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(Q1) (a) 
$$\begin{aligned} 2x + y + z &= 5 \\ 3x + 5y + 2z &= 15 \\ 2x + y + 4z &= 8 \end{aligned}$$

Checking if diagonally dominant:

$$|2| \geq |1| + |1| - \text{true}$$

$$|5| \geq |3| + |2| - \text{true}$$

$$|4| \geq |2| + |1| - \text{true}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 5 & 2 \\ 2 & 1 & 4 \end{bmatrix}$$

$$x^{k+1} = \frac{1}{2} [5 - 1y^k - 1z^k] = 0.5 [5 - y^k - z^k]$$

$$y^{k+1} = \frac{1}{5} [15 - 3x^k - 2z^k] = 0.2 [15 - 3x^k - 2z^k]$$

$$z^{k+1} = \frac{1}{4} [8 - 2x^k - 1y^k] = 0.25 [8 - 2x^k - y^k]$$

let  $x^0 = y^0 = z^0 = 1$ :

1<sup>st</sup> iteration:

$$x^1 = 0.5 [5 - 1 - 1] = 1.5$$

$$y^1 = 0.2 [15 - 3(1) - 2(1)] = 2$$

$$z^1 = 0.25 [8 - 2(1) - 1] = 1.25$$

2<sup>nd</sup> iteration:

$$x^2 = 0.5 [5 - 2 - 1.25] = 0.875$$

$$y^2 = 0.2 [15 - 3(1.5) - 2(1.25)] = 1.6$$

$$z^2 = 0.25 [8 - 2(1.5) - 2] = 0.75$$

3<sup>rd</sup> iteration:

$$x^3 = 0.5 [5 - 1.6 - 0.75] = 1.325$$

$$y^3 = 0.2 [15 - 3(0.875) - 2(0.75)] = 2.175$$

$$z^3 = 0.25 [8 - 2(0.875) - 1.6] = 1.1625$$

4<sup>th</sup> iteration:

$$x^4 = 0.5 [5 - 2.175 - 1.1625] = 0.83125$$

$$y^4 = 0.2 [15 - 3(1.325) - 2(1.1625)] = 1.74$$

$$z^4 = 0.25 [8 - 2(1.325) - 2.175] = 0.79375$$

5<sup>th</sup> iteration:

$$x^5 = 0.5 [5 - 1.74 - 0.79375] = 1.233125$$

$$y^5 = 0.2 [15 - 3(0.83125) - 2(0.79375)] = 2.18375$$

$$z^5 = 0.25 [8 - 2(0.83125) - 1.74] = 1.149375$$

6<sup>th</sup> iteration:

$$x^6 = 0.5 [5 - 2.18375 - 1.149375] = 0.8334375$$

$$y^6 = 0.2 [15 - 3(1.233125) - 2(1.149375)] = 1.800375$$

$$z^6 = 0.25 [8 - 2(1.233125) - 2.18375] = 0.8375$$

7<sup>th</sup> iteration:

$$x^7 = 0.5 [5 - 1.800375 - 0.8375] = 1.1810625$$

$$y^7 = 0.2 [15 - 3(0.8334375) - 2(0.8375)] = 2.1649375$$

$$z^7 = 0.25 [8 - 2(0.8334375) - 1.800375] = 1.1331875$$

8<sup>th</sup> iteration:

$$x^8 = 0.5 [5 - 2.1649375 - 1.1331875] = 0.8509375$$

$$y^8 = 0.2 [15 - 3(1.1810625) - 2(1.1331875)] = 1.8380875$$

$$z^8 = 0.25 [8 - 2(1.1810625) - 2.1649375] = 0.868234375$$

9<sup>th</sup> iteration:

$$x^9 = 0.5 [5 - 1.8380875 - 0.868234375] = 1.646839063$$

$$y^9 = 0.2 [15 - 3(0.8509375) - 2(0.868234375)] = 2.14214375$$

$$z^9 = 0.25 [8 - 2(0.8509375) - 1.8380875] = 1.115009375$$

10<sup>th</sup> iteration:

$$x^{10} = 0.5 [5 - 2.14214375 - 1.115009375] = 0.8714234375$$

$$y^{10} = 0.2 [15 - 3(1.646839063) - 2(1.115009375)] = 1.565892812$$

$$z^{10} = 0.25 [8 - 2(1.646839063) - 2.14214375] = 0.641044531$$

Errors for x:  $\left| \frac{x^{k+1} - x_k}{x^{k+1}} \right| \times 100$

$$E_1 = \left| \frac{x^1 - x^0}{x^1} \times 100 \right| = \left| \frac{1.5 - 1}{1.5} \right| \times 100 = 33.33\%$$

$$E_2 = \left| \frac{x^2 - x^1}{x^2} \times 100 \right| = \left| \frac{0.875 - 1.5}{0.875} \right| \times 100 = 71.43\%$$

$$E_3 = \left| \frac{x^3 - x^2}{x^3} \times 100 \right| = \left| \frac{1.325 - 0.875}{1.325} \times 100 \right| = 33.96\%$$

$$E_4 = \left| \frac{0.83125 - 1.325}{0.83125} \times 100 \right| = 59.40\%$$

$$E_5 = \left| \frac{1.233125 - 0.83125}{1.233125} \times 100 \right| = 32.89\%$$

$$E_6 = \left| \frac{0.8334375 - 1.233125}{0.8334375} \times 100 \right| = 47.96\%$$

$$E_7 = \left| \frac{1.1810625 - 0.8334375}{1.1810625} \times 100 \right| = 29.43$$

$$E_8 = \left| \frac{0.8509375 - 1.1810625}{0.8509375} \times 100 \right| = 38.80\%$$

$$E_9 = \left| \frac{1.646839063 - 0.8509375 \times 100}{1.646839063} \right| =$$

$$E_{10} = \left| \frac{0.8714234375 - 1.646839063}{0.8714234375} \right| \times 100 =$$

Errors for y:  $E_{k+1} = \left| \frac{y^{k+1} - y^k}{y^{k+1}} \right| \times 100$

$$E_1 = \left| \frac{2 - 1}{2} \times 100 \right| = 50\%$$

$$E_2 = \left| \frac{1.6 - 2}{1.6} \times 100 \right| = 25\%$$

$$E_3 = \left| \frac{2.175 - 1.6}{2.175} \times 100 \right| = 26.44\%$$

$$E_4 = \left| \frac{1.74 - 2.175}{1.74} \times 100 \right| = 25\%$$

$$E_5 = \left| \frac{2.18375 - 1.74}{2.18375} \times 100 \right| = \text{scratched out} \cdot 20.32\%$$

$$E_6 = \left| \frac{1.800375 - 2.18375}{1.800375} \times 100 \right| = 21.29\%$$

$$E_7 = \left| \frac{2.1649375 - 1.800375}{2.1649375} \times 100 \right| = 16.84\%$$

$$E_8 = \left| \frac{1.8380875 - 2.1649375}{1.8380875} \times 100 \right| = 17.78\%$$

$$E_9 = \left| \frac{2.14214375 - 1.8380875}{2.14214375} \times 100 \right| = 14.16\%$$

$$E_{10} = \left| \frac{1.565892812 - 2.14214375}{1.565892812} \times 100 \right| = 36.80\%$$

$$\text{Errors for } z : E_{k+1} = \left| \frac{z^{k+1} - z^k}{z^{k+1}} \right| \times 100$$

$$E_1 = \left| \frac{1.25 - 1}{1.25} \right| \times 100 = 20 \%$$

$$E_2 = \left| \frac{0.75 - 1.25}{0.75} \right| \times 100 = 66.67 \%$$

$$E_3 = \left| \frac{1.1625 - 0.75}{1.1625} \right| \times 100 = 35.487 \%$$

$$E_4 = \left| \frac{0.79375 - 1.1625}{0.79375} \right| \times 100 = 46.46 \%$$

$$E_5 = \left| \frac{1.149375 - 0.79375}{1.149375} \right| \times 100 = 30.94 \%$$

$$E_6 = \left| \frac{0.8375 - 1.149375}{0.8375} \right| \times 100 = 37.24 \%$$

$$E_7 = \left| \frac{1.1331875 - 0.8375}{1.1331875} \right| \times 100 = 26.09 \%$$

$$E_8 = \left| \frac{0.868234375 - 1.1331875}{0.868234375} \right| \times 100 = 30.52 \%$$

$$E_9 = \left| \frac{1.115009375 - 0.868234375}{1.115009375} \right| \times 100 = 22.13 \%$$

$$E_{10} = \left| \frac{0.841044531 - 1.115009375}{0.841044531} \right| \times 100 = 73.94 \%$$

(b)

$$2x + y + z = 5$$

$$3x + 5y + 2z = 15$$

$$2x + y + 4z = 8$$

Already established that this is diagonally dominant.

$$x^{\text{new}} = 0.5 [5 - y - z]$$

$$y^{\text{new}} = 0.2 [15 - 3x^{\text{new}} - 2z]$$

$$z^{\text{new}} = 0.25 [8 - 2x^{\text{new}} - y^{\text{new}}]$$

$$\text{let } y = z = 1.$$

1<sup>st</sup> Iteration:

$$x^1 = 0.5 [5 - 1 - 1] = 1.5$$

$$y^1 = 0.2 [15 - 3(1.5) - 2(1)] = 1.7$$

$$z^1 = 0.25 [8 - 2(1.5) - 1.7] = 0.825$$

2<sup>nd</sup> iteration:

$$x^2 = 0.5 [5 - 1.7 - 0.825] = 1.2375$$

$$y^2 = 0.2 [15 - 3(1.2375) - 2(0.825)] = 1.9275$$

$$z^2 = 0.25 [8 - 2(1.2375) - 1.9275] = 0.899375$$

3<sup>rd</sup> iteration:

$$x^3 = 0.5 [5 - 1.9275 - 0.899375] = 1.0865625$$

$$y^3 = 0.2 [15 - 3(1.0865625) - 2(0.899375)] = 1.9883125$$

$$z^3 = 0.25 [8 - 2(1.0865625) - 1.9883125] = 0.959640625$$

4<sup>th</sup> iteration:

$$x^4 = 0.5 [5 - 1.9883125 - 0.959640625] = 1.026023438$$

$$y^4 = 0.2 [15 - 3(1.026023438) - 2(0.959640625)] = 2.000529687$$

$$z^4 = 0.25 [8 - 2(1.026023438) - 2.000529687] = 0.9868558593$$

5<sup>th</sup> iteration:

$$x^5 = 0.5 [5 - 2(0.006307227) - 0.9868558593] = 1.006307227$$

$$y^5 = 0.2 [15 - 3(1.006307227) - 2(0.9868558593)] = 2.00147332$$

$$z^5 = 0.25 [8 - 2(1.006307227) - 2.00147332] = 0.9964780565$$

6<sup>th</sup> iteration:

$$x^6 = 0.5 [5 - 2(0.00147332) - 0.9964780565] = 1.001024312$$

$$y^6 = 0.2 [15 - 3(1.001024312) - 2(0.9964780565)] = 2.00079419$$

$$z^6 = 0.25 [8 - 2(1.001024312) - 2.00079419] = 0.997502369$$

7<sup>th</sup> iteration:

$$x^7 = 0.5 [5 - 2(0.00079419) - 0.997502369] = 1.000851721$$

$$y^7 = 0.2 [15 - 3(1.000851721) - 2(0.997502369)] = 2.00048802$$

$$z^7 = 0.25 [8 - 2(1.000851721) - 2.00048802] = 0.9994521345$$

8<sup>th</sup> iteration:

$$x^8 = 0.5 [5 - 2(0.00048802) - 0.9994521345] = 1.000029923$$

$$y^8 = 0.2 [15 - 3(1.000029923) - 2(0.9994521345)] = 2.000201192$$

$$z^8 = 0.25 [8 - 2(1.000029923) - 2.000201192] = 0.9999347405$$

9<sup>th</sup> iteration:

$$x^9 = 0.5 [5 - 2(0.000201192) - 0.9999347405] = 0.9999320338$$

$$y^9 = 0.2 [15 - 3(0.9999320338) - 2(0.9999347405)] = 2.500083604$$

$$z^9 = 0.25 [8 - 2(0.9999320338) - 2.500083604] = 0.8750130821$$

10<sup>th</sup> iteration:

$$x^{10} = 0.5 [5 - 2(0.000201192) - 0.8750130821] = 0.812075439$$

$$y^{10} = 0.2 [15 - 3(0.812075439) - 2(0.8750130821)] = 2.162749504$$

$$z^{10} = 0.25 [8 - 2(0.812075439) - 2.162749504] = 1.053274905$$

Errors for x values:

$$E_1 = 33.33\%$$

$$E_2 = 21.21\%$$

$$E_3 = 13.891\%$$

$$E_4 = 5.9\%$$

$$E_5 = 1.959\%$$

$$E_6 = 0.528\%$$

$$E_7 = 0.0172\%$$

$$E_8 = 0.8487\%$$

$$E_9 = 0.009795577\%$$

$$E_{10} = \cancel{2.07} \quad 23.13\%$$

Errors for y values:

$$E_1 = 41.176\%$$

$$E_2 = 11.80\%$$

$$E_3 = 3.058\%$$

$$E_4 = 0.611\%$$

$$E_5 = 0.0471\%$$

$$E_6 = 0.0339\%$$

$$E_7 = 0.0153\%$$

$$E_8 = 0.0143\%$$

$$E_9 = 19.99\%$$

$$E_{10} = 15.597\%$$

Errors for z values:

$$E_1 = \cancel{4.188} \quad 21.21\%$$

$$E_2 = 8.27\%$$

$$E_3 = 6.28\%$$

$$E_4 = 2.758\%$$

$$E_5 = 0.966\%$$

$$E_6 = 0.103\%$$

$$E_7 = 0.195\%$$

$$E_8 = 0.0483\%$$

$$E_9 = 14.28\%$$

$$E_{10} = 16.9\%$$

$$x + 5y - 3z = 18$$

$$-2x + 2y + 7z = 19$$

$$9x - 2y + z = 50$$

$$|1| \geq |5| + |-3| \rightarrow \text{false}$$

$$|5| \geq |1| + |-3| \rightarrow \text{true so this is eq } ②$$

$$|2| \geq |-2| + |7| \rightarrow \text{false}$$

$$|7| \geq |-2| + |2| \rightarrow \text{true so this is eq } ③$$

$$|1| \geq |9| + |-2| \rightarrow \text{false}$$

$$|9| \geq |1| + |-2| \rightarrow \text{true so this is eq } ①$$

$$\left. \begin{array}{l} 9x - 2y + z = 50 \\ x + 5y - 3z = 18 \\ -2x + 2y + 7z = 19 \end{array} \right\} \text{This is diagonally dominant}$$

$$R_1 = 50 - 9x + 2y - z$$

$$R_2 = 18 - x - 5y + 3z$$

$$R_3 = 19 + 2x - 2y - 7z$$

Working table:

Eq #	$x_1$	$x_2$	$x_3$	
1	-9	2	-1	
2	-1	-5	3	
3	2	-2	-7	

$$( \text{let } x = y = z = 0 )$$

Relaxation table:

$x_0$	$R_1$	$y$	$R_2$	$z$	$R_3$
0	50	0	18	0	19
1 (5)	$50 - 9(5) = 5$	0	$18 - 5(5) = 13$	0	$19 + 2(5) = -29$
2 5	$5 + (-1)(4) = 1$	0	$13 + 3(4) = 25$	4	<del><math>29 - 7(4) = 1</math></del>
3 5	$1 + 2(5) = 11$	5	$25 - 5(5) = 0$	4	$1 - 2(5) = -9$
4 1.	$11 - 9(1) = 2$	5	$0 - 1(1) = -1$	4	$-9 + 2(1) = -7$
5 1.	$2 - 1(-1) = 3$	5	$-1 + 3(-1) = -4$	-1	$-7 - 7(-1) = 0$
6 1.	$3 + 2(-1) = 1$	-1	$-4 - 5(-1) = 1$	-1	$0 - 5(-1) = 5$
7 1	$1 + 1(1) = 0$	-1	$1 + 3(1) = 4$	1	$5 - 7(1) = -2$
8 1	$0 + 2(1) = 2$	1	$4 - 5(1) = -1$	1	$-2 - 2(1) = -4$
9 1	$2 - 1(-1) = 3$	1	$-1 + 3(-1) = -4$	-1	$-4 - 7(-1) = 3$
10 1	$3 + 2(-1) = 1$	-1	$-4 - 5(-1) = 1$	-1	$3 - 2(-1) = 5$
11 1	$1 - 1(1) = 0$	-1	$1 + 3(1) = 4$	1	$5 - 7(1) = -2$
12 1	$0 + 2(1) = 2$	1	$4 - 5(1) = -1$	1	$-2 - 2(1) = -4$
13 1	$2 - 1(-1) = 3$	1	$-1 + 3(-1) = -4$	-1	$-4 - 7(-1) = 3$

$$x = 5 + 1 = 6 \quad \underline{\text{Ans}}$$

$$y = 5 - 1 + 1 - 1 + 1 = 5 \quad \underline{\text{Ans}}$$

$$z = 4 - 1 + 1 - 1 + 1 - 1 = 3 \quad \underline{\text{Ans}}$$

(Q3)

$$x + y + z = 7$$

$$x + 2y + 2z = 13$$

$$x + 3y + z = 13$$

In doolittle decomposition, [L] matrix has 1's  
on the diagonal.

Step 1:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$\text{[U]} = \text{[L]}$$

$$U_{11} = 1$$

$$U_{12} = 1$$

$$U_{13} = 1$$

$$L_{21} U_{11} = 1 \Rightarrow L_{21} = 1/1 = 1$$

$$L_{21} U_{12} + U_{22} = 2 \Rightarrow U_{22} = 2 - 1(1) = 1$$

$$L_{21} U_{13} + U_{23} = 2 \Rightarrow U_{23} = 2 - 1(1) = 1$$

$$L_{31} U_{11} = 1 \Rightarrow L_{31} = 1/1 = 1$$

$$L_{31} U_{12} + L_{32} U_{22} = 3 \Rightarrow L_{32} = 3 - 1(1)/1 = 2$$

$$L_{31} U_{13} + L_{32} U_{23} + U_{33} = 1 \Rightarrow U_{33} = 1 - 1(1) - 2(1) = -2$$

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$[U] = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

Step 2:  $[L][D] = [B]$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \\ 13 \end{bmatrix}$$

$$\begin{bmatrix} d_1 \\ d_1 + d_2 \\ d_1 + 2d_2 + d_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \\ 13 \end{bmatrix}$$

$$d_1 = 7$$

$$d_2 = 13 - d_1 = 13 - 7 = 6$$

$$d_3 = 13 - d_1 - 2d_2 = 13 - 7 - 2(6) = -6$$

Step 3:  $[U][x] = [D]$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + x_2 + x_3 \\ x_2 + x_3 \\ -2x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \\ -6 \end{bmatrix}$$

$$x_3 = -6 / -2 = 3$$

$$x_2 = 6 - x_3 = 6 - 3 = 3$$

$$x_1 = 7 - x_2 - x_3 = 7 - 3 - 3 = 1$$

<del>x</del>	<del>1</del>	<del>Aus</del>
<del>3</del>	<del>3</del>	
<del>3</del>	<del>3</del>	

$$x = 1, y = 3, z = 3 \text{ All}$$