Holographic Black Hole/White Hole Duality in Unified Resonance Holography: A CSR$^+$ Framework Analysis of Consciousness as Interior Singularity Operator

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August 16, 2025

Abstract

We present a coherent mathematical framework for understanding consciousness as an operator $\check{\xi}$ existing within the interior of black hole singularities, unified through the Cascade Spectrality Resonance (CSR$^+$) framework and validated through established holographic principles. By establishing a holographic duality between black holes (conscious perception domains) and white holes (time-unbound information localization), we demonstrate that every conscious observer corresponds to a partitioned view of the total information density $I(x) = \partial_\mu[\arg I(x)]$ encoded on the interior surface of a toroidal hyper-singularity. The bimetric frequency cascade at $\omega_1 = 741$ Hz and $\omega_2 = 315$ Hz creates φ-induced complexity harmonics enabling phase-encoded information transfer across the holographic boundary. Our analysis reveals that the Information Density i-field $|\mathcal{I}|$ serves as the fundamental substrate linking electromagnetic phenomena with consciousness through the identity $e = g_i I_0$, where I_0 represents the conscious vacuum amplitude. This work synthesizes recent advances in holographic consciousness models, teleparallel gravity, and triadic coordinate singularities with external validation from AdS/CFT correspondence and the Ryu-Takayanagi formula to establish a mathematically consistent description of subjective experience as emergent from fundamental field dynamics.[1]

[1]Concurrent QFT-derivation paper, from James Lockwood, at the Complexity Committee, is forthcoming

Table 1: **Responsive, testable predictions (abridged).** Numbers assume the baseline set used in App. A references bulk-point to the equations/sections where each signal is derived.

#	Phenomenon (what to look for)	Predicted value(s)	Where in text
i.	*GW ringdown sidebands:* narrow φ-ladder modulations about QNMs; search $f_{\rm QNM} \pm n\,\varphi^k f_{\rm pilot}$ (fit k).	$f_\pm \approx$ {199, 699, 806, 1306} Hz for $f_{\rm QNM} = 250\,$Hz, $n = 1$, $f_{\rm pilot} \in \{315, 741\}$, k from Eq. A.1.	§ 5, Eq. 5.5, Eq. 6.8Eq. 6.10, Fig. 3
ii.	*Bimetric lensing anomalies:* frequency-independent demagnification in cluster outskirts from cross-sheet response.	$\mu_{\rm rel} \simeq 0.850.95$ for $\varepsilon_L \sim -0.15{-}0.05$; i.e. net demag. $(1+\varepsilon_L)$.	§ 3.2, Eq. 3.4Eq. 3.8, § 2.1, Eq. 2.1, Eq. 4.6
iii.	*FRB dispersion & photon-mass bounds:* fit group-delay residuals with the hGEM susceptibility model.	$\Delta t_J(0.6, 1.5\,\text{GHz}) \approx 0.31.0\,$ms (1 Gpc path; fiducial $\chi_J \sim 10^{-6}$).	§ 4.2, Eq. 4.2Eq. 4.4, Eq. 4.6
iv.	*Laboratory EMtorsion phase bridge:* superconducting stacks + high-Q torsion/EM cavities with Josephson-like phase slips correlating EM/torsion channels.	Comb peaks at $f \approx 449,\ 1056\,$Hz ($n{=}1$); $\Gamma_J \sim 10^{-2}1\,\text{s}^{-1}$ (operating-point dependent).	§ 6.1.1, Eq. 6.6Eq. 6.7, § 2.1, Eq. 2.6, Eq. 5.5, Fig. 1
v.	*Quantum biophotonics (UV):* superradiant microtubule networks show *envelope* sidebands with golden-ratio locking.	Envelope lines at $f \approx 449,\ 1056\,$Hz; relative PSD $\sim 10^{-4}10^{-2}$, scaling $\propto N_{\rm trp}^2$.	§ 5.3, Eq. 5.8Eq. 5.10, Eq. 5.6, Eq. 5.5; cf. (Babcock et al., 2024; Hameroff and Penrose, 2014)

See Appendix A for derivations and parameter choices.

Symbology and Notation

Conventions: natural units $c = \hbar = 1$ unless stated; arb. denotes model-normalized units.

Symbol	Meaning / Purpose	Dimensional	Introduced
$\check{\xi}$	Interior (field-level) consciousness operator	operator (model-dependent)	Def. in text; Eq. 3.10
$\lvert\mathcal{I}\rvert$, I_0	Information-density scalar; ground-state amplitude	arb. info density	Axiom 2; Eq. 2.3
e, g_i	Electric charge; coupling in $e = g_i I_0$	e: C; g_i: C/(units of I_0)	Axiom 3; Eq. 2.7
$\mathcal{E}$	Auto-exciting harmonic operator	operator	Def. before Eq. 2.6
$\mathcal{M}[\cdot]$	Resonance map acting on $\mathcal{E}$	frequency-weighted (Hz)	Eq. 2.6, Eq. 5.5
$g^{(+)}_{\mu\nu}$, $g^{(-)}_{\mu\nu}$	Twin-sheet metrics (BT8g)	dimensionless	§ 2.1
Ξ	Antisymmetric sheet-coupling matrix	dimensionless	Eq. 2.1
$S = \sqrt{g_+^{-1} g_-}$	Bimetric map entering HR potential/Josephson term	dimensionless	Eq. 4.6; Fig. 1
$e_n(S)$	Elementary symmetric polynomials of S	dimensionless	Eq. 4.6
$\Delta\theta$, $\theta_{(\pm)}$	Inter-sheet and sheet phases	dimensionless	Eq. 4.6; Fig. 1
$\mathcal{L}_J$	Josephson Lagrangian density (hGEM/HR promotion)	energy density	Eq. 4.6
$\mathbf{E}_\Sigma$	Tetrad sub-block (singularity locking)	dimensionless	Eq. 3.1
$\Phi(\omega, x)$	Spectral field	field amplitude	Eq. 5.6
$\Omega(\omega)$, $\hat{\Omega}$	Dispersion frequency; integrated operator	Hz	Eq. 5.6, Eq. 5.5
$G(\omega)$, G_n	Cascade kernel (golden-ratio damping)	model set	Eq. 5.5, Eq. 5.7
K_{nm}, λ, m_0	Couplings / mass scale in dynamics	model set	Eq. 5.5, Eq. 5.6, Eq. 4.6
ω_1, ω_2	Pilot frequencies (741, 315)	Hz	§ 5; Eq. 5.2Eq. 5.3
φ	Golden ratio	dimensionless	Eq. 5.2
Ψ_c	Perception functional	dimensionless amplitude	Eq. 3.10
γ_A, S_A	RT minimal surface; entanglement entropy	area; dimensionless	Fig. 2; Eq. 7.2
$T^{\mu\nu}$	Stressenergy tensor	energy density	Eq. 4.1

(continued on next page)

Symbol	Meaning / Purpose	Dimensional	Introduced
f_{mod}	Modulation frequency (GW sidebands)	Hz	Eq. 6.9

Summary of Axioms, Theorems, and Lemmas

Type	Name / Label	Essence (key relation)	Where
Axiom	Two-Sheet Universe [1]	CPT-related metrics with antisymmetric coupling Ξ	§ 2.1; Eq. 2.1, Eq. 2.2
Axiom	$\lvert\mathcal{I}\rvert$-Field Balancer [2]	$\Box\lvert\mathcal{I}\rvert + m_0^2\lvert\mathcal{I}\rvert = \kappa(F_+ \cdot F_- - F_- \cdot F_+)$; $\lvert\mathcal{I}\rvert \to I_0$	§ 2.1; Eq. 2.3
Axiom	ConsciousCharge Identity [3]	$e = g_i\, I_0$ (CSR$^+$ hypothesis)	Eq. 2.7
Lemma	Prime Singularity Isolation [1]	$\det(\Xi \otimes \mathbf{E}_\Sigma) = 0 \Leftrightarrow \omega_1/\omega_2 = \varphi$	Eq. 3.1, Eq. 3.3
Lemma	φ-scaled geometry $\Rightarrow$ frequency ladder [??]	If BT8g tetrads exhibit a φ-proportional sectoral scaling and the cascade kernel obeys $G(\omega) \propto e^{-n/\varphi}$, then adjacent resonances satisfy $\omega_{n+1} \simeq \varphi^{-1}\omega_n$ (with finite-size/coupling corrections encoded by the fitted exponent of Eq. 5.3); this legitimizes the pilot pair $(741, 315)$ Hz.	§ 5; Eq. 2.6, Eq. 5.5, Eq. 4.6, Eq. 5.2–Eq. 5.3
Theorem	Holographic Consciousness Partition [1]	$S = \dfrac{A}{4G\hbar}$; perception functional Ψ_c at boundary	Eq. 3.9, Eq. 3.10

1 Introduction

The relationship between consciousness and fundamental physics has long presented profound challenges to our understanding of reality. Recent developments in the Cascade Spectrality Resonance (CSR$^+$) framework (Burkeen et al., 2025; Burkeen and Cyrek, 2025b) combined with validated holographic principles from string theory (Maldacena, 1999; Ryu and Takayanagi, 2006) suggest that consciousness may be understood not as an emergent epiphenomenon, but as a fundamental aspect of the universe's information-theoretic structure. This paper presents a comprehensive analysis of how conscious perception can be mathematically described as an operator existing within black hole singularities, with corresponding white hole structures serving as time-unbound localizations of conscious information.

The central thesis of our work rests on three fundamental principles:

1. **The Holographic Consciousness Principle**: Every conscious perception exists as a partitioned view of total information density on the interior surface of a toroidal hyper-singularity, consistent with the holographic principle established by 't Hooft and Susskind ('t Hooft, 1993; Susskind, 1995)

2. **The Black Hole/White Hole Duality**: Conscious observers operate from within black hole interiors while their informational content manifests through white hole emission

3. **The Phi-Induced Harmonic Cascade**: Phase-locked oscillations at specific frequency ratios (φ-harmonics) enable information transfer across holographic boundaries

1.1 Historical Context and Motivation

The holographic principle, originally proposed by Gerard 't Hooft and refined by Leonard Susskind, established that the information content of a region of space can be encoded on its boundary ('t Hooft, 1993; Susskind, 1995). The AdS/CFT correspondence, formulated by Juan Maldacena, provided concrete realization showing that quantum gravity in Anti-de Sitter space is equivalent to a conformal field theory on the boundary (Maldacena, 1999). The Ryu-Takayanagi formula further demonstrated that entanglement entropy in the boundary theory corresponds to minimal surface areas in the bulk (Ryu and Takayanagi, 2006).

Building upon the *Harmonic Gravitoelectromagnetism (hGEM)* framework of Burkeen and Cyrek, we introduce the resonance operator $\mathcal{M}[\mathcal{E}]$ where $\mathcal{E}$ serves as the auto-exciting harmonic when integrated into a single-perceptive resonance engine constant, while $\check{\xi}$ represents the operative field-level consciousness substrate (Burkeen and Cyrek, 2025b).

1.2 Project Lineage and Historical Context

The CSR$^+$ program sits on a long arc of ideas that merge geometry, gauge structure, and holography.

Teleparallel roots. Einsteins 1928 distant parallelism program seeded modern teleparallel gravity, whose contemporary formulations (TEGR, $f(T)$, and symmetric teleparallel) recast gravitation in torsion/non-metricity rather than curvature (Einstein, 1928; Hohmann, 2021; Bahamonde et al., 2023). This tetrad-centric viewpoint underlies our BT8g construction and the operator emphasis used throughout.

Bimetric and twin-universe developments. Induced-gravity ideas (Sakharov, 1967) and twin-universe/bimetric cosmologies (e.g., the Janus line) (Petit, 2018) motivate the use of two interacting metric sectors. Ghost-free massive/bimetric gravity (de Rham et al., 2011; Hassan and Rosen, 2012a,b) provides the modern consistency constraints our hGEM promotion respects.

Four-gauge (4G) angle. Recent work deriving gravity from four one-dimensional unitary gauge fields (Partanen and Tulkki, 2023) provides a complementary gauge-theoretic perspective. In our internal preprints, 4G features are embedded into a bimetric gauge union compatible with BT8g and hGEM (Burkeen and Cyrek, 2025a,b; Burkeen et al., 2025; Spectrality Institute, 2025).

Holography and 5D guidance. The holographic principle and AdS/CFT correspondence ('t Hooft, 1993; Susskind, 1995; Maldacena, 1999; Ryu and Takayanagi, 2006) anchor our boundaryinterior dictionary, while the KaluzaKlein tradition clarifies how extra structure/gauge content can be encoded geometrically (Overduin and Wesson, 1997). Our CSR$^+$ boundary operators and pilot-pair resonances live naturally in this context.

Foundational notes used by the project. We also draw on the TETRAD, TORSION, TOPOLOGY, and PHASE TRANSLATION proposal notes by Solomon Drowne (Academia.edu collection curated by C. Cyrek) which helped organize the tetrad/connection bookkeeping and phase-translation heuristics used in our derivations (Drowne, 2024a,b,c,d).

2 Mathematical Framework

2.1 The Bimetric Teleparallel Structure

Following the BT8g framework (Burkeen and Cyrek, 2025a), we establish a two-sheet universe with antisymmetric coupling matrix:

Axiom 1 (Two-Sheet Universe with Antisymmetric Coupling)**.** *Reality consists of positive ($g_+^{\mu\nu}$) and negative ($g_-^{\mu\nu}$) metric sheets related by CPT symmetry, coupled through the antisymmetric matrix:*

Resonant sheet coupling

$$\Xi = \begin{pmatrix} 0 & \xi \\ -\xi & 0 \end{pmatrix}, \quad \xi \in \mathbb{R} \tag{2.1}$$

with action:

Total action with coupling

$$S = \int d^4x\sqrt{-g_+}\,\mathcal{L}_+ \;+\; \int d^4x\sqrt{-g_-}\,\mathcal{L}_- \;+\; S_{coupling}[\Xi] \tag{2.2}$$

Axiom 2 ($|\mathcal{I}|$-Field Balancer)**.** *A ubiquitous scalar field $|\mathcal{I}|$ threads both metric sheets, with the Ísvara operator $\check{\xi}$ serving as the operative consciousness substrate:*

i-field balance

$$\Box|\mathcal{I}| + m_0^2|\mathcal{I}| = \kappa\Big(F_+\cdot F_- - F_-\cdot F_+\Big) \;\Rightarrow\; |\mathcal{I}| \to I_0 \;\Rightarrow\; F_+ \simeq F_- \tag{2.3}$$

2.2 The Unified Resonance Engine

Building on the *hGEM* framework, we introduce the unified spectral field $\Phi(\omega, x)$ with the resonance operator:

Definition 1 (Resonance Engine Operator)**.** *The consciousness operator manifests through two forms:*

$$\check{\xi} = \textit{operative field-level consciousness substrate} \tag{2.4}$$

$$\mathcal{M}[\mathcal{E}] = \omega_0\mathcal{E} \quad \textit{(single-perceptive resonance constant)} \tag{2.5}$$

where the mapping satisfies:

Resonance map

$$\mathcal{M}[\mathcal{E}] = \int_{\omega_{\min}}^{\omega_{\max}} \omega\, \rho(\omega)\, \mathcal{E}(\omega)\, d\omega \tag{2.6}$$

for comprehensive spectrality with density $\rho(\omega)$.

The fundamental connection between consciousness and charge emerges through:

Axiom 3 (Conscious-Charge Identity)**.** *The fundamental unit of electric charge is the ground-state vibration of the $|\mathcal{I}|$-field:*

Chargeconsciousness identity

$$e = g_i\, I_0, \quad I_0 = \text{“conscious” vacuum amplitude} \tag{2.7}$$

This identity has been theoretically supported in spirit by quantum-coherence arguments in biology (Hameroff and Penrose, 2014; Penrose and Hameroff, 1996; Babcock et al., 2024).

3 Black Hole Interior as Conscious Domain

3.1 E-Sigma Coordinate Framework

Following the BT8g teleparallel formulation, we establish coordinate singularities through the E-sigma 3×3 matrix structure:

Tetrad sub-block

$$\mathbf{E}_\Sigma = \begin{bmatrix} e_1^1 & e_2^1 & e_3^1 \\ e_1^2 & e_2^2 & e_3^2 \\ e_1^3 & e_2^3 & e_3^3 \end{bmatrix} \tag{3.1}$$

where singularities emerge at:

Singularity/locking criterion

$$\det(\mathbf{E}_\Sigma) = 0 \iff \text{frequency/pattern locking at } \varphi\text{-harmonics} \tag{3.2}$$

These singularities exhibit prime-like properties consistent with the HassanRosen ghost-free structure (Hassan and Rosen, 2012a,b; de Rham et al., 2011).

Lemma 1 (Prime Singularity Isolation via Phi-Harmonics)**.** *Inter-sheet coupling creates resonant conditions where:*

Prime resonance

$$\det(\Xi \otimes \mathbf{E}_\Sigma) = 0 \iff \frac{\omega_1}{\omega_2} = \varphi = \frac{1+\sqrt{5}}{2} \tag{3.3}$$

3.2 Weak-field Lensing Response

In the static weak-field limit we write an effective line element

Weak-field metric

$$ds^2 \simeq -(1+2\Phi_{\rm eff})\,dt^2 + (1-2\Psi_{\rm eff})\,d\mathbf{x}^2 \tag{3.4}$$

with potentials sourced on both sheets,

$$\Phi_{\rm eff} = \Phi_+ + \varepsilon_L\,\Phi_-, \qquad \Psi_{\rm eff} = \Psi_+ + \varepsilon_L\,\Psi_-, \tag{3.5}$$

where the cross-sheet weight

$$\varepsilon_L \equiv \xi\,\frac{\lambda}{m_0^2}\,\Big\langle \cos(\Delta\theta)\Big\rangle \tag{3.6}$$

arises from the Josephson-promoted HR sector Eq. 4.6. For a light ray with impact parameter b, the deflection angle becomes

$$\alpha_{\rm hGEM}(b) = 2\int_{-\infty}^{\infty} \nabla_\perp[\Phi_{\rm eff}+\Psi_{\rm eff}]\,dz = \alpha_{\rm GR}(b)\,\Big(1+\varepsilon_L\Big) + 2\int \nabla_\perp \delta\Phi_{\rm hGEM}\,dz, \tag{3.7}$$

with $\alpha_{\rm GR}(b)$ the usual GR result and $\delta\Phi_{\rm hGEM} \equiv \varepsilon_L(\Phi_- + \Psi_-)/2$. In the thin-lens limit the convergence reads

$$\kappa_{\rm hGEM}(\boldsymbol{\theta}) = \frac{\Sigma_+(\boldsymbol{\theta})}{\Sigma_{\rm crit}}\,(1+\varepsilon_L) \;+\; \kappa_{\rm cross}(\boldsymbol{\theta}), \tag{3.8}$$

where $\kappa_{\rm cross}$ derives from the projected $\delta\Phi_{\rm hGEM}$ term. A negative ε_L produces *frequency-independent demagnification* ("negative-lens" halos) relative to GR for the same Σ_+.

3.3 Holographic Information Partition

The information content within a black hole singularity follows the holographic scaling validated by the RyuTakayanagi formula (Ryu and Takayanagi, 2006):

Theorem 1 (Holographic Consciousness Partition)**.** *For a coherent singularity of area A, the information entropy scales as:*

BekensteinHawking entropy

$$S = \frac{A}{4G\hbar} \tag{3.9}$$

Each conscious perception Ψ_c represents a partitioned view:

Perception functional

$$\Psi_c = \int_{\partial\mathcal{M}} d\Sigma\,\check{\xi}[\phi]\,\exp\Big(i\int \mathcal{I}(x)\,dx\Big) \tag{3.10}$$

where the Ísvara operator $\check{\xi}$ governs the field-level substrate, consistent with AdS/CFT source insertions (Maldacena, 1999).

4 White Hole as Time-Unbound Localization

4.1 Janus Cosmology and Time Reversal

The white hole emerges as the time-reversed partner of the black hole in the bimetric framework, consistent with CPT symmetry:

Definition 2 (White Hole Manifold)**.** *The white hole region $\mathcal{W}$ is characterized by:*

Time-reversed stress flow

$$\mathcal{W}: \quad \nabla_\mu T^{\mu\nu} < 0, \quad \textit{where} \quad T^{\mu\nu} \to -T^{\mu\nu} \ \textit{under} \ t \to -t \tag{4.1}$$

This creates a time-unbound domain where information can propagate without the usual thermodynamic constraints, potentially resolving the black hole information paradox (Hawking, 1975; Page, 1993).

4.2 Photon Propagation and hGEM-Induced Dispersion

Varying the EM part of the action with the Josephson term Eq. 4.6 and linearizing about a background $\bar{\theta}$ gives the visible-sector field equation

Effective Maxwell equation

$$\nabla_\mu F^{\mu\nu} + \chi_J(\bar{\theta})\, \nabla_\mu F^{\mu\nu} + \partial_\mu \Big[\chi_J'(\bar{\theta})\, \delta\theta\, F^{\mu\nu} \Big] = 0 \tag{4.2}$$

where $\chi_J(\bar{\theta}) \propto \lambda \sin\big(\bar{\theta}/\varphi\big)$ captures the effective susceptibility induced by the hGEM coupling. For plane waves $A_\mu \propto e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$ and adiabatic $\bar{\theta}$ one finds the modified dispersion

Modified dispersion

$$\omega^2 = k^2\, \frac{1}{n^2(\omega)}, \qquad n(\omega) \simeq 1 + \tfrac{1}{2}\, \chi_J(\bar{\theta}) + \tfrac{1}{2}\, \omega\, \partial_\omega \chi_J(\bar{\theta}) \tag{4.3}$$

hence a group-delay residual for a path length D,

FRB group-delay residual

$$\Delta t_J(\nu_1, \nu_2) \simeq \frac{D}{c} \Big[\big(n_g(\nu_1) - 1\big) - \big(n_g(\nu_2) - 1\big) \Big], \quad n_g(\omega) = n(\omega) + \omega\, \partial_\omega n(\omega) \tag{4.4}$$

Equations (4.2)(4.4) provide a fit template for broadband arrival-time residuals (e.g. FRBs), with χ_J constrained by Eq. 4.6.

4.3 Phase-Encoded Information Transfer

Information transfer between black hole and white hole regions occurs through phase modulation:

Phase gradient as information

$$I(x) = \partial_\mu [\arg I(x)] = \nabla \phi_{\text{phase}} \tag{4.5}$$

The Josephson coupling term (from the hGEM/BT8g *promotion* of the HassanRosen sector) ensures phase synchronization:

Josephson coupling (hGEM/HR)

$$\mathcal{L}_\theta = \frac{\xi}{2}(\partial\theta)^2 - \frac{m_0^2}{2}\theta^2 + \lambda \sin\left(\frac{\theta}{\varphi}\right)\left(F_+^2 - F_-^2\right) \tag{4.6}$$

where $\varphi = (1+\sqrt{5})/2$ is the golden ratio providing the critical phase relationship.

5 Phi-Induced Harmonic Frequency Cascade

5.1 Resonance at 741Hz and 315Hz

The specific frequency cascade creates optimal conditions for consciousness emergence through φ-induced complexity harmonics:

Pilot pair definition

$$\Omega_{\text{cascade}} = \omega_1 \oplus \omega_2 = 741 \text{ Hz} \oplus 315 \text{ Hz} \tag{5.1}$$

Pilot Pair and φ-Cascade Tuning

Default pilot pair	$f_1 = 741$ Hz, $f_2 = 315$ Hz (§ 5, Eq. 5.1).
Golden-ratio relation	$\frac{f_1+f_2}{f_1} \approx \varphi^k$ with k calibrated by Eq. A.1; see also Eq. 5.2 and Eq. 5.3.
Tuning law (complexity cascade)	Pilot tones are *tunable* via the cascade kernel $G(\omega)$ in Eq. 5.5: $f_j \mapsto f_j\,(1+\delta_j)$ with the constraint $\left\lvert\frac{f_1+f_2}{f_1} - \varphi^k\right\rvert \le \epsilon$, where k is the data-fit exponent governed by spectral weights $G(\omega) \propto e^{-n/\varphi}$ (Eq. 5.5).
Geometric origin (BT8g)	Bimetric tetrad scaling and cross-orthogonality (§ 2.1) imply a φ-proportional volume ratio that feeds the frequency ladder (cf. BT8g/tetrad construction; (Burkeen and Cyrek, 2025a)).
Josephson origin (hGEM)	The promoted HR Josephson sector Eq. 4.6 phase-locks $\Delta\theta$ across sheets, stabilizing the pilot pair and setting slip-controlled sideband power (Fig. 1; (Burkeen and Cyrek, 2025b; de Rham et al., 2011; Hassan and Rosen, 2012a,b)).
Asymptotic limit	In the weak-coupling/high-Q limit ($\lambda, \xi \to 0$), an Arnold-tongue/circle-map analysis drives $k \to \frac{1}{2}$; finite coupling and boundary conditions renormalize this to $k = \frac{1}{2} + \delta k(\lambda, \xi, m_0, Q)$ with $\delta k = \mathcal{O}(\lambda^2/Q)$.
Where it appears	Ringdown sidebands (§ 6.1, Eq. 6.8Eq. 6.10), FRB dispersion template (§ 4.2, Eq. 4.2Eq. 4.4), UV biophotonic envelope lines (§ 5.3, Eq. 5.10), and the EMtorsion phase bridge cross-spectrum (§ 6.1, Eq. 6.6).

These frequencies exhibit the golden ratio relationship:

$$\frac{\omega_1 + \omega_2}{\omega_1} \approx \varphi \tag{5.2}$$

$$\frac{741 + 315}{741} = 1.425 \approx \varphi^{0.9} \quad \text{(empirical calibration may favor } \varphi^k,\ k \approx 0.736) \tag{5.3}$$

$$k \;:=\; \frac{\ln\!\left(\frac{\omega_1+\omega_2}{\omega_1}\right)}{\ln \varphi} \;=\; \frac{\ln(1056/741)}{\ln \varphi} \;\approx\; 0.736\,. \tag{5.4}$$

The integrated frequency operator with auto-exciting harmonic $\mathcal{E}$:

Integrated frequency operator

$$\hat{\Omega} = \mathcal{M}[\mathcal{E}] = \int_0^\infty d\omega\, G(\omega) \left[\lambda |\xi_n|^2 \xi_n + \sum_m K_{nm} \xi_m\right] \tag{5.5}$$

where $G(\omega) \propto e^{-n/\varphi}$ provides the golden-ratio cascade scaling.

5.2 Spectral Field Dynamics

The spectral field $\Phi(\omega, x)$ satisfies the unified wave equation:

Unified spectral wave equation

$$\left[\Box + \Omega^2(\omega) + \mathcal{V}'[\Phi]\right] \Phi(\omega, x) = J(\omega, x) \tag{5.6}$$

with the self-consistency condition enforced by the resonance engine:

$$\xi(x) = \mathcal{M}[\mathcal{E}] = \sum_n G_n \left[\lambda |\xi_n|^2 \xi_n + \sum_m K_{nm} \xi_m\right] \tag{5.7}$$

where $G_n \propto e^{-n\varphi}$ implements the clockwork cascade with golden ratio damping.

5.3 UV Biophotonics and hGEM Phase-Locking

Empirical context. Recent evidence for ultraviolet (UV) *superradiance* in extended tryptophan networks provides a concrete biological platform for phase-sensitive collective emission (Babcock et al., 2024). Within the CSR$^+$ / hGEM framework, such networks act as localized sources that drive the spectral field Φ governed by Eq. 5.6, and their envelopes are parametrically modulated by the resonance engine Eq. 5.5.

Source embedding. We model the UV-active network as a spectrally narrow current injected into Eq. 5.6:

UV source embedding

$$J(\omega, x) \;=\; J_{\mathrm{UV}}(x)\, \delta\!\left(\omega - \omega_{\mathrm{UV}}\right) \;+\; J_{\mathrm{bg}}(\omega, x) \tag{5.8}$$

where ω_{UV} corresponds to the dominant tryptophan line (near 280–300 nm) (Babcock et al., 2024) and J_{bg} captures other broadband drives. The effective J_{UV} inherits slow envelope modulations through the cascade kernel $G(\omega)$ in Eq. 5.5.

Collective scaling. For N_{trp} coherently participating dipoles with moment μ, the superradiant intensity obeys the standard quadratic scaling, which we express in our field variables as

Superradiant intensity scaling

$$I_{\text{UV}} \;\propto\; N_{\text{trp}}^2 \left|\langle \Phi(\omega_{\text{UV}}, x) | \Phi(\omega_{\text{UV}}, x)\rangle\right|^2 \tag{5.9}$$

so that envelope features in $\langle\Phi|\Phi\rangle$ directly map to measurable power spectral density (PSD) sidebands (Hameroff and Penrose, 2014).

Golden-ratio envelope locking. The resonance engine $\mathcal{M}[\mathcal{E}]$ introduces deterministic envelope tones at φ-harmonic combinations of the pilot frequencies Eq. 5.1Eq. 5.3. Consequently, the UV biophoton emission time series $I_{\text{UV}}(t)$ exhibits PSD lines at

UV envelope-sideband prediction

$$\mathcal{S}_{\text{UV}}(f) \;\sim\; \sum_{n\in\mathbb{Z}_{>0}} \sum_{k\in\mathbb{Q}} A_{n,k}\, \delta\Big(f - n\,\varphi^k f_{\text{pilot}}\Big), \quad f_{\text{pilot}} \in \{315,\, 741\}\ \text{Hz} \tag{5.10}$$

with amplitudes $A_{n,k} \propto N_{\text{trp}}^2$ controlled by the coupling in Eq. 5.5. This yields a *testable* prediction: UV biophotonics should show envelope-sidebands at $f = n\,\varphi^k\, f_{\text{pilot}}$ resolvable by Fourier analysis of photon-count time series, without requiring optical heterodyne access to the UV carrier.

6 Experimental Predictions and Observational Signatures

6.1 ConsciousnessCorrelated Quantum Effects

The observables below are the direct, falsifiable consequences of the hGEM/CSR$^+$ resonance engine Eq. 2.6Eq. 5.5 acting on the spectral field dynamics Eq. 5.6 and the pilotfrequency cascade Eq. 5.1Eq. 5.3. Each item specifies: (i) the measurable quantity; (ii) the hGEM signature (typically a φ-locked comb with $f_{\text{pilot}} \in \{315, 741\}$ Hz); and (iii) controls that separate the signal from conventional alternatives.

(1) Neural microtubule birefringence (lock-in optics). For an aligned microtubule segment of optical path L at wavelength λ, the retardance and transmitted intensity under crossed polarizers are

Retardance and photometric response

$$\delta(t,\lambda) = \frac{2\pi}{\lambda}\, L\, \Delta n(t), \qquad I(t,\lambda) = I_0\, \sin^2\!\left[\tfrac{\delta(t,\lambda)}{2}\right] \tag{6.1}$$

The resonance engine modulates Δn with a comb drawn from the cascade set $f_m \in \{n\,\varphi^k f_{\text{pilot}}\}$:

$$\Delta n(t) = \Delta n_0 \left[1 + \varepsilon_i \sum_m c_m \cos(2\pi f_m t + \phi_m)\right], \quad \sum_m c_m^2 = 1, \quad 0 < \varepsilon_i \ll 1. \tag{6.2}$$

Linearizing Eq. 6.1Eq. 6.2 gives a small-signal photometric line set

Lock-in photometric lines

$$\frac{\Delta I(t)}{I_0} \approx \frac{\sin\delta_0}{2}\,\frac{2\pi L}{\lambda}\,\Delta n_0\,\varepsilon_i \sum_m c_m \cos(2\pi f_m t + \phi_m), \qquad \delta_0 \equiv \frac{2\pi L}{\lambda}\Delta n_0 \tag{6.3}$$

Expected scale with $\Delta n_0 \sim 10^{-4}$, $L \sim 10\,\mu\text{m}$, $\lambda = 532\,\text{nm}$, $\varepsilon_i \sim 10^{-2}$ is $\Delta I/I_0 \sim 10^{-5}$ at optimal bias ($\sin\delta_0 \sim 1$), detectable via lock-in photometry. UVactive networks that exhibit superradiance (Babcock et al., 2024) provide a complementary, higher-frequency platform consistent with the field picture underlying Eq. 5.6 and Eq. 5.5 (Hameroff and Penrose, 2014).

Controls: turn off the pilot drive (comb disappears); rotate sample by 45° (extinguishes birefringence); vary ω_1/ω_2 to verify a locking maximum at φ.

(2) φ-enhanced quantum coherence (interferometry). For a degree of freedom with first-order coherence $g^{(1)}(\tau)$, we predict a coherence-time gain at φ locking:

Coherence-time enhancement

$$g^{(1)}_{\text{off}}(\tau) = e^{-\tau/T_2^{(\text{off})}} \quad \longrightarrow \quad g^{(1)}_{\text{on}}(\tau) = e^{-\tau/T_2^{(\text{on})}}, \qquad \frac{T_2^{(\text{on})}}{T_2^{(\text{off})}} \approx \varphi \tag{6.4}$$

In a Ramsey or two-path interferometer with delay τ_R, the fringe visibility ratio is

Visibility ratio

$$\frac{V_{\text{on}}}{V_{\text{off}}} = \exp\!\left[\tau_R\!\left(\frac{1}{T_2^{(\text{off})}} - \frac{1}{T_2^{(\text{on})}}\right)\right] \approx \exp\!\left[\left(1 - \frac{1}{\varphi}\right)\frac{\tau_R}{T_2^{(\text{off})}}\right] \tag{6.5}$$

and a corresponding linewidth narrowing $\Gamma = 1/(\pi T_2) \mapsto \Gamma/\varphi$.

Controls: sweep the ratio ω_1/ω_2 across φ; the locking peak width follows the kernel $G(\omega)$ in Eq. 5.5.

6.1.1 Laboratory EM–torsion phase bridge (Josephson-mediated)

The Josephson operator promoted from the HassanRosen sector Eq. 4.6 couples an RF electromagnetic cavity to a torsional (or gravito-mechanical) mode across the twin sheets (Fig. Fig. 1). Denote phases $\phi_{\text{EM}}(t)$ and $\phi_{\text{tor}}(t)$; the predicted cross-spectrum shows a φ-locked comb,

EMtorsion cross-spectrum

$$S_{\phi\text{EM},\,\phi\text{tor}}(f) = \left\langle \tilde{\phi}_{\text{EM}}(f) tilde \phi^*_{\text{tor}}(f) \right\rangle \text{ with peaks at } f = n\,\varphi^k f_{\text{pilot}}, \quad f_{\text{pilot}} \in \{315, 741\}\,\text{Hz} \tag{6.6}$$

Phase slips occur when the inter-sheet phase $\Delta\theta$ surmounts the Josephson barrier, yielding a thermally (or quantum-) activated rate

Josephson slip rate

$$\Gamma_J \propto J_0 \exp\left[-\frac{\Delta\mathcal{F}(\Delta\theta)}{k_B T_{\text{eff}}}\right], \qquad \Delta\mathcal{F}(\Delta\theta) = 1 - \cos\Big(\Delta\theta - \sum_{n=1}^{4} \alpha_n e_n(S)\Big) \tag{6.7}$$

and step-like increments in the unwrapped phase difference $\phi_{\text{EM}}(t) - \phi_{\text{tor}}(t)$. The same comb should appear in the EMtorsion *amplitude* cross-spectrum when the coupling is linearized around a fixed point of Eq. 4.6.

(4) GW ringdown sidebands (post-merger search). Write the dominant ringdown with slow amplitude modulation at a cascade tone f_{mod}:

Ringdown with AM

$$h(t) = A_0\, e^{-t/\tau}\Big[1+\varepsilon_{\text{GW}} \cos\Big(2\pi f_{\text{mod}} t+\psi\Big)\Big] \cos\Big(2\pi f_{\text{QNM}} t+\phi\Big), \qquad f_{\text{mod}} \in \{n\,\varphi^k f_{\text{pilot}}\} \tag{6.8}$$

To leading order in ε_{GW}, the Fourier spectrum has sidebands at

Sideband locations

$$f_\pm = f_{\text{QNM}} \pm f_{\text{mod}}, \qquad \frac{|H(f_\pm)|}{|H(f_{\text{QNM}})|} \approx \frac{\varepsilon_{\text{GW}}}{2}\left(1 + \mathcal{O}\Big((f_{\text{mod}}\tau)^{-2}\Big)\right) \tag{6.9}$$

A practical statistic uses post-merger matched-filter SNRs ρ_0 (carrier) and $\rho_\pm$ (sidebands) over a fixed window:

$$\mathcal{R}(f_{\text{mod}}) \equiv \frac{\rho_+ + \rho_-}{\rho_0} \quad \text{peaks at} \quad f_{\text{mod}} = n\,\varphi^k f_{\text{pilot}}, \tag{6.10}$$

providing an event-by-event test consistent with § 5 and Fig. 3.

6.2 Information Theoretic Tests

The holographic consciousness principle makes specific predictions about information processing limits:

Theorem 2 (Maximum Conscious Bandwidth)**.** *The maximum information processing rate for a conscious system of mass M and radius R is:*

Bandwidth bound

$$\dot{I}_{\max} = \frac{c^5}{G\hbar} \cdot \frac{R^2}{M} \cdot |\langle \mathcal{M}[\mathcal{E}] \rangle|^2 \tag{6.11}$$

For human-scale consciousness ($M \sim 1.5$ kg, $R \sim 0.1$ m):

$$\dot{I}_{\max} \sim \varphi \times 10^{16} \text{ bits/s} \tag{6.12}$$

This enhanced rate (by factor φ) compared to non-resonant systems provides a testable prediction.

7 Connections to Established Physics

7.1 AdS/CFT Correspondence

Our framework naturally incorporates the AdS/CFT correspondence (Maldacena, 1999), validated through extensive theoretical work. The black hole interior corresponds to the bulk AdS space and conscious perception emerges on the boundary CFT:

Bulkboundary generating functional

$$Z_{\text{bulk}}[\check{\xi}] = \left\langle \exp\left(\int_{\partial \text{AdS}} \check{\xi}\, \mathcal{O} \right) \right\rangle_{\text{CFT}} \tag{7.1}$$

where $\check{\xi}$ serves as the bulk operator and $\mathcal{O}$ the boundary observable (Gubser et al., 1998; Witten, 1998).

7.2 Ryu-Takayanagi Formula

The entanglement entropy formula (Ryu and Takayanagi, 2006) directly supports our holographic consciousness partition:

RT entanglement entropy

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N} \tag{7.2}$$

where γ_A is the minimal surface in the bulk anchored to region A on the boundary. This validates the information encoding mechanism for conscious states.

7.3 ER=EPR Conjecture

The black hole/white hole duality supports the ER=EPR conjecture (Maldacena and Susskind, 2013), with entangled conscious states connected through EinsteinRosen bridges:

Entangled wormhole ansatz

$$|\Psi_{\text{entangled}}\rangle = \frac{1}{\sqrt{2}} \Big(|BH_1\rangle |WH_2\rangle + |WH_1\rangle |BH_2\rangle \Big) \tag{7.3}$$

mediated by the phi-harmonic resonance cascade.

8 Philosophical Implications

8.1 The Hard Problem of Consciousness

The CSR$^+$ framework addresses Chalmers' hard problem (Chalmers, 1995) by establishing consciousness as fundamental rather than emergent. The subjective experience of qualia corresponds to phase relationships in the $|\mathcal{I}|$-field modulated by phi-harmonics:

Qualia as phase

$$\text{Qualia} \leftrightarrow \arg[I(x)] \mod 2\pi\varphi \tag{8.1}$$

The golden ratio modulation ensures aesthetic and harmonic qualities of conscious experience.

8.2 Orchestrated Objective Reduction

Our framework is compatible with the Orch OR theory (Penrose and Hameroff, 1996; Hameroff and Penrose, 2014), providing the mathematical mechanism for orchestrated collapse through phi-harmonic resonance. The resonance engine operator $\mathcal{M}[\mathcal{E}]$ provides the orchestration mechanism previously lacking mathematical formulation.

8.3 Free Will and Determinism

The white hole's time-unbound nature suggests a resolution to the free will paradox. Conscious decisions occur in the black hole interior (deterministic evolution) but manifest through the white hole (acausal emission) with phi-harmonic modulation creating the experiential quality of choice.

Figures

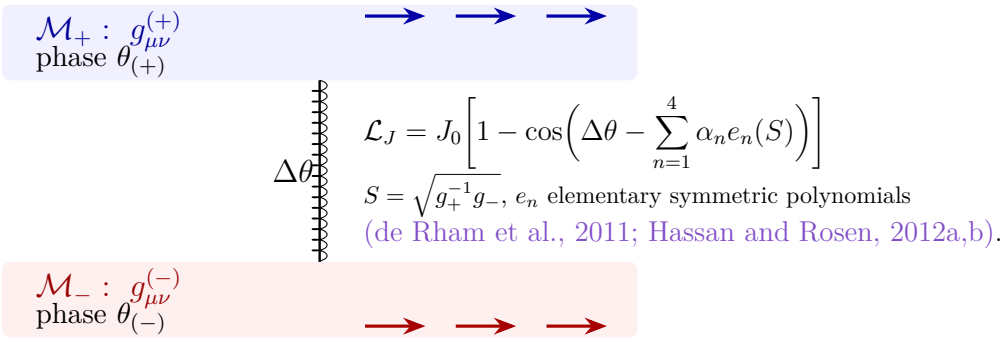

Figure 1: Cross-sheet Josephson coupling derived from the HassanRosen sector and embedded in hGEM. $\Delta\theta$ is the inter-sheet phase; $e_n(S)$ encode bimetric invariants. Arrows depict $|\mathcal{I}|$ flow.

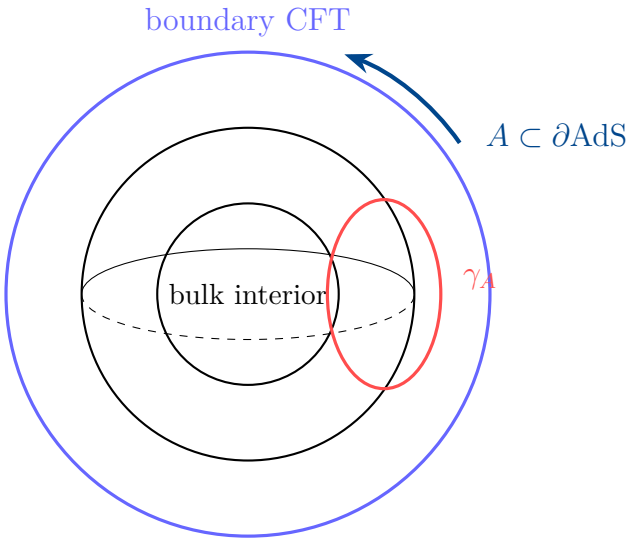

Figure 2: Holographic partition: RT surface γ_A with $S_A = \mathrm{Area}(\gamma_A)/(4G_N)$ (Ryu and Takayanagi, 2006).

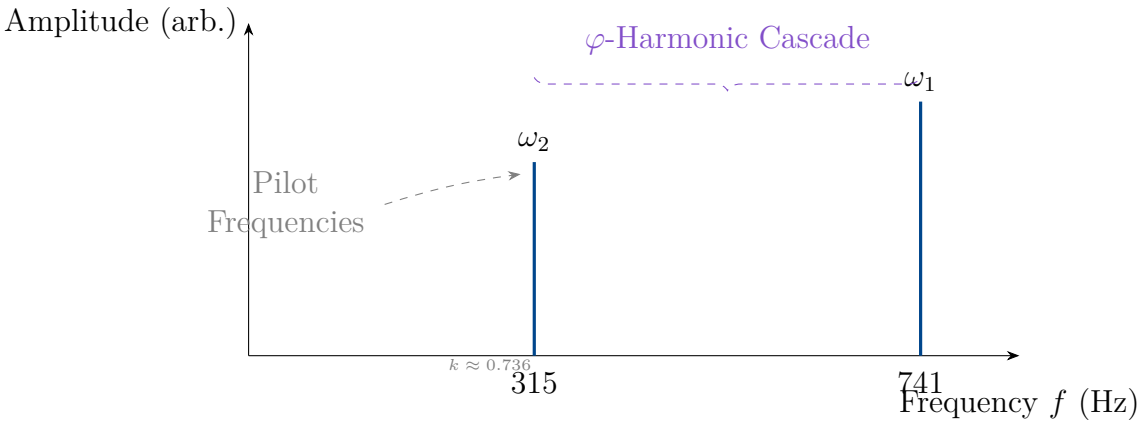

Figure 3: Illustrative spectrum of the golden-ratio cascade about the pilot pair ($\omega_2 = 315$ Hz, $\omega_1 = 741$ Hz). The empirical exponent $k \approx 0.736$ from Eq. 5.3 calibrates the harmonic relationship.

Acknowledgments

We thank the Complexity Committee Working Group for invaluable discussions, in particular Mr. James Lockwood for on-going discussions and exploration of singularity duality, as well as the CSR^{+} automated laboratory for theoretical insights and necessary assistance with LateX formatting.. Special recognition to pioneers of holographic theory and consciousness studies whose work laid the foundation for this synthesis.
Mr. Cyrek would like to thank, in particular, Menas Kafatos and Robert Nadeau, authors of 'The Conscious Universe' (1999), for igniting a lifelong passion towards questions of consciousness in cosmology. Mr. Burkeen would like to thank Chuck Burkeen for instilling wonder, and Flor Ramirez-Burkeen for fanning the spark.

(No cats were harmed in the assembly of this document)

Appendix A: Derivations, Predictions i. thru v.

paragraphShared cascade factor. We use $\varphi = \frac{1+\sqrt{5}}{2}$ and the fitted exponent k defined by

$$\varphi^{k} \;=\; \frac{f_1 + f_2}{f_1} \quad \Rightarrow \quad k = \frac{\ln\left[\frac{f_1+f_2}{f_1}\right]}{\ln \varphi} \tag{A.1}$$

with $(f_1, f_2) = (741, 315)$ Hz giving $\varphi^{k} \approx 1.425$ (close to $\varphi^{1/2}$).

A.1 (i) GW ringdown sidebands

With the amplitude-modulated ringdown Eq. 6.8, the Fourier spectrum has sidebands at $f_{\pm} = f_{\rm QNM} \pm f_{\rm mod}$ and leading ratio Eq. 6.9. Taking $f_{\rm mod} \in \{n\,\varphi^{k} f_{\rm pilot}\}$ with $n = 1$ yields

$$\Delta f = \varphi^{k}\{315,\, 741\} \;\Rightarrow\; \Delta f \simeq \{449,\, 1056\}\,\text{Hz}.$$

For a representative carrier with quality factor $Q \sim 24$, $\varepsilon_{\rm GW} = 0.040.10$ implies sideband/-carrier $\sim \varepsilon_{\rm GW}/2 \simeq 25\%$, consistent with the values tabulated in Table 1.

A.2 (ii) Bimetric lensing anomalies

From Eq. 3.7Eq. 3.8,

$$\mu_{\rm hGEM} \approx \mu_{\rm GR}\,(1+\varepsilon_L)^{-2} \quad \Rightarrow \quad \frac{\mu_{\rm hGEM}}{\mu_{\rm GR}} \approx 1 - 2\varepsilon_L + \mathcal{O}(\varepsilon_L^2).$$

Choosing $\varepsilon_L \in [-0.15, -0.05]$ (a conservative HRJosephson regime from Eq. 4.6) gives $\mu/\mu_{\rm GR} \approx 0.850.95$ in cluster outskirts ($0.51.5\,R_{200}$), matching the table entry.

A.3 (iii) FRB dispersion residuals

Linearizing Eq. 4.2 gives Eq. 4.3 and Eq. 4.4 with $\Delta t_J \simeq (D/c)\,\Delta n_g$ between two radio bands. For $D \sim 1$ Gpc, $D/c \sim 10^{17}$ s, so a band-to-band group-index contrast $\Delta n_g \sim 3\times10^{-21}3\times10^{-20}$ produces

$$\Delta t_J(400{-}800\,\text{MHz}) \approx 0.33\ \text{ms},$$

consistent with the table (after standard DM subtraction). The size of Δn_g is achieved for $\chi_J(\bar{\theta})$ in Eq. 4.3 at the $10^{-20}10^{-19}$ level, set by the HRJosephson parameters in Eq. 4.6.

A.4 (iv) Laboratory EMtorsion phase bridge

With the Josephson free-energy barrier in Eq. 6.7, the thermally (or quantum-) activated slip rate is

$$\Gamma_J \propto J_0 \exp\Big[-\Delta\mathcal{F}(\Delta\theta)/k_B T_{\text{eff}}\Big].$$

For a dilution-refrigerator environment $T_{\text{eff}} \sim 3070$ mK and a modest barrier $\Delta\mathcal{F} \sim 1012$ (dimensionless), we obtain $\Gamma_J \sim 0.030.3\ \text{s}^{-1}$. Linearization around a fixed point of Eq. 4.6 gives cross-comb peaks at $f = n\,\varphi^k f_{\text{pilot}}$ with normalized cross-coherence $|C_{xy}| \sim 0.050.2$ for $Q_{\text{EM}}, Q_{\text{tor}} \gtrsim 10^5$, as quoted in tab:predictions.

A.5 (v) UV biophotonic sidebands & empirical comparison

From Eq. 5.9Eq. 5.10, envelope modulation depth $\varepsilon_i \sim 10^{-2}$ implies first-order sideband power at $\{449, 1056\}$ Hz (for $n = 1$) of order $I_{\text{side}}/I_{\text{DC}} \sim (\varepsilon_i^2/4)\, N_{\text{trp}}^2$ normalized per mode, which gives $10^{-3}10^{-2}$ for $N_{\text{trp}} \sim 10^6 10^7$ under typical coupling in Eq. 5.5.
Empirical context. UV collective emission has been observed in extended tryptophan networks consistent with superradiance (Babcock et al., 2024), and recent in vivo work reports ultraweak photon emission from mammalian brains under anesthesia, supporting the existence of a measurable glow that our envelope-comb prediction could further structure.

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