Standard VIII

MATHEMATICS

Part - 1

Government of Kerala
Department of Education

THE NATIONAL ANTHEM

Jana-gana-mana adhinayaka, jaya he

Bharatha-bhagya-vidhata.

Punjab-Sindh-Gujarat-Maratha

Dravida-Utkala-Banga

Vindhya-Himachala-Yamuna-Ganga

Uchchala-Jaladhi-taranga

Tava subha name jage,

Tava subha asisa mage,

Gahe tava jaya gatha.

Jana-gana-mangala-dayaka jaya he

Bharatha-bhagya-vidhata.

Jaya he, jaya he, jaya he,

Jaya jaya jaya, jaya he!

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give respect to my parents, teachers and all elders and treat everyone with courtesy.

I pledge my devotion to my country and my people. In their well-being and prosperity alone lies my happiness.

Dear children,

We have travelled much

In the world of math

Let's continue

Our explorations

Our findings

We have much more to go ahead

In Mathematics

To the ever widening world of Numbers

To the precise reasoning of Geometry

To the higher levels to Algebra

Let's continue our search...

Dr. S. Raveendran Nair
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TEXTBOOK DEVELOPMENT

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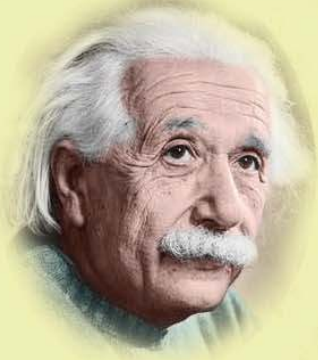Govt. College of Teacher Education,
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Contents

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1

Equal Triangles

Draw triangle ABC and mark two points D, E. Choose the **Translate by Vector** tool and click first on the triangle ABC and then on the points D, E. Didn't you get a new triangle A′B′C′? What is the relation between these two triangles? Change the sides and angles of ∆ABC. Does A′B′C′ also change?

Change the position of E. What happens when E coincides with D?
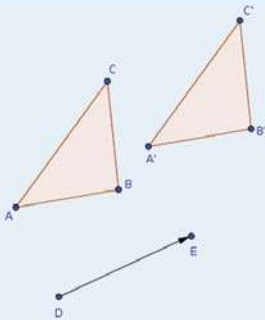
Draw triangles ABC and line DE. Choose the **Reflect about Line** tool and click on the triangles and the line. You get triangle A′B′C′.

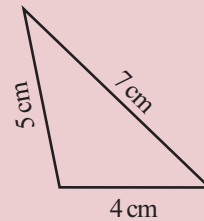Try changing the triangle ABC and the position and slant of the line DE.
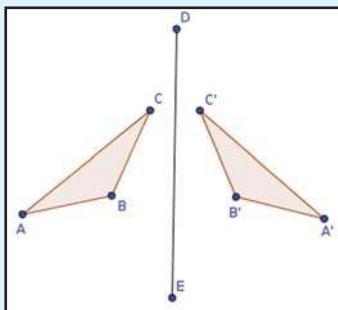
Sides and angles

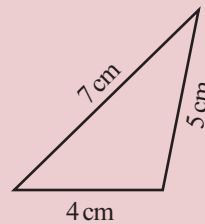If the lengths of the sides of a triangle are given, you know how to draw it.

The sides are 4 centimetres, 5 centimetres, 7 centimetres.
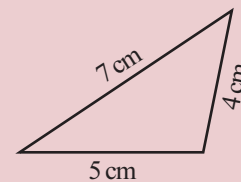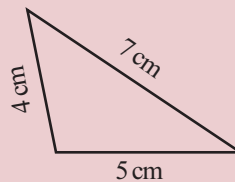
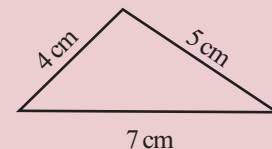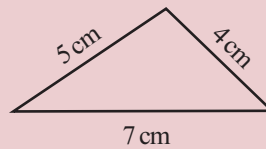Can you draw the triangle?

We can draw like this.

Or like this:

We can draw with the 5 centimetres side as base:

Or with the 7 centimetre side as base:

In all these six triangles, sides are equal. What about the angles?

All these are got by turning and flipping the first one, right?

Cut out the first triangle in thick paper and see if you can make it coincide with all the other triangles by turning or flipping.

When equal sides coincides, angles also coincide, don't they?

Take a different set of lengths and draw triangle like this in various positions.

They too have the same angles, right?

Let's write down as a general principle, what we have seen here:

> **If the sides of a triangle are equal to the sides of another triangle, then the angles of the triangles are also equal.**

Look at these triangles:
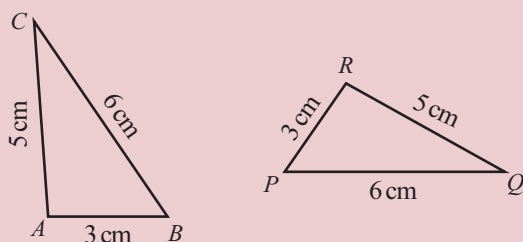
Since sides are equal, angles must also be equal.

That is, each angle of ΔABC is equal to some angle of ΔPQR. But how do the angles match?

Which angle is equal to $\angle A$?

$\angle A$ is the largest angle in ΔABC.

Which is the largest angle in ΔPQR.

So

$\angle A$ = ...

Now which are the smallest angles in the two triangles?
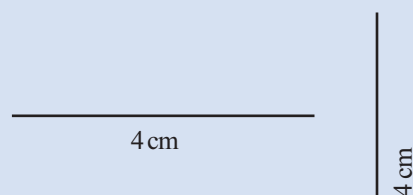
$\angle C$ = ...
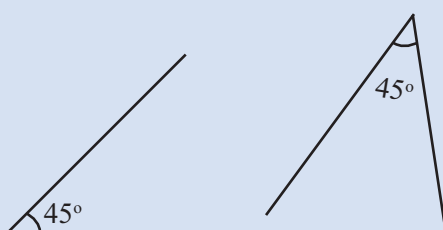
And the medium sized angles?

$\angle B$ = ...

Equality

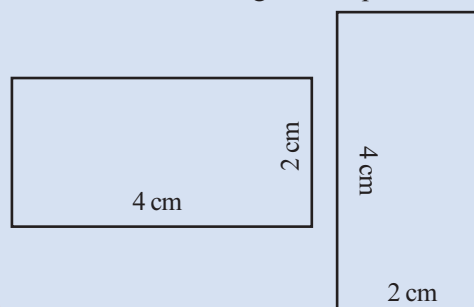Lines, angles, rectangles, triangles, ... so many geometrical shapes.

We often say that two lines of the same length are equal, however they are drawn.

4 cm

Similarly for angles of the same size.

We can also say that rectangles of the same width and height are equal.

We can see this in a different manner. The longest side of ∆*ABC* is *BC*; and the angle opposite this side is ∠*A*, which is the largest angle.
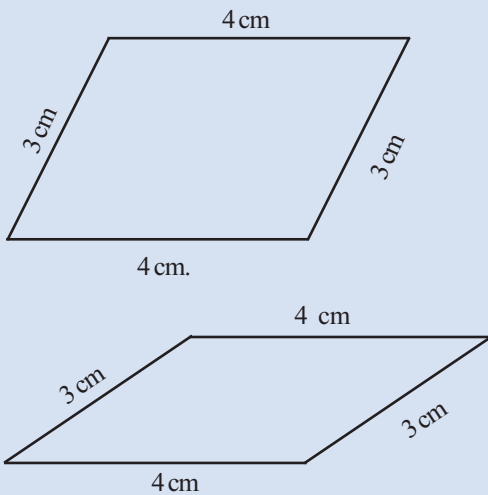
Again, the smallest angle is ∠*C*, which is opposite the shortest side, *AB*; the medium angle is ∠*B*, which is opposite the medium sized side, *AC*.

It is the same in ∆*PQR*.

So, we can write our earlier observation in some more detail:

If the sides of a triangle are equal to the sides of another triangle, then the angles opposite to the equal sides of these triangles are equal.

Let's look at an illustration of this idea. Draw the triangle shown below.

Now draw the same triangle below *AB*, with right and left
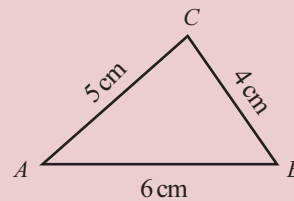
flipped.

The sides *AC* and *BC* of ∆*ABC* are equal to the sides *BD*
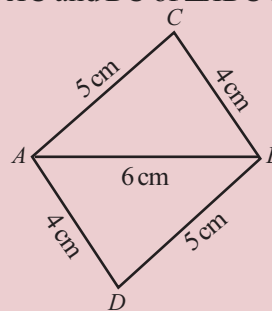
and *AD* of ∆*ABD*.

And the third side of both triangle is *AB*.

Since the lengths of all three sides are equal, the angles must also be equal. That is,

∠*CAB* = ∠*DBA*   ∠*CBA* = ∠*DAB*

Geometric equality

Look at these parallelograms.

Both have sides of lengths 4 centimetre and 3 centimetre. Still, it doesn't seen right to say that the parallelograms are equal.

This is what Euclid says about equality of geometric figures.

Things which coincide are equal.

The lines, angles and rectangles in the last page do coincide if we turn them around, right?

∠*CAB* and ∠*DBA* are alternate angles made by the line *AB* meeting the pair *AC*, *BD* of lines. Since their angles are equal, the lines *AC* and *BD* are parallel.

Similarly, *BC* and *AD* are parallel. (can you explain why?)

So, *ACBD* is a parallelogram. (The section **same direction** of the lesson, **Parallel lines** in the class 7 textbook)

Now can you draw a prallelogram of sides 5 centimetres, 6 centimetres and one diagonal 8 centimetres.

(1) In each pair of triangles below, find all pairs of matching angles and write them down.

i)
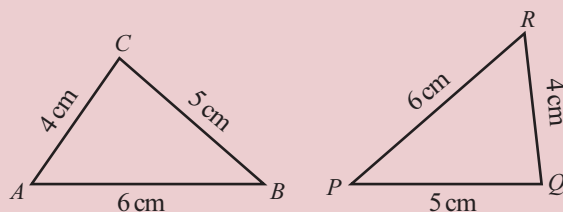
ii)
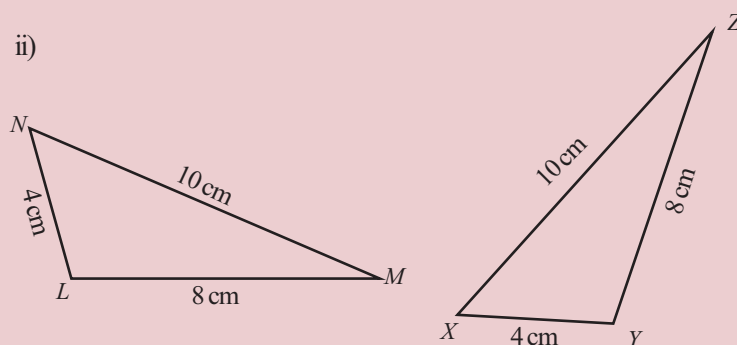
(2) In the triangles below,

$AB = QR$ $BC = RP$ $CA = PQ$
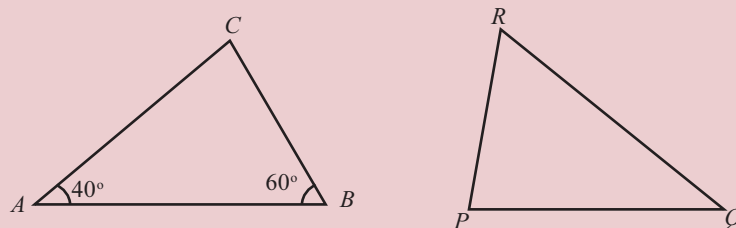
Compute ∠*C* of Δ*ABC* and all angles of Δ*PQR*.

Word and meaning

We saw that if the sides of one triangle are equal to the sides of another triangle, then they can be made to concide. In Euclid's words,

If the sides of a triangle are equal to the sides of another triangle, then these triangle are equal.

Euclid wrote the book, *Elements* in Greek and it was translated to Latin in Renaissance Europe. The word for "coincide" in Latin is *congruent*. From the nintenth century the word "congruent" was used in English also for equality of geometric figures.

(3) In the triangles below,

$$AB = QR \qquad BC = PQ \qquad CA = RP$$
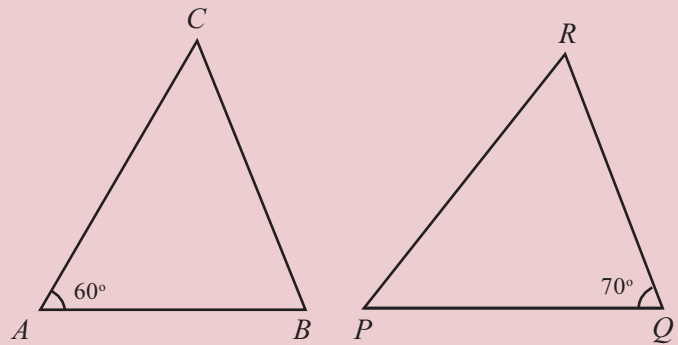
Compute the remaining angles of both triangles.

(4)
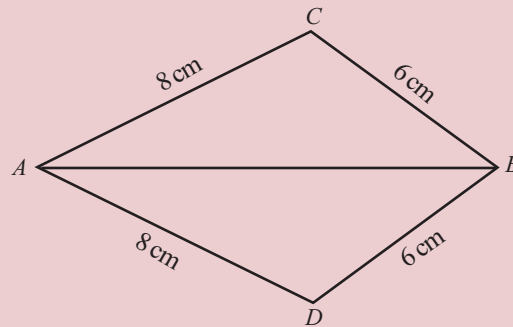
Are the angles of ΔABC and ΔABD equal in the figure above? why?

(5) In the quadrilateral $ABCD$ shown below,

$$AB = AD \qquad BC = CD$$
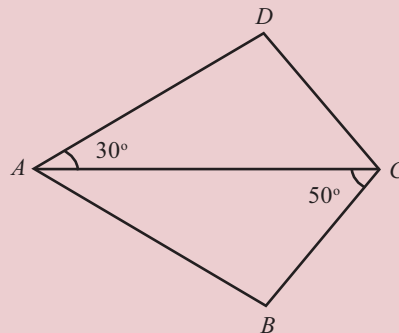
Compute all the angles of the quadrilateral.

If the angles of a triangle are equal to the angles of another triangle, would their sides also be equal?

Two sides and an angle

Given the length of three sides, we can draw the triangle. What if two sides and the angle made by them are given?

The lengths of two sides are 5 centimetres and 7 centimetres; and they meet at an angle of 40°.

Can you draw the triangle?

It can be drawn like this:
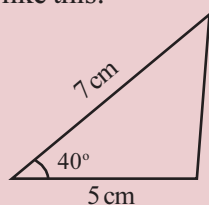
Or like this:
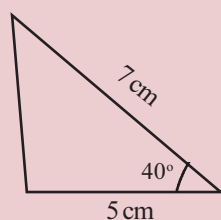
> Create a slider **a** with min = 0 and max 5. Draw a triangle with sides 4, 5, 6 and another with sides 4a, 5a, 6a. Look at the angles of the triangle (choose the **Angle** tool and click triangle to get its angles). Slide **a** to another value and see what happens. What if we set **a** = 1?

We can also draw with the 7 centimetres long side as base:
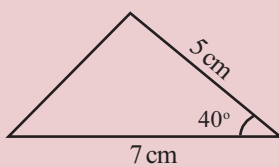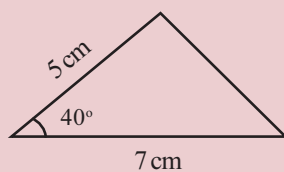
Any other way?

Are the third sides of the triangle also equal?

As we did earlier, we can cut out one triangle and place it in different positions over the other.

Can you make them coincide?

Change the sides and angle and check.

Let's write our observations as a general principle:

> **If two sides of a triangle and the angle made by them are equal to two sides of another triangle and the angle made by them, then the third sides of the triangle are also equal; the other two angles are also equal.**

See these triangles:

Determining a triangle

Bend a long piece of *eerkkil* to make an angle:
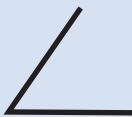
We want to make a triangle, placing another piece of *eerkkil* over the sides of this angle. We can do this in several ways.
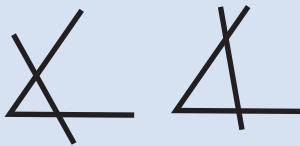
Suppose we mark a spot on the upper side of the angle and insist that the second *eerkkil* must pass through this?
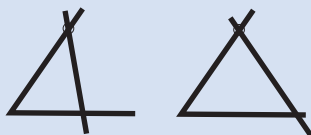
Now let's mark spots on both the upper and lower sides of the angle what if we want the second *eerkkil* to pass through both these spots.

How many triangles can we make?

Once we fix an angle and the lengths of its sides, the triangle also is fixed, isn't it?
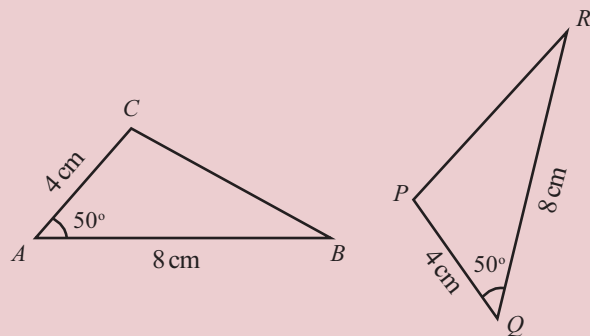
The sides *AB, CA* and $\angle A$ made by them in ΔABC are equal to the sides *QR* and *PQ* and $\angle Q$ made by them in ΔPQR.

So, by what we have seen now, the third sides *BC* and *PR* of ΔABC and ΔPQR are also equal; $\angle B$ and $\angle C$ are equal to the remaining two angles of ΔPQR.

Which angle is equal to $\angle B$?

Equal angles are opposite equal sides.

$\angle B$ is opposite the side *AC* of ΔABC.

The side in ΔPQR are equal to *AC* is *PQ*; and the angle opposite is $\angle R$.

So, $\angle B = \angle R$.

Similarly, we can see that $\angle C = \angle P$ (can you explain?)

Now look at these figures:
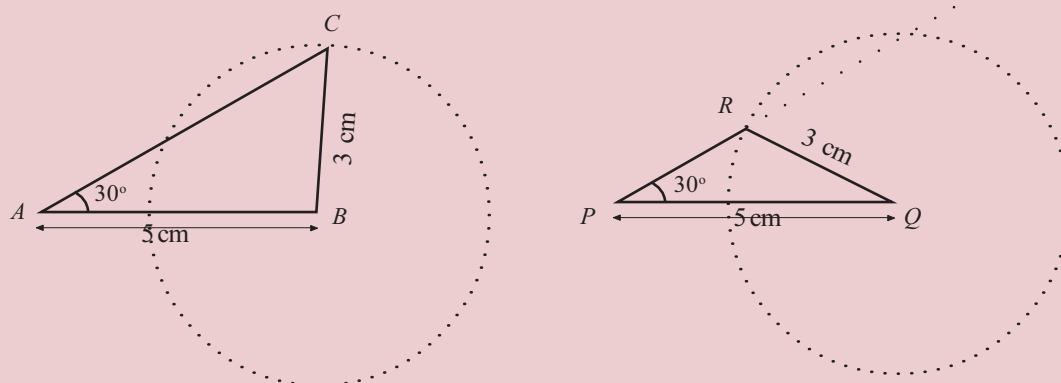
Remember drawing such triangles? (The section, **Another angle** of the lesson, **Drawing Triangles** in the class 7 textbook)

In $\triangle ABC$ and $\triangle PQR$,

$$AB = PQ = 5 \text{ centimetre}$$
$$BC = QR = 3 \text{ centimetre}$$
$$\angle A = \angle P = 30°$$

Are the sides AC and PR equal?

Why is it that even though two sides and an angle are equal, the third sides are not equal?
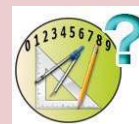
(1) In each pair of triangles below, find the pairs of matching angles and write them down.

i)
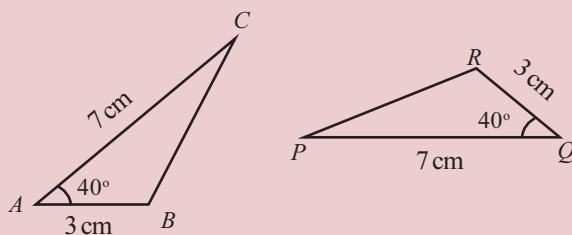
ii)
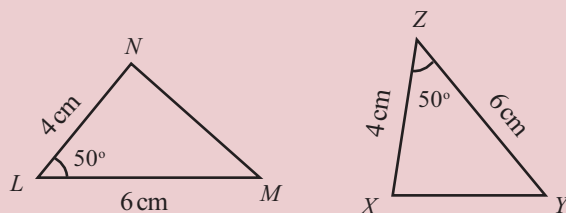
Geometry in action

Thales was a mathematician and philosopher, who lived in Greece, during the sixth century BC. Here's a trick he is supposed to have used to compute the distance from shore of the ship anchored at sea.

First he stuck a pole on the shore, directly in line with the ship. He stuck a second pole on the shore, some distance away from the first. A third pole he stuck, right in the middle.

He then drew a line perpendicular to the shore, and walked backwards along the line, keeping the ship in sight. Just when the middle stick came in his line of sight, he stopped and marked this spot.
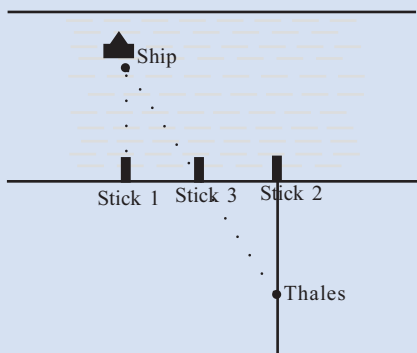
Now the triangle on sea and the triangle on shore are equal (why?) So, the distance of the ship from shore is equal to the distance between his final position and the second stick.

(2) In the figure below, *AC* and *BE* are parallel lines:
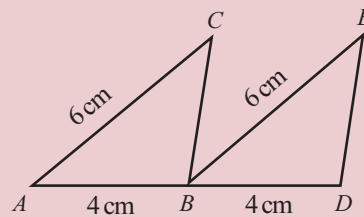
i) Are the lengths of *BC* and *DE* equal? Why?

ii) Are *BC* and *DE* parallel? Why?

(3) Is *ACBD* in the figure, a parallelogram? Why?
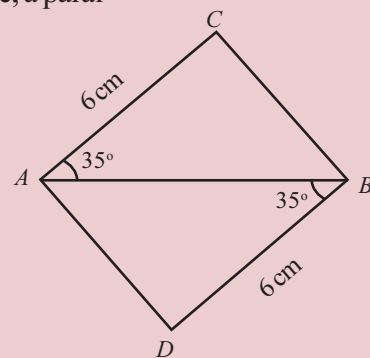
(4) In the figure below, *M* is the mid point of the line *AB*. Compute the other two angles of ΔABC.
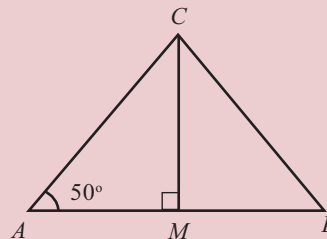
(5) In the figure below, the lines *AB* and *CD* are parallel and M is the mid point of *AB*.
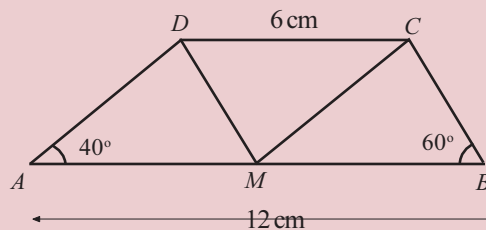
i) Compute the angles of ΔAMD, ΔMBC and ΔDCM?

ii) What is special about the quadrilaterals $AMCD$ and $MBCD$?

One side and two angles

If all sides of a triangle are specified, we can draw it; if two sides and the angle made by them are specified, then also we can draw the triangle.

What if the length of one side and the angles at the both its ends are specified?

One side is 8 centimetres and the angles at its ends are 40° and 60°. Can you draw the triangle?

It can be drawn like this:
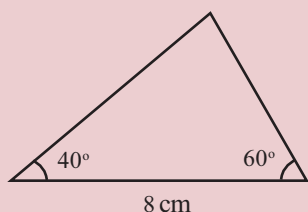
Changing the positions of the angles, we can also draw like this:
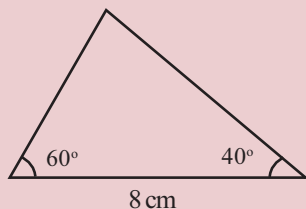
There are other ways:
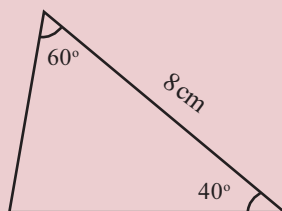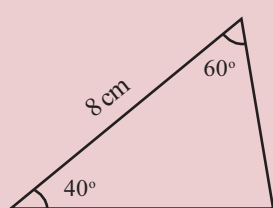
Any other way?

In all such triangles, the third angle is 80° (why?)

What about the other two sides?

Cut out one of these triangles and try to make it coincide with others. The other two sides are also equal, right?

So, we have a third general principle:

> **If one side of a triangle and the angles at its ends are equal to one side of another triangle and the angles at its ends, then the third angles are also equal and the sides opposite equal angles are equal.**

In any triangle, the sum of all three angles is 180°. So, if we know two angles of a triangle, then we can calculate the third.

Thus if any two angles of a triangle are equal to any two angles of another triangle, then the third angles are also equal.

Suppose one pair of sides are also equal. Would the other sides also be equal?

Draw two triangles like these:
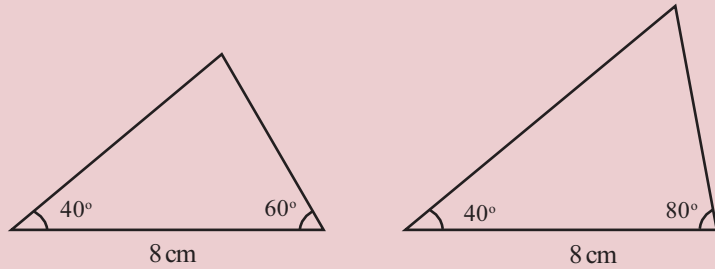
What is the third angle of each triangle?
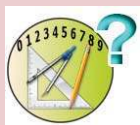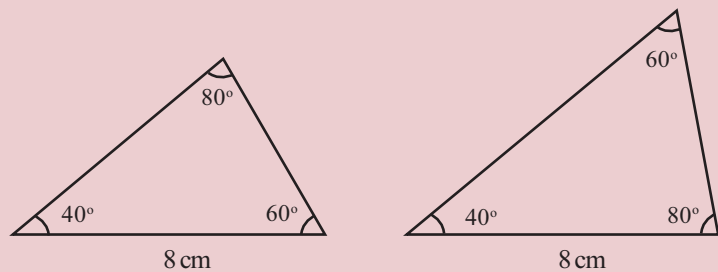
> **Why is it that, even though one side and all angles are equal, the other two sides are not equal?**

Let's see an application of the general principle stated above? In the figure below, *ABCD* is a parallelogram:
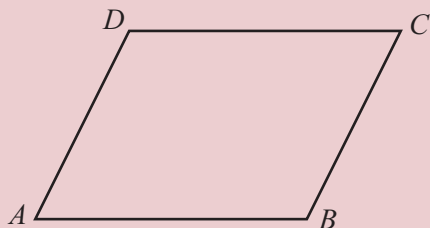
That is, the opposite sides *AB, CD* and the opposite sides *AD, BC* are parallel.

Drawing the diagonal *AC*, we can split it into two triangles:
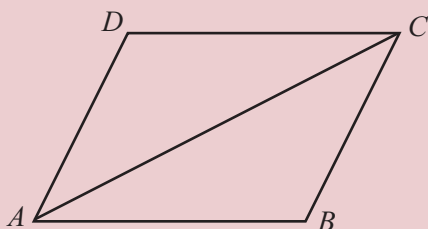
In both $\triangle ABC$ and $\triangle ADC$, one side is *AC*.

Are the angles at its ends equal?

$\angle CAB$ and $\angle DCA$ are alternate angles made by the line *AC* meeting the pair *AB, CD* of parallel lines.

So,

$$\angle CAB = \angle DCA$$

Similarly, we can also see that

$$\angle ACB = \angle DAC$$

(How?)

Thus in $\triangle ABC$ and $\triangle ADC$, the side *AC* and the angles at its ends are equal. So, the sides opposite equal angles are also equal. This means

$$AB = CD \qquad AD = BC$$

This is true for any parallelogram.

In any parallelogram, opposite sides are equal.

Wrong match

A triangle has three sides and three angles - six measures altogether. If in two triangles three specific measures (three sides, two sides and angle made by them, one side and angles at its two ends) are equal, then the triangles are congruent (that is, the remaining three measures are also equal), as we have seen.

Now draw a triangle of sides 4, 6, 9 centimetres.
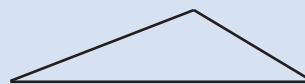
And another triangle of sides 6, 9, 13 centimetres.
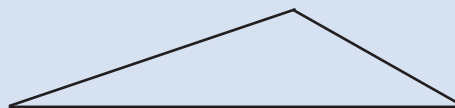
Measure the angles of both triangles. Aren't they the same? (You can also check this by making out one triangle and placing it over the other)

Thus in these triangles, five measures (three angles and two sides) are equal. But the triangles are not congruent.

Let's draw the second diagonal *BD* also of our parallelogram and name the intersection of the diagonals as *P*.
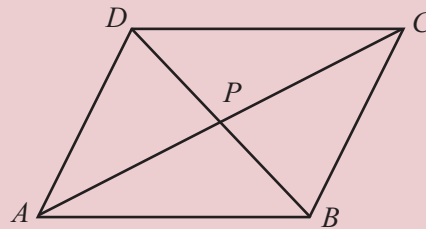
Draw lines AB and AC. Choose the **parallel line** tool and draw the lines parallel to AC through BC and the line parallel to AB trough C. Mark their point of intersection as D. Draw the diagonals AD, BC of the parallelogram ABCD.
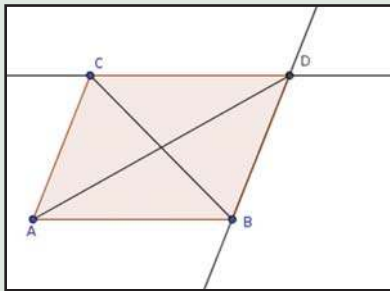
Check if the diagonals bisect each other (choose the **Midpoint or center** tool and click on a digonal to get its mid point.) Changing the positions of A, B, C we get different parallelograms.

Look at $\triangle APB$ and $\triangle CPD$. We have already seen that the sides *AB* and *CD* are equal. What about the angles at their ends?

We have seen that $\angle CAB$, $\angle DCA$ are equal.

That is, $\angle PAB = \angle PCD$

Are $\angle PBA$ and $\angle PDC$ equal?

They are alternate angles made by the line *BD* meeting the pair *AB*, *CD* of parallel lines. So they are also equal.

Thus in $\triangle APB$ and $\triangle CPD$, the sides *AB* and *CD* are equal; and the angles at their ends are also equal. So the sides opposite equal angles are also equal.

That is $AP = CP \quad BP = DP$

In other words, P is the mid point of both the diagonals *AC* and *BD*.

> **In any parallelogram, the point of intersection of the diagonals is the midpoint of both.**

When a line passes through the midpoint of another, we say that the first line bisects the other. So, we can state the above principle like this:

> **In any parallelogram, the diagonals bisect each other.**

(1) In each pair of triangles below, find matching pairs of sides and write their names.

i)
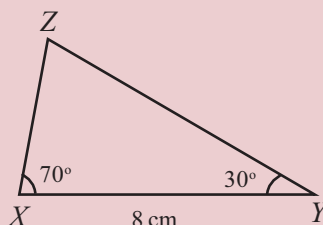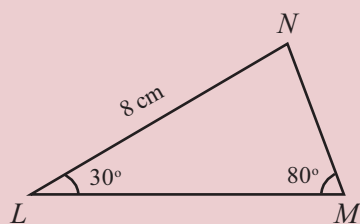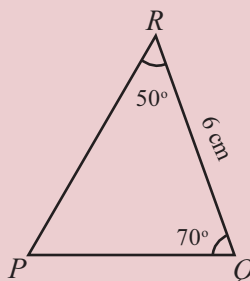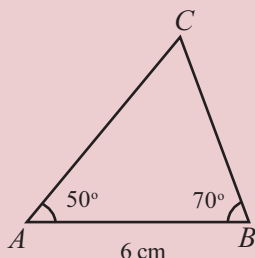
(2) In the figure, *AP* and *BQ* equal and parallel are lines drawn at the ends of the line *AB*. The point of intersection of *PQ* and *AB* is marked as *M*.
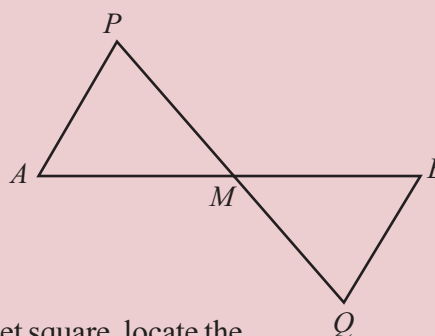
i) Are the sides of ΔAMP equal to the sides of ΔBMQ? Why?

ii) What is special about the position of *M* on *AB*?

iii) Draw a line 5.5 centimetres long. Using a set square, locate the midpoint of this line.

(3) In the figure, *ABCDE* is a pentagon with all sides of the same length and all angles of the same size. The sides *AB* and *AE* extended, meet the side *CD* extended at *P* and *Q*.
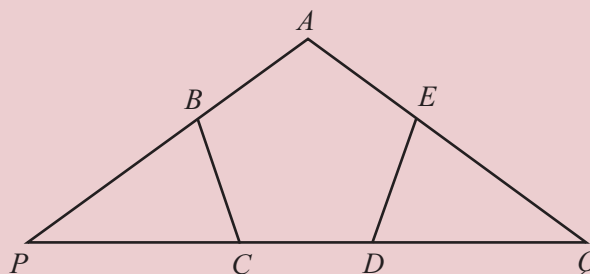
i) Are the sides of ΔBPC equal to the sides of ΔEQD? Why?

ii) Are the side of *AP* and *AQ* of ΔAPQ equal? Why?

(4) In ΔABC and ΔPQR shown below.

$$AB = QR \qquad BC = RP \qquad CA = PQ$$
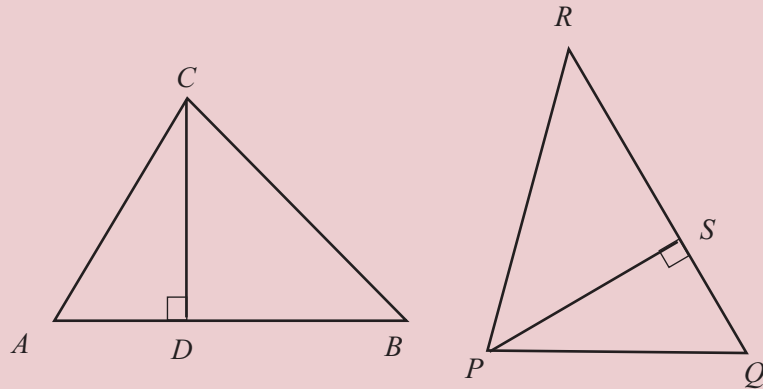
i) Are *CD* and *PS* equal? Why?

ii) What is the relation between the areas of ΔABC and ΔPQR?

(5) In the quadrilateral *ABCD* shown below, the sides *AB* and *CD* are parallel. M is the midpoint of the side *BC*.
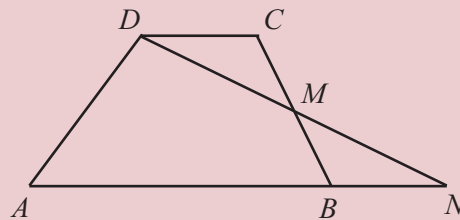
The lines *DM* and *AB* extended, meet at *N*.

i) Are the areas of ΔDCM and ΔBMN equal? Why?

ii) What is the relation between the areas of the quadrilateral *ABCD* and the triangle ADN?

(6) Are the two diagonals of a rectangle equal? Why?

Isosceles triangles

See this triangle. Two of its sides are equal.

Don't the angle at the bottom also seen to be equal?

Cut out a paper triangle like this and fold along the middle. So that the equal sides coincide. The angles also coincide, right?

Why are the angles equal?

Let's draw the line of folding in our figures. That is, join the top corner and the midpoint of the bottom side:
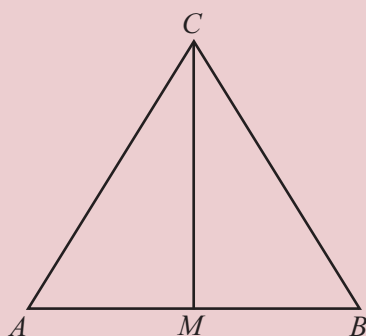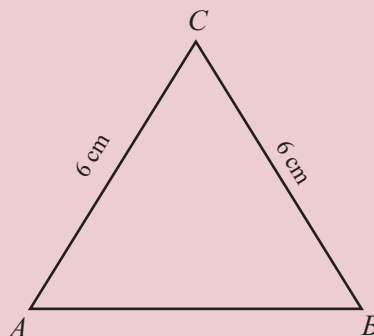
Now we have two triangles *AMC* and *BMC*. The sides *AC* and *BC* are equal.

Create slider **a** with min = 3 and max = 10. Draw line *AB* of length 6. Draw circles centred at A and B, of radius **a**. Mark their point of intersection C. Now the circles can be hidden. By changing a, we get different triangles and all of them have two equal sides. What about the angles? What are the angles when a = 6?

Since *M* is the midpoint of *AB*, we have *AM* and *BM* equal.

In both triangles, the third side is *CM*.

Since the sides of the triangles are the same, angles opposite equal sides are also equal.

So the ∠A and ∠B opposite the side CM in the triangles are equal.

Let's write this as a general principle.

> **If two sides of a triangle are equal, the angles opposite these sides are also equal.**

We also note another thing ∠*AMC* and ∠*BMC* in Δ*AMC* and Δ*BMC* are opposite the equal sides AC and BC; so these angles are also equal.

Since these angles are on either side of the line CM, their sum is 180°.

So each of these angles is 90°

That is, the line CM is perpendicular to AB.

Now we can ask : is our general principle true in reverse?

That is, if two angles of a triangle are equal, are the sides opposite them also equal?

Let's draw a picture.
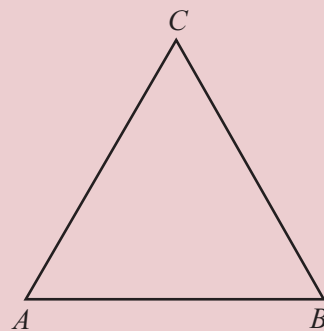
In the GeoGebra construction in the last page, choose Trace on for C. Not the path of C.

a = 4.38
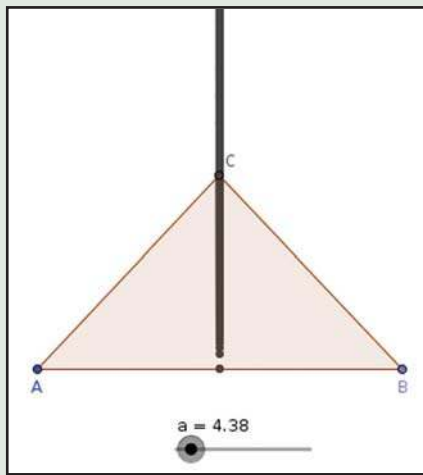
In $\triangle ABC$, we have $\angle A = \angle B$. The question is whether $AC = BC$

Let's split the triangle into two as before. Here, it is more convenient to draw the perpendicular from C to AP, rather than joining C and the midpoint of AB as before.
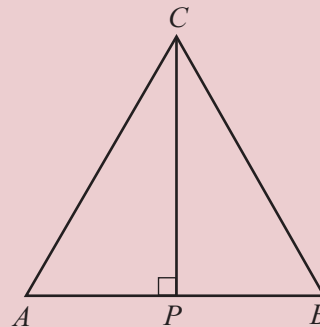
CP is a side of both $\triangle APC$ and $\triangle BQC$; and the angles at the end P are both right. What about the angles at the other end?

We know that $\angle A = \angle B$

And also $\angle APC = 90^\circ = \angle BPC$

So, the third angles ACP and BCP are also equal (why?)

Thus in these triangles, one side and angles at its ends are equal.

So, the sides opposite equal angles are also equal. Thus we see that AC and BC are equal.

> **If two angles of a triangle are equal, then the sides opposite the equal angles are equal.**

A triangle with two sides equal, is called an isosceles triangle. According to the general principle seen now, triangles with two angles equal are also isosceles.

See this triangle.

Take the **slider** tool and select **Angle**.

This gives α. Put min $= 0^\circ$ and max $= 90^\circ$. Draw line AB of length 6.

Draw lines through A and B with $\angle A = \angle B = \alpha$ and mark their point of intersection C. Draw $\triangle ABC$.
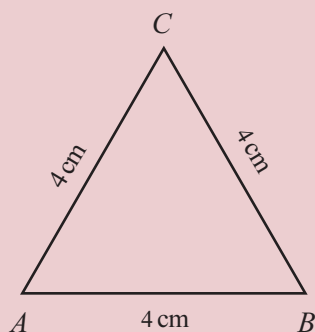
We know that a triangle like this, with all three sides equal, is called an equilateral triangle.

In $\triangle ABC$ shown above, $AC = BC$ and so $\angle B$ and $\angle A$ opposite these sides are equal. Also AB = AC, So that $\angle C$ and $\angle B$ opposite these sides are also equal. Thus all three angles of this triangle are equal. Since the sum of the angles is 180°, we see that each angle is $180^\circ \div 3 = 60^\circ$
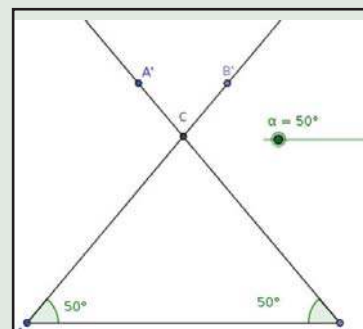
Now hide the lines A′C, B′C and the points A′, B′ . See how the triangle changes on changing α. What is special about the triangle, when $\alpha = 60^\circ$? What about $\alpha = 45^\circ$?

> **In any equilateral triangle all angles are 60°**

On the other hand, any triangle with all angles 60°, is an equilateral triangle (can you explain why?).

(1) Some isosceles triangles are drawn below. In each, one angle is given. Find the other angles.
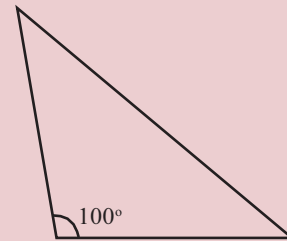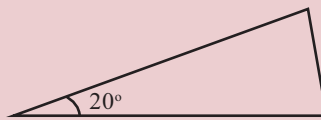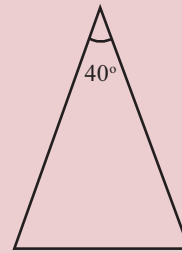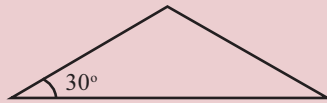
(2) One angle of an isosceles triangle is 90°. What are the other two angles?

(3) One angle of an isosceles triangle is 60°. What are the other two angles?

(4) In figure below, O is the centre of the circle and A, B are points on the circle.
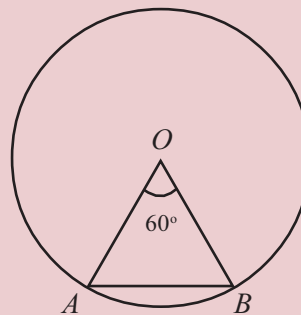
Compute $\angle A$ and $\angle B$.

5. In the figure below, O is the centre of the circle and A, B, C are points on the circle.
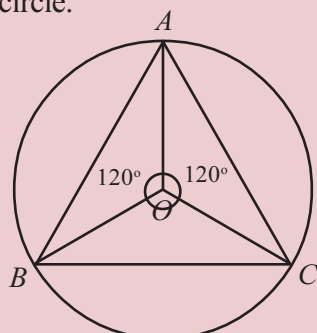
What are the angles of $\triangle ABC$?

Bisectors
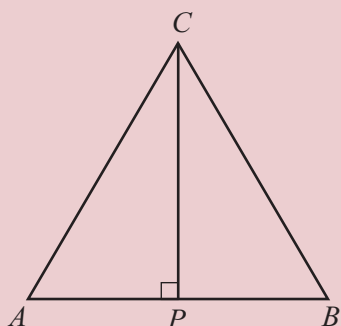
See this figure.

In $\triangle ABC$, the sides AC and BC are equal. CP is the perpendicular from C to AB.

We have seen that the sides and angles of $\triangle APC$ and $\triangle BPC$ are the same. So $AP = BP$. That is, CP bisects AB. Again $\angle ACP$ and $\angle BCP$ are equal. So we can say that the line CP bisects $\angle C$ also.

> **In any isosceles triangle the perpendicular from the point joining equal sides to the opposite side bisects the angle at this point and the side opposite.**

A line dividing a line or an angle into two equal parts is called a bisector. Thus in the figure above, the line CP is a bisector of AB and $\angle C$. Since it is perpendicular to AB also, it is called the perpendicular bisector of AB.

Perpendicular bisector

We can draw several perpendicular, to a line.
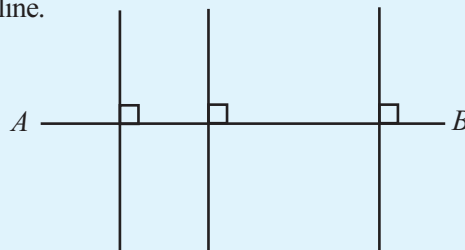
Also, many bisectors of the line
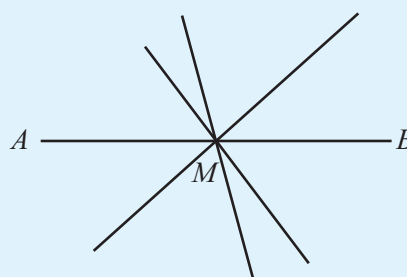
But only one perpendicular bisector.
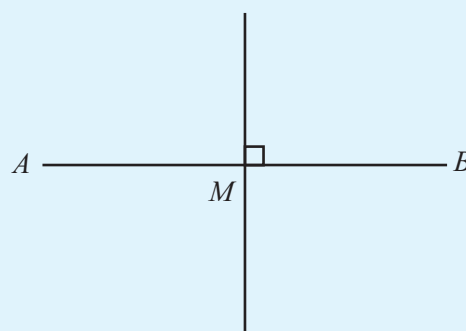
Internal perpendicular

How do we draw a perpendicular to a line, from a specific point on it?

A — P — B
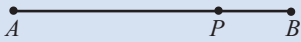
First mark two points C, D on AB, at equal distances from P.

A — C P D B
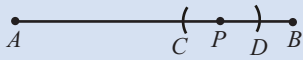
Next mark Q at the same distance from C and D.

Q

A — C P D B
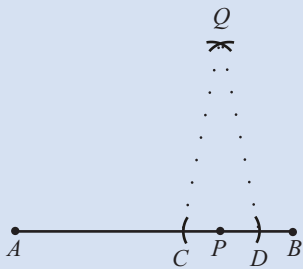
Now ΔCQD is an isosceles triangle. So the line QP is perpendicular to CD. Since CD is a part of the line AB, the line QP is perpendicular to AB.

Q

A — C P D B
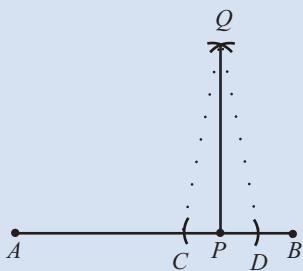
We can also say this in a slightly different way: The perpendicular bisector of AB passes through C.

We can draw other isosceles triangles on AB.

And the perpendicular bisector of AB passes through the top vertex of all these triangles.
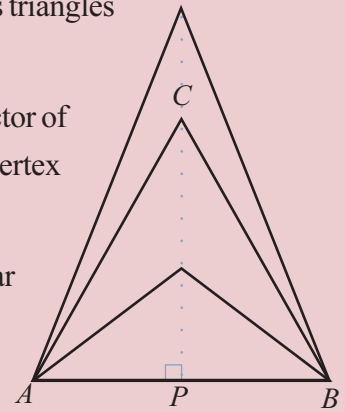
So to draw the perpendicular bisector of AB. We need only join all these points and extend to P.

To draw a line, we need only two points, right? So, to draw the perpendicular bisector, we need only two such triangles. And we need not draw the whole triangles even; we need only the top vertices.

This means two points at the same distances from A and B.
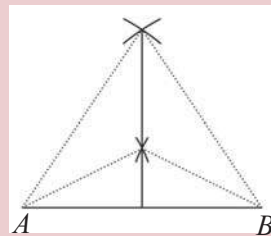
If we want the bisector to be extended below, we can draw like this:
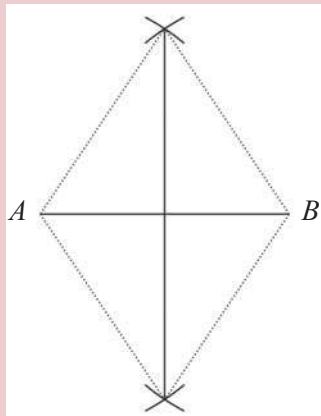
We can use this idea to draw the bisector of an angle also.

First we draw an isosceles triangle including this angle like this:
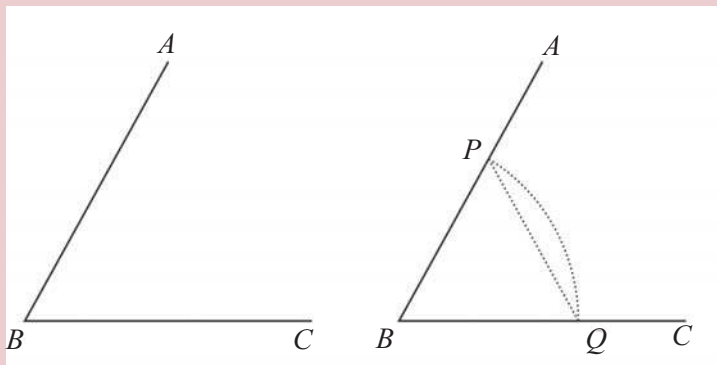
Now we need only draw the perpendicular bisector of the side PQ of ΔPBQ.

Here we can save some work, by noting that this bisector passes through B (why?). So we need only one more point on this bisector.
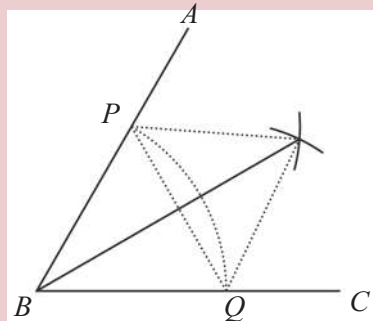
(1) Draw a line 6.5 centimetres long and draw its perpendicular bisector.

(2) Draw a square, each side 3.75 centimetres long.

(3) Draw an angle of 75° and draw its bisector.

(4) Draw a circle of radius 2.25 centimetres.

(5) Draw ΔABC, with $AB = 6$ cm, $\angle A = 22\frac{1}{2}^{\circ}$, $\angle B = 67\frac{1}{2}^{\circ}$.

External Perpendicular

We can draw a perpendicular to a line, from a point on it, using compass. How do we draw a perpendicular from a point not on the line? $\bullet P$
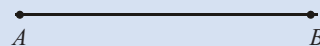
For this, we must draw an isosceles triangle with P as the top vertex and base along AB. To do this, we need only mark two points on AB, at the same distance from P. Draw a circle with P as centre and cut AB at Q and R
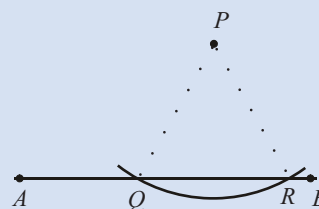
Now we need only draw the perpendicular bisector of QR.
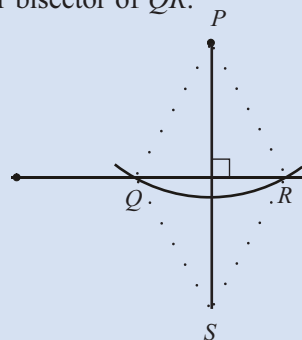
(6) Draw a triangle and the perpendicular bisectors of all three sides. Do all three bisectors intersect at the same point?

(7) Draw a triangle and the bisectors of all three angles, do all three bisectors intersect at the same point?

(8) Prove that if both pairs of opposite sides of a quadrilateral are equal, then it is a parallelogram.

(9) In the figure below. *ABCD* is a parallelogram and *AP = CQ*.
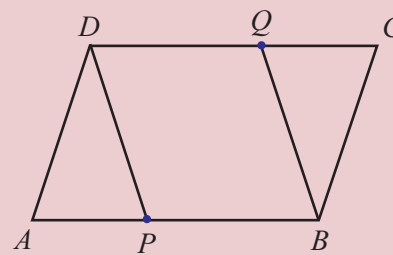
Prove that *PBQD* is a parallelogram.

(10) Prove that if all sides of a parallelogram are equal, then each diagonal is the perpendicular bisector of the other.

(11) In the figure below, *O* is the centre of the circle and *AB* is a diameter. *C* is a point on the circle.
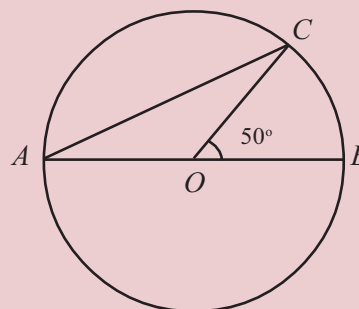
i) Compute ∠*CAB*.

ii) Draw another figure like this, with a different number for the size of ∠*COB*. Calculate ∠*CAB*.

Rope Math

We have mentioned **Elements**, the authoritative text on ancient geometry. In it, Euclid considers only figures made of straight lines and circles. In other words only shapes that can be drawn using an unmarked ruler (straightedge) and compass. Why is this so?

In ancient times, people used strings or ropes to measure and draw.

Straight lyrics and circles are the shapes which can be drawn with rope: As a piece of rope stretched between two fixed pegs makes a straight line; detaching one peg and relating about the other makes a circle.

Now that we can make tests to draw different shapes, such constructions have only historical and theoretical importance.

(12) In the figure below, O is the centre of the circle and AB is a diameter. C is a point on the circle.

   i) Compute $\angle ACB$.

   ii) Draw another figure like this, changing the size of $\angle COB$ and calculate $\angle ACB$.

> **In any circle, if the ends of a diameter are joined to another point on the circle, what would be the angle got?**
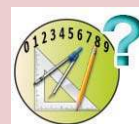
(13) How many different isosceles triangles can be drawn with one angle $50°$ and one side 7 centimetres?

(14) Draw $\triangle ABC$ with $AB = 7$ cm, $\angle A = 67\frac{1}{2}^{\circ}$, $\angle B = 15°$ without using a protractor.
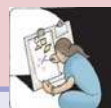
> If the four sides of a quadrilateral are equal to the four sides of another quadrilateral, would the angles of the two quadrilaterals also be equal?
>
> Draw some figures and check. If in addition to the four sides of the quadrilaterals, some other lengths are equal, would the angles be equal?

Looking back

Learning outcomes	What I can	With teacher's help	Must imporve
• Explaining the various cases in which the equality of some measures of a triangle imply the equality of other measures.			
• Forming some other geometric principles from such principles about triangles.			
• Describing the various methods to draw the perpendicular bisector of a line and the bisectors of an angle.			
• Describing methods to draw perpendiculars to a line from a point on the line and from a point not on the line.			

2
Equations

Addition and Subtraction

Suhara opened her money-box and started counting. "How much do you have?', mother asked. "If you give me seven rupees, I'd have a round fifty", suhara looked up hopefully.

How much does she have in her box?

7 rupees more would make 50 rupees, which means 7 less than 50, that is, $50 - 7 = 43$.

Unni spent 8 rupees out of his *Vishukkaineettam* to buy a pen. Now he has 42 rupees left. How much is his *Kaineettam* ? 8 rupees less made it 42 rupees.

So, what he got is 8 more than 42; that is, $42 + 8 = 50$.

(1) "Six more marks and I would've got full hundred marks in the math test", Rajan was sad. How much mark did he actually get?

(2) Mother gave 60 rupees to Lissy for buying books. She gave back the 13 rupees left. For how much money did she buy books?

(3) Gopalan bought a bunch of bananas. 7 of them were rotten which he threw away. Now there are 46. How many bananas were there in the bunch?

(4) Vimala spent 163 rupees shopping and now she has 217 rupees. How much money did she have at first?

(5) 264 added to a number makes it 452. What is the number?

(6) 198 subtracted from a number makes it 163. What is the number?

Multiplication and division

In an investment scheme, the amount deposited doubles in six years. To get back ten thousand rupees finally, how much money should be deposit now?

10000 is double the investment; so the investment should be half of 10000; that is, 5000.

Four people divided the profit they got from vegetable business and Jose got one thousand five hundred rupees. What is the total profit?

1500 is $\frac{1}{4}$ of the profit; so total profit is 4 times 1500;

that is, $= 1500 \times 4 = 6000$.

(1) In a company, the manager's salary is five times that of a peon. The manager gets 40000 rupees a month. How much does a peon gets a month?

(2) The travellers of a picnic split equally, the 5200 rupees spent. Each gave 1300 rupees. How many travellers were there?

(3) A number multiplied by 12 gives 756. What is the number ?

(4) A number divided by 21 gives 756. What is the number ?

Different Changes

Look at this problem:

23 rupees was spent in buying two note books and a pen of three rupees. What is the price of a notebook?

Let's look at it like this. The total cost became 23, when a pen of 3 rupees was also bought, Suppose the pen was not bought.

The cost would have been only 20 rupees. This 20 rupees is the price of two books. So, the price of a book is 10 rupees. Now let's look at this in reverse. Two books, 10 rupees each, cost 20 rupees; and 3 rupees for the pen.

Altogether, 23 rupees.

Look at another problem:

When a number is tripled and then two added, it became 50. What is the number?

An unknown number, first multiplied by 3 and then 2 added gives 50.

$$?\ \xrightarrow{\times 3}\ ?\ \xrightarrow{+2}\ 50$$

To get the original number back, what all should we do?

2 added finally gave 50; so before that it must be $50 - 2 = 48$

$$?\ \xrightarrow{\times 3}\ 48\ \underset{-2}{\overset{+2}{\longleftrightarrow}}\ 50$$
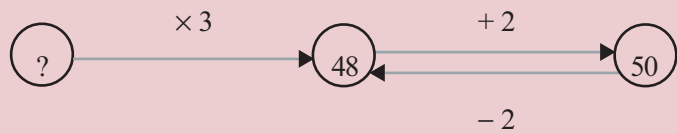
Now how do we get back to the original number from 48?

It was multiplication by 3 that gave 48; so, the original number is $48 \div 3 = 16$

$$16\ \underset{\div 3}{\overset{\times 3}{\longleftrightarrow}}\ 48\ \underset{-2}{\overset{+2}{\longleftrightarrow}}\ 50$$
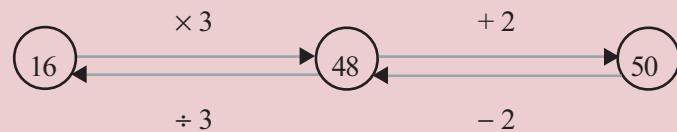
Let's change the problem:

When a number is tripled and two subtracted, it became 40. what is the number?

Here the number, before subtracting 2 finally must have been $40 + 2 = 42$.

This was got on multiplying by 3; so before that it must have been $42 \div 3 = 14$. Thus original number is 14.

$$14\ \underset{\div 3}{\overset{\times 3}{\longleftrightarrow}}\ 42\ \underset{+2}{\overset{-2}{\longleftrightarrow}}\ 40$$
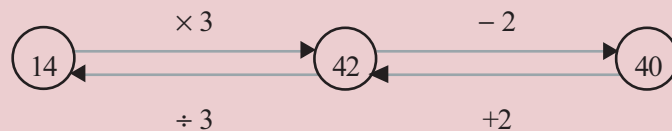
Look at another problem:

When a fourth of a number is added to the number, 30 is got. What is the number?

Inversion

If we know the result of adding 2 to a number, then to get the number back, we must subtract 2. What if we know the result of subtracting 2? To get the number back, we must add 2. Like this, to get a number back from the result of multiplying by 2, we must divide by 2; and to get back a number from the result of dividing by 2, we must multiply by 2.

The Indian mathematcian Bhaskara-charya discusses this in his book *Lilavathi*. He describes what he calls in the method of inversion. Thus:

> *To get the number if we know the result, change division to multiplication and multiplication to division, square root to square; positive numbers to negative numbers and negative numbers to positive.*

When a fourth is added, we get $\frac{5}{4}$ of the number. Thus $\frac{5}{4}$ of the number is 30.

So the number is $\frac{4}{5}$ of 30.

That is $30 \times \frac{4}{5} = 24$

(1) Anita and her friends bought pens. For five pens bought together, they got a discount of three rupees and it cost them 32 rupees. Had they bought the pens separately, how much would each have to spend?

(2) The perimeter of a rectangle is 25 metres and one of its side is 5 metres. How many metres is the other side?

(3) In each of the problems below, the result of doing some operations on a number is given. Find the number.

    (i)   three added to double is 101.

    (ii)   two added to triple is 101.

    (iii)   three subtracted from double is 101.

    (iv)   two subtracted from triple is 101.

(4) Half a number added to the number gives 111. What is the number?

(5) A piece of folk math: a child asked a flock of birds, "How many are you?"

A bird replied.

    We and us again,
    With half of us
    And half of that
    With one more,
    Would make hundred"

How many birds were thre?

In this bird problem, what other numbers can be the final sum, instead of 100?

Ancient Math

Even as early as the third millennium BC, Egyptians used to keep written records. They used to write on sheets made from flattened stems of plant called papyrus. Lots of such records, also called papyrus, are discovered by archaeologists.

Some of these discuss mathematical problems and methods of solution. One such papyrus, estimated to be written around 1650 BC gives the name of the scribe as Ahmose and that it is copied from another, two hundred years older. It is called the Ahmose papyrus and now preserved in the British Museum. (Since it was discovered by the researcher Alexander Rhind, it is also called the Rhind papyrus.)

It discusses problems on arithmetic and geometry.

Algebraic method

What is the common feature of all the problems we have done so far?

The result of doing some operations on an unknown number is given; and we find the original number.

How did we do it? The inverse of all operations done are done in the reverse order, last to the first. For example, look at this problem:

Rashida bought 4 kilograms of okra, and curry leaves and coriander leaves for 10 rupees. she had to pay 130 rupees. What is the price of one kilogram of okra?

First we write this in math language:

When a number is multiplied by 4 and 10 added, we get 130. What is the number?

How do we find the original number? First subtract the final 10 added; then divide by the 4, by which it was multiplied first. That is

$$(130 - 10) \div 4 = 120 \div 4 = 30$$

Thus we see that the price of one kilogram okra is 30 rupees.

Now look at this problem:

A ten metre long rod is to be bent to make a rectangle. Its length should be one metre more than the breadth. What should be the length and breadth?

First, let's write the problem using only numbers. The perimeter of a rectangle is twice the sum of its length and breadth. Here, the length should be 1 more than the breadth. So sum of the length and breadth means the sum of breadth and 1 added to the breadth.

Thus the problem is this :

The sum of a number and 1 added to it, multiplied by 2 is 10. What is the number?

Getting rid of the last multiplication by 2, it can be put like this:

The sum of a number and 1 added to it is 5. What is the number?

Old method

A puzzle in Ahmose papyrus is this one:

A number added to its one fourth gives 15. Which is the number? The method to solve is like this: 4 is added with its one fourth gives 5. We need 15. It is 3 times of 5. So have the anser is 12 which is three times of 4.

Why this logic is working here? Is it true for any quesions?

Whatever be the number, the sum of itself and one added to it is equal to one added to twice the number. Remember seeing this in class 7? (The section, **Number relations** of the lesson **unchanging relations**)

We also noted that it is more convenient to write it in algebra.

$$x + (x + 1) = 2x + 1, \text{ for every number } x.$$

Let's use this in the problem we are discussing now. If we denote the unknown number in this problem as x, then the problem becomes this.

If $2x + 1 = 5$, what is x?

What is the meaning of this? When a number is doubled and 1 added to it, it becomes 5. what is the number?

We can find the number by inversion.

$$(5 - 1) \div 2 = 2.$$

So, the breadth of the rectangle is 2 metres and the length, 3 metres.

Sometimes, it is convenient to do such problems using algebra from the very beginning. Look at this problem.

> The price of a chair and a table together is 4500 rupees. The price of the table is 1000 rupees more than that of the chair. What is the price of each?

Let's take the price of the chair as x rupees. Since the price of the table is 1000 rupees more, it is $x + 1000$ rupees. So what is the algebraic form of the problem?

If $x + (x + 1000) = 4500$, what is x?

How can we rewrite $x + (x + 1000)$?

$$x + (x + 1000) = 2x + 1000$$

So the problem becomes

If $2x + 1000 = 4500$, what is x?

What is its meaning?

Addition and subtraction

If we add a number to a number, and then subtract the added number, we get the number we started with. We can write this in algebra as,

$(x + a) - a = x$, for all numbers x, a.

We can write it like this also.

If $x + a = b$, then $x = b - a$.

This is the algebraic form of the method of finding an unknown number, given the result of adding a known number to it similarly,

if $x - a = b$, then $x = b + a$.

This is the algebraic form of the method of finding an unknown number, given result of subtracting a known number from it.

When a number is multiplied by 2 and then 1000 added gives 4500. What is the number?

This is just the last problem with numbers changed, right?

We can find the number by inversion. Let's write that also in algebra.

We first get twice the number as $4500 - 1000 = 3500$.

That is,

$$2x = 4500 - 1000 = 3500$$

Then we find the number itself as $3500 \div 2 = 1750$.

Writing this in algebra,

$$x = 3500 \div 2 = 1750$$

Now we can go back to the original problem and say that the price of the chair is 1750 rupees and the price of the table is 2750 rupees.

Let's look at one more problem:

A hundred rupee note was changed to ten and twenty rupee notes, seven notes in all. How many of each?

Let's take the number of twenty rupee notes as x; then the number of ten rupee notes is $7 - x$.

x twenty rupee notes make $20x$ rupees. $7 - x$ ten rupee notes make $10(7 - x)$ rupees.

Altogether $20x + 10(7 - x)$ rupees and this we know is 100 rupees.

Multiplication and division

Starting from the product of a number by another, to get the original number back, we have to divide by the multiplier. Similarly, to get a number back, from one of its quotients, we must multiply by the divisor. These we write in algebraic language like this:

If $ax = b\ (a \neq 0)$ then $x = \dfrac{b}{a}$

If $\dfrac{x}{a} = b$ then $x = ab$

These are the algebraic forms of the rules of inversion to recover a number from a product and a quotient.

So the problem, in algebra, is this:

If $20x + 10(7 - x) = 100$, what is x?

In this, we can simplify $20x + 10(7 - x)$

$$20x + 10(7 - x) = 20x + 70 - 10x = 10x + 70$$

Using this, we can rewrite the problem.

If $10x + 70 = 100$, what is x?

The means, the number x multiplied by 10 and 70 added to the product gives 100. So to get the number x, we have to subtract 70 from 100 and divide by 10. In algebraic terms,

$$x = (100 - 70) \div 10 = 30 \div 10 = 3$$

Thus the answer to the original problem is 3 twenty –rupee notes, 4 ten –rupee notes.

Al-khwarizmi

(1) The perimeter of a rectangle is 80 metres and its length is one metre more than twice the breadth. What are its length and breadth?

(2) From a point on a line, another line is to be drawn such that the angle on one side is 50° more than the angle on the other side. How much is the smaller angle?

(3) The price of a book is 4 rupees more than the price of a pen. The price of a pencil is 2 rupees less than the price of the pen. The total price of 5 books, 2 pens and 3 pencils is 74 rupees. What is the price of each?

(4) i) The sum of three consecutive natural numbers is 36. What are the numbers?

ii) The sum of three consecutive even numbers is 36. What are the numbers?

iii) Can the sum of three consecutive odd numbers be 36? Why?

iv) The sum of three consecutive odd numbers is 33. What are the numbers?

v) The sum of three consecutive natual numbers is 33. What are the numbers?

(5) i) In a calendar, a square of four numbers is marked. The sum of the numbers is 80. What are the numbers?

What 's in a name?

Algebra was introduced to Europe during Renaissance through the translation of Arab texts. The most important among these were the works of Mohammed Al-khwarizmi.

He lived during the eighth century AD. To indicate an unknown number, he used an Arab word meaning "thing".

Given that 2 subtracted from a number gives 5, we add 2 and 5 to get the number back. Al–khwarizmi calls such operations by the Arab word "*al jabr*". The word means joining or restoring. The English word *algebra* is derived from this.

In English, the word *algorithm* is used for a step-by-step procedure to solve a problem (especially in computers). This word is derived from the word al-khwarizmi.

ii) A square of nine numbers is marked in a calendar. The sum of all these numbers is 90. What are the numbers?

Different Problems

See this problem:

Ten added to thrice a number makes five times the number. What is the number?

Here we can't find the number by inversion, right? But we can think like this: to get five times any number from thrice the number, we have to add double the number.

(the section, **Number relations** in the lesson, **Unchanging relations** of the class 7 textbook)

In our problem, what is added is ten. So, double the number is ten, and thus compute the number as five.

How about writing these in algebra?

If we take the original number as x, the problem says.

$$3x + 10 = 5x$$

We know that to get $5x$ from $3x$, we have to add $2x$.

That is,

$$3x + 2x = 5x.$$

In our problem. What is added to $3x$ to get 5 is 10:

Thus $2x = 10$ and so $x = 5$.

Let's change the problem slightly:

36 added to 13 times a number gives 31 times the number. What is the number?

To get 31 times a number from 13 times, how many times the number must be added?

$31 - 13 = 18$ times, right?

In our problem, what is added is 36. So, 18 times the number is 36 and the number is 2.

How about the algebra? Taking the number as x, the problem and the method of solution we can write like this:

> **Equations**
>
> What is the meaning of
>
> $2x + 3 = 3x + 2$? x is a number such that 3 added to 2 times the number, and 2 added to 3 times the number gives the same result.
>
> This is true only if $x = 1$. Algebraic statements like this, which say that two operations on numbers give the same result, are generally called equations.

$$13x + 36 \;=\; 31x$$
$$31x - 13x \;=\; 18x$$
$$18x \;=\; 36$$
$$x \;=\; 2$$

Now look at this problem:

12 added to 3 times a number is equal to 2 added to 5 times the number. what is the number?

Taking the number as x, what the problem says can be written,

$$3x + 12 = 5x + 2$$

We know that $2x$ added to $3x$ makes $5x$. To get $5x + 2$, we must add 2 more, right?

That is,

$$3x + (2x + 2) = 5x + 2$$

In the problem, what is added is 12.

So,

$$2x + 2 = 12$$

Now we can compute x by inversion:

$$x = (12 - 2) \div 2 = 5$$

Let's look at some more problems:

The age of Appu's mother is nine times that of Appu. After nine years, it would be three times. What are their ages now?

We start by taking Appu's age now as x. So according to the problem his mother's age is $9x$ now.

After 9 years?

Appu's age would be $x + 9$.

Mother's age would be $9x + 9$.

By what is said in the problem, mother's age then would be 3 times Appu's age; that is $3(x + 9) = 3x + 27$.

Now we can write what the problem says, in algebra.

$$3x + 27 = 9x + 9$$

What all should be added to $3x$ to get $9x + 9$?

Trick of nine

Take any two-digit number ending in 9 and add the sum and product of the digits. For example if we take 29, sum of the digits is $2 + 9 = 11$ and product of the digits is $2 \times 9 = 18$, and their sum is $11 + 18 = 29$.

Is this true for all two-digit numbers ending in 9?

Take the number as $10x + 9$ and check.

Does two digit numbers ending in any other digit have this peculiarity?

Can you compute y from the equation $10x + y = x + y + xy$?

In algebraic terms,

$$(9x + 9) - 3x = 6x + 9$$

In the problem, what is added is 27. So,

$$6x + 9 = 27$$

From this, we get $6x = 27 - 9 = 18$ and $50x = 3$. Thus Appu's age is 3 and mother 's 27.

Folk math

There are some lotus flowers in a pond. Some birds in flight thought of resting on them. When each bird sat on a flower, one bird had no seat. When a pair of birds sat on each flower, one flower had no bird. How many flowers? How many birds?

(1) Ticket rate for the science exhibition is 10 rupees for a child and 26 rupees for an adult. 740 rupees was got from 50 persons. How many children among them?

(2) A class has the same number of girls and boys. Only eight boys were absent on a particular day and then the number of girls was double the number of boys. What is the number of boys and girls?

(3) Ajayan is ten years older than Vijayan. Next year, Ajayan's age would be double that of Vijayan. What are their ages now?

(4) Five times a number is equal to three times the sum of the number and 4. What is the number?

(5) In a co-operative society, the number of men is thrice the number of women. 29 women and 16 men more joined the society and now the number of men is double the number of women. How many women were there in the society at first?

Looking back

Learning outcomes	What I can	With teacher's help	Must imporve
• Solving simple problems on numbers by inversion			
• Using algebra, according to need, in problems which cannot be directly solved through inversion.			

3
Polygons
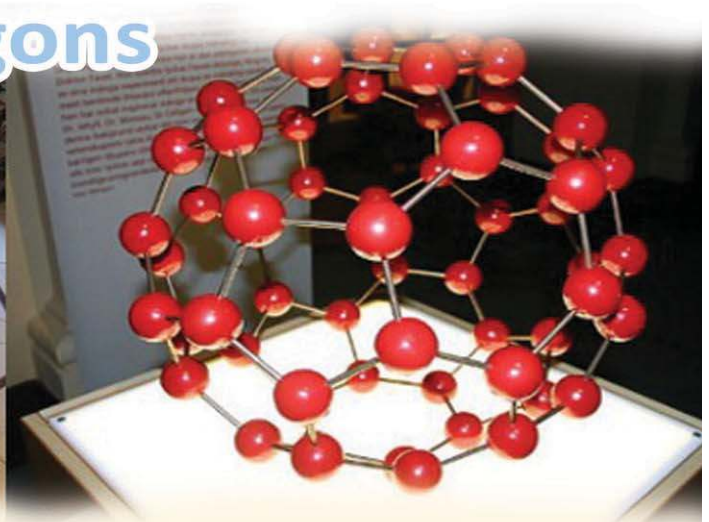
Shapes

See this picture:
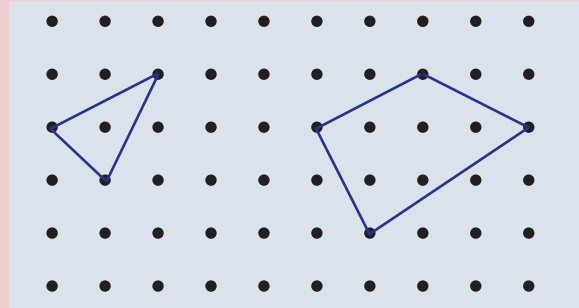
Different shapes by joining dots.

Three dots joined to make a triangle.

For a quadrilateral?

Now see the shape on joining five dots:

How many vertices? How many sides?
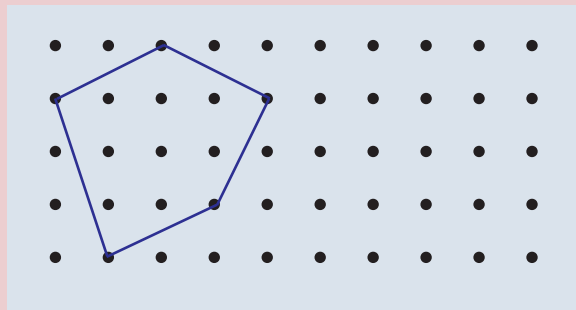
Strange Polygons

See these figures:

They are also drawn with straight lines only. So figures like these are also considered polygons sometimes. But in our lessons, we don't include such figures among polygons, where the vertices dip inwards or the sides cross each other. This is because many of the general results we wish to study do not hold for these.

Draw a shape with six vertices.

How many sides?

A figure with five vertices and five sides is called a pentagon. A figure with six vertices and six sides is named hexagon. (The section, **Polygons** of the chapter, **when lines join**, in the class 5 textbook). Such figures with three or more sides are generally called polygons.

Sum of angles

We have seen in class 7 that the sum of the angles of a triangle is 180°.

Do we get the same sum for the angles of all quadrilaterals?

Draw a quadrilateral and a diagonal.
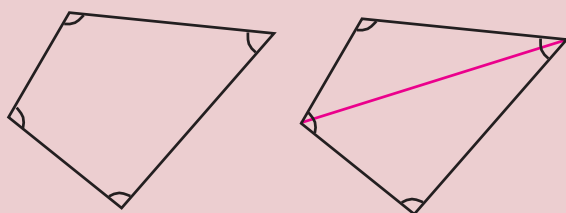
Now the quadrilateral is split into two triangles. The diagonal splits the angles at two corners into two parts each; one of these in one triangle and the other angle in the other triangle. Thus the angles of the quadrilateral are now angles of two triangles. So, the sum of the angles of the quadrilateral is equal to the sum of the angles of two triangles.

That is, $2 \times 180° = 360°$.

Thus we can see that in any quadrilateral, the sum of all angles is 360°.

How about a pentagon?

Taking a vertex, skipping a nearby vertex and joining with the next, we can divide it into a quadrilateral and a triangle:
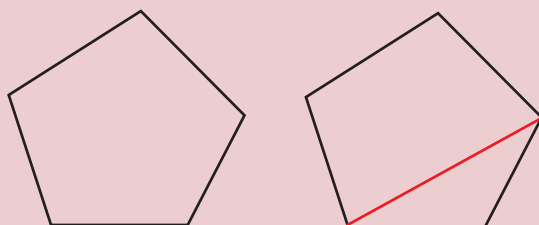
And the sum of the angles of the pentagon is the sum of the angles of this quadrilateral and triangle.

That is,

$$360° + 180° = 540°$$

In other words, the pentagon can be divided into three triangles, and the sum of the angles of the pentagon is the sum of the angles of all these triangles.
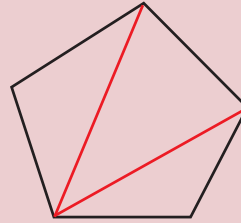
Next, let's look at a polygon of eight sides (octagon)
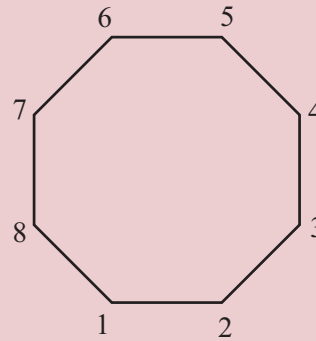
How many triangles can we divide this into, as before?

Vertex 1 can be joined to vertices 3, 4, 5, 6, 7
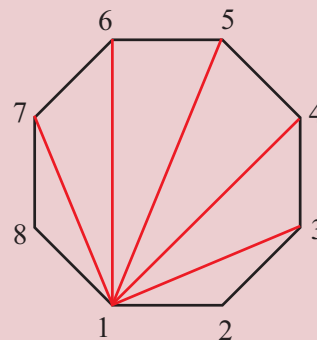
Five lines, six triangles.

Sum of angles is $6 × 180° = 1080°$

What about a polygon of 12 sides?

Let's think about it, without drawing any picture.

Around a point

Look at this figure:

Many angles at the same point.

What is the sum of all these angles?

Making all sides equal, we can draw a circle like this:
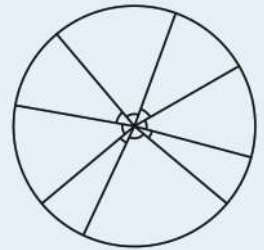
So we can make a full circle by putting these angles together without overlap. In other words, these angles are got by slicing the circle. So, the sum of all these is 360° by definition of the degree measure.

What we have seen now can be put like this: The sum angles around a point is 360°.

We can think like this : starting with one vertex , we can join it with all other vertex except the ones just next to it on either side, making 9 lines and 10 triangles; so the sum of angles is

$$10 \times 180° = 1800°$$

We can say this using algebra. In a polygon of n vertices (and sides), if we choose one vertex, there are $n - 1$ vertices remaining. If we join with all these, except the ones just next to the chosen one on either side, we get $(n - 1) - 2 = n - 3$ lines.

We get one new triangle and a remaining polygon on drawing each of these lines; the final line gives a new triangle and what remains is another triangle. Altogether $(n - 3) + 1 = n - 2$ triangles and the sum of the angles is $(n - 2) \times 180°$.

> **The sum of the angles of an *n*-sided polygon is**
> **$(n - 2) \times 180°$**

Now a question:

Is the sum of the angles of any polygon equal to 2700°?

The sum of the angles of any polygon is a multiple of 180°; right?

So, we need only check whether 2700 is a multiple of 180; for this, we divide by 180.

$$2700 \div 180 = 15$$

So, $\quad\quad\quad 2700 = 180 \times 15$

According to our general principle, for a polygon of $15 + 2 = 17$ sides, the sum of the angles is 2700°.

(1) What is the sum of the angles of a 52 sided polygon?

(2) The sum of the angles of a polygon is 8100°. How many sides does it have?

Another division

We can divide a polygon into triangles, by joining all vertices to a point within also:
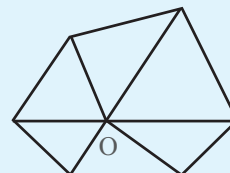
An *n*-sided polygon is divided into *n* triangles this way. Thus sum of all the angles is $n \times 180°$.

Of these angles the sum of all except those at *O*, is the sum of the angles of the polygon. We have seen that the sum of the angles around *O* is 360°. So the sum of the angles of the polygon is

$(n \times 180°) - (2 \times 180°) = (n - 2) \times 180°$.

(3) Is the sum of the angles of any polygon 1600°? How about 900°?

(4) All the angles of a 20 sided polygon are the same. How much is each?

(5) The sum of the angles of a polygon is 1980°. What is the sum of the angles of a polygon with one side less? What about a polygon with one side more?

Outer angles

Draw a triangle and extend one side. We then get a new angle outside the triangle:
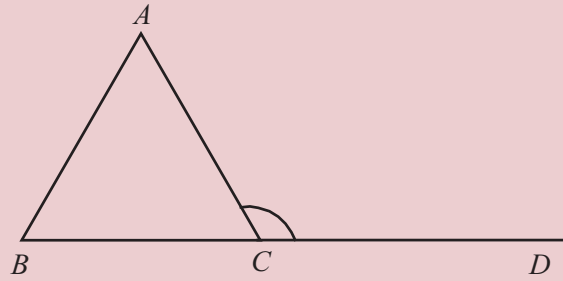
This angle is called an outer angle or exterior angle of the triangle.

There is also an angle of the triangle at C itself. We call it the inner angle or interior angle at C.

What is the relation between the outer angle ACD and the inner angle ACB? Since they form a linear pair, $\angle ACD = 180° - \angle ACB$. Now if we extend the side AC, we get another outer angle at C itself.

.
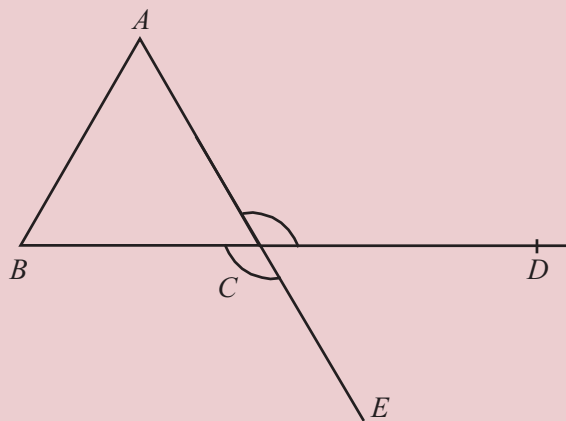
Is there any relation between these outer angles? They are a pair of opposite angles formed when the lines *AE* and *BD* cross each other. So ∠*ACD* = ∠*BCE*.

Thus the two outer angles at the same vertex are equal.

So, when we talk about the size of an outer angle at a vertex, it doesn't matter which we means.

We can draw outer angles at all three vertices of a triangle:
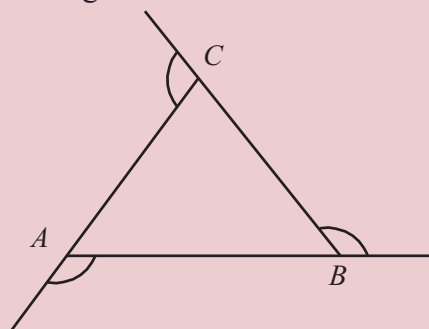
Similarly, we can draw the outer angles at all vertices of a quadrilateral or a pentagon:
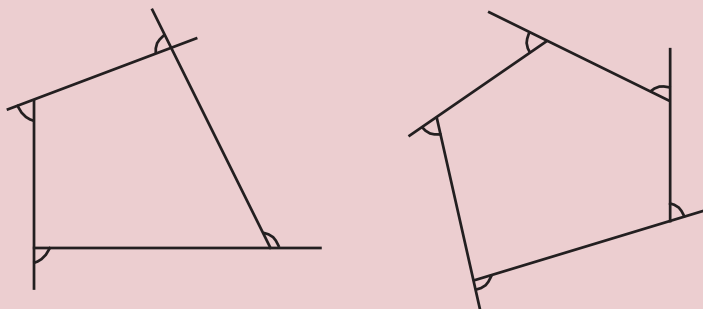
The inner and outer angles at each vertex form a linear pair, right?

(1) Two angles of a triangle are 40º and 60º. Calculate all its outer angles.

(2) Compute all angles in the figure below.

105º

35º
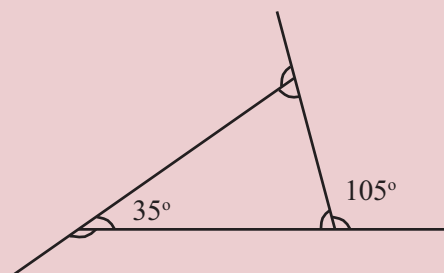
(3) Compute all outer angles of the quadrilateral shown below.

130°   70°

60°
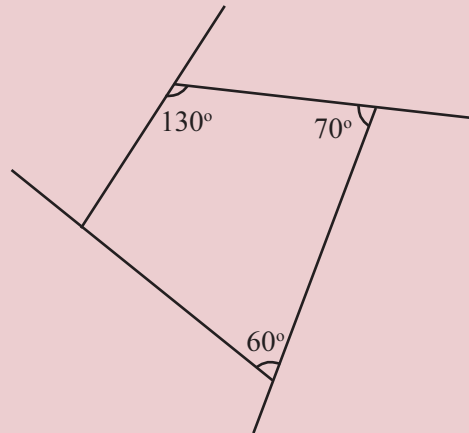
(4) Compute all angles of each of the figures below:

70°

145°

100°

75°

115°

85°

80°

95°
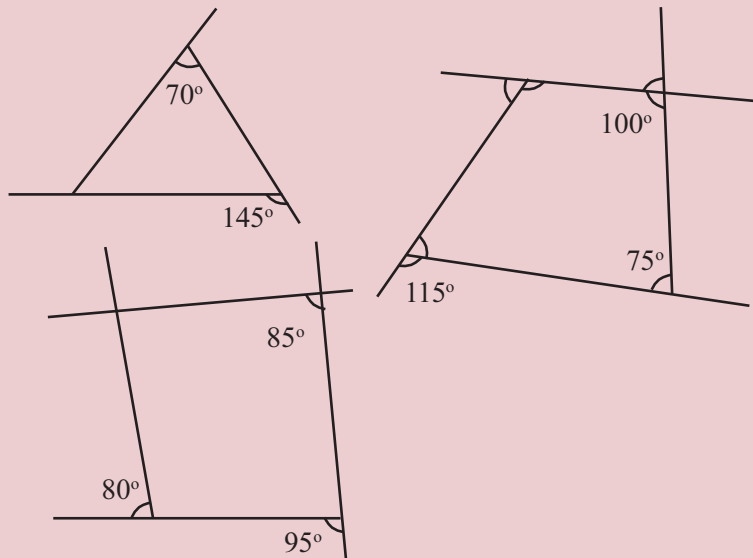
(5) Prove that in any triangle, the outer angle at a vertex is equal to the sum of the inner angles at the other two vertices.

Unchanging sum

To calculate the sum of the inner angles of any polygon we need only know the number of sides. What about the sum of the outer angle. Let's start with triangle.

Can you compute all outer angles of the triangle shown alongside?

C

A   65°   35°   B
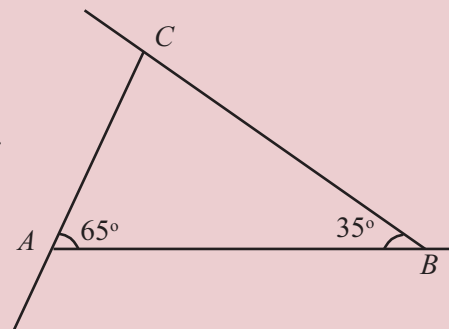
Outer angle at A is $180° – 65° = 115°$

At B, $180° – 35° = 145°$

Inner angle at C is $180° – (65° + 35°)$ $= 180° – 100°$
                                            $= 80°$

Outer angle at C $180° – 80° = 100°$

Sum of outer angles is

   $115° + 145° + 100° = 360°$

Is the sum of outer angles
$360°$ for all triangles?
see the figure
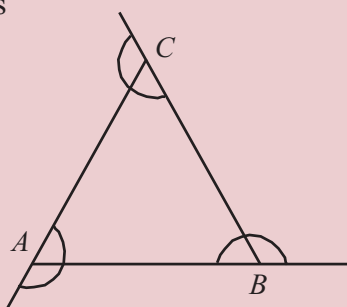
The sum of the inner and outer angles at A is 180°. Simi-
larly we get the sum 180° at B and C. So, adding all inner
and outer angles at the three vertices, we get,

$$3 × 180° = 540°$$

In this, the sum of the angles of the triangle is 180°.

So, the sum of the outer angles alone is
$540° – 180° = 360°$.

What about a quadrilateral? The sum of the inner and outer
angles at each vertex is again 180°.

So altogether we have,

   $$4 × 180° = 720°$$

To get the sum of outer angles, we
subtract the sum of the angles of the
quadrilateral, which is 360°.

   $720° – 360° = 360°$
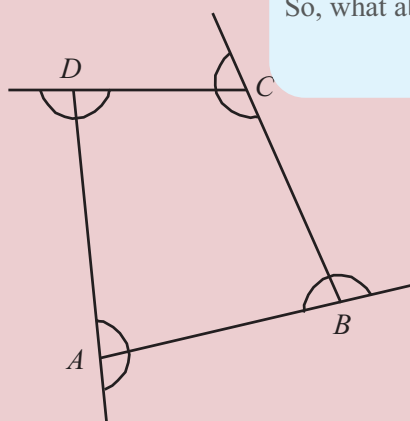
The sum of the outer angles of a quadrilateral is also 360°.

Compute like this for a pentagon and a hexagon.

Shrinking Polygon

Just as we shrank a triangle without altering angles, we can shrink any polygon.

See these figures.

Let's think about an n-sided polygon, in general n vertices and a linear pair formed by inner and outer angles at each vertex. Altogether n linear pairs. The sum of all these angles is $n \times 180°$. In this, the sum of inner angles is $(n-2) \times 180°$. So the sum of outer angles is

$$n \times 180° - (n-2) \times 180°$$

$$= 2 \times 180°$$

$$= 360°$$

Thus we have the following result:

The sum of the outer angles of any polygon is 360°
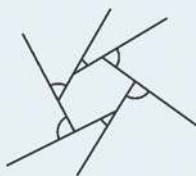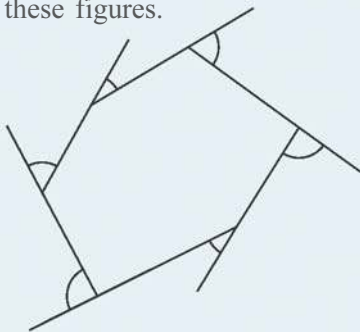
What if the polygon shrinks finally to a point ?

What about the sum of the outer angles of the outer angles of the polygon?

(1) All angles in an 18 sided polygon are equal. How much is each outer angle?

(2) The sides PQ, RS of the quadrilateral shown below are parallel. Compute all inner and outer angles of the quadrilateral.

(3)
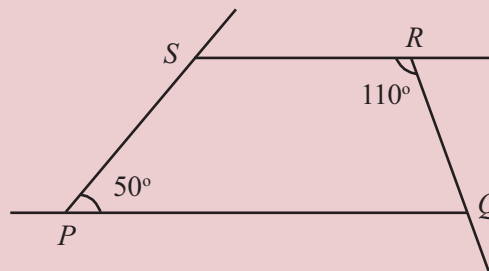
Draw a quadrilateral and mark any two outer angles. Is there any relation between the sum of these two and the inner angles at the other two vertices?

(4) In a polygon with all angles equal, one outer angle is twice an inner angle.

    i) How much is each of its angles?

    ii) How many sides does it have?

(5) The sum of the outer angles of polygon is twice the sum of the inner angles. How many sides does it have? What if the sum of outer angles is half the sum of inner angles? And if the sums are equal?

Regular Polygons

If all angles of a triangle are equal, how much is each angle?

Since the angles of this triangle are equal, so are the sides (the section, **Isosceles triangles** of the lesson **Equal Triangles**)

On the other hand, if the sides of a triangle are equal, so are the angles; and it is such triangles we call equilateral.

If the angles of a quadrilateral are equal, is it necessary that the sides are also equal?

The angles of a rectangle are equal, but sides may not be equal, If sides are also equal. then it is a square.
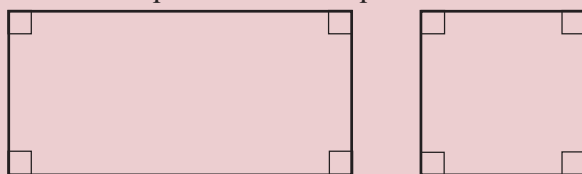
On the other hand, if the sides of a quadrilateral are equal, should the angles also be equal?

A parallelogram with equal sides may not have equal angles.
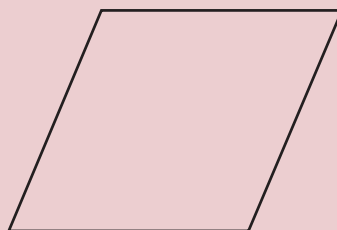
If the angles are also equal, it also must be a square.

Make a slider a with min = 0.01, max = 2, increment = 0.01. Draw a circle of radius a and mark five or six points on it. Join them as shown in the picture (the Ray tool can be used)

a = 1.55
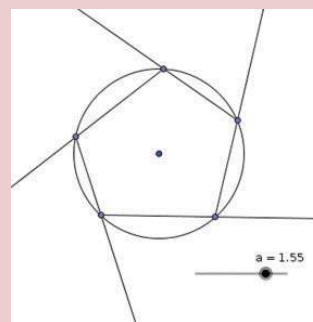
Now the circle can be hidden. Use the Angle tool to mark the outer angles. Change a and see what happens.

Circle and regular polygons

Remember drawing regular pentagons and regular hexagons inside circles?

We can draw a regular pentagon by drawing 72° angles at the centre.
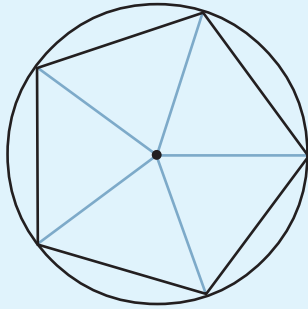
To draw a regular hexagon like this, what angle should be taken at the centre? For a regular Octagon?

Using the set squares, we can split the circle into equal parts in so many different ways.

What regular polygon can we draw using set squares? How about a 24-sided regular polygon?

Choose the Regular Polygon tool and click at two places in the screen. In the dialogue window which come up, give the number of vertices (sides) you want and click ok.

In other words, a square is a quadrilateral with equal sides and equal angles.

If the angles of a quadrilateral are equal, how much is each?

The sum of the angles of a pentagon is $3 \times 180° = 540°$

So each angle is $\dfrac{540°}{5} = 108°$. So to draw a pentagon with equal angles, we need only draw an angle of 108° at each vertex. Is it necessary that the sides are equal?
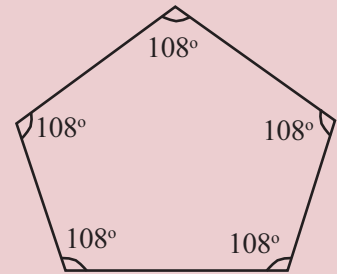
We can also draw a pentagon with sides and angles equal. Such a pentagon if called a regular pentagon
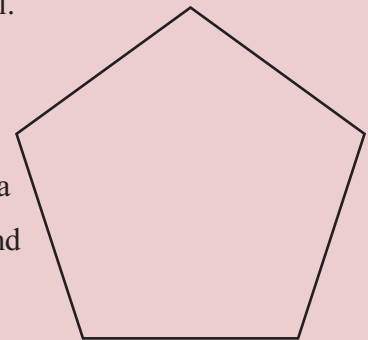
Similarly, can't you draw a hexagon with equal sides and angles?

A polygon with equal sides and angles is called a regular polygon.

Look at this picture.

ABCD is a regular pentagon. Can you calculate the three angles at the vertex *D*?

Since it is a regular pentagon, all angles are 108°.
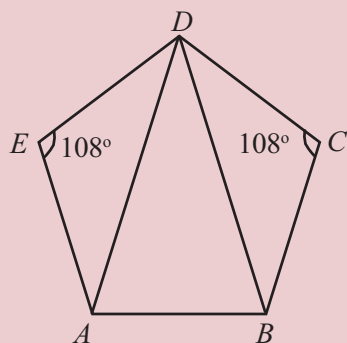
△*AED* and △*BCD* are isosceles (why?) So, we can calculate their other two angles (how?)
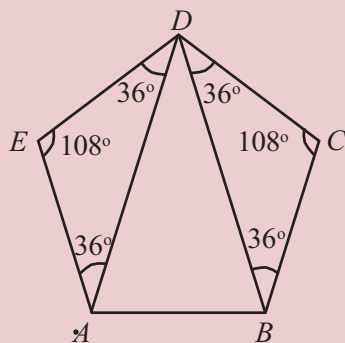
The sum of the three angles at *D* is again 108°. So the remaining angle *ADB* is

$$108° - (36° + 36°) = 36°.$$
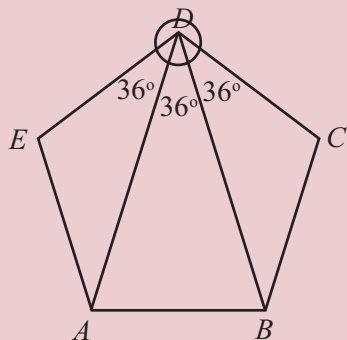
Thus the lines AD and BD divide the angle at D into three equal parts.

Putting together

See six equilateral triangles put together around a point :
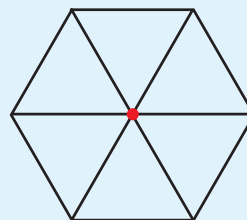
Likewise, what other equal regular polygons can we put together around a point? The sum of the angles around a point is 360°. So to fit equal regular polygons around a point, an angle of the polygon must be a factor of 360°.

See this picture:
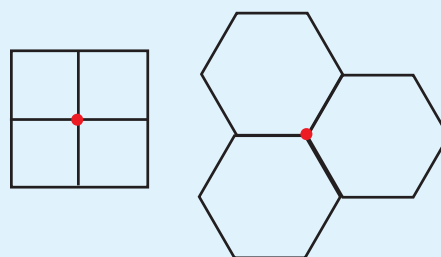
Any other regular polygons?

What about non-regular Polygons?

Next in this figure, draw a line through *D*, parallel to *AB*.

Choose slider and select the Integer option (Integer means whole number). Take min = 3, max =8. Make a regular polygon, giving the number of sides as *n* sliding *n* to 8 makes a regular octagon. Take Reflect about line and click within the octagon and on each side to make the figure below.
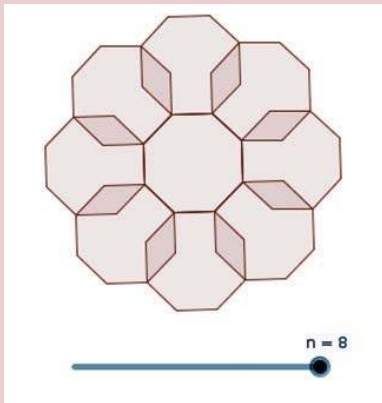
What happens when n is less than 6? And when n is greater than 6? What about *n* = 6?
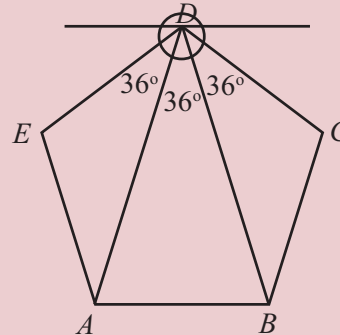
Now the two new angles at D are also 36° each, right? Why?

Another question:

One angle of a regular polygon is 144°.

So each outer angle is 36°.

Since the sum of the outer angles is 360°, the number of sides is

$$\frac{360°}{36°} = 10$$

Thus the polygon has 10 sides.

(1) Draw a hexagon of equal sides and unequal angles.

(2) Draw a hexagon of equal angles and unequal sides.

(3) How much is each angle of a 15 sided regular polygon? How much is each outer angle?

(4) One angle of a regular polygon is 168°. How many sides does it have?

(5) Can we draw a regular polygon with each outer angle 6°? What about 7°?

(6) The figure shows a regular pentagon and a regular hexagon put together. How much is ∠*PQR*?
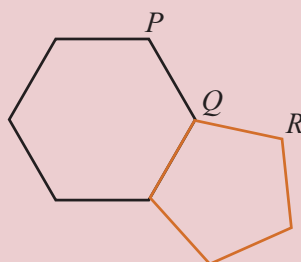
(7) The figure shows a square, a regular pentagon and a regular hexagon put together. How much is ∠*BAC*?

Compass

Without measuring angles using set square or protractor, we can use a compass to draw regular polygons. We have seen how we can draw equilateral triangles, squares and regular hexagons like this, in various classes. There are various ways to draw a regular pentagon using compass. One simple method can be found at the webpage.

www.cut-the-knot.org/pythagoras/PentagonConstruction

The famous mathematician Gauss proved that a 17 sided polygon can be drawn using compass, this he did when he was just 19.

More details of this can be found at

en.wikipedia.org/wiki/Heptadecagon

(8) In the figure, *ABCDEF* is a regular hexagon. Prove that Δ*BDF*, drawn joining alternate vertices is equilateral.
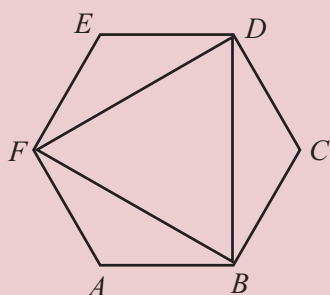
(9) In the figure, *ABCDEF* is a regular hexagon. prove that *ACDF* is a rectangle.

Looking back

Learning outcomes	What I can	With teacher's help	Must imporve
• Explaining various methods of computing the sum of the angles of a polygon.			
• Explaining the relation between inner and outer angles of a polygon			
• Explaining the method to calculate the sum of outer angles			
• Distinguishing regular polygons among polygons.			
• Calculating the number of sides of a regular polygon, given one of its angles.			

4
Identities

$$7 = 4 + 3$$

4×2	3×2
4×1	3×1

$$3 = 2 + 1$$

(i) $983^2 - 17^2$

(a) $(a+b)^2 = a^2 + 2ab + b^2$

(b) $(a-b)^2 = a^2 - 2ab + b^2$

(c) $(a+b)(a-b) = a^2 - b^2$

(d) $(x+a)(x+b) = x^2 + (a+b)x + ab$

$$a^2 - b^2 = (a+b)(a$$
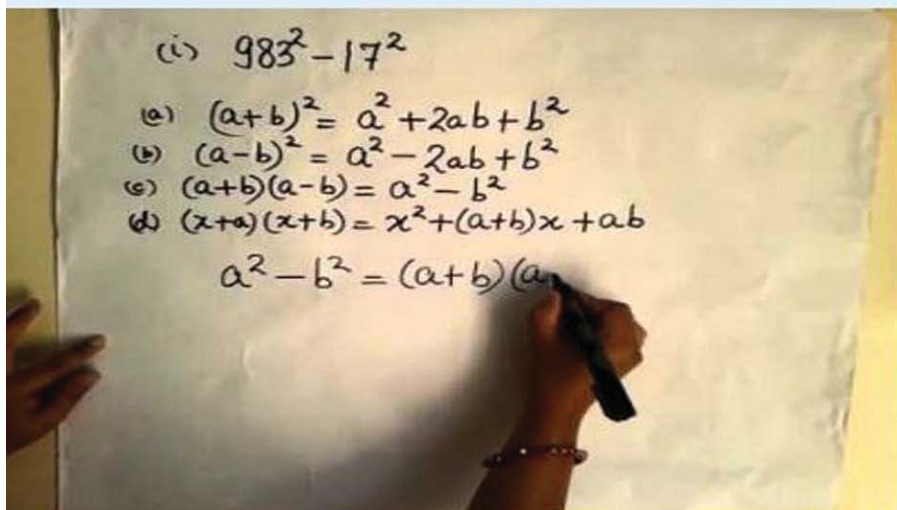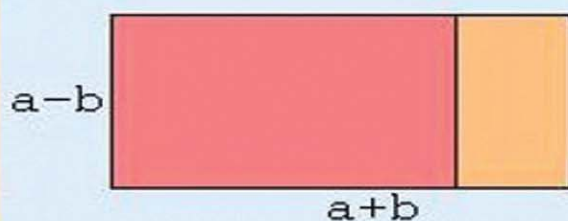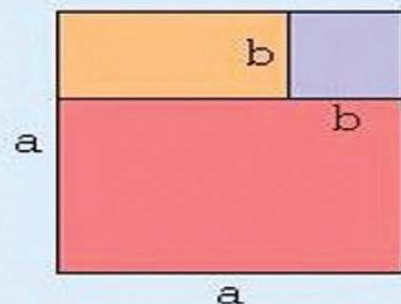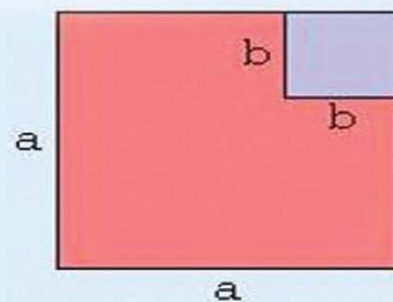
Product of sums

The length of a rectangle is 12 centimetres and its breadth is 7 centimetres. What is its area?
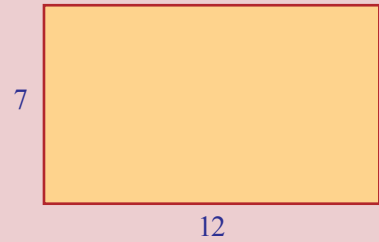
The rectangle is enlarged, increasing its length by 3 centimetres.
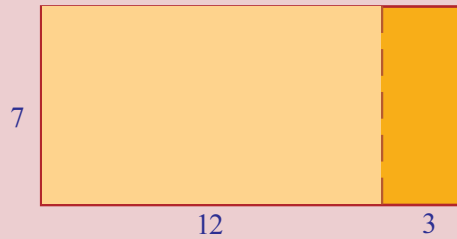
By how much is the area increased?

We can calculate like this: the original area was 84: the enlarged area is $15 \times 7 = 105$ and so the increase is $105 - 84 = 21$

We can also do it without calculating the products separately:.

$$(12 + 3) \times 7 = (12 \times 7) + (3 \times 7) = (12 \times 7) + 21$$

From this, we can see that the increase is 21.

Identities

The equation $2x + 3 = 3x + 2$ is true only for $x = 1$. What about the equation

$$x + (x + 1) = 2x + 1$$

It is true, whatever number we take as x. Such Equations, which are true for all numbers are called identities.
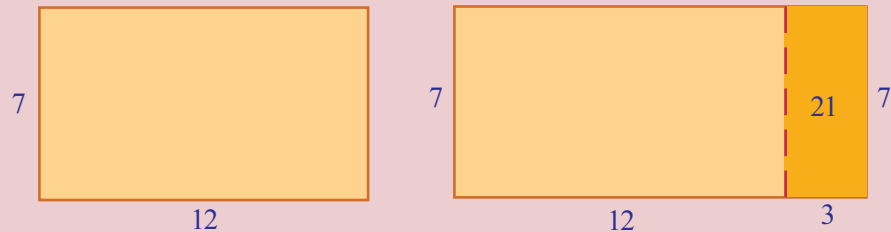
Now suppose in the original rectangle, we increase the breadth by 2 centimetres instead:
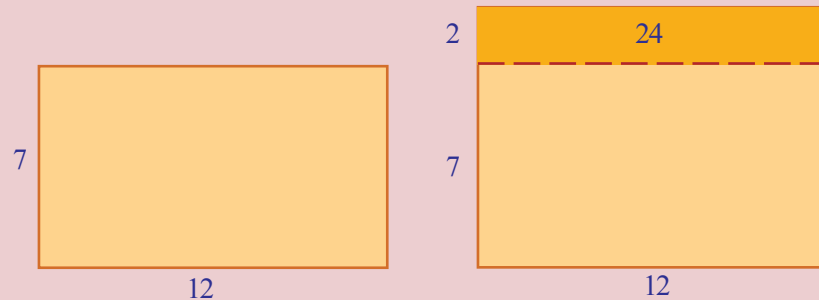
We can compute the increase in area like this:

$$12 \times (7 + 2) = (12 \times 7) + (12 \times 2) = (12 \times 7) + 24$$

And this gives the increase as 24.

Now suppose we increase the length by 3 centimetres and also the breadth by 2 centimetres.
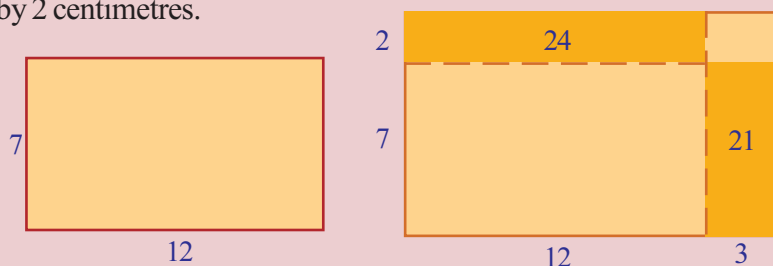
As we have seen, the increase in area on increasing the length is 21; and the increase in area on increasing the breadth is 24, so the total increase is $21 + 24 = 23$.

But the shape is not a rectangle. To make it so, we need one more small rectangle at one corner.
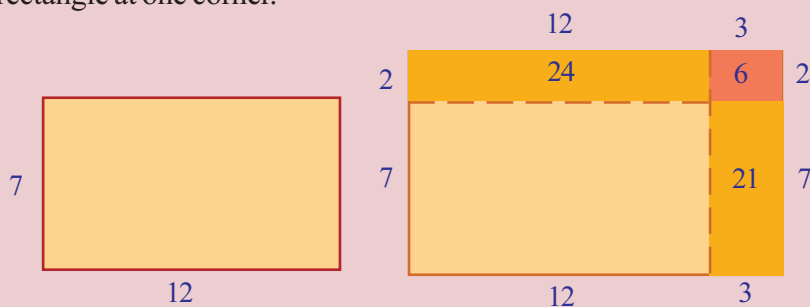
For this enlarged rectangle, the increase in area is $21 + 24 + 6 = 51$.

Let's look at this in another manner. The area of the original rectangle is 12×7 and the area of the enlarged rectangle is 15×9. What all we added to reach the second product from the first product?

$$15 \times 9 = (12 \times 7) + 24 + 21 + 6$$

Let's write the numbers added also as products:

$$15 \times 9 = (12 \times 7) + (12 \times 2) + (3 \times 7) + (3 \times 2)$$

Thus,

$$(12 + 3) \times (7 + 2) = (12 \times 7) + (12 \times 2) + (3 \times 7) + (3 \times 2)$$

What did we do here?

To get 15×9 from 12×7

- 15×9 is split as $(12 + 3) \times (7 + 2)$
- 7 and 2 are multiplied by 12

- 7 and 2 are multiplied by 3

- All these products are added

Let's see how we can get 14×16 from 13×15.

$$14 \times 16 = (13 + 1) \times (15 + 1)$$
$$= (13 \times 15) + (13 \times 1) + (1 \times 15) + (1 \times 1)$$

In both problems, we multiplied a sum by another sum. What is the general method to do this?

> **To multiply a sum of positive numbers by a sum of positive numbers, multiply each number in the second sum by each number in the first sum and add.**

Let's do 26×74 using this:

$$26 \times 74 = (20 + 6) \times (70 + 4)$$
$$= (20 \times 70) + (20 \times 4) + (6 \times 70) + (6 \times 4)$$
$$= 1400 + 80 + 420 + 24$$
$$= 1924$$

How about 103×205?

$$103 \times 205 = (100 + 3)(200 + 5)$$
$$= (100 \times 200) + (100 \times 5) + (3 \times 200) + (3 \times 5)$$
$$= 20000 + 500 + 600 + 15$$
$$= 21115$$

Multiplication

How do we usually calculate 24×36.

$$
\begin{array}{r}
24 \ \times \\
36 \\
\hline
144 \\
720 \\
\hline
864
\end{array}
$$

How did we get each product here?

$$
\begin{array}{r}
24 \ \times \\
36 \\
\hline
144 \\
720 \\
\hline
864
\end{array}
$$

$\to 6 \times (4 + 20) = 24 + 120$

$\to 30 \times (4 + 20) = 120 + 600$

Now let's write this idea using algebra.

Let's take the first sum as $x + y$ and the second sum as $u + v$. To find their product, we must add the products xu, xv, yu, yv.

The general principle above translates to this:

> **$(x + y)(u + v) = xu + xv + yu + yv,$**
> **for any four positive numbers x, y, u, v**

One more problem:

$$6\frac{1}{2} \times 8\frac{1}{3} = \left(6 + \frac{1}{2}\right) \times \left(8 + \frac{1}{3}\right)$$

$$= (6 \times 8) + \left(6 \times \frac{1}{3}\right) + \left(\frac{1}{2} \times 8\right) + \left(\frac{1}{2} \times \frac{1}{3}\right)$$

$$= 48 + 2 + 4 + \frac{1}{6}$$

$$= 54\frac{1}{6}$$

Let's look at another thing. We have seen in class 7, several interesting connections between sum of numbers of a calendar. (the section, **Calendar math** and **Another trick** of the lesson, **Unchanging Relations**). Now let's see something about their products.

In the calender for any month, mark four numbers in a square:

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			1	2	3	4
5	⑥	⑦	8	9	10	11
12	⑬	⑭	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

Multiply the diagonal pairs:

$$14 \times 6 = 84$$
$$13 \times 7 = 91$$

Their difference is

$$91 - 84 = 7$$

Now take another four numbers in a square and do this :

$$22 \times 30 = 660$$
$$23 \times 29 = 667$$
$$667 - 660 = 7$$

Why do we get 7 always?

Let's use algebra to see this.

Taking the first number in the square as x, the others can be filled in as below:

x	$x + 1$
$x + 7$	$x + 8$

(We have seen this in class 7, in the section **Calendar math** of the lesson, **Unchanging Relations**).

Let's find the diagonal products.

$$x (x + 8) = x^2 + 8x$$

How do we split the product $(x + 1) (x + 7)$?

$$(x + 1) (x + 7) = x^2 + 7x + x + 7 = x^2 + 8x + 7$$

Look at the two products. The difference is 7, isn't it?

Here, we can take any number as x; which means this holds in any part of the calendar.

Another idea. Make a multiplication table like this:

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

As in the calendar, mark four numbers in a square:

$$12 \quad 15$$

$$16 \quad 20$$

Instead of diagonal products, find the sums:

$$12 + 20 = 32$$

$$16 + 15 = 31$$

Let's test another four numbers:

$$35 \quad 40$$

$$42 \quad 48$$

$$35 + 48 = 83$$

$$40 + 42 = 82$$

Why is the difference 1 always?

In the table, all numbers in one row are multiples of the same number.

Generally, we can write the numbers in a row like this:

$$x \quad 2x \quad 3x \quad 4x \quad 5x \quad 6x \quad 7x \quad 8x \quad 9x$$

Let's look at the next row also:

x	$2x$	$3x$	$4x$	$5x$	$6x$	$7x$	$8x$	$9\,x$
$x+1$	$2(x+1)$	$3(x+1)$	$4(x+1)$	$5(x+1)$	$6(x+1)$	$7(x+1)$	$8(x+1)$	$9(x+1)$

We can take a general number from the first row as yx. Then next number in this row is the next multiple of x; that is, $(y+1)x$.

What is the number below yx in the next row?

It's a multiple of $x+1$. Which multiple?

What is the next number in this row?

So the general form of four numbers in a square is this:

$$yx \qquad\qquad (y+1)\,x$$

$$y\,(x+1) \qquad\qquad (y+1)\,(x+1)$$

In this,

$$(y + 1)\, x \;=\; yx + x$$
$$y\,(x + 1) \;=\; yx + y$$

and their sum is

$$(y + 1)\, x + y\,(x + 1) \;=\; 2yx + y + x$$

In the other two products, we need not do anything with yx. How do we expand $(y + 1)\,(x + 1)$,

$$(y + 1)\,(x + 1) \;=\; yx + y + x + 1$$

So the sum of the second pair of products is

$$yx + (y + 1)\,(x + 1) \;=\; 2yx + y + x + 1$$

Thus one diagonal sum is $2yx + y + x$ and the other is $2yx + y + x + 1$; and their difference is 1.
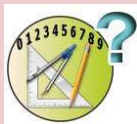
In the course of the above discussion, we found

$(y + 1)\,(x + 1) = yx + y + x + 1$.

How do we state this as a general principle in ordinary language? Can we do some multiplication in head using this?

What if we take 2 instead of 1 in this?

(1)   Write numbers like this:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

i)   As in the calendar; mark four numbers in a square and find the difference of diagonal products. Is it the same for all squares of four numbers?

ii)   Explain why this is so, using algebra.

iii) Instead of a square of four numbers, take a square of nine numbers and mark only the numbers at the four corners.

⑧	9	⑩
13	14	15
⑱	19	⑳

What is the difference of diagonal products? Explain using algebra.

(2) In the multiplication table made earlier, take a square of nine numbers, instead of four, and mark the numbers at the four corners:

⑥	8	⑩
9	12	15
⑫	16	⑳

i) What is the difference of diagonal sums?

ii) Explain using algebra, why this difference is the same for all such squares.

iii) What if we take a square of sixteen numbers?

(3) Look at these:

$$1 \times 4 = (2 \times 3) - 2$$
$$2 \times 5 = (3 \times 4) - 2$$
$$3 \times 6 = (4 \times 5) - 2$$
$$4 \times 7 = (5 \times 6) - 2$$

i) Write the next two lines in this pattern.

ii) If we take four consecutive natural numbers, what is the relation between the products of the first and the last, and the product of the middle two?

iii) Write this as a general principle in algebra and explain it.

(4) Shown below is a method to the product 46×28.

$$4 \times 2 = 8 \qquad\qquad 8 \times 100 \qquad\qquad 800$$

$$(4 \times 8) + (6 \times 2) = 44 \quad 44 \times 10 \qquad\qquad 440$$

$$\underline{6 \times 8 \qquad\qquad\qquad\qquad\qquad\qquad 48\ }$$

$$46 \times 28 \qquad\qquad\qquad\qquad\qquad 1288$$

   i)   Check this method for some other two digit numbers.

   ii)   Explain why this is correct, using algebra. (Recall that any two-digit number can be written $10m + n$, as seen in the section, **Two-digit numbers** of the lesson, **Numbers and Algebra,** in the class 7 textbook)

Square of a Sum

What is 51^2?

We have seen how we can find this without actual multiplication, in class 7 (The section, **Next square** of the lesson**, Square and Square Root**).

According to this, we need only add 50 and 51 to 50^2.

$$51^2 = 50^2 + 50 + 51 = 2601$$

Why is this right?

To see it, let's split 51^2:

$$51^2 = 51 \times 51 = (50 + 1)(50 + 1)$$

We can write this as the sum of four products:

$$
\begin{aligned}
(50 + 1)(50 + 1) &= (50 \times 50) + (50 \times 1) + (1 \times 50) + (1 \times 1)\\
&= 2500 + 50 + 50 + 1\\
&= 2500 + 50 + 51
\end{aligned}
$$

We can split any square like this.

How do we write this in algebra?

To get $(x + 1)^2$ from x^2, we must add x and the next number $x + 1$ to x^2.

To see why this is so, we use our general result on multiplication of sums:

$$(x + 1)^2 = (x + 1)(x + 1)$$
$$= (x \times x) + (x \times 1) + (1 \times x) + (1 \times 1)$$
$$= x^2 + x + (x + 1)$$

We know that $x + (x + 1) = 2x + 1$. So,

$$(x + 1)^2 = x^2 + 2x + 1$$

Using this, we can calculate

$61^2 = (60 + 1)^2 = 60^2 + (2 \times 60) + 1 = 3600 + 120 + 1 = 3721$

Now suppose we want to compute 75^2. If we try to do this by writing it as $(74 + 1)^2$, we could have to compute 74^2.

Suppose we write it as $(70 + 5)^2$?

We can split like this:

$$75^2 = (70 + 5)(70 + 5)$$
$$= 70^2 + (70 \times 5) + (5 \times 70) + 5^2$$
$$= 4900 + 350 + 350 + 25$$
$$= 5625$$

How about 103^2?

$$103^2 = (100 + 3)(100 + 3)$$
$$= 10000 + 300 + 300 + 9$$
$$= 10609$$

We have seen in class 7, that the square of a product is the product of squares.

Is the square of a sum equal to the sum of squares?

Let's write the general idea seen in all these:

The square of sum of two positive numbers is sum of the squares of the two numbers and twice their product.

For example,

$$\left(10\frac{1}{2}\right)^2 = \left(10 + \frac{1}{2}\right)^2 = 10^2 + \left(2 \times 10 \times \frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 = 100 + 10 + \frac{1}{4} = 110\frac{1}{4}$$

We can write it in algebra like this:

$(x + y)^2 = x^2 + y^2 + 2xy$, for any two positive numbers x, y
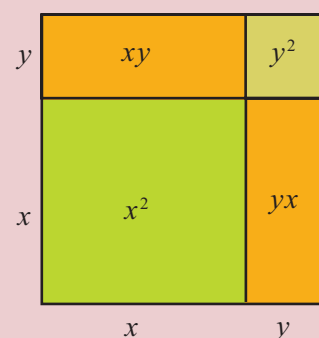
(1) Now compute the squares of these numbers in head:

    (i) 52   (ii) 105   (iii) $20\frac{1}{2}$ (iv) 10.2

Using this principles, we can also see how some patterns of perfect squares are got.

For example, look at these:

$$1 \times 3 = 3 = 2^2 - 1$$
$$2 \times 4 = 8 = 3^2 - 1$$
$$3 \times 5 = 15 = 4^2 - 1$$

Write down the next few lines. Does the pattern continue like this?

In the sequence of natural numbers, if we take the product any two alternate numbers, would it be one less than the square of the skipped number?

Let's see using algebra.

The alternate numbers can be taken as x and $x + 2$.

Their product is $x(x + 2) = x^2 + 2x$.

Here, the skipped number is $x + 1$. What is 1 less than its square?

$$(x + 1)^2 - 1 = (x^2 + 2x + 1) - 1 = x^2 + 2x$$

So,

$$x(x + 2) = (x + 1)^2 - 1$$

In this if we take the numbers 1,2,3, ... as x, we get the above pattern.

Let's look at another pattern.

$$3 = 2^2 - 1^2$$
$$5 = 3^2 - 2^2$$
$$7 = 4^2 - 3^2$$

Can we write all odd numbers greater than 1 as the difference of the squares of consecutive natural numbers?

We have seen in class 7, that we can write all odd numbers greater than 1 in the form $2x + 1$. (the section **General forms**, of the lesson **Numbers and Algebra**)

How do we write it as the difference of the squares of two consecutive natural numbers?

We know that to get the number $(x + 1)^2$ from x^2, we have to add $2x + 1$.

So, to get $2x + 1$, we need only subtract x^2 from $(x + 1)^2$.

$$2x + 1 = (x + 1)^2 - x^2$$

In this, if we take was 1, 2, 3 .... as x, we get all odd numbers greater than 1 as $2x + 1$.

Thus we see that any odd number greater than 1 can be written as the difference of consecutive perfect squares.

We can use the square of sum principle to explain some general properties of squares also.

For example, the squares of all odd numbers are again odd numbers.

Why is this so?

The square of any odd number is of the from $(2x + 1)^2$.

$$(2x + 1)^2 = (2x)^2 + (2 \times 2x \times 1) + 1^2 = 4x^2 + 4x + 1$$

In this, we can write

$$4x^2 + 4x = 4(x^2 + x) = 4x(x + 1)$$

So,

$$(2x + 1)^2 = 4x(x + 1) + 1$$

In this $4x(x + 1)$ is a multiple of 4 and hence is an even number. So 1 added to it is an odd number.

Here we get another fact.

$4x(x + 1) + 1$ divided by 4 gives 1 as remainder. Thus we see that the square of any odd number, divided by 4, gives remainder 1.

Let's think a bit more.

Since x, $x + 1$ here are consecutive natural numbers, one of them must be even. Whichever it is, the product $x(x + 1)$ is even.

So $4x(x + 1)$ is a multiple of 8.

Thus we can see that the square of any odd number, divided by 8, leaves remainder 1.

> **Trick with 76**
>
> $$76^2 = 5776$$
> $$176^2 = 30976$$
> $$276^2 = 76176$$
>
> Calculate the squares of some more numbers ending in 76.
>
> What do you see?
>
> Why is this so?
>
> Any number ending in 76 can be written in the form $100x + 76$.
>
> $(100x + 76)^2 = 10000x^2 + 15200x + 5776$.
> Whatever number be x, the last two digits of $10000x^2 + 15200x$ are zeros. When 5776 is also added, the last two digits become 76.
>
> Does any other two-digits number have this property?

(1) Is there a general method to compute the squares of numbers like $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$ ...? Explain it using algebra.

(2) Given below is a method to calculate 37^2?

$3^2 = 9$	9×100	900
$2 \times (3 \times 7) = 42$	42×10	420
7^2		49
37^2		1369

   i)   Check this for some more two-digit numbers.

   ii)   Explain why this is correct, using algebra.

   iii)   Find an easy method to compute squares of number ending in 5.

(3) Look at this pattern

$$1^2 + (4 \times 2) = 3^2$$
$$2^2 + (4 \times 3) = 4^2$$
$$3^2 + (4 \times 4) = 5^2$$

   i)   Write the next two lines.

   ii)   Explain the general principle using algebra.

(4) Explain using algebra, the fact that the square of any natural number which is not a multiple of 3, leaves remainder 1 on division by 3.

(5) Prove that for any natural number ending in 3, its square ends in 9.

What about numbers ending in 5? And numbers ending in 4?

Product of differences

We have seen how some products can be split into sums.

For example,

$$302 \times 205 = (300 + 2) \times (200 + 5) = 60000 + 1500 + 400 + 10$$
$$= 61910$$

Now suppose we want to calculate 298×195?

We can split it as

$$298 \times 195 = (300 - 2) \times (200 - 5)$$

How do we split this as before, into four products?

First we write only,

$$298 \times 195 = (300 - 2) \times 195$$

This we can split as

$$(300 - 2) \times 195 = (300 \times 195) - (2 \times 195)$$

Now let's write 195 = 200 − 5, and split each product.

$$300 \times 195 = 300 \times (200 - 5) = 60000 - 1500 = 58500$$
$$2 \times 195 = 2 \times (200 - 5) = 400 - 10 = 390$$

Putting all these together

$$
\begin{aligned}
298 \times 195 &= (300 - 2) \times 195 \\
&= (300 \times 195) - (2 \times 195) \\
&= 58500 - 390
\end{aligned}
$$

A quick way to subtract 390 is to subtract 400 and add 10.

That is,

$$58500 - 390 = 58500 - 400 + 10 = 58\,110$$

(The section, **Less and more** of the lesson, **Unchanging Relations**, in the class 7 textbook)

Let's try the multiplication of 397 by 199 like this:

$$
\begin{aligned}
397 \times 199 &= (400 - 3) \times 199 \\
&= (400 \times 199) - (3 \times 199) \\
400 \times 199 &= 400 \times (200 - 1) \\
&= (400 \times 200) - (400 \times 1) \\
&= 80000 - 400 \\
&= 79600 \\
3 \times 199 &= 3 \times (200 - 1) \\
&= 600 - 3 \\
&= 597
\end{aligned}
$$

What do we get on putting these together?

Cutting down

Suppose we shorten by 3 centimetres, the length of a rectangle of length 12 centimetres and breadth 7 centimetres
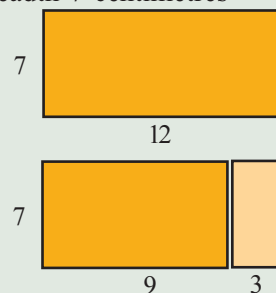
By how much is the area decreased? What is the calculation done here?

$$(12 - 3) \times 7 = (12 \times 7) - (3 \times 7)$$

Geometrical explanation

Look at this picture of reducing a rectangle by shortening both sides.
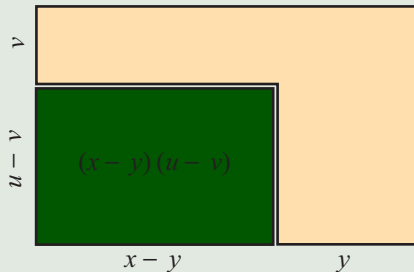
Look at this pictures:
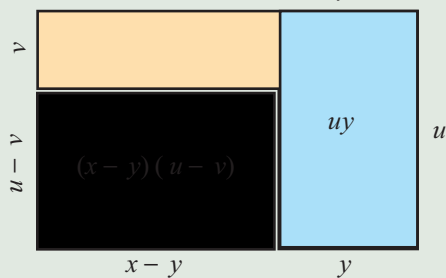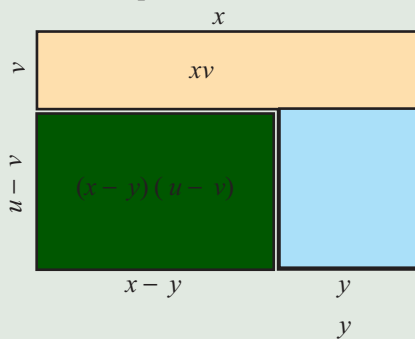
If we subtract the areas of both the rectangle on the top and on the right, the rectangle at the top corner would be subtracted twice.
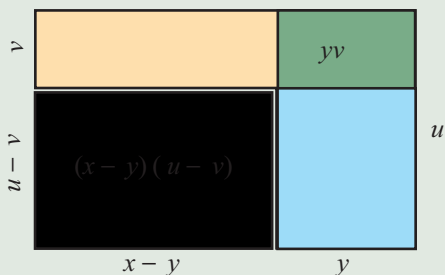
To compensate, we have to add this rectangle once.

$(x - y)(u - v) = xu - xv - yu + yv$

$397 \times 199 = 79600 - 597$

Instead of subtracting 597 directly, it is easier to subtract 600 and add 3. So,

$$397 \times 199 = 79600 - 600 + 3 = 79003$$

Let's write all computations together:

$$397 \times 199 = 80000 - 400 - 600 + 3$$

Writing it out in full,

$$397 \times 199 = (400 \times 200) - (400 \times 1) - (3 \times 200) + (3 \times 1)$$

Similarly,

$$
\begin{aligned}
398 \times 197 &= (400 - 2) \times (200 - 3) \\
&= (400 \times 200) - (400 \times 3) - (2 \times 200) + (2 \times 3) \\
&= 80000 - 1200 - 400 + 6 \\
&= 78400 + 6 \\
&= 78406
\end{aligned}
$$

It is not easy to write in ordinary language, the general principle here, How about algebra?

> $(x - y)(u - v) = xu - xv - yu + yv$ **for all positive numbers** x, y, u, v **with** $x > y$ **and** $u > v$

We can also find the general method to compute the square of a difference using this:

$$
\begin{aligned}
(x - y)^2 &= (x - y) \times (x - y) \\
&= (x \times x) - (x \times y) - (y \times x) + (y \times y) \\
&= x^2 - xy - yx + y^2 \\
&= x^2 - xy - xy + y^2
\end{aligned}
$$

In this, we first subtract xy from x^2 and then subtract itself once more.

Instead of subtracting one after another like this, we need only subtract the sum $xy + xy = 2xy$. (The section, **Addition and subtraction** of the lesson. **Unchanging Relation** in the class 7 textbook)

That is,

$$x^2 - xy - xy = x^2 - (xy + xy) = x^2 - 2xy$$

Now let's start again from where we left off:

$$(x - y)^2 = x^2 - xy - xy + y^2 = x^2 - 2xy + y^2$$

Let's write this down as a general principle.

> $$(x - y)^2 = x^2 + y^2 - 2xy$$
> **for all positive numbers x, y with $x > y$**

This we can say in ordinary language also:

> **The square of the difference of two positive numbers is twice their product subtracted from the sum of their squares.**

For example,

$$99^2 = (100 - 1)^2 = 100^2 - (2 \times 100 \times 1) + 1^2$$
$$= 10000 - 200 + 1 = 9800 + 1 = 9801$$

Now look at this pattern:

$$2(2^2 + 1^2) = 10 = 3^2 + 1^2$$
$$2(3^2 + 2^2) = 26 = 5^2 + 1^2$$
$$2(5^2 + 1^2) = 52 = 6^2 + 4^2$$
$$2(4^2 + 6^2) = 104 = 10^2 + 2^2$$

Take some pairs of natural numbers and calculate the sum of the squares; Can you write twice this sum again as a sum of a pair of perfect squares?

What is the relation between the starting pair and the final pair?

Find the sum and difference of the starting pair

What is the reason for this?

Let's use algebra. Starting with x, y, the square of the sum is,

$$(x + y)^2 = x^2 + y^2 + 2xy$$

If we take the larger number of the pair as x, the square of the difference is

$$(x - y)^2 = x^2 + y^2 - 2xy$$

What if we add these? x^2 and y^2 occur twice. Since $2xy$ is added and subtracted, it vanishes. That is,

$$(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$$

If we write this in reverse as $2(x^2 + y^2) = (x + y)^2 + (x - y)^2$ we get the explanation for our original pattern.

Thus if we add the squares of the sum and difference of two numbers, we get twice the sum of their squares.

What if we subtract the square of the difference from the square of the sum?

$$(x + y)^2 - (x - y)^2 = (x^2 + y^2 + 2xy) - (x^2 + y^2 - 2xy)$$

This means from the sum of the numbers $x^2 + y^2$ and $2xy$ we have to subtract their differnces. It is double the number $2xy$, isn't it? (The section, **Sum and difference**, of the lesson, **Unchanging Relations** in the calss 7 textbook)

$$(x + y)^2 - (x - y)^2 = 2 \times 2xy = 4xy$$

Writing this in reverse,

$$4xy = (x + y)^2 - (x - y)^2$$

For example,

$$8 = 4 \times 2 \times 1 = 3^2 - 1^2$$
$$12 = 4 \times 3 \times 1 = 4^2 - 2^2$$
$$16 = 4 \times 4 \times 1 = 5^2 - 3^2$$
$$20 = 4 \times 5 \times 1 = 6^2 - 4^2$$

Like this, all multiples of 4, starting with 8, can be written as the difference of two perfect squares.

Pythagoras triples

If three natural numbers are such that the sum of the squares of two of them is equal to the square of the third, we call them a Pythagoras triple, as mentioned in the class 7 textbook. For example, since.

$$3^2 + 4^2 = 5^2$$

3, 4, 5 form a Pythagoras triple. A clay tablet of ancient Babylon dated to be around 2000 BC gives a table of some such triples.

There is a method to find all such triples. Take any two natural numbers m, n and compute x, y, z as follows:

$$x = m^2 - n^2$$
$$y = 2mn$$
$$z = m^2 + n^2$$

It is not difficult to see that

$$x^2 + y^2 = z^2$$

for, we have

$$
\begin{aligned}
x^2 + y^2 &= (m^2 - n^2)^2 + (2mn)^2 \\
&= m^4 + n^4 - 2m^2n^2 + 4m^2n^2 \\
&= m^4 + n^4 + 2m^2n^2 \\
&= (m^2 + n^2)^2 \\
&= z^2
\end{aligned}
$$

Greek mathematicians knew this technique as early as 300 BC

(1) Compute the squares of these numbers.

   i) 49   ii)   98   iii) $7\frac{3}{4}$ iv)   9.25

(2) Look at this pattern:

$$\left(\frac{1}{2}\right)^2 + \left(1\frac{1}{2}\right)^2 = 2\frac{1}{2} \qquad 2 = 2 \times 1^2$$

$$\left(1\frac{1}{2}\right)^2 + \left(2\frac{1}{2}\right)^2 = 8\frac{1}{2} \qquad 8 = 2 \times 2^2$$

$$\left(2\frac{1}{2}\right)^2 + \left(3\frac{1}{2}\right)^2 = 18\frac{1}{2} \qquad 18 = 2 \times 3^2$$

Explain the general principle using algebra.

(3) Some natural numbers can be written as a difference of two perfect squares in two ways. For example.

$$24 = 7^2 - 5^2 = 5^2 - 1^2$$
$$32 = 9^2 - 7^2 = 6^2 - 2^2$$
$$40 = 11^2 - 9^2 = 7^2 - 3^2$$

   i) Explain using algebra, the method of writing all multiples of 8, starting with 24 as the difference of two perfect squares in two ways.

   ii) In how many different ways can we write multiples of 16, starting with 48 as the difference of two perfect squares?

Sum and difference

We have calculated products by splitting numbers as sums or differences. For example,

$$203 \times 302 = (200 + 3) \times (300 + 2) = 60000 + 400 + 900 + 6 = 61306$$

$$197 \times 298 = (200 - 3) \times (300 - 2) = 60000 - 400 - 900 + 6 = 58706$$

What is the convenient way to split 203×298?

$$203 \times 298 = (200 + 3) \times (300 - 2)$$

To compute this, we first split only 203, as before:

$$203 \times 298 = (200 + 3) \times 298 = (200 \times 298) + (3 \times 298)$$

Now we split 298 and compute the two products separately:

$$200 \times 298 = 200 \times (300 - 2) = 60000 - 400 = 59600$$

$$3 \times 298 = 3 \times (300 - 2) = 900 - 6 = 894$$

Putting these together,

$$203 \times 298 = (200 + 3) \times 298$$
$$= (200 \times 298) + (3 \times 298)$$
$$= 59600 + 894$$
$$= 60494$$

To understand the general principle, let's write all computations together

$$203 \times 298 = 60000 - 400 + 900 - 6$$

In more detail,

$$(200 + 3) \times (300 - 2) = (200 \times 300) - (200 \times 2) + (3 \times 300) - (3 \times 2)$$

Like this,

$$105 \times 197 = (100 + 5) \times (200 - 3)$$
$$= (100 \times 200) - (100 \times 3) + (5 \times 200) - (5 \times 3)$$
$$= 20000 - 300 + 1000 - 15$$
$$= 20000 + 700 - 15$$
$$= 20685$$

We can write the algebraic form of this as follows:

> **$(x + y)\ (u - v) = xu - xv + yu - yv$ for all positive numbers x, y, u, v with $u > v$**

Using this, we can also find a general method to compute the product of the sum and difference of two numbers.

$$(x + y)\ (x - y) = (x \times x) - (x \times y) + (y \times x) - (y \times y)$$
$$= x^2 - xy + yx - y^2$$
$$= x^2 - y^2$$

> **$(x + y)\ (x - y) = x^2 - y^2$**
> **for all positive numbers x, y wih $x > y$**

How about writing this in ordinary language?

> **The product of the sum and difference of two positive numbers is the difference of their squares.**

For example,

$205 \times 195 = (200 + 5) \times (200 - 5) = 200^2 - 5^2 = 40000 - 25 = 39975$

$9\frac{1}{2} \times 8\frac{1}{2} = \left(9 + \frac{1}{2}\right) \times \left(9 - \frac{1}{2}\right) = 9^2 - \left(\frac{1}{2}\right)^2 = 81 - \left(\frac{1}{4}\right) = 80\frac{3}{4}$

We can also apply this in reverse:

> **The difference of the squares of two positive numbers is the product of their sum and difference.**

For example,

$168^2 - 162^2 = (168 + 162) \times (168 - 162) = 330 \times 6 = 1980$

We have seen that some natural numbers can be written as the difference of two perfect squares. The above principle can be used to do it .

For example, consider 45. We want to find numbers x, y such that $x^2 + y^2 = 45$.

This we can write

$$45 = (x + y)(x - y)$$

This means $(x + y)$ and $(x - y)$ must be factors of 45.

45 can be written as a product of two factors in various ways.

$$45 = 45 \times 1$$
$$45 = 15 \times 3$$
$$45 = 9 \times 5$$

Taking the factors 45 and 1, let's write.

$$x + y = 45$$
$$x - y = 1$$

We have seen a method to calculate two numbers, if their sum and difference are known. (The section, **Sum and difference,** of the lesson, **Unchanging Relations of** class 7)

So x is half the sum of 45 and 1, and y is half their difference.

$$x = 23 \qquad y = 22$$

So,

$$45 = 23^2 - 22^2$$

Difference of squares

● **Identities**
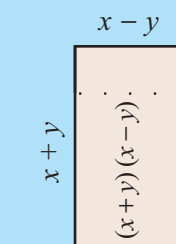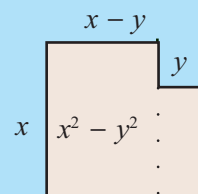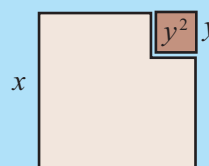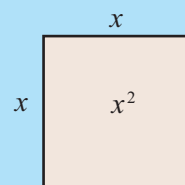
Another method
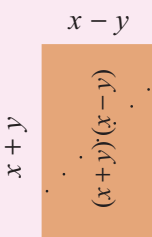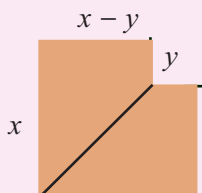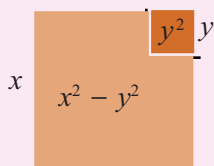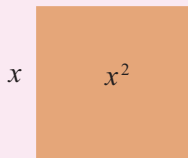
Similarly let's take $45 = 15 \times 3$. Can't we think without x and y?

Half the sum of 15 and 3 is 9; half the difference is 6.

So,

$$45 \ = \ 9^2 - 6^2$$

How about starting with $45 = 9 \times 5$?

$$45 \ = \ 7^2 - 2^2$$

Can we write any natural number as the difference of two squares, using this method?

For example, let's take 10. We have $10 = 10 \times 1$.

Half the sum of factors give $5\frac{1}{2}$; half the difference gives $4\frac{1}{2}$. So we can write,

$$10 \ = \ \left(5\frac{1}{2}\right)^2 - \left(4\frac{1}{2}\right)^2$$

But these are not squares of natural numbers; that is, not perfect squares.

How about using $10 = 5 \times 2$?

What kind of natural numbers cannot be written as the difference of two perfect squares?

Sometimes, writing a product as the difference of two squares makes the computation easier.

For example, consider 26.5×23.5. Can we write it as the difference of two squares?

We need only find two numbers whose sum is 26.5 and product is 23.5, right?

And for that we need only find half the sum and half the difference of 26.5 and 23.5. That is 25 and 1.5, So,

$$26.5 = 25 + 1.5 \qquad 23.5 = 25 - 1.5$$

Using this,

$26.5 \times 23.5 = (25 + 1.5)(25 - 1.5) = 25^2 - 1.5^2 = 625 - 2.25 = 622.75$

(1) Compute the following in head:

i)   a) $68^2 - 32^2$ b) $\left(3\frac{1}{2}\right)^2 - \left(2\frac{1}{2}\right)^2$ c) $3.6^2 - 1.4^2$

iv)   a) 201×199 b) $2\frac{1}{3} \times 1\frac{2}{3}$ c) 10.7×9.3

(2) Look at this pattern:

$$\left(1\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = 2$$

$$\left(2\frac{1}{2}\right)^2 - \left(1\frac{1}{2}\right)^2 = 4$$

$$\left(3\frac{1}{2}\right)^2 - \left(2\frac{1}{2}\right)^2 = 6$$

Explain the general principle using algebra.

(3) Find out the larger product of each pair below, without actual multi-plication.

    i) 25×75, 26×74

    ii) 76×24, 74×26

    iv) 10.6×9.4, 10.4×9.6

(4) Compute the following differences:

    i) (125×75) $-$ (126×74)

    ii) (124×76) $-$ (126×74)

    iii) (224×176) $-$ (226×174)

    iv) (10.3×9.7) $-$ (10.7×9.3)

    v) (11.3×10.7) $-$ (11.7×10.3)

Take some pairs of numbers with the same sum and find their products. How does the product change with the difference of the number? What is an easy method to find the largest product?

(1) Mark four numbers forming a square in a calendar:

4	5
11	12

Add the squares of the diagonal pair and find the difference of these sums:

$$4^2 + 12^2 = 160 \qquad 11^2 + 5^2 = 146 \qquad 160 - 146 = 14$$

i) Do this for other four numbers.

ii) Explain using algebra, why the difference is 14 always.

(2) Take nine numbers forming a square in a calendar and mark the four numbers at the corners.

③	4	⑤
10	11	12
⑰	18	⑲

Add the squares of diagonal pairs and find the difference of the sums.

$$3^2 + 19^2 = 370 \qquad 17^2 + 5^2 = 314 \qquad 370 - 314 = 56$$

i) Do this for other such nine numbers.

(ii) Explain using algebra, why the difference is always 56. (it is convenient to take the number at the centre of the square as x – see the section, **Another trick** of the lesson, **Unchanging Relation** of the Class 7 textbook)

(3) Take nine numbers forming a square in a calendar and mark the four numbers at the corners.

③	4	⑤
10	11	12
⑰	18	⑲

Multiply the diagonal pairs and find the difference of these products.

$3 \times 19 = 57$ $17 \times 5 = 85$ $85 - 57 = 28$

i) Do this for other such squares.

ii) Explain using algebra, why the difference is always 28 (It is convenient to take the number at the centre as x).

Looking back

Learning outcomes	What I can	With teacher's help	Must imporve
• Explaining the method of multiplying a sum by a sum.			
• Explaining the square of a sum geometrically and algebraically.			
• Explaining the square of a difference geometrically and algebraically.			
• Explaining properties of perfect squares using algebra.			
• Explaining the characteristics of natural numbers which can be expressed as the differences of perfect squares.			
• Finding out the pair with the largest product from pairs of numbers with the same sum.			
• Expressing number relations in general using algebra.			

5
Money Maths

$$A = P\left(1 + \frac{r}{100}\right)^n$$

Interest for interest

Look at the ads of two banks.

> ## 10% interest
> **1 lakh rupees grows to 1.20 lakhs in 24 months.**

> ## 10% Interest
> **1 lakh rupees grows to 1.21 lakhs in 24 months**

Both banks give interest at the same rate. If the same amount is deposited for the same period of time, how come the amounts got are different?

There are different ways of computing interest. Remember one method learnt in class 7?

For example, suppose 15000 rupees is deposited for 2 years at 10% interest.

What is the actual interest each year?

Look at another problem:

Anu and Manu deposit 15,000 rupees each in a bank giving 10% interest annually. After one year, Anu withdraw the entire amount including interest and redeposited it the same day. After one more year, both withdrew the total amounts. Who got more? How much more?

Manu gets interest for 2 years. That is.

$$15000 \times \frac{10}{100} \times 2 = 3000$$

So how much does he get in all after two years?

$$15000 + 3000 = 18000 \text{ rupees}$$

What about Anu?

How much did he get as first year's interest?

$$15000 \times \frac{10}{100} = 1500$$

So how much did he withdrew first?

$$15000 + 1500 = 16500$$

It is this amount that he re-deposited.

So how much interest does he get after two years?

$$16500 \times \frac{10}{100} = 1650$$

What is the total amount?

$$16500 + 1650 = 18150$$

How much more does Anu get?

The amount he got more is the interest for the first year's interest for 15000 rupees.

In many schemes, interest for each year is added to the current amount in calculating interest for the next year, without actual withdrawal and re-investment. Interest calculated thus is called compound interest. Interest calculated only on the original investment each year is called simple interest.

Now you know why the second bank gives better returns.

Sumesh deposited 10000 rupees in a bank which gives 5% interest compounded annually. How much would he get after 2 years?

$$
\begin{aligned}
\text{First year's balance} \quad &= \quad 10000 \text{ rupees} \\
\text{First year's interest} \quad &= \quad 10000 \times \frac{5}{100} \\
&= \quad 500 \text{ rupees} \\
\text{Second year's balance} \quad &= \quad 10000 + 500 \\
&= \quad 10500 \\
\text{Second year's interest} \quad &= \quad 10500 \times \frac{5}{100} \\
&= \quad 525 \text{ rupees}
\end{aligned}
$$

Amount Sumesh gets after two years

$$= 10500 + 525$$

$$= 11025 \text{ rupees}$$

(1) Sandeep deposited 25000 rupees in a bank which pays 8% interest compounded annually. How much would he get back after two years?

(2) Thomas took out a loan of 15000 rupees from a bank which charges 12% interest, compounded annually. After 2 years, he paid back 10000 rupees. To settle the loan, how much should he pay at the end of three years?

(3) The simple interest at 5% got for a certain amount after 2 years is 200 rupees. If interest is compounded annually, what would be the interest for same amount at the same rate after 2 years?

Another Method

We have seen that if interest is compounded annually, then 10000 rupees grows to 11025 rupees after 2 years. Let's look at the method of computation again. First years interest is $\frac{5}{100}$ of 10000 rupees. The 500 rupees got thus is added to 10000 rupees to make 10500 rupees and the second year's interest is $\frac{5}{100}$ of this. This interest of 525 rupees is added to 10500 rupees to make 11025 rupees, which is the amount got after two years.

Suppose the account is continued for one more year. To compute how much would be got after three years, $\frac{5}{100}$ of 11025 rupees is to be added to it.

Thus after each year, $\frac{5}{100}$ of the current balance must be added to it. Using algebra, if the current balance is x, then $\frac{5}{100}$ of x must be added to it.

We can write

$$x + \frac{5}{100} x = \left(1 + \frac{5}{100}\right) x$$

So, instead of adding $\frac{5}{100}$ of the balance each year, we need only multiply by $1 + \frac{5}{100}$. That is

Amount got after one year is $10000 \left(1 + \frac{5}{100}\right)$

After 2 years $10000 \left(1 + \frac{5}{100}\right)^2$

After 2 years $10000 \left(1 + \frac{5}{100}\right)^3$

and so on. Using algebra, amount got after n years is $10000 \left(1 + \frac{5}{100}\right)^n$.

We can find the final returns like this, when the numbers are different.

In general, we can say this:

> **If p rupees is invested in a scheme giving $r\%$ interest compounded annually. The amount got after n years is $p \left(1 + \frac{r}{100}\right)^n$.**

Now look at this problem:

Nancy deposited 15000 rupees in a bank which pays 9% interest compounded annually. How much would she get after 2 years?

As seen now, this can be computed directly.

$$15000 \left(1 + \frac{9}{100}\right)^2 = 15000 \left(\frac{100 + 9}{100}\right)^2$$

$$= 15000 \times \left(\frac{109}{100}\right)^2 = 15000 \times (1.09)^2$$

$$= 15000 \times 1.1881$$

$$= 17821.5 = 17821 \text{ rupees } 50 \text{ paise}$$

$$109 \times 109 = (100 + 9)^2$$
$$= 10000 + 1800 + 81$$
$$= 11881$$
$$1.09^2 = 1.1881$$

In financial transactions, amounts between 50 paise and 1 rupee is rounded to 1 rupee and amounts less than 50 paise are ignored.

So, Nancy would get 17822 rupees after 2 years.

(1) Anas deposited 20000 rupees in a bank which pays 6% interest compounded annually. How much would he get back after 3 years?

(2) Diya deposited 8000 rupees in a bank, which gives 10% interest compounded annually. After 2 years, she withdraw 5000 rupees. After one more year, how much would she have in her account?

(3) Varun took out a loan of 25000 rupees from a bank, which charges 11% interest compounded annually. He paid back 10000 rupees after 2 years. How much should he pay after one more year to settle the loan?

Changing times

There are schemes in which interest is added every six months, instead of every year. It is half yearly compounding.

Ambili deposited 12000 rupees in a bank, which pays interest compounded half-yearly. The annual rate of interest is 8%. How much would she get back after one year?

Since interest is compounded half yearly, interest has to be calculated twice a year. Since the interest 8% each year, it is 4% for 6 months.

$$\text{Interest for the first 6 months} = 12000 \times \frac{4}{100}$$

$$= 480 \text{ rupees}$$

This is added to 12000 and the interest for the next 6 months is calculated,

$$12000 + 480 = 12480$$

$$\text{Interest for the next 6 months} = 12480 \times \frac{4}{100}$$

$$= 499.20 \text{ rupees} = 499 \text{ rupees } 20 \text{ paise}$$

Suppose in the problem, we want to find out how much Ambili gets after $1\frac{1}{2}$ years?

We have to add $\frac{4}{100}$ of the balance every six months. That is, we have to multiply by $1 + \frac{4}{100}$.

So what she gets after $1\frac{1}{2}$ years can be computed directly as

$$12000 \times \left(1 + \frac{4}{100}\right)^3 = 12000 \times \left(\frac{104}{100}\right)^3 = 12000 \times (1.04)^3$$

Using a calculator, this can be found as 13498. 368. So what she actually gets is 13498 rupees

We can similarly calculate the amounts after different periods of time.

There are also schemes in which interest is compounded every three month.

Such a method is called quarterly compounding.

Suppose Ambili made her deposit in a bank which compounds interest quarterly?

She would get 2% interest every 3 months

So after one year, she would get.

$$12000 \times \left(1 + \frac{2}{100}\right)^4 = 12000 \times \left(\frac{102}{100}\right)^4 = 12000 \times (1.02)^4$$

Compute this with a calculator.

(1) Arun deposited 5000 rupees in a bank which compounds interest half yearly and Mohan deposited the same amount in another bank which compounds interest quarterly. The annual rate of interest is 6% at both the banks. How much more would Mohan get after one year?

(2) A person took out a loan of 16,000 rupees from a bank which charges interest compounded quarterly. The annual rate of interest is 10%. How much should he pay back after 9 months to settle the loan?

(3) Manu deposited 15,000 rupees in a financial establishment which pays interest compounded every 3 months, at 8% annual rate. How much would he get back after one year?

(4) John deposited 2500 rupees on the first of January in a bank where interest is compounded half-yearly at 6% annual rate. On the first of July, he deposits 2500 rupees more. How much would he have in his account at the end of the year?

(5) Ramlat deposits 30000 rupees in a financial establishment which pay interest at 9% annual rate, compounded every four months. How much would she get back after one year?

Increasing and decreasing

The production of some things increases annually at a fixed rate. Likewise, the price of certain things also increase or decrease at a fixed annual rate. We can use our method of computing compound interest in such cases also, to calculate the number of units produced each year or the price each year.

Most people use mobile phones now. Let's look at a related problem.

> It is estimated that a company manufacturing mobile phones increases production by 20% every year. In 2014 it made about 7 crore phones. How many handsets does it expect to produce in 2018?

It aims for 20% annual increase.

Let's use the method of computing total amount under compound interest.

Number of phones made in 2014 $=$ 7 crores

Number of phones to be made in 2018 $= 70000000 \left(1 + \dfrac{20}{100}\right)^4$

Use a calculator to compute this.

(1) A report estimates e-waste increasing by 15% every year and the e-waste in 2014 is about 9 crore tons . What is the expected amount of e-waste in 2020?

(2) A TV manufacturer decreases the price of a particular model by 5% each year. The current price of this model is 8000 rupees. What would be the price after 2 years?

(3) Tiger is our National Animal. Their number decreases every year. Figures show 3% annual decrease. According to the census of National Tiger conservation Authority, there were 1700 tigers in India in 2011. If the trend continues, how many tigers would be there in 2016?

Looking back

Learning outcomes	What I can	With teacher's help	Must imporve
• Explaining the method of computing interest by compounding interest for interest.			
• Explaining the method of computing amounts under interest compounded, half yearly, quarterly or in any frequencies.			
• Solving other practical problems, using the method of compound interest.			