Assignment 8

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

Question 1

derivative of unevenly spaced data

$$f^{'}(x) = f(x_0)\frac{2x - x_1 - x_2}{(x_0 - x_1)(x_0 - x_2)} + f(x_1)\frac{2x - x_0 - x_2}{(x_1 - x_0)(x_1 - x_2)} + f(x_2)\frac{2x - x_0}{(x_2 - x_0)(x}$$

In [2]:
```
# given data set
z = np.array([3.75 , 1.25 , 0 ])
T = np.array( [10 , 12 , 13.5 ])
k = 3.5e-7
r = 1800
c = 840
```

In [3]:
```
# calculating the derivative at Z = 0
x = 0
dtdz = 0
for i1 in range( 3 ):
    i2 = (i1+1)%3
    i3 = (i1+2)%3
    dtdz += T[i1]*(2*x - z[i1] - z[i2])/((z[i1]-z[i2])*(z[i1]-z[i3]))
```

In [4]:
```
# calculating q( x = 0 )
q = - k * r * c * dtdz
print( f'q(x=0) = {q}')
```

```
q(x=0) = 5.997599999999999
```

Question 2

theory : given $\frac{dy}{dx} = f(x, y(x))$ and $y(x_0) = y_0$ we have
$y(x_0 + h) = y_0 + h * (f(x_0, y_0))$

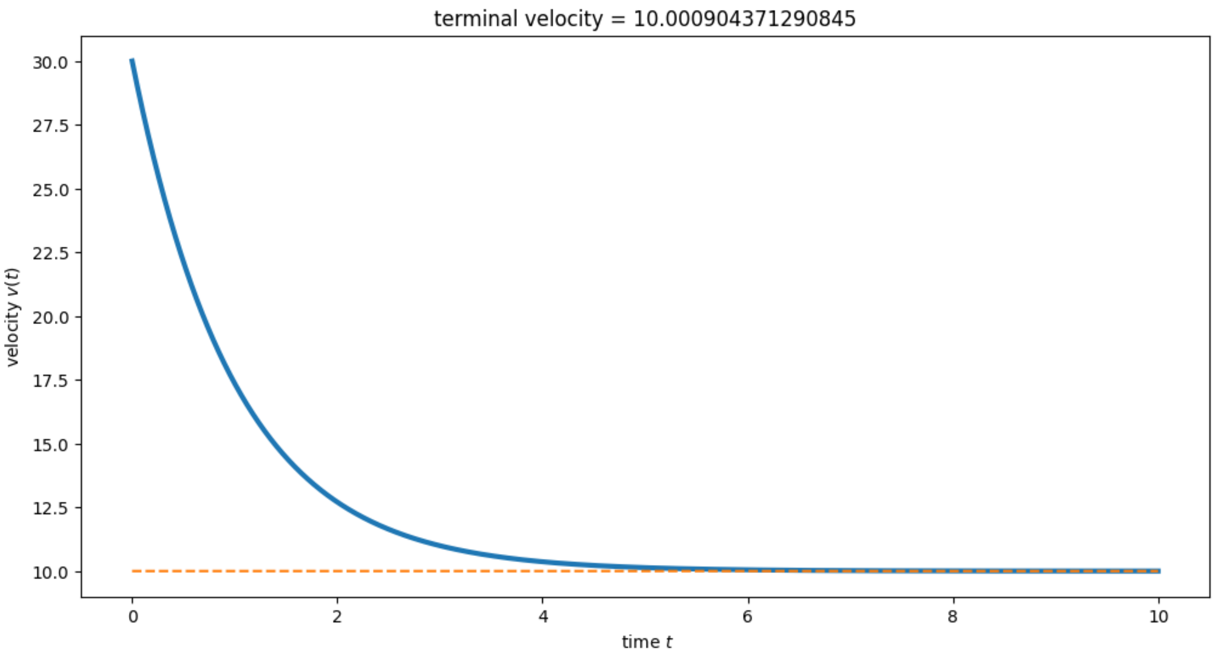
question :

$$\frac{\mathrm{d}v}{\mathrm{d}t} = a - bv$$

In [5]:
```python
# defining parameters
a = 10
b = 1.0

# defining initial conditions
# at t = 0
v0 = 30 # m/s
```

In [6]:
```python
def euler_ode( dvdt , v0 ,  t0 , tn , n = 10000):
    t = np.linspace( t0 , tn , n )
    h = ( tn - t0 )/n
    v = np.zeros( t.size )
    v[0] = v0
    for i in range( 1 , t.size ):
        v[i] = v[i-1] + h*dvdt( v[i-1] )
    return v , t
```

In [7]:
```python
dvdt = lambda v : a - b*v
v , t = euler_ode( dvdt , v0 , 0 , 10 )

fig , ax = plt.subplots( figsize =( 12  , 6 ))
plt.plot( t , v  , linewidth = 3 )
plt.plot( t , np.zeros( t.size ) + a , '--')
plt.xlabel(r' time $t$')
plt.ylabel(r'velocity $v(t)$')
plt.title('terminal velocity = '  + str( v[-1]))
plt.show()
```



Question 3

$$\frac{dN_A}{dt} = N_b/\tau - N_a/\tau$$

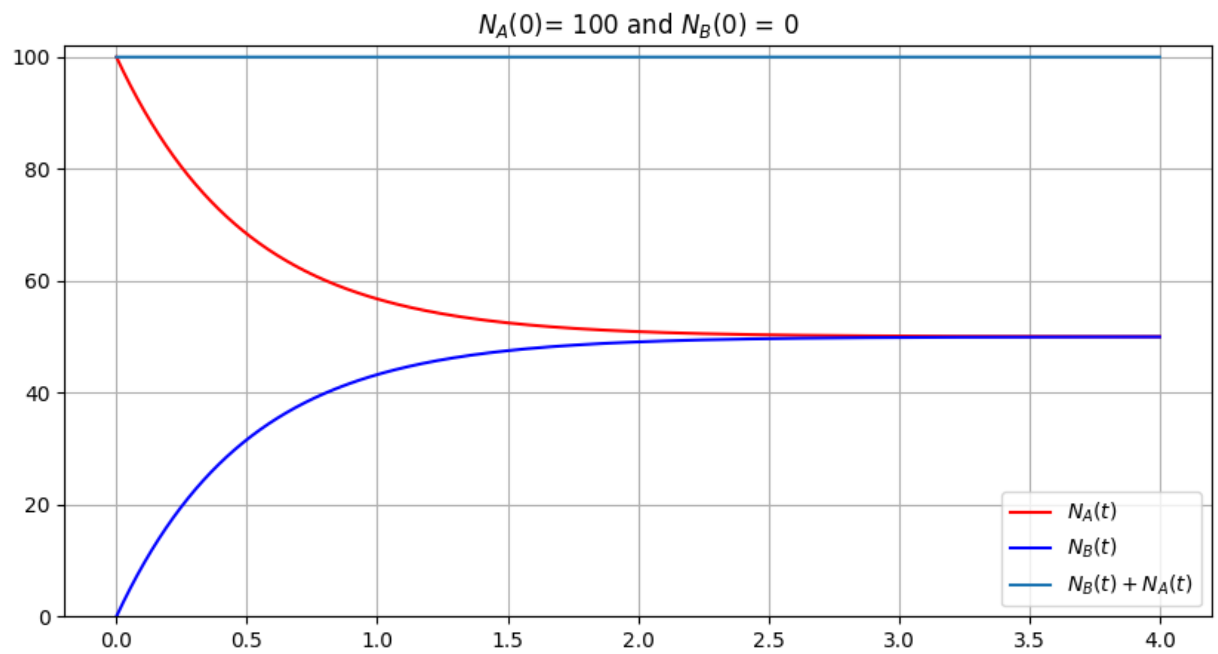
$$= N_A/\tau - N_B/\tau$$

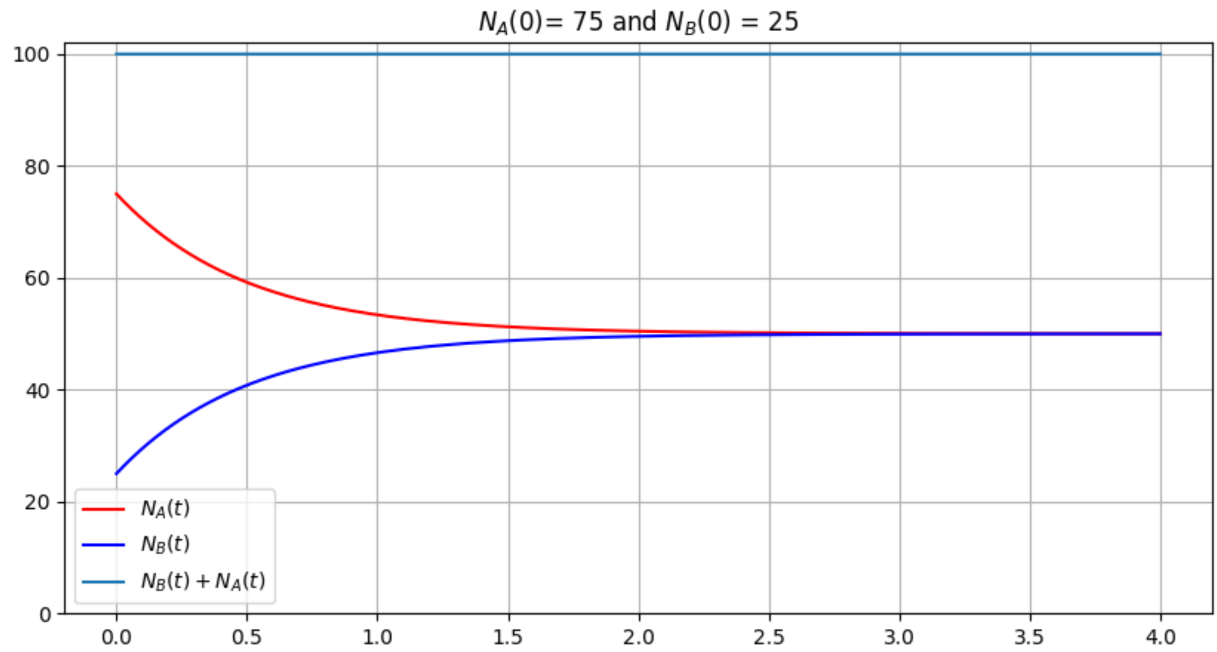
In [8]:
```python
def euler_ode_2( dnadt , dnbdt , na0 , nb0 , t0 , tn , n = 10000 ):
    t = np.linspace( t0 , tn , n )
    h = ( tn - t0 )/n
    na = np.zeros( t.size )
    nb = np.zeros( t.size )
    na[0] = na0
    nb[0] = nb0
    for i in range( 1 , t.size ):
        na[i] = na[i-1] + h*dnadt( na[i-1] , nb[i-1])
        nb[i] = nb[i-1] + h*dnbdt( na[i-1] , nb[i-1])
    return na , nb , t
```

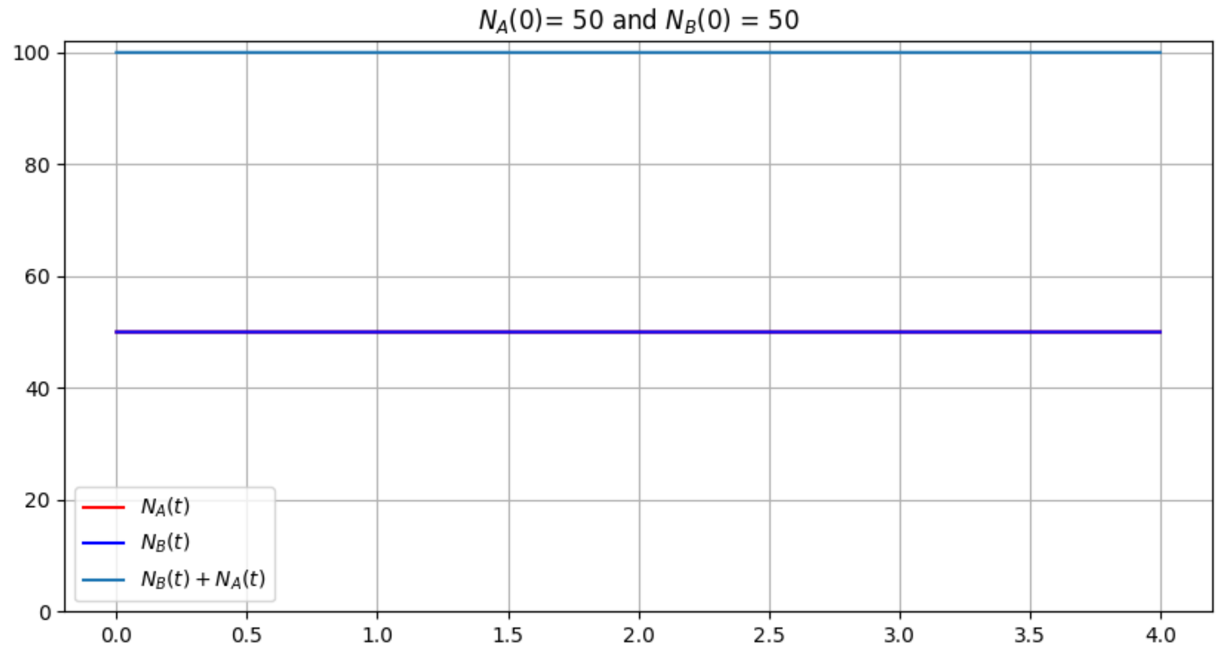
In [9]:
```python
tau = 1
def dnadt( na , nb ):
    return ( nb - na )/tau
def dnbdt( na , nb ):
    return ( na - nb )/tau
```

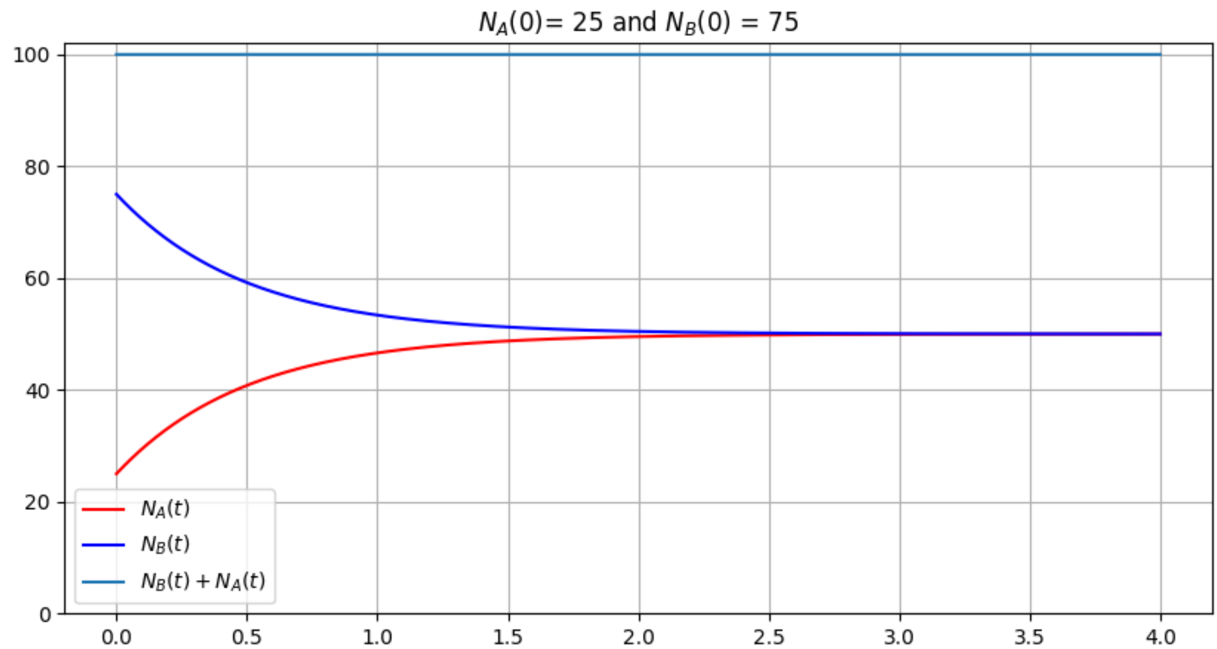
In [33]:
```python
# trying with na = 100 , nb = 0 at t = 0
def plotq3(  na0 , nb0 , tn ):
    fig , ax = plt.subplots( figsize =( 10  , 5 ))
    na , nb , t = euler_ode_2( dnadt , dnbdt , na0 , nb0 , 0 , tn )
    ax.set_title(rf'$N_A(0)$= {na0} and $N_B(0)$ = {nb0}')
    ax.plot( t , na , label = r'$N_A(t)$' ,color =  'r')
    ax.plot( t , nb , label = r'$N_B(t)$' ,color = 'b')
    ax.plot( t , nb + na , label = r'$N_B(t) + N_A(t)$')
    plt.ylim([0,102])
    plt.legend()
    plt.grid()
    plt.show()
```

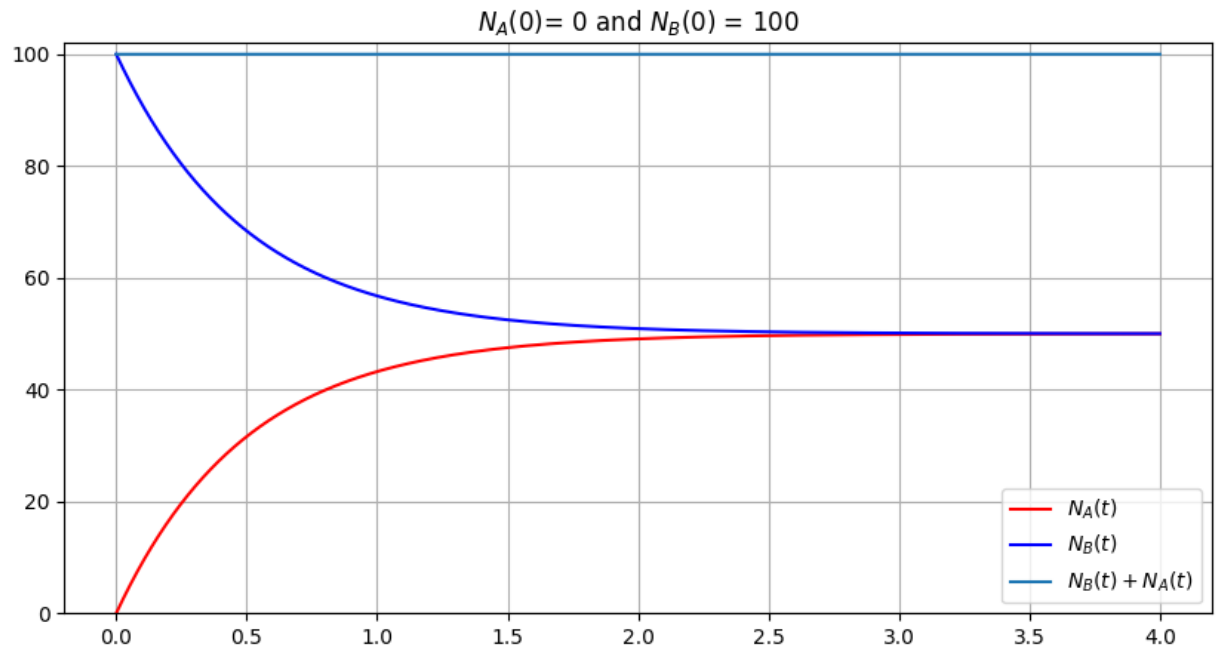
In [34]:
```python
na0 = [ 100 , 75 , 50 , 25 , 0 ]
nb0 = [ 0 , 25 , 50 , 75 , 100 ]

for i in range( len( na0 )):
    plotq3( na0[i] , nb0[i] , 4  )
```




$N_A(0)$= 75 and $N_B(0)$ = 25
0
20
40
60
80
100
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
$N_A(t)$
$N_B(t)$
$N_B(t) + N_A(t)$


$N_A(0)$= 50 and $N_B(0)$ = 50
0
20
40
60
80
100
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
$N_A(t)$
$N_B(t)$
$N_B(t) + N_A(t)$


$N_A(0)$= 25 and $N_B(0)$ = 75
0
20
40
60
80
100
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
$N_A(t)$
$N_B(t)$
$N_B(t) + N_A(t)$



$N_A(0)$= 0 and $N_B(0)$ = 100
100
80
60
40
20
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
$N_A(t)$
$N_B(t)$
$N_B(t) + N_A(t)$