**JK Lakshmipat University, Jaipur**
**Institute of Engineering and Technology**
Mid Term Examination I, September 2025
B. Tech., Odd Semester, 2025-26

**Roll No.** 2023BTECH[redacted]

AS1209: Matrix Computations — Time: 1 hour 30 minutes — Max. Marks: 20

**Instructions to students:**
1. Do not write anything other than your roll number on the question paper.
2. Mention all the assumptions for your answers clearly.
3. Use of scientific calculator is allowed.

| | | |
|---|---|---|
| Q. 1 | Which of the following transformations are not linear? For each non-linear case, briefly justify why it fails linearity. (No need to prove linearity for the others.)<br>a. $T: M_{mn} \to M_{nm}$ defined by $T(A) = -A^T$<br>b. $T: M_{nn} \to M_{nn}$ defined by $T(A) = A^{-1}$, where $A$ is invertible.<br>c. $T: M_{nn} \to \mathbb{R}$ defined by $T(A) = \det(A)$<br>d. $T: M_{nn} \to \mathbb{R}$ defined by $T(A) = \text{trace}(A)$<br>e. $T: M_{mn} \to \mathbb{R}$ defined by $T(A) = \text{rank}(A)$<br>f. $T: M_{mn} \to \mathbb{R}$ defined by $T(A) = \lVert A \rVert_2$. | 3.0<br>*(CO 1)* |
| Q. 2 | Given a diagonal matrix $D = diag\{2, -3, 4, 0, -2\}$.<br>a. Find the rank of $D$.<br>b. Determine the eigenvalues and singular values of $D$.<br>c. Compute the spectral norm and Frobenius norm of $D$.<br>d. Compute the $cond_2(D)$. Discuss whether $D$ is well-conditioned or ill-conditioned for solving systems of equations $Dx = b$.<br>e. For $b = [1, 1, 1, 1, 1]^T$, discuss the consistency of the linear system $Dx = b$.<br>f. Consider the perturbed diagonal matrix $D_\varepsilon = diag\{2, -3, 4, \varepsilon, -2\}$, $\varepsilon > 0$. Find $cond_2(D_\varepsilon)$ and analyze its behaviour as $\varepsilon \to 0$. | 0.5+1.0+<br>1.5+1.0+<br>1.0+1.0<br>*(CO 1, 2, 4)* |
| Q. 3 | Prove the following properties of the Frobenius norm:<br>a. $\lVert A \rVert_F^2 = trace(A^T A)$<br>b. $\lVert AQ \rVert_F = \lVert A \rVert_F$, where $Q$ is an orthogonal matrix (with multiplication defined). | 3.0<br>*(CO 1)* |
| Q. 4 | If the determinant of a matrix is small, does this mean the matrix is close to singular? Justify. | 1.0<br>*(CO 1)* |
| Q. 5 | Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that first performs a horizontal shear that leaves $e_1$ unchanged and maps $e_2$ into $e_2 - 2e_1$ and then reflects points through the line $x_2 = -x_1$. Find the standard matrix of $T$. | 3.0<br>*(CO 1)* |
| Q. 6 | Let $U = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{2}{3} \\ \frac{1}{\sqrt{2}} & -\frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$ be a matrix with orthonormal columns and $x = \begin{bmatrix} \sqrt{2} \\ 3 \end{bmatrix}$. Verify that $\lVert Ux \rVert_2 = \lVert x \rVert_2$. | 2.0<br>*(CO 1)* |

| | | |
|---|---|---|
| Q. 7 | Estimate the upper bound on the relative error in solution $x$, given by $\frac{\|\delta x\|}{\|x\|}$, of the linear system $Ax = b$. Assume a relative error in $b$, given by $\frac{\|\delta b\|}{\|b\|} = 10^{-3}$ and $cond(A) = 10^4$. | 2.0 (CO 1, 2) |

:::END:::