JK Lakshmipat University, Jaipur
Institute of Engineering and Technology
Mid Term Examination II, October 2024
B. Tech., Odd Semester, 2024-25

Roll No. 2023BTech[redacted]

AS1209: Matrix Computations | Time: 1 hour 40 minutes | Max. Marks: 15

Instructions to students:
1. Do not write anything other than your roll number on the question paper.
2. Mention all the assumptions for your answers clearly.
3. Scientific calculator is allowed.

Q. 1	a. Given a matrix A of order 11×5, determine the orders of the matrices Q and R in the QR factorization of A. Also, specify the orders of matrices involved in the reduced QR factorization of A. b. Let A be a tridiagonal matrix and B be a pentadiagonal matrix, comment on the structural properties of the matrix AB. c. Given a banded matrix A of order 7 with lower bandwidth 2 and upper bandwidth 1, what will be the structure of factors L and U in the LU factorization of A? d. Let A be a toeplitz matrix with the first row $[2 \quad -1 \quad 0 \quad 0]$ and the first column $[2 \quad -1 \quad 0 \quad 0]^T$. Construct the complete 4×4 toeplitz matrix A.	4×1 *(CO 1,2,3)*
Q. 2	Given a Householder matrix $H = \begin{bmatrix} -\frac{1}{6} & -\frac{1}{2} & \frac{1}{6} & -\frac{5}{6} \\ -\frac{1}{2} & \frac{11}{14} & \frac{1}{14} & -\frac{5}{14} \\ \frac{1}{6} & \frac{1}{14} & \frac{41}{42} & \frac{5}{42} \\ -\frac{5}{6} & -\frac{5}{14} & \frac{5}{42} & \frac{17}{42} \end{bmatrix}$ and a vector $x = \begin{bmatrix} 1 \\ 3 \\ -1 \\ 5 \end{bmatrix}$, find $\\|Hx\\|_2$. Also, find $H(Hx)$. You are encouraged to use the properties of Householder matrix.	2 *(CO 4)*
Q. 3	a. Estimate the upper bound on the relative error in solution x, given by $\frac{\\|\delta x\\|}{\\|x\\|}$, of the linear system $Ax = b$. Assume a relative error in b, given by $\frac{\\|\delta b\\|}{\\|b\\|} = 10^{-3}$ and $cond(A) = 10^4$. b. Consider solving the linear system $Ax = b$. Assuming you are using IEEE double precision floating point numbers, how many digits of accuracy can you expect in your solution if $cond(A) = 1000$?	2 *(CO 1,2)*
Q. 4	The *Spectral norm* of a matrix $A \in \mathbb{R}^{m \times n}$ is defined as $\\|A\\|_2 = \sqrt{\lambda_{max}(A^T A)}$. For the matrix $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$, calculate the spectral norm.	2 *(CO 1,2)*

Q. 5	Let $A = \begin{bmatrix}0 & 1 & 1\\ 1 & 2 & 3\\ 1 & 1 & 1\end{bmatrix}$, we want to find the QR factorization of A using Givens rotations. We follow the following steps: **Step 1.** Compute $A_2 = G_1(1,2,\theta)^T A = \begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix} A_1 = \begin{bmatrix}-1 & -2 & -3\\ 0 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$. **Step 2.** Compute $A_3 = G_2(1,3,\theta)^T A_2 = \begin{bmatrix}? & 0 & ?\\ 0 & 1 & 0\\ ? & 0 & ?\end{bmatrix} A_2 = \begin{bmatrix}\sqrt{2} & ? & ?\\ 0 & 1 & 1\\ 0 & ? & ?\end{bmatrix}$. **Step 3.** Compute $A_4 = G_3(?,?,\theta)^T A_3 = \begin{bmatrix}1 & 0 & 0\\ 0 & ? & ?\\ 0 & ? & ?\end{bmatrix} A_3 = \begin{bmatrix}1.4142 & 2.1213 & 2.8284\\ 0 & ? & ?\\ 0 & 0 & 0.5774\end{bmatrix}$. a. Complete the missing values in the steps provided above. b. Find the orthogonal matrix Q and the upper triangular matrix R such that $A = QR$.	3 (CO 1,4)
Q. 6	Given $x = \begin{bmatrix}1\\ 2\\ 2\end{bmatrix}$, construct a Householder matrix H such that $Hx = \begin{bmatrix}*\\ 0\\ 0\end{bmatrix}$.	2 (CO 1,4)

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