3、现在假设我们有一个长度为8的信号f=[1 3 5 7 4 3 2 1], 利用哈尔小波进行两层的快速小波变换分解，计算各层的滤波器输出，然后再进行完美重建，请利用与书中例子相同的框图进行计算。

$$W_{\varphi}(2,n) = f(n) = [1,3,5,7,4,3,2,1]$$
$$\varphi(n) = [1/\sqrt{2}, 1/\sqrt{2}]$$
$$\psi(n) = [1/\sqrt{2}, -1/\sqrt{2}]$$

各层滤波器输出：

$$W_{\psi}(1,n) = \{1,3,5,7,4,3,2,1\} * \{-1/\sqrt{2}, 1/\sqrt{2}\}\Big|_{\text{down }2} = 1/\sqrt{2}\{-1,-2,-2,-2,3,1,1,1,0\}\Big|_{\text{down }2} = 1/\sqrt{2}\{-2,-2,1,1\}$$

$$W_{\phi}(1,n) = \{1,3,5,7,4,3,2,1\} * \{1/\sqrt{2}, 1/\sqrt{2}\}\Big|_{\text{down }2} = 1/\sqrt{2}\{1,4,8,12,11,7,5,3,0\}\Big|_{\text{down2}} = 1/\sqrt{2}\{$$

$$W_{\psi}(0,n) = 1/\sqrt{2}\{4,12,7,3\} * \{-1/\sqrt{2}, 1/\sqrt{2}\}\Big|_{\text{down2}} = \{-4,2\}$$

$$W_{\phi}(0,n) = 1/\sqrt{2}\{4,12,7,3\} * \{1/\sqrt{2}, 1/\sqrt{2}\}\Big|_{\text{down }2} = \{8,5\}$$

重建：

$$W_{\phi}(1,n) = \{-4,0,2,0\} * 1/\sqrt{2}\{1,-1\} + \{8,0,5,0\} * 1/\sqrt{2}\{1,1\} = 1/\sqrt{2}\{-4+8,4+8,2+5,-2+$$
$$1/\sqrt{2}\{4,12,7,3\}$$

$$f(n) = W_{\phi}(2,n) = 1/\sqrt{2}\{-2,0,-2,0,1,0,1,0\} * 1/\sqrt{2}\{1,-1\} + 1/\sqrt{2}\{4,0,12,0,7,0,3,0\} * 1/\sqrt{2}\{1$$
$$1/2\{-2+4,2+4,-2+12,2+12,1+7,-1+7,1+3,-1+3\} = \{1,3,5,7,4,3,2,1\}$$