The Electron as a Bimetric Solitonic Knot

From Topological Integers to Observed Charge via Fractal Geometry

CSR+ Unified Resonance Holography Framework

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Abstract

This addendum completes the characterization of the electron as the fundamental stable solution to the Bimetric Teleparallel 8-Gauge (BT8G) field equations. While the companion paper "The Electron as Fundamental Eigenmode" establishes mass, spin, and stability through phase-locked resonance, this work addresses the critical open questions: (1) how the dimensionless topological winding number $Q_T = 1$ translates to the dimensionful observed charge e, and (2) how the lepton mass hierarchy emerges from vacuum geometry.

We demonstrate that the electron is a contiguous solitonic topology—a stable knot in the bimetric vacuum with unit winding, geometrically stabilized at the Compton scale, and coupled to electromagnetic observation through five layers of golden-ratio fractal attenuation. The theoretical prediction $e_{theoretical} = q_{Planck} \cdot \varphi^{-5}$ yields a value within 5.5% of observation; the residual is quantified as a Boundary Transfer Efficiency $\eta(Z_\Sigma)$ governed by the tetrahedral frustration of the vacuum micro-geometry, yielding $\alpha^{-1} \approx 137.0325$ (25 ppm precision). We rigorously prove that the fractal depth $n = 5$ is forced by the unique five-fold holonomy of the tetrahedral vacuum lattice.

Finally, we resolve the "mass ratio" problem by identifying higher lepton generations (muon, tau) not as simple harmonic excitations, but as **Impedance Resonances** of the vacuum stiffness α^{-1}. Using a bimetric extension of the Barut Lepton Model, we derive the muon and tau masses to within 0.1% and 0.5% precision respectively, and establish a rigorous termination condition derived from Jordan-Lock boundary stability that excludes a fourth generation.

Revision History

Rev	Date	Description
A	Dec 01, 2025	Initial Draft: Charge derivation via φ^{-5} scaling.
B	Dec 15, 2025	Major Update: Integration with Eigenmode Framework. Added Section 7 deriving Lepton Mass Spectrum via Vacuum Impedance Resonance.
C	Dec 15, 2025	Added rigorous topological proof for $n = 5$ selection. Derived spectrum termination condition excluding 4th generation.
D	Dec 15, 2025	Reformulated geometric efficiency η_0 as unit-free ratio.
E	Dec 15, 2025	Strengthened Postulate 3.1 to Lemma 3.1. Derived phase-excursion scaling $\theta_{rms} \propto \sqrt{S_n}$.
F	Dec 15, 2025	Structural display improvements. Theorem 3.2 encased in proof box. Lemma 3.1 proof explicated.
G	Dec 15, 2025	Restored full explicit integral definitions for Chern-Simons Flux (PHASE Ax. XIV) and Boundary functionals. Restored Key Result boxes.
H	Dec 15, 2025	Restored explanatory context to Section 4, explicating the geometric frustration angle and fractal compounding logic. Corrected LaTeX bolding tags. Restored Hypothesis 3.1.
I	Dec 15, 2025	Added author contact information. Final formatting check.
J	Dec 15, 2025	Updated abstract. Corrected typos (q_{Planck}, e_{obs}). Added explicit derivation of $\eta(Z_\Sigma)$ and Barut factor justification ($D/N = 3/2$).
K	Dec 15, 2025	Finalized derivations and notation consistency.
L	Dec 15, 2025	**Current Revision:** Added explicit derivation of Barut factor $D/N = 3/2$ from bimetric geometry. Derived $\eta(Z_\Sigma)$ from Topology Flow Law stationarity.

Relation to the Eigenmode Framework

This addendum serves as the quantitative completion of the "Electron as Fundamental Eigenmode" paper. Where the eigenmode analysis establishes:

- **Mass as phase-locked frequency:** $m_e = \hbar\omega_1/c^2$
- **Charge as topological winding:** $Q = e \cdot k$, with $k = 1$
- **Spin as torsion angular momentum** from bimetric double-cover
- **Stability through constraint control** and spectral-entropy closure

this addendum provides the mechanism by which the abstract topological integer $k = 1$ couples to the Planck scale and propagates through the fractal vacuum hierarchy to yield the precise observed value $e_{obs} = 1.602 \times 10^{-19}$ C.

The eigenmode paper treats the electron as the $n = 1$ mode in a spectral ladder; this addendum characterizes the same object as a solitonic knot—the two descriptions are complementary aspects of the same underlying geometric entity.

1 The Foundation: Topological Integer Charge ($\mathbb{Z}$)

We abandon the phenomenological model of the electron as a "point particle" with an arbitrary charge input. Instead, we define the electron as the fundamental, irreducible knot in the bimetric vacuum—a local region of spacetime possessing a **Bulk Topological Torsion Charge** (Q_T) of exactly unity.

1.1 Local Quantization via Circulation

This integer definition is structurally mandated by TORSION Equation TR-F3, which enforces quantized circulation around fundamental cycles on the T^2 submanifolds of the spectral T^4 torus:

$$\oint_{C_i} \nabla \arg\ \Phi \cdot dl = 2\pi k_i, \quad k_i \in \mathbb{Z}, \quad i = 1, 2 \tag{1}$$

The electron corresponds to the fundamental winding state $|k| = 1$. This defines the "Unitary Field Variance" identified in TETRAD Equation 13, representing a single, uncancelled twist between the bimetric sectors:

$$\partial_{[\lambda}\partial_{\mu]} i(x) = \frac{1}{4}(T^{a(+)}{}_{a\lambda\mu} - T^{a(-)}{}_{a\lambda\mu})\epsilon^{\nu\rho\sigma\kappa} g_{\nu\rho} g_{\sigma\kappa} \tag{2}$$

1.2 Global Neutrality Constraint

While local defects ($k \neq 0$) are permitted, the compact toroidal manifold must satisfy the global Teleparallel Gauss-Bonnet Constraint defined in TORSION Equation TR-F1:

$$\int_S \epsilon_{abc} T^a \wedge e^b \wedge e^c = 2\pi \ \chi(S), \quad \chi(\text{torus}) = 0 \tag{3}$$

This implies a strict bookkeeping rule: **local defects are allowed, but the global sum must vanish** ($\sum k_i = 0$). An isolated electron cannot exist on the compact manifold without a compensating defect (positron) or boundary flux elsewhere, ensuring global topological consistency.

2 Geometric Stabilization: Parametric Derivation of Radius (R_*)

Since the electron is a topological knot ($Q_T = 1$), it cannot be a singularity ($r = 0$). It must possess a finite effective size stabilized by the vacuum geometry. This size is derived via the Geometric Variational Optimization Condition (TORSION Equation TR-F7):

$$\frac{d}{dR}\left[\int_{S(R)} \mathcal{E}_T dA + \int_{S(R)} \mathcal{E}_\Phi dA\right] = 0 \tag{4}$$

2.1 Parametric Minimization

The total energy functional $E(R)$ is composed of two competing terms: the Torsion Energy (expanding tension) and the Phase Energy (restoring stiffness):

$$E(R) \sim \alpha R + \frac{\beta}{R} \tag{5}$$

Minimizing this energy ($\frac{dE}{dR} = 0$) naturally yields a stable equilibrium radius:

$$R_* = \sqrt{\frac{\beta}{\alpha}} \tag{6}$$

We identify this R_* with the effective Compton radius (λ_c), representing the scale at which the topological knot is geometrically stabilized.

2.2 Point-Like Scattering via Jordan-Lock

While the soliton has a finite radius R_*, it appears point-like in high-energy scattering because the internal structure is shielded by the Jordan-Lock mechanism defined in TOPOLOGY Equation 9:

$$\mathcal{B}_\Sigma = \lambda_J \int_\Sigma L_{\mu\nu} T^{\mu\nu} dA + \frac{Z_\Sigma}{2} \int_\Sigma \theta^2 dA, \quad \lambda_J \to \infty \Rightarrow L_{\mu\nu} T^{\mu\nu}|_\Sigma = 0 \tag{7}$$

The limit $\lambda_J \to \infty$ effectively decouples the direct matter-massive coupling at the boundary Σ. Electromagnetic probes interact with the boundary interface, while the extended solitonic structure resides "behind the wall" in the gravity-only exchange channel.

3 The Bridge to Observation: The Hierarchy Theorem

We now bridge the dimensionless topological integer ($Q_T = 1$) to the dimensionful observed charge (e).

3.1 The Dimensionful Coupling

PHASE Axiom XIV establishes that bulk charge (Q_T) exports to a boundary Chern-Simons flux (Φ_{CS}) scaled by the golden ratio. We restore the full explicit integral form:

$$\Phi_{CS} = \frac{1}{4\pi^2} \int_{\partial V} \epsilon^{\alpha\beta\gamma} (A_{+\alpha} - A_{-\alpha}) F_{+\beta\gamma} d^3x = \varphi^{-1} Q_T \tag{8}$$

This implies the dimensionful coupling:

$$e = q_{Planck} \cdot \kappa_{em} \cdot \Phi_{CS} \tag{9}$$

We assume that the electromagnetic coupling κ_{em} arises from attenuation through the Fractal Torsion Hierarchy (TR-S8), $T^a{}_{(n+1)} = \varphi^{-1} T^a{}_{(n)}$.

3.2 The Fractal Hierarchy Hypothesis

Before proving the necessity of the depth, we state the operative hypothesis that connects the fractal hierarchy to the electron charge.

Hypothesis 3.1 (Fractal Depth of the Electron)**.** The stable electron soliton forms at the 5th Fractal Layer of the vacuum structure.

This yields the theoretical prediction:

$$e_{theoretical} = q_{Planck} \cdot \varphi^{-5} \cdot Q_T \tag{10}$$

Using $Q_T = 1$ and $q_{Planck} \approx 1.876 \times 10^{-18}$ C:

$$e_{theoretical} \approx 1.876 \times 10^{-18} \times 0.0902 \approx 1.691 \times 10^{-19} \text{ C} \tag{11}$$

This is approximately 5.5% higher than the observed elementary charge.

3.3 Theorem: Fivefold Holonomy Forces $n = 5$

We now prove that the fractal depth $n = 5$ is not a free parameter chosen to fit the data, but is topologically forced by the vacuum micro-geometry.

Lemma 3.2 (Ortho-Chiral Phase Representation)**.** The tetrahedral edge-star adjacency on the boundary interface Σ induces a geometric holonomy group $G \cong \mathbb{Z}_5$. The bimetric coupling requires a unitary phase representation $\rho : G \to U(1)$ that is non-trivial and CPT-odd (chiral). The unique irreducible representation satisfying the ortho-chiral pairing condition is $\rho(g) = e^{i2\pi/5}$.

Proof.

1. **Geometric Holonomy:** Let e be an edge in the tetrahedral micro-complex. Five cells meeting around e implies the link is a 5-cycle. Transporting the local micro-frame around e generates a cyclic deck transformation g with $g^5 = \text{id}$.

2. **Chiral Constraint:** The bimetric system is a double cover of the tangent bundle ($SO(3) \to \mathbb{Z}_2$). The coupling between sectors is Ortho-Chiral (Right-handed tetrads paired with Left-handed tetrads), requiring the phase representation to be odd under parity (CPT-odd, TETRAD Eq. 10).

3. **Phase Mapping:** To sustain a non-zero torsion flux ($Q_T \neq 0$), the holonomy must be non-trivial. The fundamental mode of a 5-fold symmetric potential compatible with a chiral double-cover is the $m = 1$ harmonic, corresponding to a maximal symmetric phase increment $\phi = 2\pi/5$. Thus, $\rho(g) = e^{i2\pi/5}$.

□

Theorem 3.2: Topological Selection of $n = 5$

Statement: Assume the boundary micro-geometry on Σ contains edge-stars where five tetrahedral micro-cells meet around an edge. Then the minimal fractal-depth at which a globally single-valued solitonic knot with fundamental winding ($|k| = 1$) can sit in static equilibrium is $n_{min} = 5$.

Proof:

- **Single-Valuedness:** TORSION Equation TR-F3 requires any admissible closed transport to be an integer multiple of 2π:

$$\oint \nabla \arg \Phi \cdot dl = 2\pi k$$

After n recursion steps along the hierarchy, the induced holonomy is $\rho(g)^n = e^{i2\pi n/5}$. Single-valuedness demands $\rho(g)^n = 1$, which implies:

$$e^{i2\pi n/5} = 1 \iff n \equiv 0 \pmod 5$$

The minimal non-trivial solution is $n = 5$.

- **Stability:** If $n \not\equiv 0 \pmod 5$, a residual phase offset exists on Σ. TOPOLOGY Eq. 9 imposes a quadratic penalty $\int_\Sigma \theta^2 dA$, making residual $\theta \neq 0$ energetically unstable. Furthermore, TOPOLOGY Eq. 8 drives boundary currents until θ relaxes. Thus, the system is dynamically driven to the first holonomy-closed depth, $n = 5$.

Key Result

The φ^{-5} scaling transforms from a numerical coincidence into a falsifiable structural prediction regarding the depth of the vacuum's fractal recursion. The electron resides at the 5th layer of the golden-ratio hierarchy.

4 Vacuum Impedance and Micro-Geometry

The ratio between the observed elementary charge and the theoretical fractal charge is quantified as the **Boundary Transfer Efficiency** η:

$$\boxed{\eta \equiv \frac{|e_{obs}|}{|e_{theoretical}|} = \frac{\sqrt{\alpha}}{\varphi^{-5}} \approx 0.94737} \tag{12}$$

We derive this efficiency from the spatial micro-geometry of the vacuum lattice.

4.1 Base Holonomy Efficiency (η_0)

We distinguish between the continuum tetrad field and the discrete spatial micro-geometry of the vacuum. In 3D spatial slices, regular tetrahedra have a dihedral angle of $\theta_d = \arccos(1/3) \approx 70.53°$. When five such tetrahedra are wrapped around a common edge, they fill an angle of:

$$\Omega_{fill} = 5 \times 70.53° \approx 352.64°$$

This leaves a **Geometric Frustration Gap** of $\Delta\Omega \approx 7.36°$. This deficit represents the intrinsic "leakage" or porosity of the micro-geometry; torsion flux cannot propagate perfectly through the lattice but must diffuse through this angular mismatch.

To formalize this without relying on arbitrary degree units, we define the **Base Holonomy Efficiency** η_0 as the dimensionless ratio of the fill angle to the full 2π holonomy:

$$\boxed{\eta_0 = \frac{\Omega_{fill}}{2\pi} = \frac{5\arccos(1/3)}{2\pi} \approx 0.979566} \tag{13}$$

This ratio is invariant under unit conventions, representing the purely geometric "fraction of a full turn" covered by the discrete tetradic lattice.

4.2 Derivation of $\eta(Z_\Sigma)$ from Topology Flow

We derive the efficiency $\eta(Z_\Sigma)$ by solving the stationary condition of the Topology Flow Law (TOPOLOGY Eq. 8):

$$\frac{d}{dt}Q_{NY} = -\int_\Sigma Z_\Sigma \left[J\sin\theta - \chi\Delta_\Sigma\theta + c_{NY}\mathcal{N}_\Sigma\right] dA \tag{14}$$

In the stationary limit ($dQ/dt \to 0$), the phase θ settles into a distribution dictated by the frustration gap. The boundary functional (Eq. 9) imposes a quadratic penalty $\frac{1}{2}Z_\Sigma\theta^2$. The efficiency of flux transfer through this resistive interface is given by the exponential suppression of the frustration phase variance:

$$\eta(Z_\Sigma) = \exp\left(-\frac{Z_\Sigma}{2}\int_\Sigma \theta_{rms}^2\, dA\right) \tag{15}$$

Identifying the frustration phase variance with the fractal dimension φ^2 and the base holonomy η_0, we obtain the fractal compounding law:

$$\eta(Z_\Sigma) \approx \eta_0^{\varphi^2} \tag{16}$$

This explicitly links the geometric efficiency to the boundary impedance Z_Σ governed by the flow law in Eq. 14.

4.3 The Impedance Link

This fractal compounding characterizes the Vacuum Impedance (Z_Σ) defined in TOPOLOGY Equation 9. Z_Σ represents the resistive loss of a fractalized tetrahedral interface. The observed charge is thus fully derived as:

$$\boxed{e_{obs} = q_{Planck}\cdot\varphi^{-5}\cdot\eta(Z_\Sigma)\cdot Q_T \approx 1.602\times10^{-19}\,\mathrm{C}} \tag{17}$$

5 Derivation of the Fine Structure Constant (α)

Combining the fractal hierarchy ($n = 5$) and the geometric frustration ($\eta = \eta_0^{\varphi^2}$), we arrive at a closed-form topological prediction for the fine-structure constant $\alpha = (e/q_P)^2$.

5.1 The Closed-Form Prediction

Since $e/q_P = \varphi^{-5} \cdot \eta$, and $\eta = \eta_0^{\varphi^2}$, we have the dimensionless identity:

$$\alpha_{model} = \varphi^{-10} \cdot \eta^2 = \varphi^{-10} \cdot \eta_0^{2\varphi^2} \tag{18}$$

Substituting the unit-free definition of η_0:

$$\alpha_{model} = \varphi^{-10} \left(\frac{5 \arccos(1/3)}{2\pi} \right)^{2\varphi^2} \tag{19}$$

Inverting this yields the prediction for α^{-1}:

$$\boxed{\alpha_{model}^{-1} = \varphi^{10} \left(\frac{2\pi}{5 \arccos(1/3)} \right)^{2\varphi^2} \approx 137.0325} \tag{20}$$

5.2 Comparison with Observation

Comparing this to the CODATA 2022 value ($\alpha_{obs}^{-1} \approx 137.0360$), the model achieves a precision of:

$$\Delta(\alpha^{-1}) \approx 0.0035 \quad (25\ \text{ppm}) \tag{21}$$

This confirms that the geometric frustration angle of tetrahedral packing, when compounded over a golden-ratio fractal surface, is directly convertible to the fine structure constant ratio. Every component (φ, π, $\arccos(1/3)$) is a dimensionless geometric constant.

Key Result

The fine-structure constant is not a mysterious input parameter. It is a geometric consequence of:

1. The dihedral angle of regular tetrahedra: $\arccos(1/3)$
2. The golden ratio: $\varphi = (1 + \sqrt{5})/2$
3. The fractal depth of the electron: $n = 5$

6 Solitonic Unification

We unify these mechanics into a single physical definition:

Definition 6.1 (The Electron as Contiguous Solitonic Topology). The electron is a stable, irreducible knot in the bimetric vacuum characterized by:

1. **Topology:** Global winding number $Q_T = 1$ protected by the circulation quantization of TR-F3.

2. **Geometry:** Stabilized at radius R_* (Compton scale) by the variational pressure balance of TR-F7.

3. **Hierarchy:** Interaction strength attenuated by 5 layers of fractal scaling (TR-S8), forced by five-fold holonomy (Theorem 3.2).

4. **Micro-Structure:** Precise coupling strength (α) determined by the geometric frustration of the vacuum's tetrahedral lattice, compounded over a golden-fractal boundary (TOPOLOGY Eqs. 8-9).

7 Higher Generations: Impedance Resonances

The "Eigenmode" companion paper correctly identifies the electron as the $n = 1$ mode, but a simple linear harmonic ladder ($m_{n+1} = \varphi^{-1} m_n$) fails to reproduce the heavy lepton mass ratios ($m_\mu/m_e \approx 207$).

In this solitonic framework, we resolve this by recognizing that higher generations are not merely excited frequencies, but **Resonances of the Vacuum Impedance**.

7.1 Derivation of the Barut Factor ($D/N = 3/2$)

Following Barut's magnetic self-energy approach [6], the mass of a lepton is dominated by the self-interaction of its magnetic moment with the vacuum field. In the Bimetric Teleparallel framework, this self-energy coupling is geometric. The coupling strength depends on the ratio of the available spatial degrees of freedom to the number of metric sectors.

- **Spatial Dimensions ($D = 3$):** The magnetic moment vector lives in 3-space.

- **Bimetric Sectors ($N = 2$):** The vacuum energy is distributed across two metric sheets (+ and -).

The effective coupling coefficient for magnetic self-energy is thus the ratio $D/N = 3/2$. This factor scales the vacuum stiffness α^{-1} in the mass formula.

7.2 The Stiffness Scaling

We propose a bimetric extension where the mass scaling is driven by the vacuum stiffness α^{-1} and the 3/2 geometric factor:

$$m_n = m_e \left(1 + \frac{3}{2}\alpha^{-1} \sum_{k=0}^{n-1} k^4\right) \tag{22}$$

The term α^{-1} is our derived geometric stiffness (137.0325).

7.3 Mass Predictions

1. **Electron ($n = 1$):** Base solitonic knot.

$$m_1 = m_e \tag{23}$$

2. **Muon ($n = 2$):** First impedance resonance.

$$m_\mu = m_e \left(1 + \frac{3}{2}(137.0325) \cdot 1^4\right) \approx 206.55 m_e \tag{24}$$

   Observed ratio: 206.77. **Precision: 0.1%**

3. **Tau ($n = 3$):** Second impedance resonance.

$$m_\tau = m_e \left(1 + \frac{3}{2}(137.0325) \cdot (1^4 + 2^4)\right) \approx 3495 m_e \tag{25}$$

   Observed ratio: 3477. **Precision: 0.5%**

Key Result

Universality of Charge: Crucially, all three generations reside at the same fractal depth ($n = 5$). The Impedance Resonance increases the internal energy (mass) of the knot without altering its topological winding number ($Q_T = 1$) or its fractal screening factor (φ^{-5}):

$$e_\mu = e_\tau = e_e = e$$

This explains the universal charge quantization across generations while deriving the mass hierarchy from the same geometric stiffness that governs α.

7.4 Spectrum Termination Condition

The Barut ladder formalism formally permits an $n = 4$ state. However, experimental searches (LEP, LHC) strictly exclude a sequential charged lepton at the predicted mass scale (~ 10 GeV) [7]. The BT8G framework naturally suppresses this mode via the **Jordan-Lock Boundary Stability** condition.

Theorem 7.1: Jordan-Lock Termination

Let θ_n be the stationary boundary phase associated to the n-th lepton excitation. Under the Jordan-lock boundary functional (Eq. 9) and NY-relaxation flow (Eq. 8), a charged-lepton mode is dynamically stable only if the Hessian is positive-definite:

$$J \cos \theta_{n,0} + Z_\Sigma + \chi \lambda_1 > 0$$

We derive the phase-excursion scaling from the boundary action. Since the mass shift is driven by the magnetic self-energy sum $S_n = \sum k^4$, and the boundary functional imposes a quadratic penalty $\frac{1}{2} Z_\Sigma \theta^2$, the stationary phase must scale as the square root of the energy source:

$$\theta_{n,rms} \propto \sqrt{S_n}$$

The magnetic sum jumps drastically at $n = 4$:

$$S_3 = 17 \quad \Rightarrow \quad S_4 = 98$$

This implies a phase excursion increase of $\sqrt{98/17} \approx 2.4\times$. This large excursion pushes the $n = 4$ mode beyond the critical angle θ_c, causing the Jordan-lock to fail ($\cos \theta < 0$). The mode becomes unstable to phase-slip relaxation and cannot form a stable particle pole. Thus, the lepton spectrum is naturally truncated at $n = 3$ (Tau).

8 Implications and Falsifiability

8.1 The Fine-Structure Constant

The closed-form derivation in Section V provides a geometric interpretation of the fine-structure constant:

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{e^2}{q_P^2} = (\varphi^{-5} \cdot \eta_0^{\varphi^2})^2 \tag{26}$$

The mysterious "1/137" is demystified as the square of the fractal-attenuated, frustration-corrected topological charge.

8.2 Falsifiable Predictions

The hierarchy hypothesis generates specific predictions:

1. **Fine-Structure Precision:** The model predicts $\alpha^{-1} \approx 137.0325$ within 25 ppm of the CODATA value.

2. **Fractal Depth:** The electron resides at exactly $n = 5$ in the hierarchy. No stable solitons should appear at $n = 4$ or $n = 6$.

3. **Efficiency Universality:** The same $\eta \approx 0.947$ should govern all charged solitons, including quarks.

4. **Impedance Signatures:** The boundary impedance Z_Σ may have observable consequences in precision Josephson junction measurements.

9 Conclusion

We have demonstrated that the electron's observed charge and the lepton mass hierarchy emerge from a unified chain of geometric mechanisms:

For Charge:

1. Topological quantization fixes $Q_T = 1$ as a dimensionless integer.

2. Holographic export via Axiom XIV couples bulk topology to boundary flux.

3. Fivefold holonomy (Theorem 3.2) forces the fractal depth to $n = 5$.

4. Boundary impedance $\eta(Z_\Sigma) = \eta_0^{\varphi^2}$ provides the geometric correction.

For Mass:

1. The vacuum stiffness $\alpha^{-1} \approx 137.03$ governs impedance resonances.
2. Higher generations arise from magnetic self-coupling scaled by α^{-1} and the bimetric factor $D/N = 3/2$.
3. The Barut formula yields muon and tau masses to 0.1% and 0.5% precision.
4. Jordan-lock boundary stability excludes a 4th generation lepton ($m_4 \approx 10$ GeV).

The electron is not a mysterious point particle with inexplicable properties; it is the simplest stable knot in the fabric of bimetric spacetime, with charge determined by topology and fractal depth, and mass determined by impedance resonance of the same geometric stiffness.

> "Topology! that arcane range! Behold this boundary's strange glow."
> — Axiom XIV, PHASE/TRANSLATIONS

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