2、根据书中对傅立叶变换的定义，证明课本165页上有关傅立叶变换的平移性质。

$$\mathrm{f(x,y)e^{j2\pi\left(u_0x/M+v_0y/N\right)}} \Leftrightarrow \mathrm{F\left(u-u_0, v-v_0\right)}$$

$$\mathrm{f\left(x-x_0, y-y_0\right)} \Leftrightarrow \mathrm{F(u,v)e^{-j2\pi\left(x_0u/M+y_0v/N\right)}}$$

证明：

对于第一个式子：

$$\begin{aligned}
&\mathrm{DFT}\ \left(\mathrm{f(x,y)e^{j2\pi(u_0x/M+v_0y/N)}}\right) \\
&= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left(\mathrm{f(x,y)e^{j2\pi(u_0x/M+v_0y/N)}}\right)\mathrm{e^{-j2\pi(ux/M,vy/N)}} \\
&= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\mathrm{f(x,y)e^{j2\pi(u_0x/M+v_0y/N)-j2\pi(ux/M,vy/N)}} \\
&= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\mathrm{f(x,y)e^{-j2\pi\left(\frac{u-u_0}{M}x+\frac{v-v_0}{N}\right)}} \\
&= \mathrm{F\left(u-u_0, v-v_0\right)}
\end{aligned}$$

对于第二个式子：

$$\begin{aligned}
&\mathrm{IDET}\left(F\mathrm{(r,v)e^{-j2\pi\left(x_0u/M+y_0v/N\right)}}\right) \\
&= \frac{1}{\mathrm{MN}}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\left(\mathrm{F(u,v)e^{-j2\pi\left(x_0u/M+y_0v/N\right)}}\right)\mathrm{e^{j2\pi(ux/M+vy/N)}} \\
&= \frac{1}{\mathrm{MN}}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\mathrm{F(u,v)e^{-j2\pi\left(x_0u/M+y_0v/N\right)+j2\pi(ux/M+vy/N)}} \\
&= \frac{1}{\mathrm{MN}}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\mathrm{F(u,v)e^{j2\pi\left(\frac{x-x_0}{M}u+\frac{y-y_0}{N}v\right)}} \\
&= \mathrm{f\left(x-x_0, y-y_0\right)}
\end{aligned}$$

于是傅立叶变换的平移不变性得证。