Rev	**Date**	**Description**
A	Dec 01, 2025	Initial Draft: Charge derivation via φ^{-5} scaling.
B	Dec 15, 2025	**Major Update:** Integration with Eigenmode Framework. Added Section 7 deriving Lepton Mass Spectrum via Vacuum Impedance Resonance (Barut-Extension). Defined universal efficiency $\eta(Z_\Sigma)$.

The Electron as a Bimetric Solitonic Knot

From Topological Integers to Observed Charge via Fractal Geometry

CSR+ Unified Resonance Holography Framework

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Abstract

This addendum completes the characterization of the electron as the fundamental stable solution to the Bimetric Teleparallel 8-Gauge (BT8G) field equations. While the companion paper "The Electron as Fundamental Eigenmode" establishes mass, spin, and stability through phase-locked resonance, this work addresses the critical open questions: (1) how the dimensionless topological winding number $Q_T = 1$ translates to the dimensionful observed charge e, and (2) how the lepton mass hierarchy emerges from vacuum geometry.

We demonstrate that the electron is a *contiguous solitonic topology*—a stable knot in the bimetric vacuum with unit winding, geometrically stabilized at the Compton scale, and coupled to electromagnetic observation through five layers of golden-ratio fractal attenuation. The theoretical prediction $e_{\text{theoretical}} = q_{\text{Planck}} \cdot \varphi^{-5}$ yields a value within 5.5% of observation; the residual is quantified as a **Boundary Transfer Efficiency** $\eta(Z_\Sigma)$ governed by the tetrahedral frustration of the vacuum micro-geometry, yielding $\alpha^{-1} \approx 137.0325$ (25 ppm precision).

Finally, we resolve the "mass ratio" problem by identifying higher lepton generations (muon, tau) not as simple harmonic excitations, but as **Impedance Resonances** of the vacuum stiffness α^{-1}. Using a bimetric extension of the Barut Lepton Model, we derive the muon and tau masses to within 0.1% and 0.5% precision respectively, unifying the charge (topology) and mass (impedance) sectors within a single geometric framework.

Relation to the Eigenmode Framework

This addendum serves as the quantitative completion of the "Electron as Fundamental Eigenmode" paper. Where the eigenmode analysis establishes:

- Mass as phase-locked frequency: $m_e = \hbar\omega_1/c^2$

- Charge as topological winding: $Q = e \cdot k$, with $k = 1$
- Spin as torsion angular momentum from bimetric double-cover
- Stability through constraint control and spectral-entropy closure

this addendum provides the *mechanism* by which the abstract topological integer $k = 1$ couples to the Planck scale and propagates through the fractal vacuum hierarchy to yield the precise observed value $e_{\text{obs}} = 1.602 \times 10^{-19}$ C.
The eigenmode paper treats the electron as the $n = 1$ mode in a spectral ladder; this addendum characterizes the same object as a solitonic knot—the two descriptions are complementary aspects of the same underlying geometric entity.

1 The Foundation: Topological Integer Charge ($\mathbb{Z}$)

We abandon the phenomenological model of the electron as a "point particle" with an arbitrary charge input. Instead, we define the electron as the fundamental, irreducible *knot* in the bimetric vacuum—a local region of spacetime possessing a **Bulk Topological Torsion Charge** (Q_T) of exactly unity.

1.1 Local Quantization via Circulation

This integer definition is structurally mandated by **TORSION Equation TR-F3**, which enforces quantized circulation around fundamental cycles on the T^2 submanifolds of the spectral T^4 torus:

$$\oint_{C_i} \nabla \arg \Phi \cdot d\ell = 2\pi k_i, \qquad k_i \in \mathbb{Z}, \quad i = 1, 2 \tag{1}$$

The electron corresponds to the fundamental winding state $|k| = 1$. This defines the "Unitary Field Variance" identified in **TETRAD Equation 13**, representing a single, uncancelled twist between the bimetric sectors:

$$\partial_{[\lambda}\partial_{\mu]} i(x) = \frac{1}{4}\left(T^{a(+)}_{a\lambda\mu} - T^{a(-)}_{a\lambda\mu}\right) \varepsilon^{\nu\rho\sigma\kappa} g_{\nu\rho} g_{\sigma\kappa} \tag{2}$$

1.2 Global Neutrality Constraint

While local defects ($k \neq 0$) are permitted, the compact toroidal manifold must satisfy the global **Teleparallel Gauss–Bonnet Constraint** defined in **TORSION Equation TR-F1**:

$$\int_S \epsilon_{abc} T^a \wedge e^b \wedge e^c = 2\pi\chi(S), \qquad \chi(\text{torus}) = 0 \tag{3}$$

This implies a strict bookkeeping rule: **local defects are allowed, but the global sum must vanish** ($\sum k_i = 0$). An isolated electron cannot exist on the compact manifold without a compensating defect (positron) or boundary flux elsewhere, ensuring global topological consistency.

2 Geometric Stabilization: Parametric Derivation of Radius (R_*)

Since the electron is a topological knot ($Q_T = 1$), it cannot be a singularity ($r = 0$). It must possess a finite effective size stabilized by the vacuum geometry. This size is derived via the **Geometric Variational Optimization Condition** (**TORSION Equation TR-F7**):

$$\frac{d}{dR}\left[\int_{S(R)} \mathcal{E}_T \, dA + \int_{S(R)} \mathcal{E}_\Phi \, dA\right] = 0 \tag{4}$$

2.1 Parametric Minimization

The total energy functional $E(R)$ is composed of two competing terms: the *Torsion Energy* (expanding tension) and the *Phase Energy* (restoring stiffness):

$$E(R) \sim \alpha R + \frac{\beta}{R} \tag{5}$$

Minimizing this energy ($\frac{dE}{dR} = 0$) naturally yields a stable equilibrium radius:

$$R_* = \sqrt{\frac{\beta}{\alpha}} \tag{6}$$

We identify this R_* with the **effective Compton radius** (λ_c), representing the scale at which the topological knot is geometrically stabilized.

2.2 Point-Like Scattering via Jordan-Lock

While the soliton has a finite radius R_*, it appears point-like in high-energy scattering because the internal structure is shielded by the **Jordan-Lock** mechanism defined in **TOPOLOGY Equation 9**:

$$\mathcal{B}_\Sigma = \lambda_J \int_\Sigma L_{\mu\nu} T^{\mu\nu}\, dA + \frac{Z_\Sigma}{2} \int_\Sigma \theta^2 \, dA, \qquad \lambda_J \to \infty \Rightarrow L_{\mu\nu} T^{\mu\nu}\big|_\Sigma = 0 \tag{7}$$

The limit $\lambda_J \to \infty$ effectively decouples the direct matter-massive coupling at the boundary Σ. Electromagnetic probes interact with the boundary interface, while the extended solitonic structure resides "behind the wall" in the gravity-only exchange channel. Thus, the electron behaves as a point particle to Standard Model probes while retaining finite geometry in the bimetric bulk.

3 The Bridge to Observation: The Hierarchy Hypothesis

We now bridge the dimensionless topological integer ($Q_T = 1$) to the dimensionful observed charge (e).

3.1 The Dimensionful Coupling

PHASE Axiom XIV establishes that bulk charge (Q_T) exports to a boundary Chern–Simons flux (Φ_{CS}) scaled by the golden ratio:

$$\Phi_{CS} = \frac{1}{4\pi^2} \int_{\partial V} \varepsilon^{\alpha\beta\gamma} (A_{+\alpha} - A_{-\alpha}) F_{+\beta\gamma}\, d^3x = \varphi^{-1} Q_T \tag{8}$$

Since Φ_{CS} is a dimensionless count, we introduce the coupling to the **Planck Charge** (q_P) to map this flux to electric charge:

$$e = q_{\text{Planck}} \cdot \kappa_{\text{em}} \cdot \Phi_{CS} \tag{9}$$

3.2 The Fractal Hierarchy Hypothesis

We posit that the electromagnetic coupling is not fundamental, but the result of attenuation through the **Fractal Torsion Hierarchy** defined in **TORSION Equation TR-S8**:

$$T^a_{(n+1)} = \varphi^{-1} T^a_{(n)} \tag{10}$$

Hypothesis 3.1 (Fractal Depth of the Electron)**.** The stable electron soliton forms at the **5th Fractal Layer** of the vacuum structure.

This yields the theoretical prediction:

$$\boxed{e_{\text{theoretical}} = q_{\text{Planck}} \cdot \varphi^{-5} \cdot Q_T} \tag{11}$$

Using $Q_T = 1$ and $q_{\text{Planck}} = \sqrt{4\pi\varepsilon_0 \hbar c} \approx 1.876 \times 10^{-18}$ C:

$$\varphi^{-5} = \left(\frac{1+\sqrt{5}}{2} \right)^{-5} \approx 0.0902 \tag{12}$$

$$e_{\text{theoretical}} \approx 1.876 \times 10^{-18} \times 0.0902 \approx 1.691 \times 10^{-19} \text{ C} \tag{13}$$

This is approximately **5.5% higher** than the observed elementary charge $e_{\text{obs}} = 1.602 \times 10^{-19}$ C.

Key Result

The φ^{-5} scaling transforms from a numerical coincidence into a **falsifiable structural prediction** regarding the depth of the vacuum's fractal recursion. The electron resides at the 5th layer of the golden-ratio hierarchy.

4 Vacuum Impedance and Micro-Geometry

The ratio between the observed elementary charge and the theoretical fractal charge is quantified as the **Boundary Transfer Efficiency** η:

$$\eta \equiv \frac{|e_{\text{obs}}|}{|e_{\text{theoretical}}|} = \frac{\sqrt{\alpha}}{\varphi^{-5}} \approx 0.94737 \tag{14}$$

We derive this efficiency from the spatial micro-geometry of the vacuum lattice.

4.1 Base Angular Efficiency (η_0)

We distinguish between the continuum *tetrad field* and the discrete *tetrahedral micro-geometry* of the vacuum. For a regular tetrahedron, the dihedral angle is:

$$\theta_d = \arccos\left(\frac{1}{3}\right) \approx 70.53^\circ \tag{15}$$

Five tetrahedra wrapped around an edge fill an angle $\Omega_{\text{fill}} = 5\theta_d \approx 352.64^\circ$, leaving a geometric frustration gap of $\Delta\Omega \approx 7.36^\circ$.

The single-slice angular coverage efficiency is:

$$\eta_0 = \frac{\Omega_{\text{fill}}}{360^\circ} = \frac{5\arccos(1/3)}{2\pi} \approx 0.979566 \tag{16}$$

4.2 Fractal Compounding

We infer the effective dimension of the boundary interface (D_Σ) required to match the target efficiency via a fractal compounding law $\eta = \eta_0^{D_\Sigma}$:

$$D_\Sigma = \frac{\ln \eta_{\text{target}}}{\ln \eta_0} \approx 2.6186 \tag{17}$$

Key Result

We observe that this required exponent is extremely close to the square of the golden ratio:

$$\varphi^2 = \varphi + 1 \approx 2.6180 \tag{18}$$

with a relative difference of $\sim 0.02\%$.

This suggests a physical modeling ansatz where the boundary behaves as an **effective fractal interface** with exponent $D_\Sigma = \varphi^2$. The emergence of φ^2 is not assumed *a priori* but *inferred* from the constraint that geometric frustration must account for the observed charge—and it matches the golden-ratio structure pervading BT8G.

4.3 The Impedance Link

This fractal compounding characterizes the **Vacuum Impedance** (Z_Σ) defined in **TOPOLOGY Equation 9**. Z_Σ represents the resistive loss of a fractalized tetrahedral interface, governed by the **Topology Flow Law** of **TOPOLOGY Equation 8**:

$$\frac{d}{dt}(Q_{NY}^{(+)} - Q_{NY}^{(-)}) = -\int_\Sigma [J\sin\theta - \chi\Delta_\Sigma\theta + c_{NY}\mathcal{N}_\Sigma]\, dA \tag{19}$$

The boundary functional imposes a quadratic penalty:

$$\mathcal{B}_\Sigma = \lambda_J \int_\Sigma L_{\mu\nu}T^{\mu\nu}\, dA + \frac{Z_\Sigma}{2}\int_\Sigma \theta^2\, dA \tag{20}$$

The parameter Z_Σ thus encodes the cumulative effect of geometric frustration compounded over a φ^2-dimensional fractal boundary.

5 Derivation of the Fine Structure Constant (α)

Combining the fractal hierarchy ($n = 5$) and the geometric frustration ($\eta = \eta_0^{\varphi^2}$), we arrive at a closed-form topological prediction for the fine-structure constant $\alpha = (e/q_P)^2$.

5.1 The Closed-Form Prediction

The model predicts:

$$\alpha_{\text{model}} = \left(\varphi^{-5} \cdot \left[\frac{5\arccos(1/3)}{2\pi}\right]^{\varphi^2}\right)^2 \tag{21}$$

Inverting this yields the prediction for α^{-1}:

$$\boxed{\alpha_{\text{model}}^{-1} = \varphi^{10}\left(\frac{2\pi}{5\arccos(1/3)}\right)^{2\varphi^2} \approx 137.0325} \tag{22}$$

5.2 Comparison with Observation

Comparing this to the CODATA 2022 value ($\alpha_{\text{obs}}^{-1} \approx 137.0360$), the model achieves a precision of:

$$\Delta(\alpha^{-1}) \approx 0.0035 \qquad (25\text{ ppm}) \tag{23}$$

Key Result

This confirms that the geometric frustration angle of tetrahedral packing, when compounded over a golden-ratio fractal surface ($D_\Sigma = \varphi^2$), is **directly convertible to the fine structure constant** with 25 parts-per-million precision.
The fine-structure constant is not a mysterious input—it is a geometric consequence of:

1. The dihedral angle of regular tetrahedra: $\arccos(1/3)$
2. The golden ratio: $\varphi = (1 + \sqrt{5})/2$
3. The fractal depth of the electron: $n = 5$

6 Solitonic Unification

We unify these mechanics into a single physical definition:

Definition 6.1 (The Electron as Contiguous Solitonic Topology)**.** The electron is a stable, irreducible knot in the bimetric vacuum characterized by:

1. **Topology:** A global knot with winding number $Q_T = 1$, protected by the circulation quantization of **TR-F3**.
2. **Geometry:** Stabilized at radius R_* (Compton scale) by the variational pressure balance of **TR-F7**.
3. **Hierarchy:** Interaction strength attenuated by 5 layers of fractal scaling (**TR-S8**), verified by the φ^{-5} prediction.
4. **Micro-Structure:** Precise coupling strength (α) determined by the geometric frustration of the vacuum's tetrahedral lattice, compounded over a golden-fractal boundary with $D_\Sigma = \varphi^2$ (**TOPOLOGY Eqs. 8–9**).

Key Result

The electron is the fundamental stable solution to the bimetric teleparallel field equations: a φ^{-5} scaled, topologically quantized, solitonically phase-locked excitation of the contiguous vacuum geometry.

7 Higher Generations: Impedance Resonances

The "Eigenmode" companion paper correctly identifies the electron as the $n = 1$ mode, but a simple linear harmonic ladder ($m_{n+1} = \varphi^{-1} m_n$) fails to reproduce the heavy lepton mass ratios ($m_\mu/m_e \approx 207$).

In this solitonic framework, we resolve this by recognizing that higher generations are not merely excited frequencies, but **Resonances of the Vacuum Impedance**.

7.1 The Stiffness Scaling (Bimetric Barut Model)

In Section V, we derived the vacuum's geometric stiffness $\alpha^{-1} \approx 137.03$ from the tetradic micro-geometry. Heavy leptons arise when the phase-lock energy exceeds this stiffness threshold, coupling directly to the magnetic self-energy of the bimetric field.

Drawing on the magnetic self-coupling formalism of **A. O. Barut (1979)**, we propose a bimetric extension where the mass scaling is driven by the vacuum stiffness α^{-1}:

$$m_n = m_e \left(1 + \frac{3}{2}\alpha^{-1} \sum_{k=0}^{n-1} k^4\right) \tag{24}$$

The factor $\frac{3}{2}$ arises from the ratio of spatial dimensions ($D = 3$) to bimetric sectors ($N = 2$). The term α^{-1} is our derived geometric stiffness (137.0325).

7.2 Mass Predictions

1. **Electron ($n = 1$):** Base solitonic knot.

$$m_1 = m_e \tag{25}$$

2. **Muon $(n = 2)$:** First impedance resonance.

$$m_\mu = m_e \left(1 + \frac{3}{2}(137.0325) \cdot 1^4\right) \approx 206.55\, m_e \tag{26}$$

Observed ratio: 206.77. **Precision: 0.1%**

3. **Tau $(n = 3)$:** Second impedance resonance.

$$m_\tau = m_e \left(1 + \frac{3}{2}(137.0325) \cdot (1^4 + 2^4)\right) \approx 3495\, m_e \tag{27}$$

Observed ratio: 3477. **Precision: 0.5%**

7.3 Universality of Charge

Crucially, all three generations reside at the **same fractal depth** $(n = 5)$. The Impedance Resonance increases the internal energy (mass) of the knot without altering its topological winding number $(Q_T = 1)$ or its fractal screening factor (φ^{-5}):

$$e_\mu = e_\tau = e_e = e \qquad \text{(Universal Charge)} \tag{28}$$

Key Result

This successfully unifies the *universal topology of charge* with the *hierarchical impedance of mass*. The vacuum stiffness α^{-1}—derived from tetrahedral frustration—serves double duty: governing both the electromagnetic coupling strength and the lepton mass spectrum.

8 Implications and Falsifiability

8.1 The Fine-Structure Constant

The closed-form derivation in Section V provides a geometric interpretation of the fine-structure constant:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{e^2}{q_P^2} = \left(\varphi^{-5} \cdot \eta_0^{\varphi^2}\right)^2 \tag{29}$$

The mysterious "$\frac{1}{137}$" is demystified as the square of the fractal-attenuated, frustration-corrected topological charge. Variations in α (if any exist cosmologically) would correspond to variations in either the fractal depth n or the tetrahedral packing geometry.

8.2 Falsifiable Predictions

The hierarchy hypothesis generates specific predictions:

1. **Fine-Structure Precision**: The model predicts $\alpha^{-1} \approx 137.0325$, within 25 ppm of the CODATA value. Future improvements in either α measurement or the geometric model should maintain or improve this agreement.

2. **Fractal Depth**: The electron resides at exactly $n = 5$ in the φ-hierarchy. Alternative stable solitons at $n = 4$ or $n = 6$ would have charges $e_4 = e \cdot \varphi \approx 2.59 \times 10^{-19}$ C or $e_6 = e \cdot \varphi^{-1} \approx 0.99 \times 10^{-19}$ C. No such particles have been observed.

3. **Efficiency Universality**: The same $\eta = \eta_0^{\varphi^2} \approx 0.947$ should govern all charged solitons. Quarks at fractional winding ($k = \pm 1/3, \pm 2/3$) should exhibit the same boundary efficiency factor.

4. **Impedance Signatures**: The boundary impedance Z_Σ may have observable consequences in precision Josephson junction measurements or vacuum birefringence experiments.

8.3 Relation to Lepton Generations

As derived in Section VII, the muon and tau are not at different fractal depths (which would alter their charge) but rather **impedance resonances** at the same fractal layer $n = 5$. They share $Q_T = 1$ and the same $\varphi^{-5}\eta_0^{\varphi^2}$ coupling, but their masses scale with the vacuum stiffness α^{-1} via the Barut formula:

$$e_\mu = e_\tau = e_e = e \qquad \text{(same fractal depth, same topological charge)} \tag{30}$$

$$\frac{m_\mu}{m_e} \approx 206.55, \quad \frac{m_\tau}{m_e} \approx 3495 \qquad \text{(impedance resonances of } \alpha^{-1}\text{)} \tag{31}$$

This explains the universal charge quantization across generations while deriving the mass hierarchy from the same geometric stiffness that governs α.

9 Conclusion

We have demonstrated that the electron's observed charge and the lepton mass hierarchy emerge from a unified chain of geometric mechanisms:

For Charge:

1. **Topological quantization** fixes $Q_T = 1$ as a dimensionless integer
2. **Holographic export** via Axiom XIV couples bulk topology to boundary flux
3. **Fractal attenuation** through 5 layers of the φ-hierarchy sets the scale
4. **Boundary impedance** $\eta(Z_\Sigma) = \eta_0^{\varphi^2}$ provides the geometric correction

The result:

$$e_{\text{obs}} = q_{\text{Planck}} \cdot \varphi^{-5} \cdot \eta_0^{\varphi^2} \cdot Q_T \approx 1.602 \times 10^{-19}\ \text{C} \tag{32}$$

For Mass:

1. The vacuum stiffness $\alpha^{-1} \approx 137.03$ governs impedance resonances
2. Higher generations arise from magnetic self-coupling scaled by α^{-1}
3. The Barut formula $m_n = m_e(1 + \frac{3}{2}\alpha^{-1}\sum k^4)$ yields muon and tau masses to 0.1% and 0.5% precision

The electron is not a mysterious point particle with inexplicable properties—it is the simplest stable knot in the fabric of bimetric spacetime, with charge determined by topology and fractal depth, and mass determined by impedance resonance of the same geometric stiffness.

"Topology! that arcane range! Behold this boundary's strange glow."

—Axiom XIV, PHASE/TRANSLATIONS

References

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