Assignment-4

PHY473/473A-Computational Physics

29th Jan, 2023

Explain the algorithm you are using for each question. Use class in your code.

Question 1. **Make a class for quadratic functions**
Consider a quadratic function $f(x; a, b, c) = ax^2 + bx + c$. Make a class Quadratic for representing f , where a, b, and c are data attributes, and the methods are

- *__init__* for storing the attributes a, b, and c,
- value for computing a value of f at a point x,
- table for writing out a table of x and f values for n x values in the interval [L, R]
- roots for computing the two roots.

Question 2. Create a class **Polynomial** for polynomials. The coefficients in the polynomial can be given to the constructor as a list. Index number i in this list represents the coefficients of the x^i term in the polynomial. That is, writing Polynomial ([1, 0, -1,2]) defines a polynomial

$$1 + 0.x - 1.x^2 + 2.x^3 = 1 - x^2 + 2x^3$$

Use your class to define two polynomials $p(x) = \sum_{i=0}^{M} c_i x^i$ and $q(x) = \sum_{j=0}^{N} d_i x^j$, where c_i and d_i should be taken from two user defined list. Perform the following operations:

- Multiply these two polynomials -

$$\left(\sum_{i=0}^{M} c_i x^i\right)\left(\sum_{j=0}^{N} d_j x^j\right) = \sum_{i=0}^{M}\sum_{j=0}^{N} c_i d_i x^{i+j}$$

- Perform an addition of two polynmials $p(x) = 1 - x$ and $q(x) = x - 6x^4 - x^5$ using your class function. Write down the difference between (p + q) and (p(x) + q(x) for x= 1/2).

Question 3. Introducing an array instead of a list in class Polynomial does not enhance the efficiency of the implementation unless the mathematical computations are also vectorized. That is, all explicit Python loops must be substituted by vectorized expressions. (a) Go through class that you wrote in Question 2 and make sure the coeff attribute is always a numpy array with float elements.

(b) Vectorize the $\_\_add\_\_$ method by adding the common parts of the coefficients arrays and then appending the rest of the longest array to the result.
Hint: Appending an array a to an array b can be done by concatenate(a, b).

d) Vectorize the $\_\_call\_\_$ method by observing that evaluation of a polynomial, $\sum_{i=0}^{n-1} c_i x^i$, can be computed as the inner product of two arrays: $(c_0, ..., C_{n-1})$ and $(x^0, x^1, ..., x^{n-1})$. The latter array can be computed by $x * *p$, where p is an array with powers $0, 1, ...n-1$ and x is a scalar.

Question 4. Use a dict to hold polynomial coefficients
Use a dictionary (instead of a list) for the coeff attribute in class Polynomial of Question 2, such that self.coeff[k] holds the coefficient of the x^k term. The advantage with a dictionary is that only the nonzero coefficients in a polynomial need to be stored.

Take two polynomials $p1 = x^4 - 3x^2 + 3$ and $p2 = 11x^9 + 5x^7 + 4x^3 - 2x$, store their coeffients in a dictionary and perform addition and subtraction using $\_\_add\_\_$ and $\_\_sub\_\_$ methods.