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Fibonacci #s:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

Prove: $\forall$ integers > 0 can be represented by a sequence of unique Fibonacci numbers.

$F_i \{$Fibonacci #s$\}$ Base Cases:

$1 = 1 \quad 2 = 2 \quad 3 = 3 \quad 4 = 1 + 1 + 2 \quad 5 = 3 + 2$

Assume k exists as sum of Fibonacci #s and all $n \leq k$ also exists...

if $k+1$ is a Fibonacci #, then $k+1$ is immediately true

OTHERWISE:

$\exists i : F_i < k+1 < F_{i+1}$

Consider

$a = k + 1 - F_i$

$\Rightarrow a \leq k$

Given $F_i + a < F_{i+1}$ and $F_{i+1} = F_i + F_{i-1}$

$\therefore a < F_{i-1}$ and can be represented as a sequence of Fibonacci # without using F_i

$\Longrightarrow$ for $k+1$: $k+1 = a + F_i$

Q.E.D.