PROBLEM SET 3

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1. The function $MULTIPLICATION$ means that our TM starts with 0^m10^n1 on its tape, and ends with $0^{m\times n}$ on its tape, where m and n may be greater than or equal to 0.
   (a) Design a **single-tape** TM that implements $MULTIPLICATION$.
   (b) Design a **multi-tape** TM that implements $MULTIPLICATION$.

2. Design a **2-tape TM** for the language L such that $L = \{\, w \mid$ w has an equal number of 1's and 0's $\}$.
   Hint: The first tape contains the input and is scanned from left to right. The second tape is used to store the excess of 0s over 1s or vice versa in the part of the input seen so far.

3. Show that the following languages are decidable:
   (a) $L_1 = \{\, \langle R, S\rangle \mid$ R and S are regular expressions and $L(R) \subseteq L(S)\,\}$.
   (b) $L_2 = \{\, \langle D\rangle \mid$ D is a DFA and L(D) = $\Sigma^*\,\}$.
   (c) $L_3 = \{\, \langle C\rangle \mid$ C is a CFG and C generates $\lambda\,\}$.

4. Let $T = \{\, \langle M\rangle \mid$ M is a TM that accepts w^R whenever it accepts w $\}$. Show that T is undecidable.

5. Let $E^{TM} = \{\, \langle M\rangle \mid$ M is a TM and $L(M) = \emptyset\,\}$. Show that $\overline{E^{TM}}$, the complement of E^{TM}, is:
   (a) Turing-recognizable, using an enumerator.
   (b) undecidable, using a reduction from another undecidable language.

6. Let $I = \{\, \langle M\rangle \mid$ M is a TM and L(M) is an infinite language $\}$. Show that I is:
   (a) undecidable, using a reduction from another undecidable language.
   (b) Turing-**un**recognizable, using a reduction from another **un**recognizable language.

7. Assess the time complexity of the following problems to determine if they are in the class P, NP and/or NP-Complete. Prove your answer to each part.
   (a) $C = \{\, \langle G\rangle \mid$ G is a connected undirected graph $\}$.
   (b) $D = \{\, \langle G, k\rangle \mid$ G has a dominating set with k nodes $\}$
   NOTE: A subset of the nodes of a graph G is a dominating set if every other node of G is adjacent to some node in the subset.
   (c) $E = \{\, \langle G\rangle \mid$ G is an undirected graph that contains a Euler circuit $\}$.
   NOTE: An Euler circuit of the graph is a simple cycle that includes all edges.