<u>Week 1: 6/10/2021 – Wednesday</u>

1. **Outline of meeting and tasks**

Meeting with supervisor, Dr. Simon Doyle, on Zoom to discuss the details of the project and to give a brief introduction to the project along with people related to the research such as Dr Sam Rowe and Dr Tom Brien who are part of the research team for this project topic and will assist me.

A brief introduction to the project was provided. In this project, we will be working on understanding the principle of superconductivity and its microwave properties, understand how superconductivity principle can be applied to create a kinetic inductance detector (KID) and thus using the above knowledge to characterize a detector array. The real-world application of the detector would be in an SFAB airport security camera. The reading material provided is Dr. Simon Doyle's Thesis titled "Lumped Element Kinetic Inductance Detectors"

The inductance of a superconducting material forms the operating basis of the detector, hence the name Kinetic Inductance Detector. As such, the task for this week consists of going through the topics related to the theory of superconductivity and how the kinetic inductance of a material can come about due to it. The recommended literature for this is the thesis mentioned above from Chapter 3.1 to Chapter 3.4, page 13-25.

2. **Notes and Materials Covered**

Not much was covered in terms of project material. General introductory session.

Week 2: 13/10/2021 – Wednesday

1. Outline of meeting and tasks

Prior to the meeting on the 12th, I had the opportunity to visit the camera in North Building on the lower ground floor. Tom Brien explained the briefly basics of the detector and how the 2 stage-active cooling of camera allowed the temperature to remain in the ranges of mK.

The meeting with my supervisor was to discuss the aims and objectives of the project and to complete the "Aims and Objectives and safety overview" sheet for submission. The discussed aims and title are shown in the following section. The risk assessment and safety section of the sheet was also discussed.

After this, seeing as there was time, my supervisor proceeded with explaining the topic of superconductivity and how it relates to the detector. The topics covered and further reading material is given in the final section of this week's diary.

My supervisor also provided the literature to read for a better understanding of the topic. The literature given was: "Lumped Element Kinetic Inductance Detectors" by Dr. Simon Doyle, and the recommended section to read was Chapter 3.1 to 3.4, page 13-25. Further outline of the materials elaborated in the final section of this diary.

2. Aims and Objectives Sheet

The title of the project is ***Characterizing Arrays of Kinetic Inductance Detectors.*** The Kinetic Inductance Detectors (KIDs) as mentioned will be used in an SFAB Security Imaging System. The discussed aims of the experiment are:

To fully characterise the SFAB Security imaging system in terms of detector sensitivity and yield. This will involve.

1) Learning and understanding the microwave properties of superconductors

2) Learning and understanding how a Lumped Element Kinetic Inductance Detector works

3) Be able to form a simple simulation of a LEKID detector

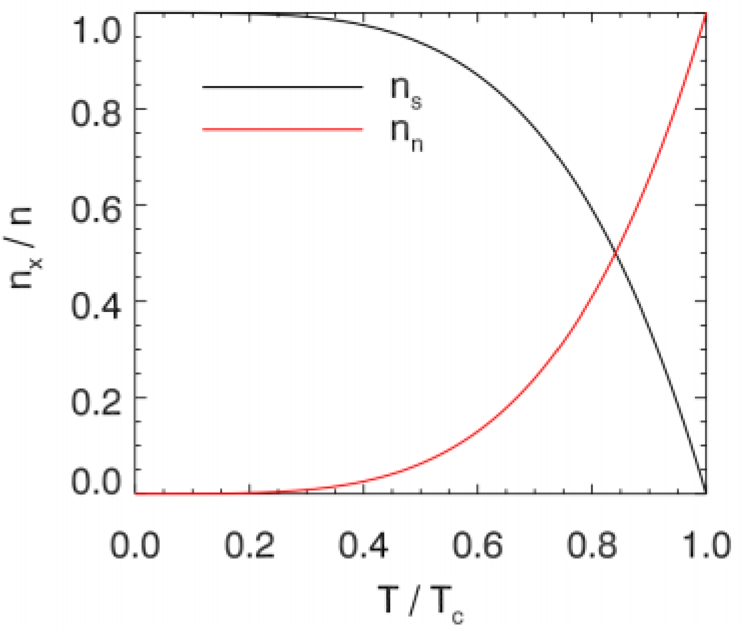
4) Understand how to analyse data from a LEKID array and to make sensitivity measurements from this data

5) Be able to compare the data of each detector in the array to the expected photon noise limit.

3. Notes and Materials Covered

Based on the literature provided in the pages outlined above, from the theory of superconductivity the phenomenon of kinetic inductance arises from it and as such is an important concept as a prerequisite to the fundamentals of the detector. A simple notes and explanation of superconductivity and kinetic inductance is given below:

In some elemental metals such as Niobium, Aluminium and Tin, the DC resistance falls to zero below a certain temperature known as the critical temperature, and as such these materials are known as superconductors. In normal metals, electrons scattering off the ions in the lattice cause a loss in kinetic energy and this leads to non-zero resistance. However, in superconductors below the critical temperature T_c, a fraction of the electron population start to "pair-up" into Cooper-pairs and these pairs are immune to scattering and as such have 0 resistance. The ratio, $\frac{n_s}{n_n}$ of the population of superconducting Cooper pairs n_s to unpaired electrons n_n is inversely proportional to the temperature T below the critical temperature. As T decreases, the number of cooper pairs increases. A plot of this relationship is shown:



Source: Simon Doyle Understanding Superconductivity and KIDs Slides

When an electrical current is applied on a superconducting material, the current simply passes through the population of superconducting electrons and thus arises zero resistance. When the temperature increases, the kinetic energy of the electrons break Cooper pairs and the population of n_n increases.

Another important concept about superconductivity is the Meissner Effect. Essentially, a superconducting material will completely expel any magnetic flux density from within the bulk by creating surface currents to cancel any flux that penetrates it. The relationship of the field into the material is characterized by the 2 London equations:

$$\frac{dJ}{dt} = \frac{n_s e^2}{\omega m}$$

$$\lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

λ_L is the London Penetration depth. It is the value at which the field decays to ${}^{1}/_{e}$ of its value at the surface within the distance $\sqrt{m/\mu_0 n_s e^2}$. The result of the London equations are essential in characterizing the inductance of the material.

The next portion touches on the two fluid model of the behaviour of the electrons in the material. As mentioned previously, below T_c, the populations of the electrons split into 2 different populations n_s and n_n which coexist within the lattice. n_n is affected by the usually scattering and exhibits loss. n_s on the other hand, does not undergo scattering and thus no loss. The Cooper pairs are bound together with an energy gap of 2Δ (1 from each electron). The model takes into account that the current in the superconductor has 2 paths to travel through, n_s and n_n. The ratio of n_s/n (n being the total conducting electrons) is given by:

$$\frac{n_s}{n} = 1 - (\frac{T}{T_c})^4$$

$$n_n = n - n_s$$

The above gives the temperature dependence of the population of the superconducting electrons, and will be essential in future calculations for the conductivity of the material. The conductivity of n_s can be denoted by σ_s and conductivity of n_n is denoted by σ_n. Taken directly from the notes: "At low frequencies, the σ_s is far greater than σ_n, thus displaying zero resistance. At higher frequencies however, especially in the microwave region, σ_n can play a considerable part in the conductivity. This is due to the kinetic inductance of the superconducting electrons. The inertia of these electrons produces a reactance giving us a large impedance at high frequencies. This effect is likened to an inductance as the energy drawn from the field E is stored in the kinetic energy of thse non-scattering electrons." As the temperature or frequency increases, more of the current will be shunted through the non-superconducting resistive path. From this, derives the temperature dependence of the London Penetration Depth, LPD:

$$\lambda_L = \lambda_L(0)\ [1 - \left(\frac{T}{T_c}\right)^4]^{-0{,}5}$$

Where $\lambda_L(0)$ is the LPD at 0 Kelvin.

By summing up the kinetic energies of all the superconducting electrons, we can use this to determine the kinetic inductance of the material given by the following expression:

$$L_k = \frac{\mu_0 \lambda^2}{Wt}$$

Where W is the width of the sheet and t is the thickness. It is also further simplified by calculating the L_k of a square of the material and thus the W term vanishes. We are also often working in the limits of t << λ and t >> λ, thus we need to perform surface integrals for current over the entire cross-sectional area to take into account variations in current density. L_k and L_m, the magnetic inductance is given by:

$$L_k = \frac{\mu_0 \lambda^2}{4}[\coth\left(\frac{t}{2\lambda}\right) + \left(\frac{t}{2\lambda}\right) cosech^2\left(\frac{t}{2\lambda}\right)$$

$$L_m = \frac{\mu_0 \lambda^2}{4}[\coth\left(\frac{t}{2\lambda}\right) - \left(\frac{t}{2\lambda}\right) cosech^2\left(\frac{t}{2\lambda}\right)$$

The total internal inductance is thus given by:

$$L_{int} = L_m + L_k = \frac{\mu_0 \lambda}{2}\coth\left(\frac{t}{2\lambda}\right)$$

Week 3: 20/10/2021 – Wednesday

1. Outline of meeting

The meeting was carried out on Zoom. The meeting was a discussion about the topics discussed previously, and as they are dense topics, a recap of the theory of superconductivity and the associated inductances. The discussed recap and outline of the task is given in section 3 of this diary. The meeting also outlined a task to recreate Figure 3.3 from Dr. Simon Doyle's thesis titled "Lumped Element Kinetic Inductance Detectors" and to calculate the internal inductance of an aluminium sheet given specific parameters. The specification of the task is given in the next section.

2. Specification of Tasks

i) Using the equations and theory discussed, recreate the plot from the thesis, Figure 3.3

ii) Calculate the internal inductance for an aluminium square with the following properties:

$$Thickness = 50nm$$
$$n_{electrons} = 18 \times 10^{28}$$
$$Critical\ temperature, T_c = 1.4\ K$$
$$Temperature, T = 0.3\ K$$

3. Outline of Theory and Methodology for Task

Using the equations 3.19 and 3.20 from last week to determine the kinetic and magnetic inductance takes the surface integral for current over the entire cross-sectional area to consider variations in current density:

$$L_k = \frac{\mu_0 \lambda^2}{4W} [\coth\left(\frac{t}{2\lambda}\right) + \left(\frac{t}{2\lambda}\right) cosech^2\left(\frac{t}{2\lambda}\right)$$
$$L_m = \frac{\mu_0 \lambda^2}{4W} [\coth\left(\frac{t}{2\lambda}\right) - \left(\frac{t}{2\lambda}\right) cosech^2\left(\frac{t}{2\lambda}\right)$$

The kinetic and magnetic inductances per square can be determined using the equations. As discussed in the meeting, the inductances can be simplified by calculating the value per square of the material instead and the W term vanishes. To find the ratio of L_k and L_m to L_{int}, the value of L_{int} was found using the equation 3.21 from the thesis:

$$L_{int} = L_m + L_k = \frac{\mu_0 \lambda}{2} \coth\left(\frac{t}{2\lambda}\right)$$

The λ term was a fixed value of 50nm. The thickness was varied for a range of 0nm to 300nm. The ratios were found and the inductances were plotted against each other, the plot is show in the section below.

The next part of the task was to calculate the internal inductance of a thin aluminium sheet using the parameters given. The first step is to calculate the London Penetration Depth at T=0K. The 2nd London Equation can be used to find the value:

$$\lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

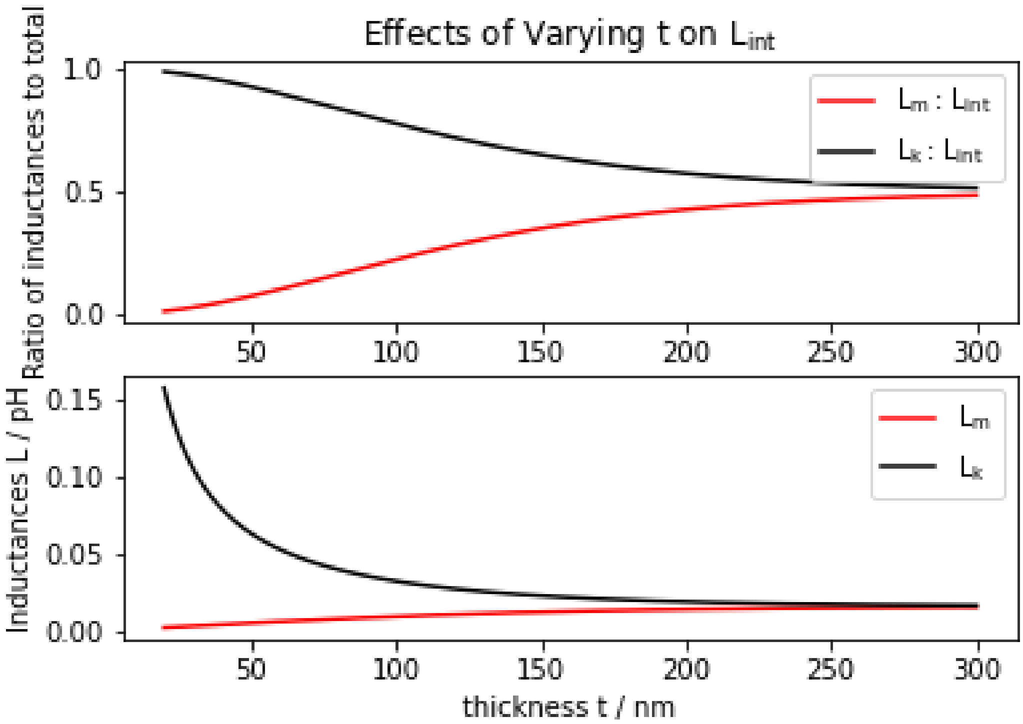
As the temperature decreases, the population of n_s increases and n_n decreases. The electrons will form Cooper-pairs as the temperature decreases. At 0K, ALL the electrons will have paired up and thus, $n_{electrons} = n_s$. Thus, we can calculate the LDP at 0K using this.

The temperature dependence of λ_L is given by equation 3.13 in the thesis:

$$\lambda_L(T) = \lambda_L(0)\ [1 - \left(\frac{T}{T_c}\right)^4]$$

From this, we can calculate the internal inductance using the parameters provided. The result is given in section 5.

4. Recreated Plots of Figure 3.3 of Thesis



5. Calculated Value for The Internal Inductance

$$L_{int} = 0.47 \frac{pH}{square}$$

6. Code for Calculating Inductances

```
'''
##########################################################################################
1. Effects of varying film thickness for $L_{m}$ and $L_{k}$ on a square film
##########################################################################################
'''
#Imports
import numpy as np
import matplotlib.pyplot as plt

def coth(x):
    return np.cosh(x)/np.sinh(x)
```

```
def cosec(x):
    return 1/np.sin(x)
def cosech(x):
    return 1/np.sinh(x)

#Defining lambda, miu_0 and lower and upper limit of t
lam = 50*10**-9
miu_0 = 1.25663706212*10**-6
lowerLimit, upperLimit = 20e-9, 300e-9

#Create array of varying thickness t
t = np.linspace(lowerLimit, upperLimit, num=1000)

#Define a fraction for neater code
fraction = t/(2*lam)

#Calculating lk
lk = (miu_0 *lam/4)*(coth(fraction) + fraction*(cosech(fraction))**2)
lm = (miu_0 *lam/4)*(coth(fraction) - fraction*(cosech(fraction))**2)
# ### The total internal inductance is given by: $L_{int} =
\frac{\mu_{0}\lambda}{2}coth(\frac{t}{2\lambda})$

#Calculating total inductance
l_int = (miu_0 * lam/2)*coth(t/(2*lam))

#Ratios
ratio_lm = lm/l_int
ratio_lk = lk/l_int

#Subplots
plt.subplot(2,1,1)
params = {'mathtext.default': 'regular' }
plt.rcParams.update(params)
plt.plot(t*10**9, ratio_lm, 'r-', label = '$L_{m}:L_{int}$')
plt.plot(t*10**9, ratio_lk, 'k-', label = '$L_{k}:L_{int}$')
plt.legend(loc='best')
plt.ylabel('Ratio of inductances to total')
plt.grid()
plt.title('Effects of varying t on $L_{int}$')

#Subplot 2
plt.subplot(2,1,2)
plt.plot(t*10**9, lm*10**12, 'r-', label = '$L_m$')
plt.plot(t*10**9, lk*10**12, 'k-', label = '$L_k$')
plt.legend(loc='best')
plt.xlabel('thickness t / nm')
plt.ylabel('Inductances L / pH')
plt.grid()
plt.savefig("Effects of Varying t on L_int")

'''
```

```
##########################################################################################
2. Internal Inductance For Aluminium Square
##########################################################################################
'''

#Define variables
n = 1.8e28
temp_c = 1.4
temp = 0.3
t_al = 50e-9
e = 1.6e-19
me = 9.11e-31

# ### We can then determine the London Penetration Depth
lam0 = (me/(miu_0*n*e**2))**0.5
lamL = lam0*(1 - (temp/temp_c)**4)**-0.5

#Calculate L_int
l_int_al = (miu_0 * lamL/2)*coth(t_al/(2*lamL))
print(l_int_al)
```

Week 4: 27/10/2021 – Wednesday

1. **Outline of meeting**

The meeting was held on Zoom. The first half of the meeting was dedicated to bug-fixing the code from the previous week and correct any misunderstandings from the topic. The second half of the meeting was dedicated to the Mattis Bardeen Theory. A discussion was had regarding the topic and a careful elaboration to the best of my understanding was given. An outline of the Mattis Bardeen Theory will be elaborated in section 3 of this diary. Essentially, Mattis Bardeen theory is derived from the fundamental principles of superconductivity and takes the band gap into consideration. From this, the task that arose from this is to create the plots of L_{int} vs T using Mattis Bardeen Approximations to find the L_{int} from the band gap energy. Then, the resistive part of the impedance can be found, and a plot of R vs T can be made. The outline of the theory and task specification is given below.

2. **Specification of Tasks**

i) Plot out L_{int} over a temperature range of 0.05 – 4 K from Mattis Bardeen Approximations using equation 3.33 for a superconducting film with the following properties:
- Normal state conductivity: $\sigma_n = 6.0 \times 10^7$
- Thickness: $t = 20 \times 10^{-9}\ m$
- Critical temperature: $T_c = 1.5\ K$
- Frequency: $f = 500 \times 10^6\ Hz$

ii) Use equation 3.38 to calculate R as a function of temperature

3. **Outline of Theory and Methodology for Task**

Mattis Bardeen Theory: *"The London Equations derived in previous weeks hold well but they are not derived from any fundamental principles of superconductivity and does not take into account the idea of a band gap. Another assumption of the London Model is that it assumes that the electrons in the superconducting state are just simply electrons which do not scatter and will all be accelerated independently if an electric field is applied."*

Source: Lumped Element Kinetic Inductance Detector – Dr. Simon Doyle Thesis

Building from Mattis Bardeen Theory, the full effects of the band gap and non-local treatment of Cooper pairs leads to the Mattis Bardeen Integrals:

$$\frac{\sigma_1}{\sigma_n} = \frac{2}{\hbar\omega}\int_{\Delta}^{\infty}[f(E) - f(E+\hbar\omega]g(E)dE + \frac{1}{\hbar\omega}\int_{\Delta-\hbar\omega}^{\Delta}[1 - f(E+\hbar\omega)]g(E)dE$$

$$\frac{\sigma_2}{\sigma_n} = \frac{1}{\hbar\omega}\int_{\Delta-\hbar\omega,-\Delta}^{\Delta}\frac{[1-2f(E+\hbar\omega)][E^2+\Delta^2+\hbar\omega E]}{[\Delta^2-E^2]^{\frac{1}{2}}[(E+\hbar\omega)^2-\Delta^2]^{\frac{1}{2}}}$$

To simplify, the integrals can be approximated when in the limits $k_B T << \Delta(0)$ and $\hbar\omega << \Delta(0)$ to the Mattis Bardeen Approximations:

$$\frac{\sigma_1}{\sigma_n} = \frac{2\Delta(\mathrm{T})}{\hbar\omega}\exp(\left(-\frac{\Delta(0)}{\mathrm{k_B T}}\right)\mathrm{K_0}(\frac{\hbar\omega}{2\mathrm{k_B T}})[2\sinh(\frac{\hbar\omega}{2\mathrm{k_B T}})]$$

$$\frac{\sigma_2}{\sigma_n} = \frac{\pi\Delta(\mathrm{T})}{\hbar\omega}[1 - 2\exp(\left(-\frac{\Delta(0)}{\mathrm{k_B T}}\right)\exp\left(\frac{-\hbar\omega}{2\mathrm{k_B T}}\right)\mathrm{I_0}(\frac{\hbar\omega}{2\mathrm{k_B T}})]$$

Where Δ is the band gap energy, I_0 and k_0 are modified Bessel functions of the first and second kind respectively. $\Delta(\mathrm{T})$ in this case was approximated to $\Delta(0)$ since $T \cong 0$.

Following this, the London Penetration depth can be found by first determining the electron density n_s using the following relation:

$$n_s = \sigma_s \omega m_e / e^2$$

Where σ_s = σ_2 in this case. Using the result, the Penetration depth can be found:

$$\lambda_{MB} = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

Plugging λ into the expression for total internal inductance:

$$L_{int} = \frac{\mu_0 \lambda}{2}\coth\left(\frac{t}{2\lambda}\right)$$

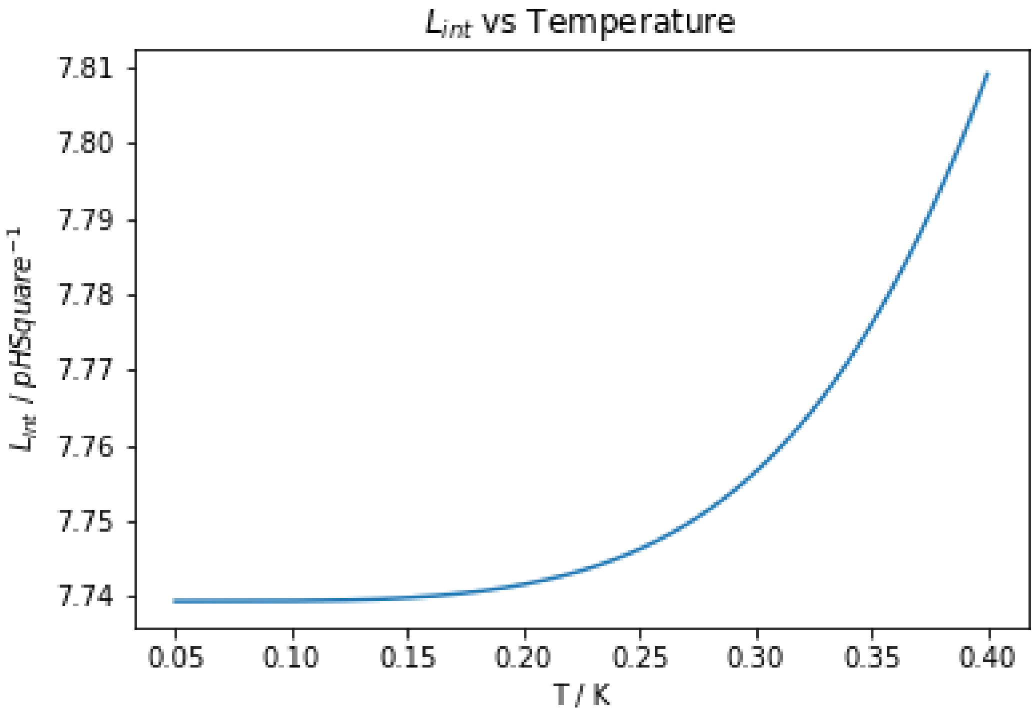
The graph of L_{int} vs T can be plotted.

Using the expression for the resistive part from equation 3.38 in the thesis:

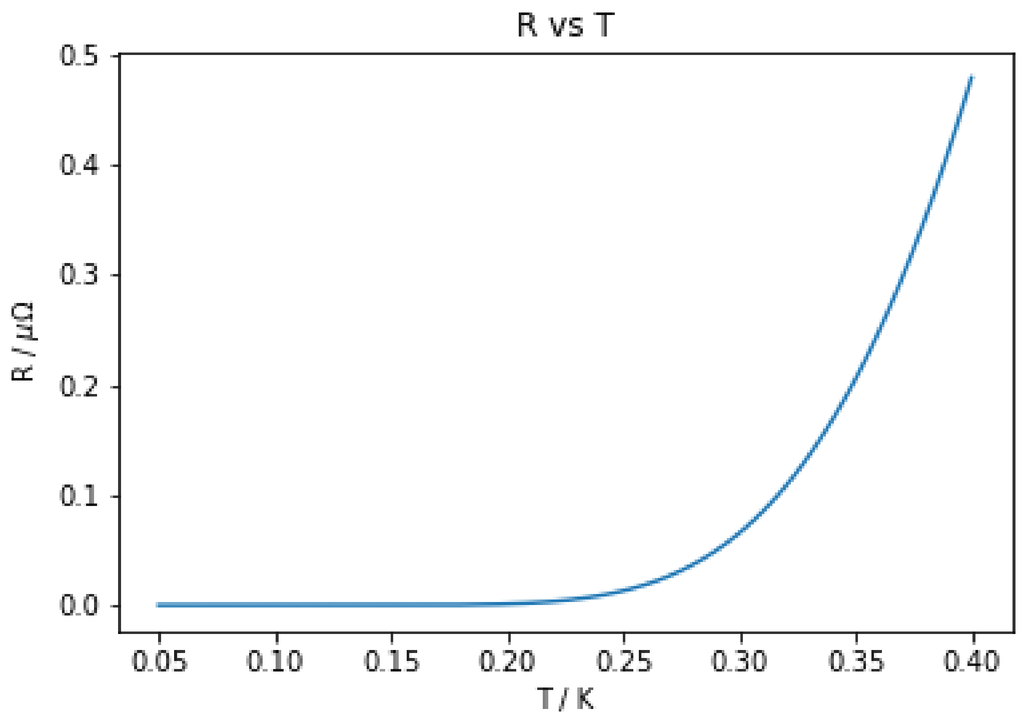
$$R = L_k\,\omega\frac{\sigma_1}{\sigma_2}$$

A plot of R vs T can also be made. The plots and code for the tasks are shown in the following sections.

4. Plot of L_{int} vs T Using Mattis Bardeen Approximations



5. Plot of R vs T Using Mattis Bardeen Approximations



6. Code for Calculating L_{int} and R

```
#imports
import numpy as np
```

```
import matplotlib.pyplot as plt
from scipy.special import iv as I0
from scipy.special import kv as K0
import scipy.constants as const

#Define function
def coth(x):
   return np.cosh(x)/np.sinh(x)

def csch(x):
   return 1/np.sinh(x)

#Define constants
TC = 1.5
Delta_0 = (3.5*const.Boltzmann*TC)/2
sigma_n = 6.0e7         # Normal stae conductvity if superconducting film
Thick = 20e-9           # Thickness of superconducting fil
f = 500e6
w = 2 * np.pi * f
me = const.m_e
miu_0 = 4*np.pi*10**-7

#Varying range of temperature
T = np.linspace(0.05, 0.4, num=500)

#An interpolation formula for delta_T (Cheating a bit by using an interpolation formula, ideally
should be integrated)
#Source: https://physics.stackexchange.com/questions/192416/interpolation-formula-for-bcs-
superconducting-gap#mjx-eqn-eq2
delta_T = Delta_0*np.tanh(1.74*np.sqrt((TC/T)-1))

#Define constants to simplify eqn
multiplying_constant = delta_T/(const.hbar * w)
e_const_1 = - Delta_0/(const.Boltzmann*T)
e_const_2 = (const.hbar*w)/(2*const.Boltzmann*T)

#Parts of the sigma1 Ratio
A = 2*multiplying_constant
B = np.exp(e_const_1)
C = K0(0, e_const_2)
D = 2*(np.sinh(e_const_2))

#Find Sigma 1 and Sigma 2
sigma1Ratio = A * B * C * D
sigma2Ratio = np.pi*multiplying_constant*(1 - (2*np.exp(e_const_1)*np.exp(-
e_const_2)*I0(0,e_const_2)))
sigma2 = sigma2Ratio * sigma_n
sigma1 = sigma1Ratio * sigma_n
```

```
# Sanity Check
# plt.subplot(2,1,1)
# plt.plot(T, sigma1Ratio)
# plt.ylabel("$\sigma_{1}/\sigma_n$")
# plt.yscale("log")
# plt.subplot(2,1,2)
# plt.plot(T, sigma2Ratio)
# plt.ylabel("$\sigma_{2}/\sigma_n$")
# plt.show()
# plt.figure()


#Depth
lower_fraction = miu_0*sigma2*w
Lambda_T_MB = (1/lower_fraction)**0.5

#Internal Inductance
fraction = Thick/(2*Lambda_T_MB)
L_int = (miu_0*Lambda_T_MB/2)*coth(fraction)
plt.plot(T, L_int*10e12)
plt.ylabel("$L_{int}$ / $pHSquare^{-1}$")
plt.xlabel("T / K")
plt.title("$L_{int}$ vs Temperature")
plt.savefig("L_int vs T Mattis Bardeen")
plt.figure()

#sigma 1 to sigma 2
sigma12Ratio = sigma1/sigma2

#Terms for lk
A = (miu_0*Lambda_T_MB)/4
B = coth(fraction)
C = fraction*(csch(fraction))**2

#R vs T
lk = A*(B+C)
R = lk * w * sigma12Ratio
plt.title("R vs T")
plt.plot(T, R*10**6)
plt.ylabel("R / $\mu\Omega$")
plt.xlabel("T / K")
plt.savefig("R vs T Mattis Bardeen")
```

Week 5: 03/11/2021 – Wednesday

1. Outline of Meeting

The meeting was carried out on Zoom. The first part of the meeting was dedicated to answering questions and clearing up any misconceptions from the previous week's topic, the Mattis Bardeen Theory. Following this, the basic concepts of KID microwave readout. In particular, the scattering parameters and the effects it has on microwave circuits. From this, the task for this week was specified which was to create a S21 plots for its amplitude and phase for a range of frequencies. More details on the theory and concepts is given in section 3 of this diary.

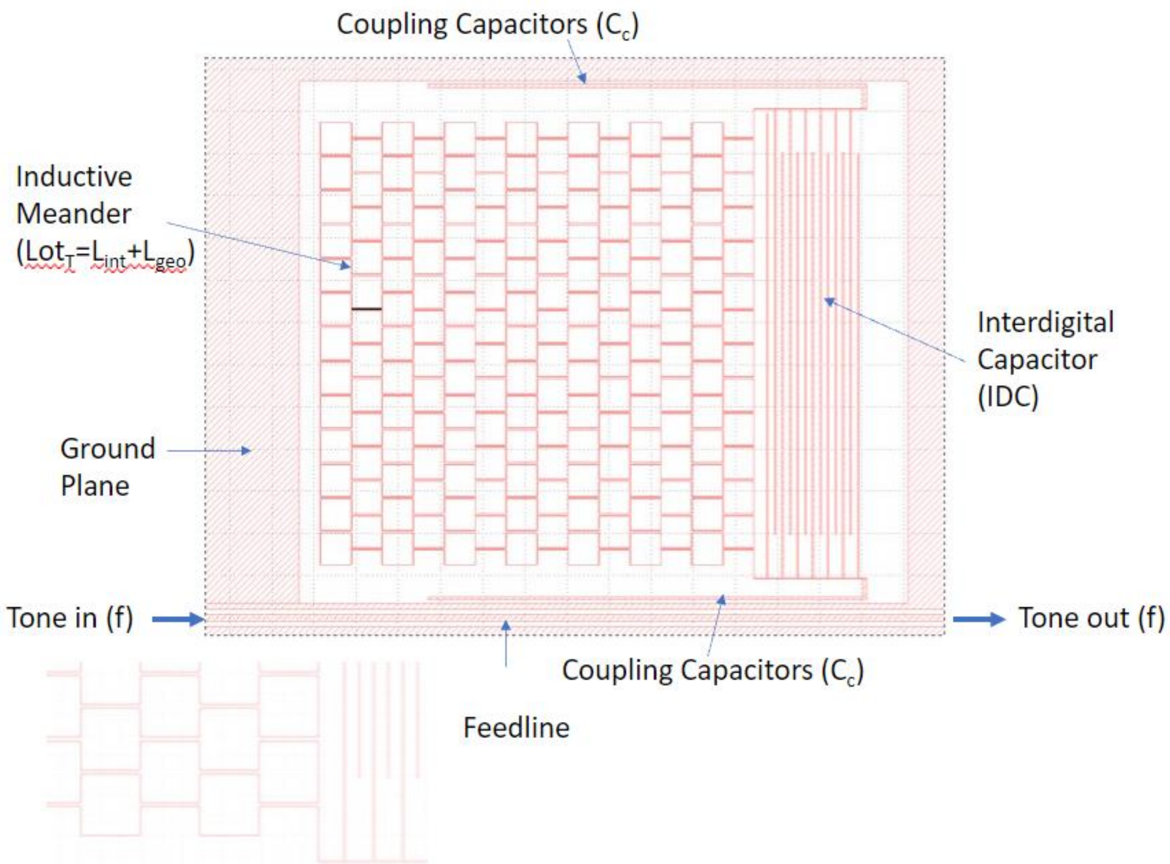
2. Specification of Tasks

i) Read through the Understanding_Kinetic_Inductance_Detector_Microwave_readout.pdf document on the one drive. Try and create the S21 plots for amplitude and phase. Use the following parameters:

- F_0 = 1 GHz
- Q_r = 9000
- Q_c = 10000

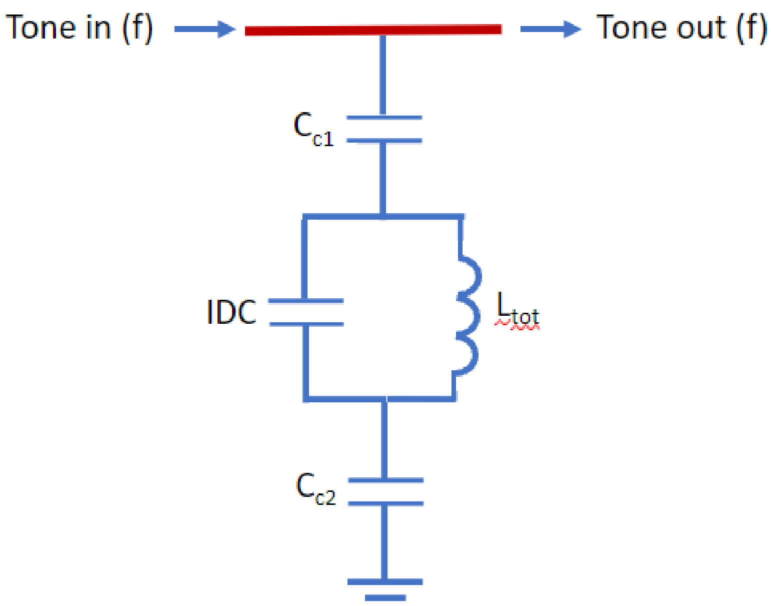
3. Outline of Theory and Task Methodology

A schematic of a single pixel on a KID is shown below:



Source: A schematic of a single pixel on a KID provided by Simon Doyle's Slides

The circuit sits on a silicon wafer (white part) and that wafer sits on an aluminium sheet (dashed lines) which is the ground plane. The inductive meander can be characterized as a single inductor with inductance L_{tot}. An equivalent circuit is given below:



Source: An equivalent circuit to the schematic of a single pixel on a KID provided by Simon Doyle's Slides

The basic mechanism of the circuit is as follows:
Incident photons will provide energy for Cooper pairs to overcome the band gap energy Δ. As such, the cooper pairs breaks and as a result the density of n_1 increases while the superconducting n_s density decreases. This increases σ_1 and as a result, the value of L_{int} will vary. The dimensions of the inductor does not change, as such will remain constant throughout. Thus, the total inductance L will change accordingly to a change in L_{int}.

For the capacitors, there are 2 coupling capacitors C_c that couple to the transmission feedline and ground. The total capacitance C_c from C_{c1} and C_{c2} is simply the series sum:

$$C_c = \frac{C_{c1}C_{c2}}{C_{c1} + C_{c2}}$$

The other capacitor is the Intedigital capacitor IDC which is coupled to the Inductive Meander L_{tot}. We can find the total capacitance of the whole circuit as a simple parallel sum of the capacitances:

$$C_{tot} = IDC + C_c$$

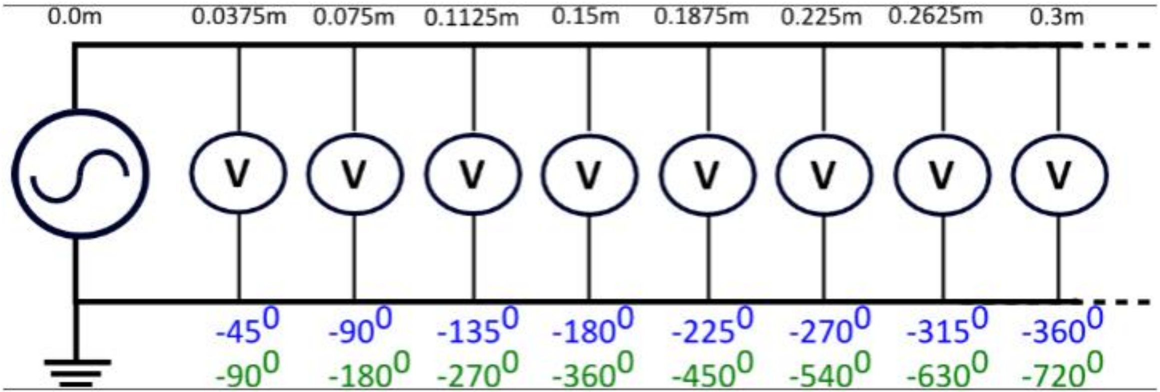
The circuit can be modelled as a basic LC circuit with a resonant frequency given by:

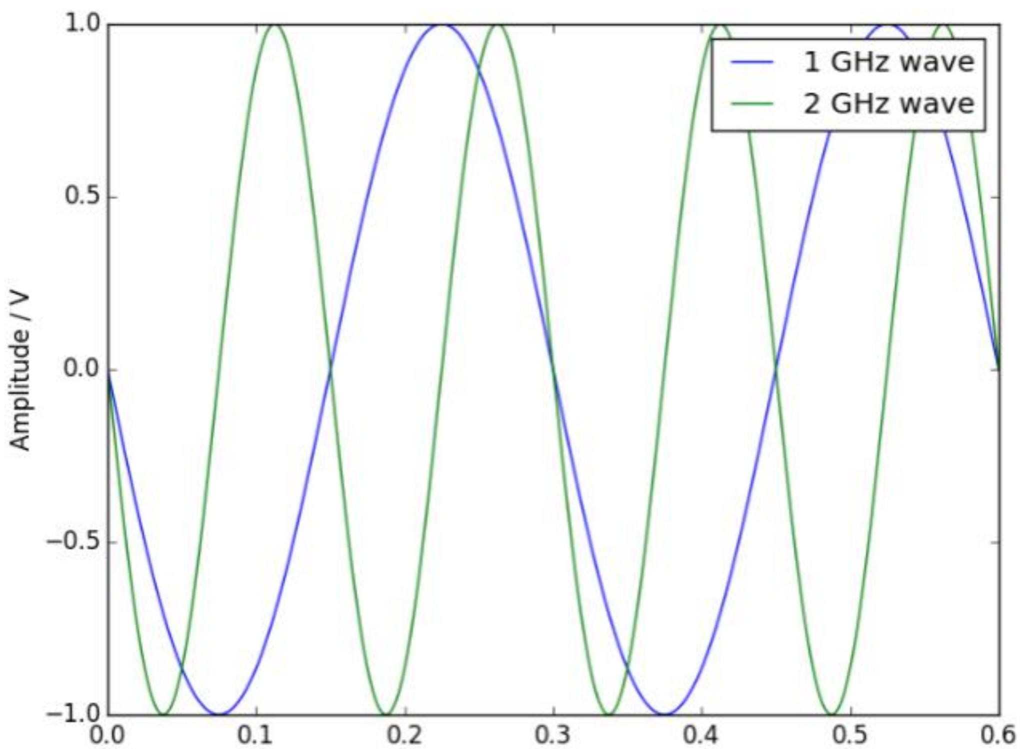
$$\omega_0 = \frac{1}{\sqrt{L_{tot}C_{tot}}}$$

Since L_{tot} varies with respect to photon intensity incident on the inductive meander, ω_0 varies as well, since the capacitors are coupled to the transmission feedline, therefore the incidence of the photons can be detected, thus a detector. More details on the relationship between incident photon energy and how the detector responds to it will be explored in future meetings and diaries. We have thus crudely characterized a single pixel of a KID.

Another phenomenon faced by the KID:
KIDs work in the microwave frequency range *"Unlike low frequency electronics, working at microwave frequencies typically requires one to treat a signal as a wave rather than simply voltages and currents that one may be used to. This is because at higher frequencies, electronic components become similar in size as the wavelength of the signals being measured."*
Source: Understanding_Kinetic_Inductance_Detector_Microwave_readout – Simon Doyle

Since the wavelengths of the signals being measured is comparable to the length of the wire, it would be useful to be able to characterize the phase at a certain point of the wire. Due to the long wavelength, the phase plays a significant role as some points of the circuit will be out of phase. This is illustrated on the diagram below:





Source: Understanding_Kinetic_Inductance_Detector_Microwave_readout – Simon Doyle

Due to this, it is useful to define a microwave circuit in terms of their scattering parameters. These define the voltage waves entering and leaving a microwave circuit via two ports. We denote the scattering parameter as S21 and the S21 for a KID is given as follows:

$$S_{21} = 1 - \frac{Q_r}{Q_c}\frac{1}{1 + 2jQ_r x}$$

Where Q_c and Q_r are known as the coupling Q and resonator Q respectively. These are set by the physical properties of the resonator. J is the complex number $-1^{0.5}$. x is given by:

$$x = \frac{F - F_0}{F_0}$$

Where F_0 is the resonant frequency of the KID and F is the wave propagated along the feedline. Using S21, the Amplitude and Phase of S21 against frequency can be plotted, where:

$$|S21| = \sqrt{S21_{Real}^2 + S21_{Imaginary}^2}$$

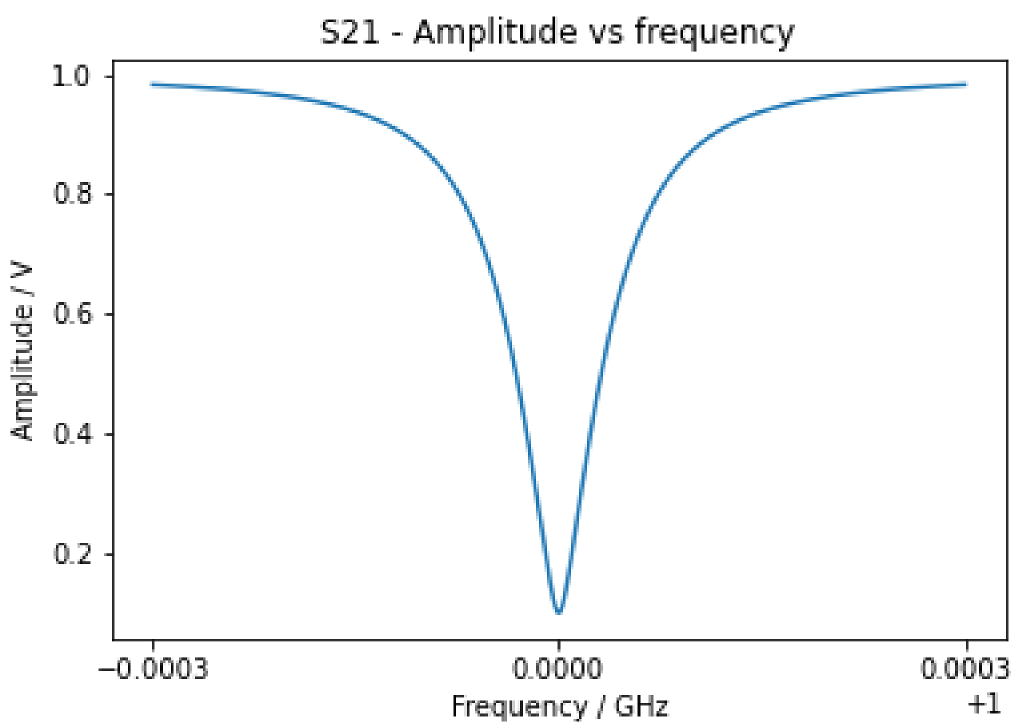
$$Phase_{S21} = Arctan(\frac{S21_{Imaginary}}{S21_{Real}})$$

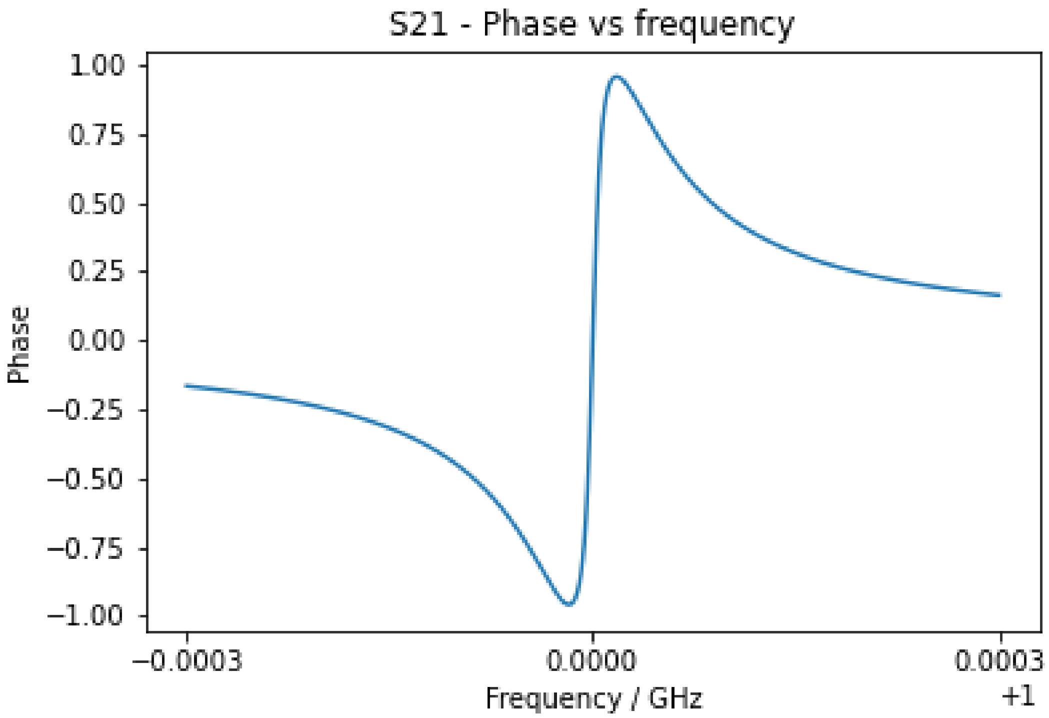
Where the imaginary and real components of S21 can be found it Python using the following code:

```
S21_Real = S21.real
S21_Imaginary = S21.imag
```

As part of the task, the plots of amplitude and phase for S21 against frequency in the bandwidth of 0.0006 GHz with F_0 at 1 GHz are given in the following section.

4. Plots of S21 for Amplitude and Phase vs Frequency





5. Code For Calculating the Components of S21

```
import numpy as np
import matplotlib.pyplot as plt
import cmath as cmath

#Function for calculating S21
def S21(Qr, Qc, f):
    x = (f-f0)/f0
    result = 1 - (Qr/Qc)*(1/(1+2j*Qr*x))
    return result

#Define values
Qr=9000
Qc=10000
f0 = 1e9

#range of frequencies
lower_f = f0-3e5
upper_f = f0+3e5
f = np.linspace(lower_f, upper_f, 1000)

#Compute S21
S21_value = S21(Qr, Qc, f)
S21_real = S21_value.real
S21_imag = S21_value.imag

#Amplitude
```

```
plt.figure()
S21_amplitude = (S21_real**2 + S21_imag**2)**0.5
plt.plot(f*10**-9, S21_amplitude)
plt.title("S21 - Amplitude vs frequency")
plt.xlabel("Frequency / GHz")
plt.ylabel("Amplitude / V")
plt.xticks([0.9997, 1.000, 1.0003])
plt.savefig("Amplitude S21 Plot")

#Phase
plt.figure()
S21_phase = np.arctan(S21_imag/S21_real)
plt.plot(f*10**-9, S21_phase)
plt.title("S21 - Phase vs frequency")
plt.xlabel("Frequency / GHz")
plt.ylabel("Phase")
plt.xticks([0.9997, 1.000, 1.0003])
plt.savefig("Phase S21 Plot")
```

Week 6: 10/11/2021 – Wednesday

1. Outline of Meeting

The meeting was held on Zoom. Following from last week's topic, the meeting was focused on characterizing a KID. Building from last week's theory, this week extends for an array of KID. By using the scattering parameters equations for the KID, by determining the ABCD values which are related to the transmission line impendence, the S21 value can be found for an array of detectors. This week's task was focused on using a Python function given by my supervisor and given parameters to model a KID for a certain temperature range. The result of which is a plot of cascading frequencies each curve corresponding to a detector.

(Current understanding so far, will change as had more time to digest the information given. Diary compiled 1 day after meeting.)

2. Specification of Task

i) Find Lint using Mattis Bardeen Approximations for Aluminium (use values given last time) at a temperature of 0.2K. Multiply this by "Squares" to get total Lint.

ii) Find the IDC value for the lowest temperature and make this fixed. Found from F0 and (Lint+Lgeo) and C_couple.

$$\omega_0 = \frac{1}{\sqrt{(L_{int}L_{Geo})(C_{IDC}C_{Couple})}}$$

iii) Run simulations code below to obtain S21 and plot this as a function of frequency

iv) Define a temperature step (say 0.02K) and calculate a new Lint using Mattis Bardeen Approximations at this slightly higher temperature.

v) Simulate again changing only the value of Lint (IDC, L_geo and C_couple remain fixed)

vi) Repeat for all temperatures up to 0.35K

3. Outline of Theory and Task Methodology

(Task to be completed by next Wednesday – 17th Nov)

4. Python Function Given:

```
def Capacitive_Res_Sim(F0, C_couple, Z0, L_geo, L_int, Res, Sweep_BW, Sweep_points,
Capacitance):
   j=complex(0,1)
   Cc=C_couple
   F_min=F0-(Sweep_BW/2.0)
   F_max=F0+(Sweep_BW/2.0)
   Sweep=np.linspace(F_min, F_max, Sweep_points)
   W=Sweep*2.0*pi
   W0=2.0*pi*F0
   L=L_geo+L_int
   C=Capacitance
   Zres= 1.0/((1./((j*W*L)+Res))+(j*W*C))   # Impedance of resonator section
   Zc=1.0/(j*W*Cc)  #impedance of coupler
   ZT=Zres+Zc
   YT=1.0/ZT
```

```
S21 = 2.0/(2.0+(YT*Z0))
I_raw=S21.real
Q_raw=S21.imag
shift=((1.0-min(I_raw))/2.0)+min(I_raw)
I_cent=I_raw-shift
Q_cent=Q_raw
Phase=Atan(abs(Q_cent/I_cent))
QU=(W0*L)/Res
QL=(C*2)/(W0*(Cc**2)*Z0)
S21_Volt=abs(S21)
I_offset=shift
return  (Sweep, S21_Volt, Phase, I_raw, Q_raw, I_cent, Q_cent, QU, QL, I_offset)
```