1、r,g,b是RGB彩色空间沿R,G,B轴的单位向量,定义向量 $\mathbf{u}=\frac{\partial R}{\partial x}r+\frac{\partial G}{\partial x}g+\frac{\partial B}{\partial x}b$ 和 $\mathrm{v}=\frac{\partial R}{\partial y}r+\frac{\partial G}{\partial y}g+\frac{\partial B}{\partial y}b, g_{xx}, g_{yy}, g_{xy}$ 定义为这些向量的点乘:

$$\begin{aligned} g_{xx} &= \mathrm{u}\cdot\mathrm{u}=\mathrm{u}^T\mathrm{u}=\left|\frac{\partial R}{\partial x}\right|^2+\left|\frac{\partial G}{\partial x}\right|^2+\left|\frac{\partial B}{\partial x}\right|^2 \\ g_{yy} &= \mathrm{v}\cdot\mathrm{v}=\mathrm{v}^T\mathrm{v}=\left|\frac{\partial R}{\partial y}\right|^2+\left|\frac{\partial G}{\partial y}\right|^2+\left|\frac{\partial B}{\partial y}\right|^2 \\ g_{xy} &= \mathrm{u}\cdot\mathrm{v}=\mathrm{u}^T\mathrm{v}=\frac{\partial R}{\partial x}\frac{\partial R}{\partial y}+\frac{\partial G}{\partial x}\frac{\partial G}{\partial y}+\frac{\partial B}{\partial x}\frac{\partial B}{\partial y} \end{aligned}$$

推导出最大变换率方向 θ 和 (x, y) 在 θ 方向上的变化率的值 $F(\theta)$.

解:

即求当 θ 为何值时,$|\vec{u}\cos\theta+\vec{v}\sin\theta|^2$ 取得最大值,先化简此式:

$$\begin{aligned} |\overrightarrow{\mathrm{u}}\cos\theta+\overrightarrow{\mathrm{v}}\sin\theta|^2 &= \mathrm{g_{xx}}\cos^2\theta+\mathrm{g_{yy}}\sin^2\theta+2\,\mathrm{g_{xy}}\sin\theta\cos\theta \\ &= \mathrm{g_{xx}}\frac{\cos 2\theta+1}{2}+\mathrm{g_{yy}}\frac{1-\cos 2\theta}{2}+\mathrm{g_{xy}}\sin 2\theta \\ &= \frac{1}{2}\left[\left(\mathrm{g_{xy}}+\mathrm{g_{yy}}\right)+\left(\mathrm{g_{xx}}-\mathrm{g_{yy}}\right)\cos 2\theta+2\,\mathrm{g_{xy}}\sin 2\theta\right] \end{aligned}$$

令:

$$\frac{\partial}{\partial\theta}|\overrightarrow{\mathrm{u}}\cos\theta+\overrightarrow{\mathrm{v}}\sin\theta|^2=\frac{\partial}{\partial\theta}\frac{1}{2}\left[\left(\mathrm{g_{xy}}+\mathrm{g_{yy}}\right)+\left(\mathrm{g_{xx}}-\mathrm{g_{yy}}\right)\cos 2\theta+2\,\mathrm{g_{xy}}\sin 2\theta\right]=0$$

得:

$$\theta=\frac{1}{2}\arctan\frac{2\,\mathrm{g_{xy}}}{\mathrm{g_{xx}}-\mathrm{g_{yy}}}$$

(x, y) 在 θ 方向上的变化率的值 $F(\theta)$:

$$\begin{aligned} \mathrm{F}(\theta) &= |\overrightarrow{\mathrm{u}}\cos\theta(\mathrm{x},\mathrm{y})+\overrightarrow{\mathrm{v}}\sin\theta(\mathrm{x},\mathrm{y})| \\ &= \sqrt{\frac{1}{2}\left[\left(\mathrm{g_{xy}}+\mathrm{g_{yy}}\right)+\left(\mathrm{g_{xx}}-\mathrm{g_{yy}}\right)\cos 2\theta(\mathrm{x},\mathrm{y})+2\,\mathrm{g_{xy}}\sin 2\theta(\mathrm{x},\mathrm{y})\right]} \end{aligned}$$