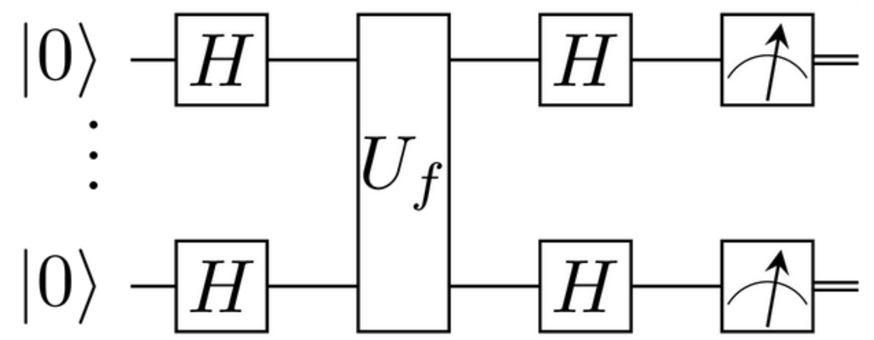
THE TEAM


IBA SMCS
Quantum Query Complexity
Our paper extends the theoretical framework of the Bernstein-Vazirani algorithm to find new function promises / orthogonal bases to develop new quantum algorithms.
State Discrimination
We find the theoretical minimum error probability to distinguish between two arbitrary quantum states, necessary for efficient quantum algorithms
Bernstein-Vazirani
We extend the theoretical framework laid down by Bernstein and Vazirani, exploring new function promises in Svetlichny-like functions
HLFP and Beyond
Using our novel technique, we look at reformulating the Recursive Bernstein-Vazirani algorithm in a more intuitive way, and connect it to the Hidden Linear Function Problem
QWorld Tutorials

Query Complexity



$$\mathbf{u}_1 = \mathbf{v}_1, \qquad \mathbf{e}_1 = \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|}$$

$$\mathbf{u}_2 = \mathbf{v}_2 - \mathrm{proj}_{\mathbf{u}_1}(\mathbf{v}_2), \qquad \mathbf{e}_2 = \frac{\mathbf{u}_2}{\|\mathbf{u}_2\|}$$

$$\mathbf{u}_3 = \mathbf{v}_3 - \mathrm{proj}_{\mathbf{u}_1}(\mathbf{v}_3) - \mathrm{proj}_{\mathbf{u}_2}(\mathbf{v}_3), \qquad \mathbf{e}_3 = \frac{\mathbf{u}_3}{\|\mathbf{u}_3\|}$$

$$\mathbf{u}_4 = \mathbf{v}_4 - \mathrm{proj}_{\mathbf{u}_1}(\mathbf{v}_4) - \mathrm{proj}_{\mathbf{u}_2}(\mathbf{v}_4) - \mathrm{proj}_{\mathbf{u}_3}(\mathbf{v}_4), \qquad \mathbf{e}_4 = \frac{\mathbf{u}_4}{\|\mathbf{u}_4\|}$$

$$\vdots \qquad\qquad \vdots$$

$$\mathbf{u}_k = \mathbf{v}_k - \sum_{j=1}^{k-1} \mathrm{proj}_{\mathbf{u}_j}(\mathbf{v}_k), \qquad \mathbf{e}_k = \frac{\mathbf{u}_k}{\|\mathbf{u}_k\|}.$$

ALGORITHM 1: Bernstein-Vazirani

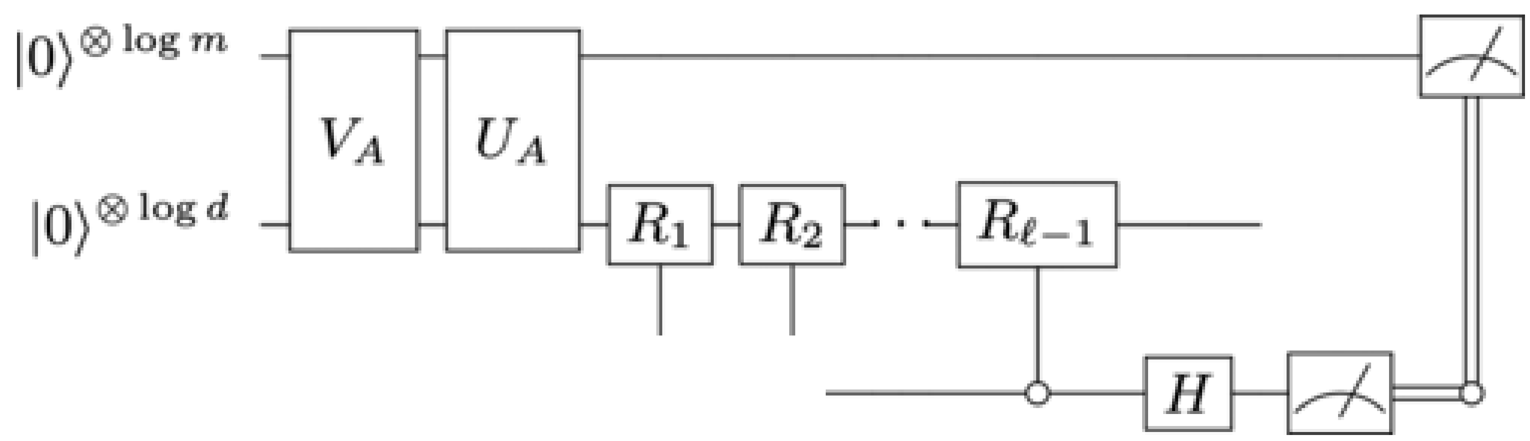
INPUT: Oracle access to f
OUTPUT : Secret string, s

1. Initialize an n-qubit register and an auxiliary qubit to $|0\rangle^{\otimes n}|1\rangle$
2. Apply the Hadamard transform to all $n+1$ qubits
3. Query the oracle, which encodes the function $f(x)$ into the phase
4. Apply the Hadamard transform again to the first n qubits
5. Measure the first n qubits, obtaining the hidden string

Svetlichny Functions

$$f(x) = \bigoplus_{\sigma \in \{0,1\}^n} s_\sigma \wedge \bigwedge_{\sigma_i = 1} x_i,$$

Quantum Gram-Schmidt Algorithm

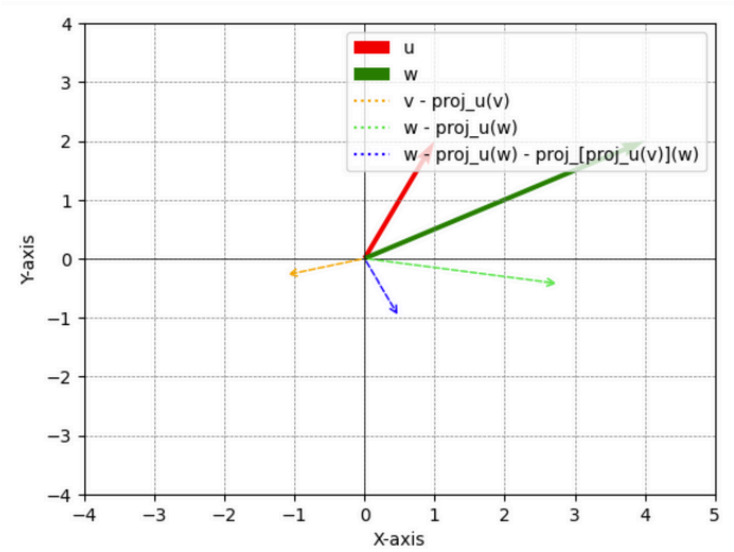


Positive Semidefinite

A matrix $A \in Herm(\mathscr{X})$ is called positive semidefinite if it satisfies $x * Ax \geq 0$ for all $x \in \mathscr{X}$.

$Pos(\mathscr{X})$ is the set of positive semidefinite matrices acting on $\mathscr{X}$.

$A \geq 0$ means $A \in Pos(\mathscr{X})$.



Positive definite

A matrix $A \in Herm(\mathscr{X})$ is called positive definite if it satisfies $x * Ax \geq 0$ for all **non-zero** $x \in \mathscr{X}$.

...he s...f positiv... efinite ... cting on $\mathscr{X}$.

A ... means ... $Pd(\mathscr{X}$

THE

Ne ative mide ite

A matrix $A \in Herm(\mathscr{X})$ is called negative semidefinite if $-A \in Pos(\mathscr{X})$.

... 0 me... $Pos(\mathscr{X})$.

IDEA

...gat... defini...

...atrix ... is call... fin... $-A \in P$...$)$.

$A < 0$ means $-A \in Pd(\mathscr{X})$.

Quantum Semidefinite Programming

Primal problem (P)

$$\sup \quad \langle M_{\text{accept}} \rho \rangle$$
$$\text{subject to} \quad \mathrm{Tr}(\rho) = 1,$$
$$\rho \in \mathrm{Pos}(\mathscr{X}),$$

$\alpha = maximize : Tr(X)$

$X = I_2$

$X \in Pos(\mathbb{C}^2)$

On Quantum Query Complexity

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ABSTRACT

This paper explores new function promises for the Bernstein-Vazirani algorithm, aiming to identify configurations that yield orthogonal basis vectors, a key factor in the efficiency of quantum algorithms. We investigate Svetlichny-like functions and demonstrate that, for these functions, the bits of s can be recovered classically in n queries. Through simulation and analysis, we find that the complexity of the function terms inversely affects the basis size. Extending our study to arbitrary functions involving bits of s, we show that recoverable bits can be classically determined in n queries, establishing that only linear complexity separation is achievable. Our results highlight the limitations and potential of new function promises in quantum query complexity, offering insights for the development of future quantum algorithms.

The project also includes interactive tutorials developed as learning material designed to supplement QWorld's[1] Quantum Annealing course module. The tutorials were focused on Quantum Gram-Schmidt (QGS) algorithm and Quantum [illegible]emi[illegible]ogramming (SDP), their practical applications and relevance to [illegible]uant[illegible] optimization techniques.

THE RESULT