JK Lakshmipat University, Jaipur

INSTITUTE OF ENGINEERING AND TECHNOLOGY

Mid Term-I Examination, September, 2025

B. Tech. (All branches), Semester V

Roll No...2023.BTECH.[redacted]..

CS1142 : Discrete Mathematics | Time: 1h 30 min | Max. Marks: 15

Instructions to students:

1. Do not write anything other than your roll number on question paper.
2. Use of calculator is not allowed.

Ques.1 Which of the following statements are true? (1mark)

(a) $\phi \in \{\{\phi\}\}$ (b) $\phi \subseteq \{\{\phi\}\}$ (c) $\{\phi\} \in \{\{\phi\}\}$ (d) $\{\phi\} \subseteq \{\{\phi\}\}$

Ques.2 Consider two functions of time (t), (1mark)

$$f(t) = 0.01t^2$$

$$g(t) = 4t; \text{ where } 0 < t < \infty$$

Now consider the two statements:

(i) For some $t > 0$, $g(t) > f(t)$.

(ii) $\exists\, T$ such that $f(t) > g(t)\ \forall t > T$.

Which one of the following options is correct-

(a) only (i) is correct (b) only (ii) is correct

(b) both (i) and (ii) are correct (d) neither (i) nor (ii) is correct

Ques.3 Construct a truth table for the following compound proposition. Check whether it is a tautology, a contradiction or a contingency? (1 mark)

$$(p \oplus q) \wedge (p \vee \neg r)$$

Ques.4 Let A, B and C be sets. Under what condition is the following statement true: (1 mark)

$$(A - B) \cup (A - C) = A$$

Ques.5 Let $f : A \to B$, where $n(A) = 25$, $B = \{x, y, z\}$ and $n(f^{-1}(x)) = 10$, $n(f^{-1}(y)) = 10$, $n(f^{-1}(z)) = 5$. If we define a relation R on A by aRb if $a, b \in A$ and $f(a) = f(b)$, how many ordered pairs are there in the relation R. [$n(A)$ = no. of elements in A] (1 mark)

Ques.6 Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$. For each of the following subsets B of $\mathbb{R}$, find $f^{-1}(B)$:

(a) $B = \{0, 1\}$ (b) $B = [0, 1]$ (1 mark)

Ques.7 Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers? (2marks)

(a) $\forall x \forall y (x^2 \neq y^3)$

(b) $\forall x \exists y (x^2 = y)$

Ques. 8 For $A = \{v, w, x, y, z\}$, the following is the (0, 1)-matrix for a relation R on A. Here the rows (from top to bottom) and the columns (from left to right) are indexed in the order v, w, x, y, z. (2 marks)

$$M(R)=\begin{bmatrix}1 & 1 & 1 & 1\\ 0 & 1 & 1 & 1\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

Check and justify whether the relation R is -

(a) reflexive (b) irreflexive (c) symmetric (d) equivalence

Ques 9. Prove the following by the principle of mathematical induction for $n \geq 1$ (2 marks)

$n^4 + 2n^3 + n^2$ is divisible by 4

Ques 10. Write the following argument in symbolic form, then establish the validity of argument using rules of inferences. (3 marks)

If the server is overloaded, then the monitoring system will raise an alert. If the firewall is disabled, then the network will be exposed. If the monitoring system raises an alert or the network is exposed, then the incident will be logged. The incident has not been logged. Consequently, the server was not overloaded and the firewall was not disabled.