

PROBLEM SET 2

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1. Let $\Sigma = \{a, b, c\}$ and $L = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1, \text{ then } j = k\}$. Show that L is not regular using the pumping lemma.

Hint: You must show both cases where when $w = xyz$ that $x = \lambda$ and where x is composed of some characters.

2. Let $\Sigma = \{0, 1, +, =\}$ and $ADD = \{u + v = w \mid u, v, w \in \{0, 1\}^* \text{ and } u + v = w \text{ is a valid addition operation}\}$. Show that ADD is not regular using the pumping lemma.

3. For each of the following grammars, determine if they are ambiguous or unambiguous. If they are ambiguous, prove that by giving two parse trees for one same string.

a) $S \rightarrow XY \mid W$
 $X \rightarrow aXb \mid \lambda$
 $Y \rightarrow cY \mid \lambda$
 $W \rightarrow aWc \mid Z$
 $Z \rightarrow bZ \mid \lambda$

b) $S \rightarrow XX$
 $X \rightarrow aXb$
 $Y \rightarrow cY \mid \lambda$

c) $S \rightarrow aXY \mid bYX \mid \lambda$
 $Z \rightarrow aZ \mid a$
 $X \rightarrow aXY \mid a\lambda$
 $Y \rightarrow bYZ \mid b \mid \lambda$

4. Construct a CFG for the following language over $\Sigma = \{a, b, c\}$:

a) $A = \{xby \mid x, y \in \{a\}^*\}$
b) $B = \{a^i b^j c^k \mid j = i + k\}$
c) $C = \{a^i b^j c^k \mid j = i \text{ or } j = k\}$

5. Construct push-down automata for the following languages:

a) $A = \{w \mid w \text{ is a palindrome i.e. } w = w^R \text{ and } w, w^R \in \{0, 1\}^*\}$
b) $B = \{0^i 1^j 2^k \mid i = j + k\}$