Assignment-5

PHY473/473A-Computational Physics

5th Feb, 2024

Explain the algorithm you are using for each question.

Question 1. BK-7 is a type of common optical crown glass. Its index of refraction n varies as a function of wavelength; for shorter wavelengths n is larger than for longer wavelengths, and thus violet light is refracted more strongly than red light, leading to the phenomenon of dispersion. The index of refraction is tabulated in the attached data file "refractive_index.txt".

Let us suppose that we wish to find the index of refraction at a wavelength of $5000°\mathring{A}$. Unfortunately, that wavelength is not found in the table, and so we must estimate it from the values in the table. We must make some assumption about how n varies between the tabular values. Presumably it varies in a smooth sort of way and does not take wild excursions between the tabulated values.

Write a function that will linearly interpolate the tabular data for the index of refraction of BK-7 and return a value for n for wavelengths between $3511°\mathring{A}$ and $23254°\mathring{A}$.

Question 2.
Imagine we have $n+1$ measurements of some quantity y that depends on $x : (x_0, y_0), (x_1, y_1), ..., (x_n, y_n)$. We may think of y as a function of x and ask what y is at some arbitrary point x not coinciding with any of the points $x_0, ..., x_n$. It is not clear how y varies between the measurement points, but we can make assumptions or models for this behavior using interpolation. One way to solve the interpolation problem is to fit a continuous function that goes through all the $n + 1$ points and then evaluate this function for any desired x. A candidate for such a function is the polynomial of degree n that goes through all the points. It turns out that this polynomial can be written as

$$p_L(x) = \sum_{k=0}^{n} y_k L_k(x), \tag{1}$$

where

$$L_k(x) = \prod_{i=0, i\neq k}^{n} \frac{x - x_i}{x_k - x_i}. \tag{2}$$

The polynomial $p_L(x)$ is known as Lagrange's interpolation formula, and the points $(x_0, y_0).....(x_n, y_n)$ are called interpolation points.

a) Make functions $p_L(x, xp, yp)$ and $L_k(x, k, xp, yp)$ that evaluate $p_L(x)$ and $L_k(x)$ by Eq. 1 and Eq. 2, respectively, at the point x. The arrays xp and yp contain the x and y coordinates of the $n+1$ interpolation points, respectively. That is, xp holds $x_0, ..., x_n$, and yp holds $y_0, ..., y_n$.

b) To verify the program, we observe that $L_k(x_k) = 1$ and that $L_k(x_i) = 0$ for $i \neq k$, implying that $p_L(x_k) = y_k$. That is, the polynomial p_L goes through all the points $(x_0, y_0)....., (x_n, y_n)$. Write a function test_p_L(xp, yp) that computes $|p_L(x_k) - y_k|$ at all the interpolation points (x_k, y_k) and checks that the value is approximately zero. Call test_p_L with xp and yp corresponding to 5 equally spaced points along the curve $y = sin(x)$ for $x \in [0, \pi]$. Thereafter, evaluate $p_L(x)$ for an x in the middle of two interpolation points and compare the value of $p_L(x)$ with the exact one.