Week 3: 20/10/2021 – Wednesday

1. Outline of meeting

The meeting was carried out on Zoom. The meeting was a discussion about the topics discussed previously, and as they are dense topics, a recap of the theory of superconductivity and the associated inductances. The discussed recap and outline of the task is given in section 3 of this diary. The meeting also outlined a task to recreate Figure 3.3 from Dr. Simon Doyle's thesis titled "Lumped Element Kinetic Inductance Detectors" and to calculate the internal inductance of an aluminium sheet given specific parameters. The specification of the task is given in the next section.

2. Specification of Tasks

i) Using the equations and theory discussed, recreate the plot from the thesis, Figure 3.3

ii) Calculate the internal inductance for an aluminium square with the following properties:

$$Thickness = 50nm$$
$$n_{electrons} = 18 \times 10^{28}$$
$$Critical\ temperature, T_c = 1.4\ K$$
$$Temperature, T = 0.3\ K$$

3. Outline of Theory and Methodology for Task

Using the equations 3.19 and 3.20 from last week to determine the kinetic and magnetic inductance takes the surface integral for current over the entire cross-sectional area to consider variations in current density:

$$L_k = \frac{\mu_0 \lambda^2}{4W} [\coth\left(\frac{t}{2\lambda}\right) + \left(\frac{t}{2\lambda}\right) cosech^2 \left(\frac{t}{2\lambda}\right)$$
$$L_m = \frac{\mu_0 \lambda^2}{4W} [\coth\left(\frac{t}{2\lambda}\right) - \left(\frac{t}{2\lambda}\right) cosech^2 \left(\frac{t}{2\lambda}\right)$$

The kinetic and magnetic inductances per square can be determined using the equations. As discussed in the meeting, the inductances can be simplified by calculating the value per square of the material instead and the W term vanishes. To find the ratio of L_k and L_m to L_{int}, the value of L_{int} was found using the equation 3.21 from the thesis:

$$L_{int} = L_m + L_k = \frac{\mu_0 \lambda}{2} \coth\left(\frac{t}{2\lambda}\right)$$

The λ term was a fixed value of 50nm. The thickness was varied for a range of 0nm to 300nm. The ratios were found and the inductances were plotted against each other, the plot is show in the section below.

The next part of the task was to calculate the internal inductance of a thin aluminium sheet using the parameters given. The first step is to calculate the London Penetration Depth at T=0K. The 2nd London Equation can be used to find the value:

$$\lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

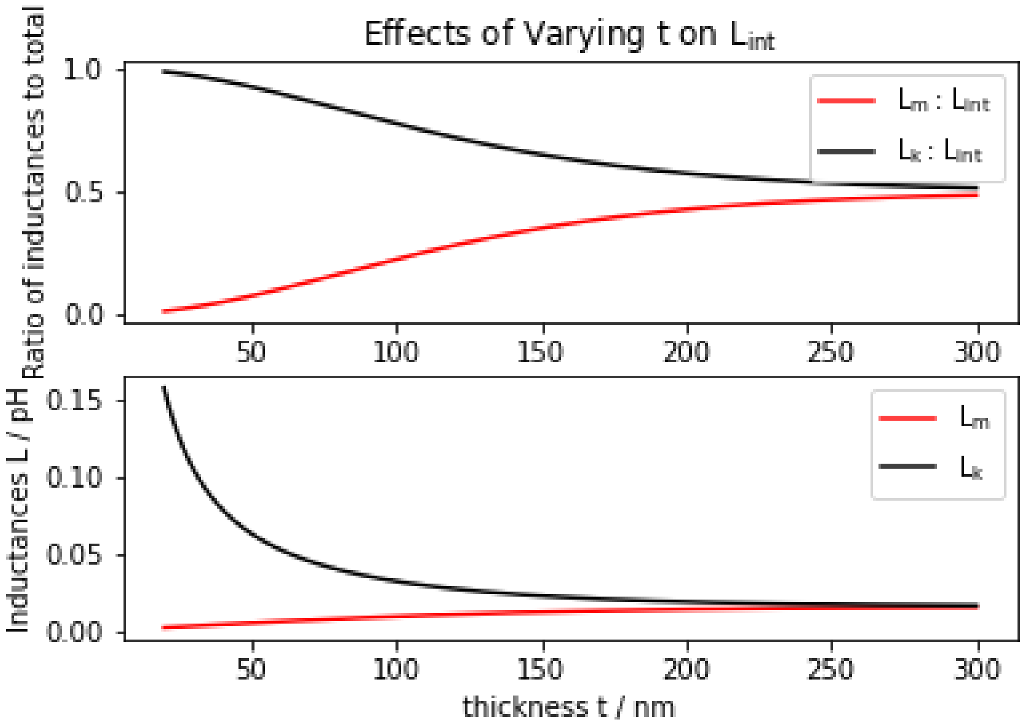
As the temperature decreases, the population of n_s increases and n_n decreases. The electrons will form Cooper-pairs as the temperature decreases. At 0K, ALL the electrons will have paired up and thus, $n_{electrons} = n_s$. Thus, we can calculate the LDP at 0K using this.

The temperature dependence of λ_L is given by equation 3.13 in the thesis:

$$\lambda_L(T) = \lambda_L(0)\ [1 - \left(\frac{T}{T_c}\right)^4]$$

From this, we can calculate the internal inductance using the parameters provided. The result is given in section 5.

4. Recreated Plots of Figure 3.3 of Thesis



5. Calculated Value for The Internal Inductance

$$L_{int} = 0.47 \frac{pH}{square}$$

6. Code for Calculating Inductances

```
'''
##########################################################################################
1. Effects of varying film thickness for $L_{m}$ and $L_{k}$ on a square film
##########################################################################################
'''
#Imports
import numpy as np
import matplotlib.pyplot as plt

def coth(x):
  return np.cosh(x)/np.sinh(x)
```

```
def cosec(x):
    return 1/np.sin(x)
def cosech(x):
    return 1/np.sinh(x)

#Defining lambda, miu_0 and lower and upper limit of t
lam = 50*10**-9
miu_0 = 1.25663706212*10**-6
lowerLimit, upperLimit = 20e-9, 300e-9

#Create array of varying thickness t
t = np.linspace(lowerLimit, upperLimit, num=1000)

#Define a fraction for neater code
fraction = t/(2*lam)

#Calculating lk
lk = (miu_0 *lam/4)*(coth(fraction) + fraction*(cosech(fraction))**2)
lm = (miu_0 *lam/4)*(coth(fraction) - fraction*(cosech(fraction))**2)
# ### The total internal inductance is given by: $L_{int} =
\frac{\mu_{0}\lambda}{2}coth(\frac{t}{2\lambda})$

#Calculating total inductance
l_int = (miu_0 * lam/2)*coth(t/(2*lam))

#Ratios
ratio_lm = lm/l_int
ratio_lk = lk/l_int

#Subplots
plt.subplot(2,1,1)
params = {'mathtext.default': 'regular' }
plt.rcParams.update(params)
plt.plot(t*10**9, ratio_lm, 'r-', label = '$L_{m}:L_{int}$')
plt.plot(t*10**9, ratio_lk, 'k-', label = '$L_{k}:L_{int}$')
plt.legend(loc='best')
plt.ylabel('Ratio of inductances to total')
plt.grid()
plt.title('Effects of varying t on $L_{int}$')

#Subplot 2
plt.subplot(2,1,2)
plt.plot(t*10**9, lm*10**12, 'r-', label = '$L_m$')
plt.plot(t*10**9, lk*10**12, 'k-', label = '$L_k$')
plt.legend(loc='best')
plt.xlabel('thickness t / nm')
plt.ylabel('Inductances L / pH')
plt.grid()
plt.savefig("Effects of Varying t on L_int")

'''
```

```
##########################################################################################
2. Internal Inductance For Aluminium Square
##########################################################################################
'''

#Define variables
n = 1.8e28
temp_c = 1.4
temp = 0.3
t_al = 50e-9
e = 1.6e-19
me = 9.11e-31

# ### We can then determine the London Penetration Depth
lam0 = (me/(miu_0*n*e**2))**0.5
lamL = lam0*(1 - (temp/temp_c)**4)**-0.5

#Calculate L_int
l_int_al = (miu_0 * lamL/2)*coth(t_al/(2*lamL))
print(l_int_al)
```