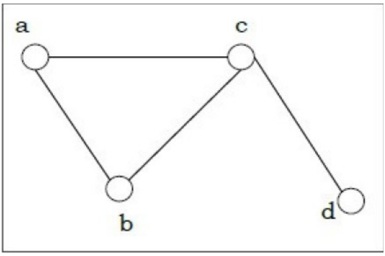
Graph:

Definition − A graph (denoted as $G = (V, E)$) consists of a non-empty set of vertices or nodes V and a set of edges E.
Example − Let us consider, a Graph is $G = (V, E)$ where $V = \{a, b, c, d\}$ and $E = \{\{a, b\}, \{a, c\}, \{b, c\}, \{c, d\}\}$



Degree of a Vertex − The degree of a vertex V of a graph G (denoted by deg (V)) is the number of edges incident with the vertex V.

Vertex	Degree	Even / Odd
a	2	even
b	2	even
c	3	odd
d	1	odd

Even and Odd Vertex : If the degree of a vertex is even, the vertex is called an even vertex and if the degree of a vertex is odd, the vertex is called an odd vertex.

Degree of a graph− The degree of a graph is the largest vertex degree of that graph. For the above graph the degree of the graph is 3.

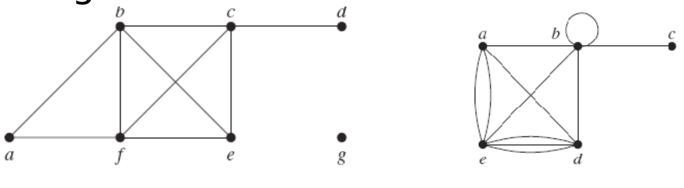
Adjacent: Two vertices u and v in an undirected graph G are called adjacent (or neighbors) in G if u and v are endpoints of an edge e of G. Such an edge e is called incident with the vertices u and v and e is said to connect u and v.

Neighborhood: The set of all neighbors of a vertex v of G = (V ,E), denoted by N(v), is called the neighborhood of v. If A is a subset of V , we denote by N(A) the set of all vertices in G that are adjacent to at least one vertex in A. So, N(A) = Uv∈A N(v).

Isolated vertex : A vertex of degree zero is called isolated.

Pendant vertex : A vertex is pendant if and only if it has degree one.

problem: What are the degrees and what are the neighborhoods of the vertices in the graphs *G* and *H* displayed in Figure 1?



G H

Solution: In G, deg(a) = 2, deg(b) = deg(c) = deg(f) = 4, deg(d) = 1, deg(e) = 3, and
deg(g) = 0. The neighborhoods of these vertices are N(a) = {b, f }, N(b) = {a, c, e, f },

N(c) = {b, d, e, f }, N(d) = {c}, N(e) = {b, c, f }, N(f) = {a, b, c, e}, and N(g) = ∅. In
H, deg(a) = 4, deg(b) = deg(e) = 6, deg(c) = 1, and deg(d) = 5. The neighborhoods of
these vertices are N(a) = {b, d, e}, N(b) = {a, b, c, d, e}, N(c) = {b}, N(d) = {a, b, e}, and
N(e) = {a, b, d}.

The Handshaking Lemma: In an undirected graph, the sum of all the degrees of all the vertices is equal to twice the number of edges.

$$2m = \sum_{v \in V} \deg(v).$$

where *m* is the number of edges.

problem: How many edges are there in a graph with 10 vertices each of degree six?

Solution: Because the sum of the degrees of the vertices is 6·10 = 60, it follows that 2*m* = 60

where *m* is the number of edges. Therefore, *m* = 30.

Theorem: An undirected graph has an even number of vertices of odd degree.

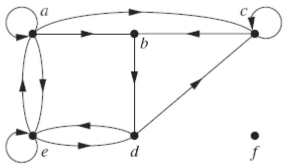
Proof: Let *V*1 and *V*2 be the set of vertices of even degree and the set of vertices of odd degree, respectively, in an undirected graph *G* = *(V ,E)* with *m* edges. Then

$$2m = \sum_{v \in V} \deg(v) = \sum_{v \in V_1} \deg(v) + \sum_{v \in V_2} \deg(v).$$

Because deg*(v)* is even for $v \in V1$, the first term in the right-hand side of the last equality is even. Furthermore, the sum of the two terms on the right-hand side of the last equality is even, because this sum is 2*m*. Hence, the second term in the sum is also even. Because all the terms in this sum are odd, there must be an even number of such terms. Thus, there are an even number of vertices of odd degree.

In-degree & out-degree: In a graph with directed edges the *in-degree of a vertex v*, denoted by deg−*(v)*, is the number of edges with *v* as their terminal vertex. The *out-degree of v*, denoted by deg+*(v)*, is the number of edges with *v* as their initial vertex. (Note that a loop at a vertex contributes 1 to both the in-degree and the out-degree of this vertex.)

problem: Find the in-degree and out-degree of each vertex in the graph *G* with directed edges shown in Figure 2.



G

Solution: The in-degrees in *G* are deg−*(a)* = 2, deg−*(b)* = 2, deg−*(c)* = 3, deg−*(d)* = 2,deg−*(e)* = 3, and deg−*(f)* = 0. The out-degrees are deg+*(a)* = 4, deg+*(b)* = 1, deg+*(c)* = 2,deg+*(d)* = 2, deg+*(e)* = 3, and deg+*(f)* = 0.