Solve the following problems by NRM:

Q1)

$$xy + x^2 - y^3 - 1 = 0$$
$$x + 2y - xy^2 - 2 = 0$$

Q2)

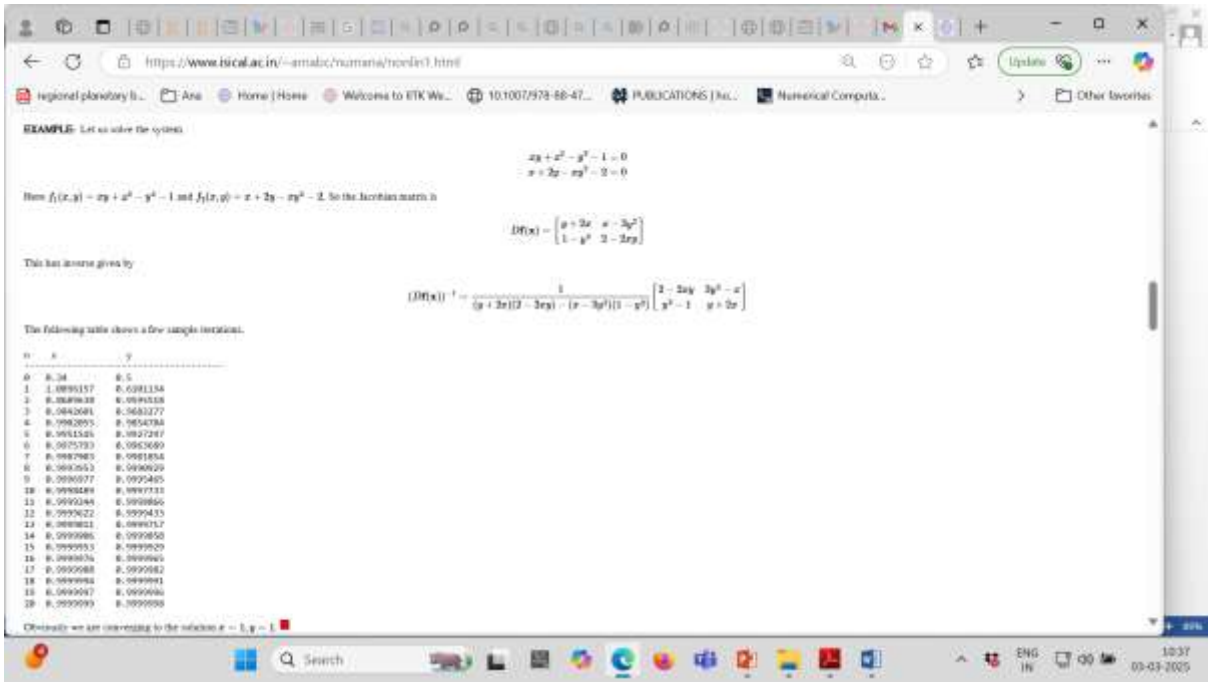
$$\sin xy + e^y = 7.10964$$
$$(x + y)^2 - \cos(xy^2) = 24.1561$$

Q3)

$$e^{xy} + x^2 + y - 1.2 = 0$$
$$x^2 + y^2 + x - 0.55 = 0$$

Q4)

$$\begin{Bmatrix} (x_1 + x_2 + x_3) \\ (x_1^2 + x_2^2 + x_3^2) \\ x_1^3 + x_2^3 + x_3^3 \end{Bmatrix} = \begin{Bmatrix} 3 \times 10^{-1} \\ .03 \\ 300 \times 10^{-5} \end{Bmatrix}$$



EXAMPLE: Let us solve the system

$$xy + x^2 - y^2 - 1 = 0$$
$$x + 2y - xy^2 - 2 = 0$$

Here $f_1(x, y) = xy + x^2 - y^2 - 1$ and $f_2(x, y) = x + 2y - xy^2 - 2$. So the Jacobian matrix is

$$Df(\mathbf{x}) = \begin{bmatrix} y + 2x & x - 2y^2 \\ 1 - y^2 & 2 - 2xy \end{bmatrix}$$

This has inverse given by

$$(Df(\mathbf{x}))^{-1} = \frac{1}{(y + 2x)(2 - 2xy) - (x - 2y^2)(1 - y^2)} \begin{bmatrix} 2 - 2xy & 2y^2 - x \\ y^2 - 1 & y + 2x \end{bmatrix}$$

The following table shows a few sample iterations.

n	x	y
0	0.34	0.5
1	1.0896157	0.6981134
2	0.8849638	0.9595338
3	0.9842681	0.9882277
4	0.9982895	0.9954784
5	0.9951536	0.9927297
6	0.9975793	0.9953689
7	0.9987985	0.9981856
8	0.9993653	0.9990029
9	0.9996577	0.9995405
10	0.9998189	0.9997733
11	0.9999044	0.9998866
12	0.9999522	0.9999433
13	0.9999801	0.9999717
14	0.9999886	0.9999858
15	0.9999953	0.9999929
16	0.9999976	0.9999965
17	0.9999988	0.9999982
18	0.9999994	0.9999991
19	0.9999997	0.9999996
20	0.9999999	0.9999998

Obviously we are converging to the solution $x = 1, y = 1$.