

# **Topology Equations**

Complexity Committee Position Paper

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# Purpose

We distill the bimetric teleparallel (BT8G) and CSR+ stack into a compact *Topology* layer: six foundational equations (bulk topology, locking, and positivity) and six extended equations (boundary flow, screening, and cosmology). Each is stated with a brief justification and a concrete application.

## I. Notation

- Tetrads  $e^a_\mu$ , torsion  $T^\lambda_{\mu\nu}$  (teleparallel)
- Phase  $\theta$  (Josephson); dual sectors  $(+/-)$
- Superpotential  $S^{\rho\mu\nu}$
- Nieh-Yan denoted as NY
- $\Sigma$  a codimension-1 interface
- $\Delta_\Sigma$  the Laplace–Beltrami on  $\Sigma$
- $\phi = (1 + \sqrt{5})/2$  (golden ratio)

## II. Foundational Topology Equations

$$\int_S \epsilon_{abc} T^a \wedge e^b \wedge e^c = 2\pi \chi(S), \quad \chi(\text{torus}) = 0. \quad (1)$$

**Justification:** Teleparallel Gauss–Bonnet reduces to a boundary invariant; torus tiles enforce zero net torsion flux.

**Application:** Baseline flux neutrality that seeds CSR+ phase-lock and the toroidal lattice.

$$\Phi(\theta, \varphi) = \Phi_0 e^{i(n\theta+m\varphi)}, \quad n, m \in \mathbb{Z}. \quad (2)$$

**Justification:** Harmonics on  $T^2$  form an integer lattice of normal modes.

**Application:** Discrete banding for the 741/315 Hz cascade and eigenmode indexing.

$$\oint_{C_i} \nabla \arg \Phi \cdot d\ell = 2\pi k_i, \quad k_i \in \mathbb{Z}. \quad (3)$$

**Justification:** Single-valuedness on  $T^2$  protects integer phase slips.

**Application:** Defect bookkeeping and phase-slip energetics across tiles.

$$\mathcal{L}_T = \frac{\kappa}{2} T^\lambda_{\mu\nu} T_\lambda^{\mu\nu} \Rightarrow \mathcal{H}_T \geq 0. \quad (4)$$

**Justification:** TEGR torsion quadratic is positive-definite with Weitzenböck connection.

**Application:** Bounded Hamiltonian enabling stable phase damping and constraint control.

$$T^{a(+)}_{\mu\nu} - T^{a(-)}_{\mu\nu} = 2i\partial_{[\mu}\partial_{\nu]}\theta. \quad (5)$$

**Justification:** Variation of bimetric tetrads related by a local phase rotation.

**Application:** Maps phase gradients to torsion stress—the geometry/phase converter.

$$de^{a(+)} = \omega^a{}_{b(+)} \wedge e^{b(+)} + i\Omega^a{}_b \wedge e^{b(-)}. \quad (6)$$

**Justification:** Extends the teleparallel connection to a bimetric mediator  $\Omega$  respecting locality.

**Application:** Gravity-only exchange channel behind the Jordan barrier.

### III. Extended Topology Equations

$$n_\rho(S^{(+)\rho\mu\nu} - S^{(-)\rho\mu\nu})t_\mu s_\nu = J \sin \theta - \chi \Delta_\Sigma \theta + c_{\text{NY}} \mathcal{N}_\Sigma. \quad (7)$$

**Justification:** Pillbox integration of the teleparallel divergence plus Josephson and NY sources.

**Application:** Tunes interface impedance to pass or pin phase; sets exchange across  $\Sigma$ .

$$\mu_T^{(+)} = \mu_T^{(-)}, \quad \mu_T \equiv \frac{\partial H}{\partial Q_{\text{NY}}}, \quad \frac{d}{dt}(Q_{\text{NY}}^{(+)} - Q_{\text{NY}}^{(-)}) = - \int_\Sigma [J \sin \theta - \chi \Delta_\Sigma \theta + c_{\text{NY}} \mathcal{N}_\Sigma] dA. \quad (8)$$

**Justification:** Quasilocal Hamiltonian  $H$  with NY charge  $Q_{\text{NY}}$  imposes chemical-potential equality at equilibrium.

**Application:** Tracks how bulk topology relaxes by boundary currents.

$$\mathcal{B}_\Sigma = \lambda_J \int_\Sigma L_{\mu\nu} T^{\mu\nu} dA + \frac{Z_\Sigma}{2} \int_\Sigma \theta^2 dA, \quad \lambda_J \rightarrow \infty \Rightarrow L_{\mu\nu} T^{\mu\nu}|_\Sigma = 0. \quad (9)$$

**Justification:** Boundary constraint enforces no direct matter–massive coupling;  $Z_\Sigma$  sets phase impedance.

**Application:** Guarantees GR/TEGR locally while allowing gravity-only exchange.

$$r_\star \sim \left( \frac{GM}{m_g^2} \right)^{1/3} \Xi_{\text{TP}}, \quad \frac{d\alpha_{\text{eff}}(r)}{d \ln r} = -p \left( \frac{r_\star}{r} \right)^p \alpha_{\text{eff}}(r), \quad p \geq 1. \quad (10)$$

**Justification:** Balance massive-branch stiffness with torsion nonlinearities gives a Vainshtein-like radius; radial RG encodes screening.

**Application:** PPN safety (inside  $r_\star$ ) with small, asymptotic Yukawa tails.

$$G_{\text{eff}}(k, a) = G \left[ 1 + \alpha_{\text{eff}}(a) \frac{k^2}{k^2 + a^2 m_g^2} \right], \quad \frac{d\alpha_{\text{eff}}}{d \ln a} = \beta_\alpha(\chi, J, c_{\text{NY}}; Z_\Sigma). \quad (11)$$

**Justification:** Integrate out  $\Sigma$  to get a Yukawa-modified Poisson operator with slow running from interface data.

**Application:** Predicts mild, testable scale-dependence in  $f\sigma_8$  and lensing.

$$\int_{\Sigma} (|\nabla_{\Sigma}\theta|^2 + m_{\Sigma}^2 \theta^2) dA \geq c_{\text{NY}} \int_{\Sigma} \theta \mathcal{N}_{\Sigma} dA, \quad m_{\Sigma}^2 \equiv \frac{J}{\chi}, \quad \xi_{\Sigma} = m_{\Sigma}^{-1}. \quad (12)$$

**Justification:** Complete-the-square/Cauchy–Schwarz on the boundary functional with NY bias.

**Application:** Sets coherence patches and stability criteria for static equilibrium.

## IV. Integration with Other Components

### Into TETRAD

TE-F5 and TE-F6 are the bridge: the phase  $\theta$  produces a torsion differential and couples via the cross-connection  $\Omega$ ; this is the minimal, local way the two tetrad stacks communicate without touching the matter frame.

### Into TOROID

TE-F1–F4 are the torus backbone: flux neutrality (F1), mode lattice (F2), integer circulation (F3), and a positive torsion reservoir (F4). These feed the CSR+ Hamiltonian and radius-optimization logic.

### Into TRANSLATIONS

The Josephson sector gives the drivers for TE-X1,X2,X6:  $J, \chi$  and the NY bias enter the boundary law, the flow of  $Q_{\text{NY}}$ , and the coherence length.

## V. Observable Signatures

Channel	Signature	Why Topology
Local PPN & pulsars	GR-like, no fifth force	TE-X3 barrier + TE-X4 screening
GWs	Tiny dispersion/leakage	Massive branch exists but decoupled from matter
LSS & lensing	Mild $k$ -dependent $G_{\text{eff}}$	TE-X5 from boundary running
Lab cascades	Golden-ratio damping/coherence	TE-F4 + Josephson $\Rightarrow$ TE-X6

## VI. Minimal Narrative

Choose the matter geometry first (Jordan-lock). Let gravity talk to itself behind a boundary that you control. The torus forces flux neutrality; the boundary sets how phase becomes geometry and how topology flows. Local physics stays GR; the sky carries the signal.