Unlimited Non-Causal Correlations And Their Relation To Non-Locality

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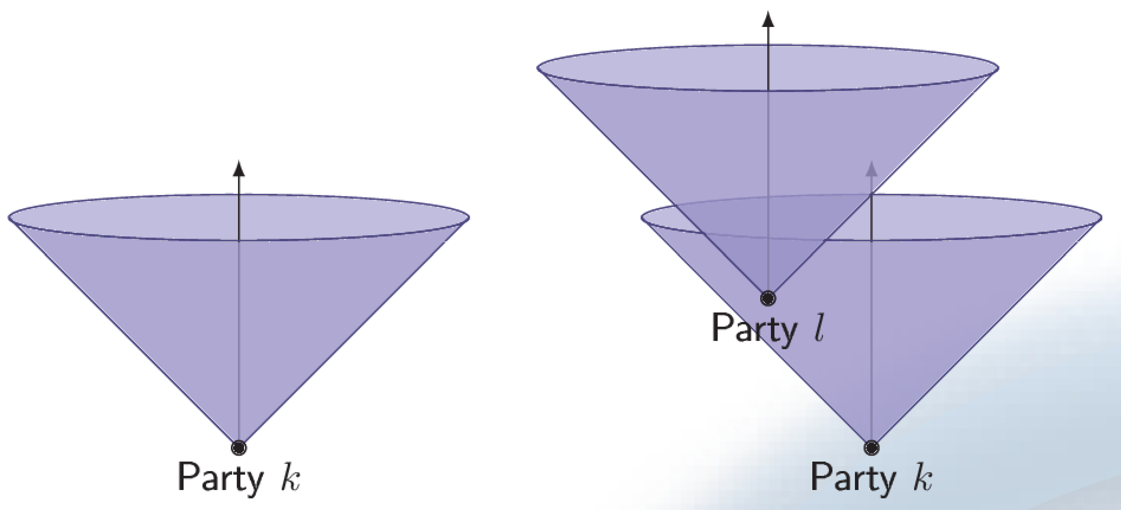
Non-Causal Correlations



The cone on the left is the future light cone of Party k. Party k can influence her future light cone and can hence influence Party l.

Definition 1 (Party) *A party k is a tuple (A_k, X_k), where A_k is a random variable that describes the output of party k, and where X_k is a uniformly distributed input random variable.*

Definition 2 (Causal Correlations) *An n-party probability distribution $P_{A|X}$ is* Causal *if and only if*

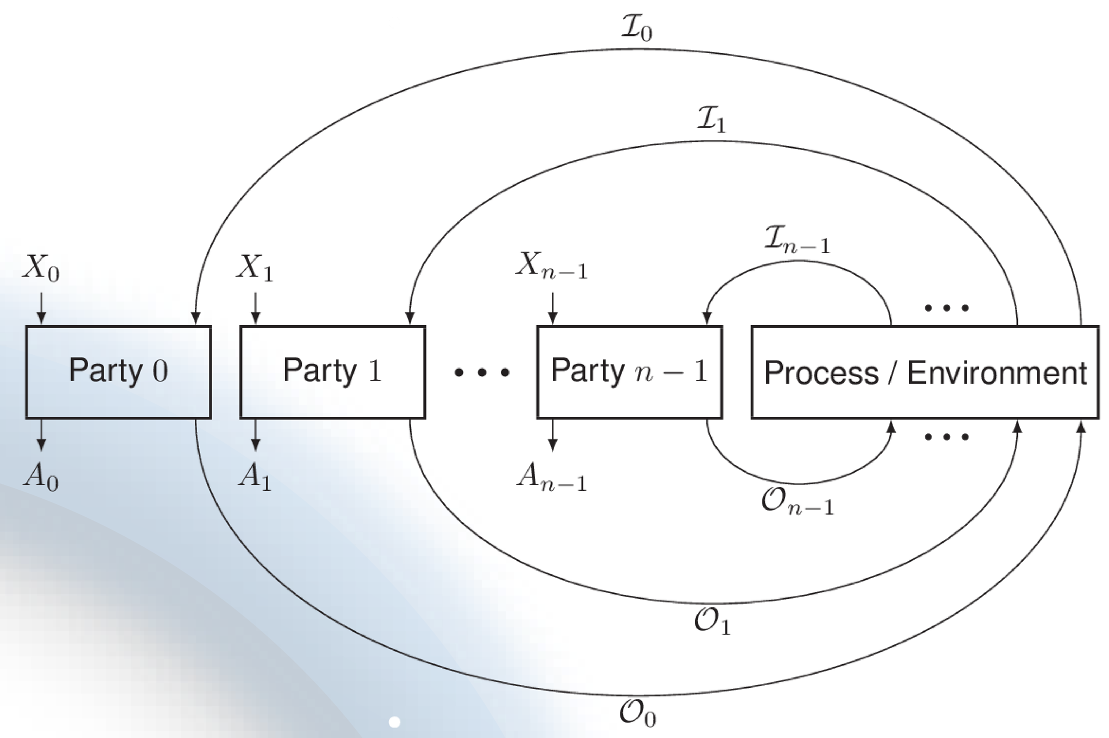
$$P_{A|X} = \sum_{k \in \mathbb{Z}_n} P_K(k) P_{A_k|X_k, K=k} P_{A_{\setminus k}|X, A_k, K=k} ,$$

where K is a random variable over the sample space and where for all $k \in \mathbb{Z}_n, a_k \in \mathcal{A}_k, x_k \in \mathcal{X}_k$ the distribution $P_{A_{\setminus k}|X_{\setminus k}, A_k=a_k, X_k=x_k, K=k}$ is causal. Distributions that lie outside of this set are called Non-Causal.

Definition 3 (Bi-causal Correlations) *Similarly, an n-party probability distribution $P_{A|X}$, for $n \geqslant 2$, is* Bi-Causal *if and only if*

$$P_{A|X} = \sum_{\emptyset \subsetneq \mathcal{K} \subsetneq \mathbb{Z}_n} P_K(\mathcal{K}) P_{A_{\mathcal{K}}|X_{\mathcal{K}}, K=\mathcal{K}} P_{A_{\setminus \mathcal{K}}|A_{\mathcal{K}}, X, K=\mathcal{K}} ,$$

where K is a random variable with sample space $\{\mathcal{K} \mid \emptyset \subsetneq \mathcal{K} \subsetneq \mathbb{Z}_n\}$. Distributions that lie outside of this set are called genuinely multi-party Non-Causal.



One way to obtain non-causal correlations is to *locally* assume the notion and have no assumptions on the global level, apart from *logical consistency*. In the *Process Matrix* framework, every party is *isolated* and interacts independently with the *Environment* only once. The environment may model general frameworks, including classical and quantum theories. It may be considered as a channel that take the outputs $\mathcal{O}_k$ of each party and produces their input $\mathcal{I}_k$.

Similar to non-locality, separation between different theories may be established by identifying bounds on maximum winning probability of games that the parties play.

Definition 4 (Maximum Winning Probability of Game $\mathcal{G}_n$) *A game $\mathcal{G}_n$ between every party $k \in \mathbb{Z}_n$ requires that each party outputs $\omega_k^n(X)$, where*

$$\omega_k^n : \mathbb{Z}_2^n \to \mathbb{Z}_2 .$$

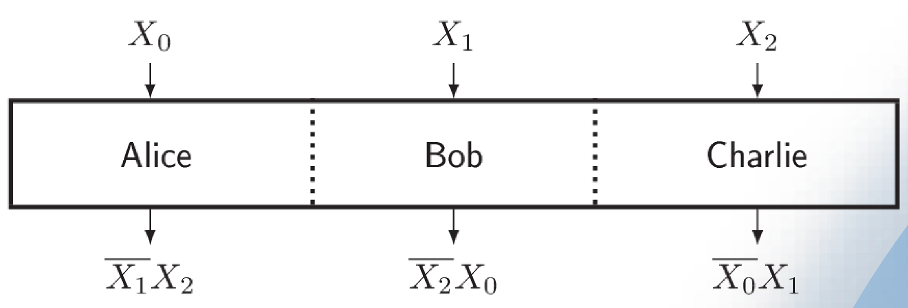
The Maximum Winning Probability *or* Value *of a game under different frameworks is given by*

$$v_{\textit{type}}(\mathcal{G}_n) := \max_{P_{A|X} \in \mathcal{C}_{\textit{type}}} \Pr[A = \omega^n(X)] ,$$

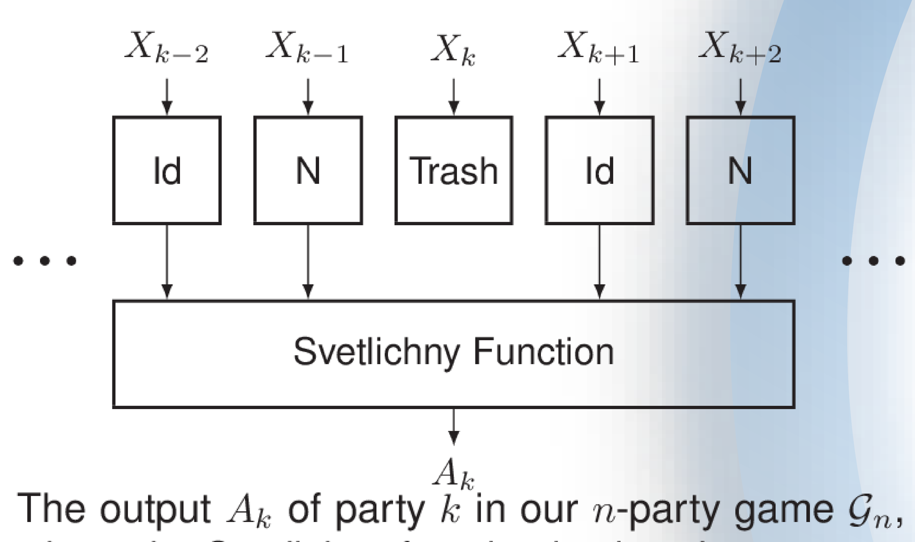
where type $\in \{$causal, bi-causal, cprocess, qprocess$\}$.

The traditional notion of causal order dictates that a space-time event cannot influence any event in its past. There exist motivations to relax this notion from the quest to combine general relativity with quantum theory. This has led to the search for physical principles that re-establish causal order. We ask whether causal order arises in the limit of multi-particle correlations and find multi-party non-causal correlations for any number of parties that are in agreement with all principles thus far posed. Moreover, these correlations violate a family of causal inequalities *maximally*, while under the assumption of causal order the corresponding game can at most be won with probability half in the limit of the number of parties. This implies that the space of logically consistent correlations is infinitely larger than the space of correlations based on the assumptions of causal order. The game is closely related to Ardehali-Svetlichny nonlocal game. We leave open the search for reasonable principles that establish causal order. It seems that as of today, the only way to establish causal order is to assume it.

Unlimited Non-Causal Correlations



The three party Svetlichny Game



The output A_k of party k in our n-party game $\mathcal{G}_n$, where the Svetlichny function is given by

$$z \mapsto \left(\bigoplus_{0 \leqslant i < j \leqslant m-1} z_i z_j \right) \oplus \left(\bigoplus_{0 \leqslant i \leqslant m-1} \lambda_i z_i \right) ,$$

where for all $i \in \mathbb{Z}_m$, $\lambda_i \in \mathbb{Z}_2$ are constants.

Main Result

Theorem 1 *For the game $\mathcal{G}_n$ given by*

$$x \mapsto \left(\bigoplus_{\substack{0 \leqslant i < j \leqslant n-1 \\ i,j \neq k}} x_i x_j \right) \oplus \left(\bigoplus_{\substack{0 \leqslant i \leqslant k-1 \\ i \not\equiv_2 k}} x_i \right) \oplus \left(\bigoplus_{\substack{k+1 \leqslant i \leqslant n-1 \\ i \equiv_2 k}} x_i \right) ,$$

with the number of parties $n \geqslant 3$, the game can be won with certainty using classical processes, i.e,

$$v_{\textit{cprocess}}(\mathcal{G}_n) = 1 .$$

For $n \geqslant 2$, the probability for winning the game with causal correlations and bi-causal correlations is upper bounded by

$$v_{\textit{causal}}(\mathcal{G}_n) \leqslant v_{\textit{bi-causal}}(\mathcal{G}_n) \leqslant \frac{1}{2} + \frac{1}{2^{\lceil \frac{n}{2} \rceil}} .$$

We also show that our game is asymptotically the hardest bi-causal game with binary outputs.

Definition (Process Function) *A function is an n-party* process function *if $\omega : \mathcal{O} \to \mathcal{I}$ such that*

$$\forall f \ \exists i : i = \omega(f(i)) ,$$

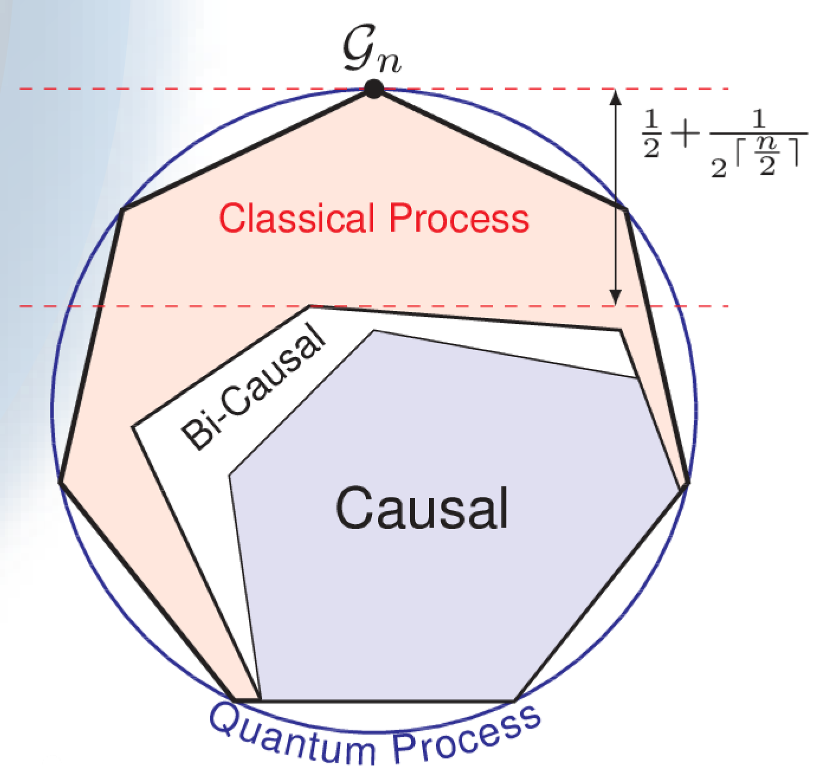
where $\mathcal{O} = \bigtimes_{j \in \mathbb{Z}_n} \mathcal{O}_j ,$

$$\mathcal{I} = \bigtimes_{j \in \mathbb{Z}_n} \mathcal{I}_j ,$$

$$f = (f_0, f_1, \ldots, f_{n-1}) ,$$

with $\forall j \in \mathbb{Z}_n : f_j : \mathcal{I}_j \to \mathcal{O}_j .$

In other words, it is a function that, when composed with arbitrary functions of the parties, admits a fixed point. We show that the function in game $\mathcal{G}_n$ is a process function.



Future Work and Connections

- We conjecture that the set of classical process correlations $\mathcal{C}_{\text{cprocess}}$ is contained in the set of quantum process correlations $\mathcal{C}_{\text{qprocess}}$.
- We conjecture that $v_{\text{causal}}(\mathcal{G}_n) \leqslant \frac{1}{4}$. Does there exist a game with $v_{\text{causal}}(\mathcal{G}_n) = 0$?
- What natural principle implies no violation of causal order?
- Is quantum computation the same as *non-causal* classical computation?

[1] M. Seevinck and G. Svetlichny, PRL 89, 060401 (2002) [2] O. Oreshkov, F. Costa, and C. Brukner, Nature Communications 3, 1092 (2012)
[3] A. A. Abbott, J. Wechs, F. Costa, and C. Branciard, Quantum 1, 39 (2017)