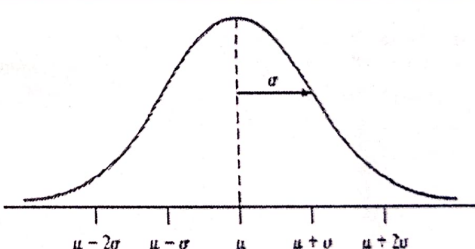Roll No. 2023.BTECH039..........

AS1117: Probability and Statistics   Time: 2 hours   Max. Marks: 20

Instructions to students:
1. Do not write anything other than your roll number on the question paper.
2. You may use non-programmable scientific calculator.
3. Unless specified, perform all the calculations till 4 decimal places with rounding.

Q. 1	a. A student takes a true-false examination consisting of 10 questions. He is completely unprepared so he plans to guess each answer. The guesses are to be made at random. Compute the probability that he guesses correctly at most five times. b. A website receives an average of 5 customer queries per hour. What is the probability that no queries are received in a 1-hour period?	1+1 (CO3)
Q. 2	A random variable X has the following probability distribution function: 	$X = x$	-2	-1	0	1	2	3	 	---	---	---	---	---	---	---	 	$f(x)$	0.1	k	0.2	$2k$	0.3	k	 Find the value of k, mean, and variance.	2 (CO3)
Q. 3	Given a normal random variable X with mean 1 and standard deviation 2, $X \sim N(1,2)$, write the density function of the normal distribution. Fill in the details below: The graph of the given probability density function is a _____ curve symmetric about _____ with its peak at _____ and height of the peak equals _____. Further, provide a title for the graph, label the axes. Mark and label the information you provided on the graph.	2 (CO3)
Q. 4	Calculate the following: a) $z_{0.025}$, where z is the z-score for the standard normal distribution. b) $b(2; 16, 0.05)$, where b is the binomial distribution function. c) $F(10; 12)$, where F is the Poisson cumulative distribution function.	3 (CO3)
Q. 5	An automatic machine fills distilled water in 500-ml bottles. Actual volumes are normally distributed about a mean of 500 ml and their standard deviation is 20 ml. (a) What proportion of the bottles are filled with water outside the tolerance limit of 475 ml to 525 ml? (b) To what value does the standard deviation need to be increased if 99% of the bottles must be within tolerance limits?	3 (CO3)

Q. 6	Let the random variable X has the probability density function $$f(x) = \begin{cases} ke^{-3x}, & x > 0 \\ 0, & x \le 0 \end{cases}.$$ a. Find the value of k and write the density function. b. Sketch the graph of the density function. c. Find $P(0.5 \le X \le 1)$. d. Find $E\big(g(X)\big)$, the expected value of the continuous random variable $g(X) = 2X + 1$.	4 × 1 (CO3)
Q. 7	If X is uniformly distributed with mean 1 and variance 4/3, find $P(X < 0)$. Given that formula for mean and variance are $\mu = \frac{a+b}{2}$ and $\sigma^2 = \frac{(b-a)^2}{12}$.	2 (CO3)
Q. 8	Consider Guwahati (G) and Delhi (D) whose temperatures can be classified as high (H), medium (M) and low (L). Let $P(H_G)$ denote the probability that Guwahati has high temperature. Similarly, $P(M_G)$ and $P(L_G)$ denote the probability of Guwahati having medium and low temperatures respectively. Similarly, we use $P(H_D)$, $P(M_D)$ and $P(L_D)$ for Delhi. The following table gives the conditional probabilities for Delhi's temperature given Guwahati's temperature. <table><tr><td></td><td>H_D</td><td>M_D</td><td>L_D</td></tr><tr><td>H_G</td><td>0.40</td><td>0.48</td><td>0.12</td></tr><tr><td>M_G</td><td>0.10</td><td>0.65</td><td>0.25</td></tr><tr><td>L_G</td><td>0.01</td><td>0.50</td><td>0.49</td></tr></table> Consider the first row in the table above. The first entry denotes $P(H_D|H_G) = 0.40$. Similarly, the next two entries are $P(M_D|M_G) = 0.48$ and $P(L_D|H_G) = 0.12$. Similarly for the other rows. If it is known that $P(H_G) = 0.2$, $P(M_G) = 0.5$, and $P(L_G) = 0.3$, then the probability (**correct to two decimal places**) that Guwahati has high temperature given that Delhi has high temperature is _____.	2 (CO2)

:::END:::