

Coursework 6

YOUR NAME

March 30, 2018

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- **Acknowledgments:** This template takes some materials from course CSE 547/Stat 548 of Washington University:
<https://courses.cs.washington.edu/courses/cse547/17sp/index.html>.
If you refer to other materials in your homework, please list here.
 - **Collaborators:** I finish this template by myself. If you finish your homework all by yourself, make a similar statement. If you get help from others in finishing your homework, state like this:
 - 1.2 (b) was solved with the help from _____.
 - Discussion with _____ helped me finishing 1.3.
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You may use `enumerate` to generate answers for each question:

6.1. Type of commonly used notations. Use another `enumerate` to start generate answers for sub-questions:

- (a) Use `$ $` to get an inline equation: $\mathbb{P}(A) = \mathbb{E}[\mathbb{1}_A(\omega)]$.
- (b) Use `equation` to have equation in display math mode:

$$\frac{a+b}{2} \geq \sqrt{ab} \tag{1}$$

- (c) Use `\eqref` to get reference for equations: (1) holds when $a \geq 0, b \geq 0$.
- (d) Now we would introduce some commonly used notations:
 - i. Use `\mathbb{P}`, `\mathbb{R}`, `\mathbb{E}` to type $\mathbb{P}, \mathbb{R}, \mathbb{E}$.
 - ii. Use `\mathcal{A}`, `\mathcal{X}`, `\mathcal{Y}`, `\mathcal{N}` to type $\mathcal{A}, \mathcal{X}, \mathcal{Y}, \mathcal{N}$.
 - iii. Use `\underline{x}`, `\underline{y}` to type vectors $\underline{x}, \underline{y}$.
 - iv. Use `\mathsf{x}`, `\mathsf{y}`, `\mathsf{z}` to type random variables x, y, z . For simplicity, I have defined several macros so you could simply type `\rvx`, `\rvy`, `\rvz`. Don't forget `$ $`!
 - v. Thanks to these macros, we could have $\mathbb{R}, \mathbb{E}[x], \text{Var}(y), \mathbb{P}(A), \perp, \mathbb{1}$ by typing `\reals`, `\E[\rvx]`, `\Var(\rvy)`, `\Prob(A)`, `\independent`, `\mathbb{1}`.
 - vi. Now you can use `\ux`, `\uy`, `\uz` to type vectors $\underline{x}, \underline{y}, \underline{z}$, and use `\urvx`, `\urvy`, `\urvz` to type random vectors $\underline{x}, \underline{y}, \underline{z}$.
 - vii. Remember that $P_{x|y}(x|y) \triangleq \mathbb{P}(x = x|y = y)$.

- α) Writing $\mathbb{P}(x)$ is wrong. \mathbb{P} should only operate on events.
- β) \mathbf{x} is a random variable, while x is a real number.
- (e) You may find <https://en.wikibooks.org/wiki/LaTeX> useful.
- (f) Writing L^AT_EX online may be easier for beginners:
 - i. ShareLaTeX: <https://www.sharelatex.com/>.
 - ii. Overleaf: <https://www.overleaf.com/>.

6.2. You may need aligned equations for your homework, here are several examples:

Total propability rule:

$$\begin{aligned}\mathbb{P}(\mathbf{x} = x) &= \sum_{y \in \mathcal{Y}} \mathbb{P}(\mathbf{x} = x, y = y) \\ &= \sum_{y \in \mathcal{Y}} \mathbb{P}(\mathbf{x} = x | y = y) \mathbb{P}(y = y),\end{aligned}$$

or

$$\begin{aligned}P_{\mathbf{x}}(x) &= \sum_{y \in \mathcal{Y}} P_{\mathbf{x}y}(x, y) \\ &= \sum_{y \in \mathcal{Y}} P_{\mathbf{x}|y}(x|y) P_y(y).\end{aligned}$$

Indicator function:

$$\mathbb{1}_A(\omega) = \begin{cases} 1, & \text{if } \omega \in A, \\ 0, & \text{if } \omega \notin A. \end{cases}$$

6.3. You may need to add figure and source codes in your homework. Figure 1 is an example that compares the empirical distribution (histogram) and probability density function of the Gaussian random variable.

The source code to plot Figure 1 could be found in Appendix A. Here are the core codes:

```
4 [cnt, x_hist] = hist(data, nbins); % not to plot, only to get
   empirical distribution.

6 cnt = cnt / n / (x_hist(2) - x_hist(1)); % normalization, be
   careful :)

7 bar(x_hist, cnt); % plot the hist using bar()
```

To understand line 6, note that if we have n samples of \mathbf{x} denoted by $x^{(i)}, i = 1, 2, \dots, n$, then the probability density function $p_{\mathbf{x}}$ could be estimated as

$$\begin{aligned}p_{\mathbf{x}}(x_0) &= \left. \frac{d}{dx} \mathbb{P}(\mathbf{x} \leq x) \right|_{x=x_0} \\ &\approx \frac{\mathbb{P}(x_0 - \Delta x < \mathbf{x} \leq x_0)}{\Delta x} \\ &\approx \frac{1}{n\Delta x} \sum_{i=1}^n \mathbb{1}_{x^{(i)} \in (x_0 - \Delta x, x_0]}.\end{aligned}$$

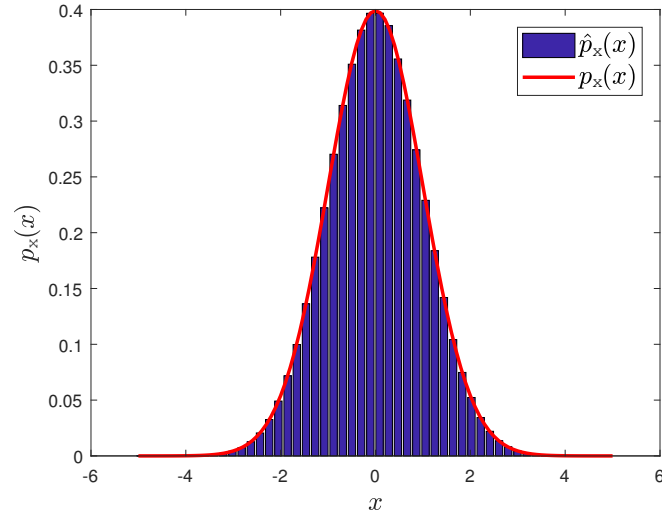


Figure 1: Gaussian PDF and histogram of samples

6.4. An example of hypothesis testing:

$$\log \frac{\mathbb{P}(H = H_1 | y = y)}{\mathbb{P}(H = H_0 | y = y)} \underset{\hat{H}=H_0}{\overset{\hat{H}=H_1}{\gtrless}} \gamma$$

A Source code

Source code for plotting Figure 1 is shown as follows.

```
1 n = 1e6; % n samples
2 data = randn(1e6, 1); % Generate n Random Gaussian samples.
3 nbins = 50; % bins in your histogram
4 [cnt, x_hist] = hist(data, nbins); % not to plot, only to get
   empirical distribution.
5 figure;
6 cnt = cnt / n / (x_hist(2) - x_hist(1)); % normalization, be
   careful :)
7 bar(x_hist, cnt); % plot the hist using bar()
8 hold on;
9 x = -5 : 0.1 : 5;
10 plot(x, normpdf(x), 'r', 'linewidth', 2);
11 legend({'$\hat{p}_{\sf{x}}(x)$', '$p_{\sf{x}}(x)$'}, 'Interpreter',
   'LaTeX', 'fontsize', 15);
12 xlabel('$x$', 'Interpreter', 'LaTeX', 'fontsize', 15); % You may
   change the size accordingly
13 ylabel('$p_{\sf{x}}(x)$', 'Interpreter', 'LaTeX', 'fontsize', 15);
14 title(your-title-here)
```

Listing 1: FigurePlot