1) a) $5n^3+2n^2+3n = O(n^3)$

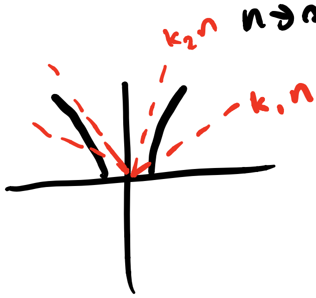
$5n^3+2n^2+3n \le c\,n^3$ for $n \ge n_0$

If $\begin{cases} C=10 \\ n_0=1 \end{cases}$ $\quad 2n^2+3n \le 5n^3 \quad 2+3 \le 5$

$\therefore 5n^3+2n^2+3n = O(n^3)$

b) $\sqrt{7n^2+2n-8} = \Theta(n)$

$$\lim_{n\to\infty} \sqrt{7n^2+2n-8} = \lim_{n\to\infty} \sqrt{7n^2}$$



$f(n) = (7n^2+2n-8)^{\frac{1}{2}}$

$f'(n) = \frac{1}{2}(7n^2+2n-8)^{-\frac{1}{2}}(14n+2)$

$f'(2) = 3.062$

If $\begin{cases} n_0 = 2 \\ k_1 = \frac{1}{2} \\ k_2 = 5 \end{cases}$ $\quad k_1 n \le f(n) \le k_2 n$

$\therefore \sqrt{7n^2+2n-8} = \Theta(n)$

c) Given: $d(n) = O(f(n))$

$e(n) = O(g(n))$

Show: $d(n)e(n) = O(f(n)g(n))$

Suppose:

$a \leq c$ and $b \leq d$.

Then: $ab \leq cd$

$\therefore \forall\, d(n) \leq f(n)$ and $\forall\, e(n) \leq g(n)$,

$$d(n)e(n) \leq f(n)g(n)$$

Thus:

$$\left.\begin{array}{l} d(n) \leq c_1 f(n) \\ e(n) \leq c_2 g(n) \end{array}\right\} \Rightarrow d(n)e(n) \leq c_1 c_2 f(n)g(n)$$

$$c_1 \cdot c_2 = C \Rightarrow d(n)e(n) \leq C(f(n)g(n))$$

$$\therefore d(n)e(n) = O(f(n)g(n))$$

$\blacksquare$

2) a) $\theta(n^2)$ where n is len(lst)

b) $\theta(n)$

c) $\theta(\sqrt{n^2}) = \theta(n)$

d) $n(n/2)(n/2) \Rightarrow$

$4 \to 2 \to 1$

$8 \to 4 \to 2 \to 1$

$16 \to 8 \to 4 \to 2 \to 1$

$\theta(\sqrt{n})$