be found by first determining the electron density n_s using the following relation:

$$n_s = \sigma_s \omega m_e / e^2$$

Where $\sigma_s = \sigma_2$ in this case. Using the result, the Penetration depth can be found:

$$\lambda_{MB} = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

Plugging λ into the expression for total internal inductance:

$$L_{int} = \frac{\mu_0 \lambda}{2}\coth\left(\frac{t}{2\lambda}\right)$$

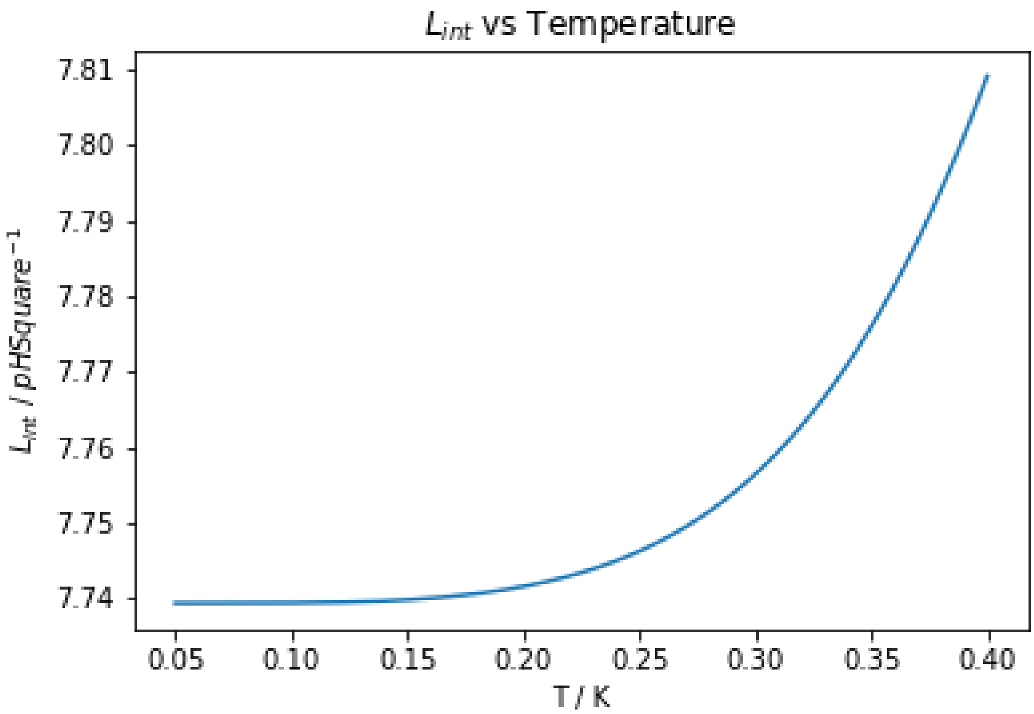
The graph of L_{int} vs T can be plotted.

Using the expression for the resistive part from equation 3.38 in the thesis:

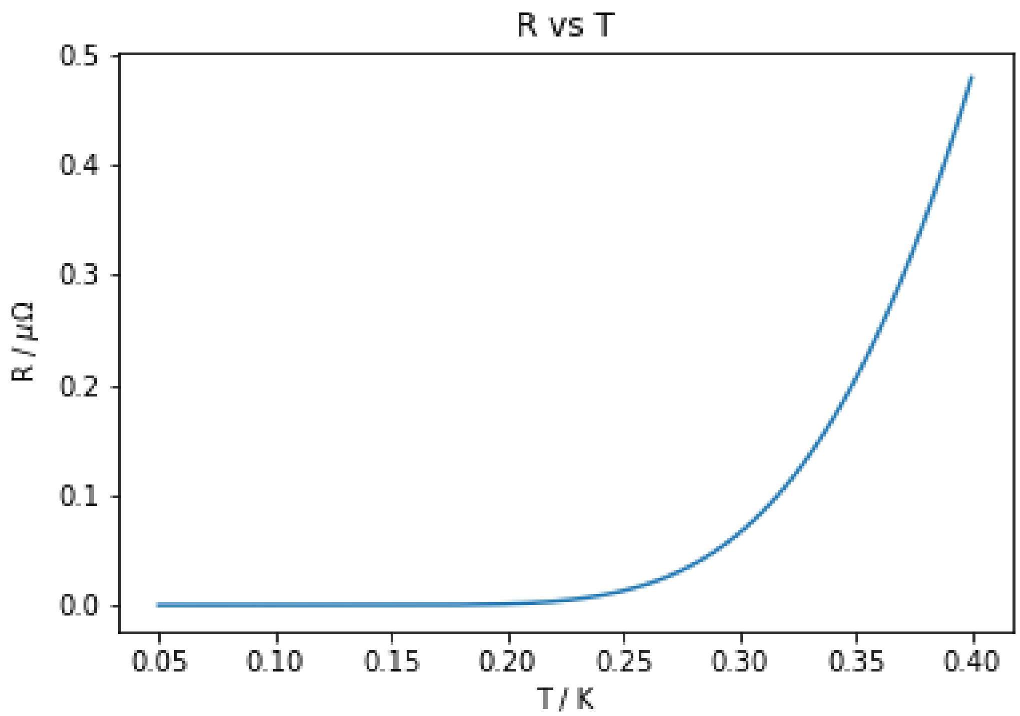
$$R = L_k\, \omega \frac{\sigma_1}{\sigma_2}$$

A plot of R vs T can also be made. The plots and code for the tasks are shown in the following sections.

4. Plot of L_{int} vs T Using Mattis Bardeen Approximations



5. Plot of R vs T Using Mattis Bardeen Approximations



6. Code for Calculating L_{int} and R

```
#imports
import numpy as np
```

```
import matplotlib.pyplot as plt
from scipy.special import iv as I0
from scipy.special import kv as K0
import scipy.constants as const

#Define function
def coth(x):
    return np.cosh(x)/np.sinh(x)

def csch(x):
    return 1/np.sinh(x)

#Define constants
TC = 1.5
Delta_0 = (3.5*const.Boltzmann*TC)/2
sigma_n = 6.0e7         # Normal stae conductvity if superconducting film
Thick = 20e-9           # Thickness of superconducting fil
f = 500e6
w = 2 * np.pi * f
me = const.m_e
miu_0 = 4*np.pi*10**-7

#Varying range of temperature
T = np.linspace(0.05, 0.4, num=500)

#An interpolation formula for delta_T (Cheating a bit by using an interpolation formula, ideally
should be integrated)
#Source: https://physics.stackexchange.com/questions/192416/interpolation-formula-for-bcs-
superconducting-gap#mjx-eqn-eq2
delta_T = Delta_0*np.tanh(1.74*np.sqrt((TC/T)-1))

#Define constants to simplify eqn
multiplying_constant = delta_T/(const.hbar * w)
e_const_1 = - Delta_0/(const.Boltzmann*T)
e_const_2 = (const.hbar*w)/(2*const.Boltzmann*T)

#Parts of the sigma1 Ratio
A = 2*multiplying_constant
B = np.exp(e_const_1)
C = K0(0, e_const_2)
D = 2*(np.sinh(e_const_2))

#Find Sigma 1 and Sigma 2
sigma1Ratio = A * B * C * D
sigma2Ratio = np.pi*multiplying_constant*(1 - (2*np.exp(e_const_1)*np.exp(-
e_const_2)*I0(0,e_const_2)))
sigma2 = sigma2Ratio * sigma_n
sigma1 = sigma1Ratio * sigma_n
```

```
# Sanity Check
# plt.subplot(2,1,1)
# plt.plot(T, sigma1Ratio)
# plt.ylabel("$\sigma_{1}/\sigma_n$")
# plt.yscale("log")
# plt.subplot(2,1,2)
# plt.plot(T, sigma2Ratio)
# plt.ylabel("$\sigma_{2}/\sigma_n$")
# plt.show()
# plt.figure()


#Depth
lower_fraction = miu_0*sigma2*w
Lambda_T_MB = (1/lower_fraction)**0.5

#Internal Inductance
fraction = Thick/(2*Lambda_T_MB)
L_int = (miu_0*Lambda_T_MB/2)*coth(fraction)
plt.plot(T, L_int*10e12)
plt.ylabel("$L_{int}$ / $pHSquare^{-1}$")
plt.xlabel("T / K")
plt.title("$L_{int}$ vs Temperature")
plt.savefig("L_int vs T Mattis Bardeen")
plt.figure()

#sigma 1 to sigma 2
sigma12Ratio = sigma1/sigma2

#Terms for lk
A = (miu_0*Lambda_T_MB)/4
B = coth(fraction)
C = fraction*(csch(fraction))**2

#R vs T
lk = A*(B+C)
R = lk * w * sigma12Ratio
plt.title("R vs T")
plt.plot(T, R*10**6)
plt.ylabel("R / $\mu\Omega$")
plt.xlabel("T / K")
plt.savefig("R vs T Mattis Bardeen")
```