DLCV Assignment 1

Due Date: 17/02/2023 11:59PM IST

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In this assignment, we will cover the following topics:

1) Training a simple Linear Model

2) Implementing Modules with Backprop functionality

3) Implementing Convolution Module on Numpy

It is crucial to get down to the nitty gritty of the code to implement all of these. No external packages (like caffe,pytorch etc), which directly give functions for these steps, are to be used.

Training a simple Linear Model

In this section, you will write the code to train a Linear Model. The goal is to classify an input X_i of size n into one of m classes. For this, you need to consider the following:

1) **Weight Matrix** $W_{n \times m}$: The Weights are multipled with the input X_i (vector of size n), to find m scores S_m for the m classes.

2) **The Loss function**:

- The Cross Entropy Loss: By interpreting the scores as unnormalized log probabilities for each class, this loss tries to measure dissatisfaction with the scores in terms of the log probability of the right class:

$$L_i = -\log\left(\frac{e^{f_{y_i}}}{\sum_j e^{f_j}}\right) \qquad \text{or equivalently} \qquad L_i = -f_{y_i} + \log\sum_j e^{f_j}$$

where f_{y_i} is the y_i-th element of the output of $W^T X_i$

3) **A Regularization term**: In addition to the loss, you need a Regularization term to lead to a more distributed (in case of L_2) or sparse (in case of L_1) learning of the weights. For example, with L_2 regularization, the loss has the following additional term:

$$R(W) = \sum_k \sum_l W_{k,l}^2$$

Thus the total loss has the form:

$$L = \underbrace{\frac{1}{N}\sum_i L_i}_{\text{data loss}} + \underbrace{\lambda R(W)}_{\text{regularization loss}}$$

4) **An Optimization Procedure**: This refers to the process which tweaks the weight Matrix $W_{n \times m}$ to reduce the loss function L. In our case, this refers to Mini-batch Gradient Descent algorithm. We adjust the weights $W_{n \times m}$, based on the gradient of the loss L w.r.t. $W_{n \times m}$. This leads to:

$$W_{t+1} = W_t - \alpha\frac{\partial L}{\partial W},$$

where α is the learning rate. Additionally, with "mini-batch" gradient descent, instead of finding loss over the whole dataset, we use a small sample B of the training data to make each learning step. Hence,

$$W_{t+1} = W_t - \alpha\frac{\partial \sum_{i \in B} L_{x_i}}{\partial W},$$

where $|B|$ is the batch size.

Question 1

Train a **Single-Layer Classifier** for the MNIST dataset.

- Use Softmax-Loss.
- Maintain a train-validation split of the original training set for finding the right value of λ for the regularization, and to check for over-fitting.
- Finally, evaluate the classification performance on the test-set.

main (loading data)

In [ ]:

```
# imports
import numpy as np
import matplotlib.pyplot as plt
import torchvision.datasets as datasets

from sklearn.model_selection import train_test_split
```

In [ ]:

```
mnist_trainset = datasets.MNIST(root='./data', train=True, download=True, transf
orm=None)
mnist_testset = datasets.MNIST(root='./data', train=False, download=True, transf
orm=None)

# train-val split
mnist_trainset, mnist_valset =  train_test_split(mnist_trainset, test_size=0.2,
random_state=42)
```

In [ ]:

```
mnist_testset
```

Out[ ]:

```
Dataset MNIST
    Number of datapoints: 10000
    Root location: ./data
    Split: Test
```

In [ ]:

```
def get_minibatch( training_set = mnist_trainset, batch_size = 64):
    """ suffle the training_set and return a generator of minibatches with tuple
of (x, y) """
    np.random.shuffle(training_set)

    for i in range(0, len(training_set), batch_size):
        yield training_set[i:i+batch_size]
```

single_layer_classifier class

In [ ]:

```
# Define the class for a Single Layer Classifier
class Single_layer_classifier():

    def __init__(self, input_size, output_size,SIGMA = 1e-5):

        self.input_size = input_size
        self.output_size = output_size
        self.W = np.random.normal(0, SIGMA, (input_size, output_size))
        self.W_grad = np.zeros(shape = (self.W.shape))
        self.x = None

        # Give the instance a weight matrix, initialized randomly
        # One possible strategy for a good initialization is Normal (0, σ) where
σ = 1e-3.
        # Try experimenting with different values of σ.
    def update_weights(self,lr = 0.001):
      self.W -= self.W_grad * lr
      self.W_grad = np.zeros(shape = (self.W.shape))

    # Define the forward function
    def forward(self, input_x):
        """ input_x : image of shape (1, 28, 28)
            returns : scores of shape (10, )
        """
        # flatten the input to 1D
        input_x = np.array(input_x).flatten()
        self.x = input_x
        # get the scores
        scores = np.dot(self.W.T, input_x)
        scores = scores.reshape(-1)
        return scores

    # Similarly a backward function
    # we define 2 backward functions (as Loss = L_data + L_reg, grad(Loss) = gra
d(L1) + grad(L2))

    def backward_Ldata(self, grad_from_loss):
        """ grad_from_loss : numpy array of 1D of shape (10, )
            returns : grad_matrix : numpy array of 2D of shape (784, 10)
        """

        # this function returns a matrix of the same size as the weights,
        # where each element is the partial derivative of the loss w.r.t. the co
rresponding element of W

        grad_matrix = np.outer( self.x, grad_from_loss )

        return grad_matrix

    def backward_Lreg(self,reg_coeff = 1):
        """
            reg_coeff : regularization coefficient
            returns : grad_matrix : numpy array of 2D of shape (784, 10)
        """
        # this function returns a matrix of the same size as the weights,
        # where each element is the partial derivative of the regularization-ter
m
        # w.r.t. the corresponding element of W

        # for L2 loss
        grad_matrix = 2 * reg_coeff * self.W
```

```
        return grad_matrix
```

loss_function

In [ ]:

```
# Implement the Softmax loss function
def loss_function(input_y,scores):
    """ don't change these. write seperate func. for Q3

    input_y : label of the image (int)
    scores : numpy array of 1D of shape (10, )
    returns : loss : float
    """

    y_i = input_y
    # to avoid overflow
    s_max = np.max(scores)
    scores_reduced = scores - s_max
    # print("s_max:",s_max) #debug
    temp = s_max + np.log( np.exp(scores_reduced).sum() )
    loss = -1* scores[y_i] + temp #np.log( np.sum( np.exp( scores )))

    return loss
# def loss_backward(loss):

def loss_backward(input_y,scores):
    """ gradient depends on scores and label_y .
    input_y : label of the image (int)
    scores : numpy array of 1D of shape (10, )
    returns : grad_from_loss : numpy array of 1D of shape (10, )
    """
    # This part deals with the gradient of the loss w.r.t the output of network
    # for example, in case of softmax loss(-log(q_c)), this part gives grad(los
s) w.r.t. q_c
    # pass this to backward_ldata

    ## gradient of softmax loss wrt scores
    # to avoid overflow
    s_max = np.max(scores)
    scores_reduced = scores - s_max

    temp = (1/np.sum( np.exp( scores_reduced )) ) * np.exp(scores_reduced )
    temp[input_y] += ( -1 )
    grad_from_loss = temp

    return grad_from_loss
```

looped_utils

In [ ]:

```
def test_loss_and_accuracy(model, test_set):
    """ get the accuracy and loss on the test set
    """
    test_loss = 0
    test_correct = 0
    for image, label in test_set:
        # forward
        scores = model.forward(image)
        # loss
        loss = loss_function(label,scores)
        test_loss += loss
        # accuracy
        if np.argmax(scores) == label:
            test_correct += 1
    test_loss /= len(test_set)
    test_accuracy = test_correct/len(test_set)
    return test_loss, test_accuracy
```

In [ ]:

```python
def train(model,train_set,val_set,epochs = 10, batch_size = 64, lr = 0.001, reg_
coeff = 1, verbose = True):
    """ train the model for given epochs and return the train and validation los
ses
    model : instance of Single_layer_classifier
    train_set : training set
    val_set : validation set
    epochs : number of epochs
    batch_size : batch size
    lr : learning rate
    reg_coeff : regularization coefficient
    returns : train_losses : list of train losses
              train_acc
              val_losses
              val_acc
    """

    train_losses = []
    train_acc = []
    val_losses = []
    val_acc = []
    for epoch in range(epochs):
        if verbose:
            print("Epoch:",epoch)
        # train
        train_loss = 0
        train_correct = 0
        for minibatch in get_minibatch(train_set, batch_size):
            for image, label in minibatch:
                # forward
                scores = model.forward(image)
                #train_accuracy
                train_correct += (np.argmax(scores) == label)

                # loss
                loss = loss_function(label,scores)
                train_loss += loss
                # backward
                grad_from_loss = loss_backward(label,scores)
                grad_Ldata = model.backward_Ldata(grad_from_loss)
                grad_Lreg = model.backward_Lreg(reg_coeff)
                grad = grad_Ldata + grad_Lreg
                # update weights
                model.W_grad += grad
            model.update_weights(lr)
        train_loss /= len(train_set)
        train_losses.append(train_loss)
        train_acc.append(train_correct/len(train_set))

        # -------------------
        # validation
        if val_set is not None:
            val_loss, val_accuracy = test_loss_and_accuracy(model, val_set)
            val_losses.append(val_loss)
            val_acc.append(val_accuracy)
        if verbose:
            print("Train loss:",train_loss)
            print("Val loss:",val_loss)
            print("Val accuracy:",val_accuracy)
            print("val loss:",val_loss)
```

```
    return train_losses, train_acc, val_losses, val_acc
```

In [ ]:

```
import matplotlib.pyplot as plt
def plot_loss_and_accuracy(train_loss, train_acc,val_loss=None,val_acc=None):
    """ plot the loss and accuracy curves

    """
    plt.figure(figsize=(10,5))
    plt.subplot(1,2,1)
    plt.plot(train_loss, label = "train_loss")

    if val_loss is not None:
        plt.plot(val_loss, label = "val_loss")


    plt.legend()
    plt.xlabel("Epochs")
    plt.ylabel("Loss")

    plt.subplot(1,2,2)
    plt.plot(train_acc, label = "train_acc")
    if val_acc is not None:
        plt.plot(val_acc, label = "val_acc")
    plt.legend()
    plt.xlabel("Epochs")
    plt.ylabel("Accuracy")

    plt.show()
```

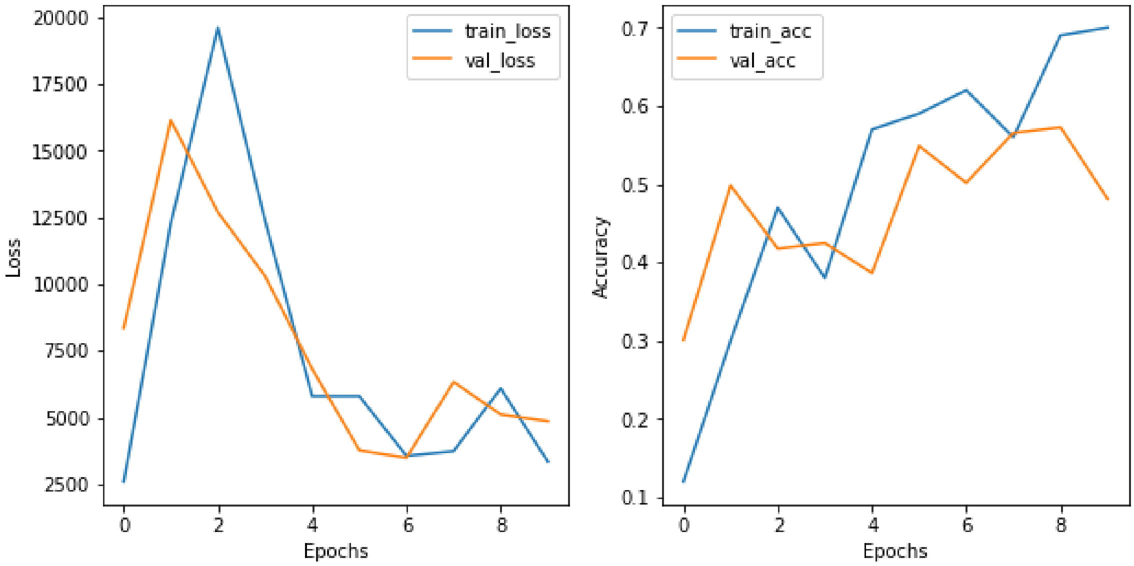
Training

In [ ]:

```
# %%capture

# try to overfit to small dataset
size = 100            # got 99.55 with 50 epochs
mini_train_set = mnist_trainset[:size]
# initialize the model
model = Single_layer_classifier( 28*28, 10)
# train the model
train_losses, train_acc, val_losses, val_acc = train(model,mini_train_set,mnist_
valset,epochs = 10, batch_size = 64, lr = 0.001, reg_coeff = 1, verbose = False)
```

In [ ]:

```
# plot the loss and accuracy curves
plot_loss_and_accuracy(train_losses, train_acc,val_losses,val_acc)
```



In [ ]:

```
# test the model on the validation set
test_loss, test_accuracy = test_loss_and_accuracy(model, mnist_valset)
print("Test loss:",test_loss)
print("Test accuracy:",test_accuracy)
```

```
Test loss: 4862.645972277221
Test accuracy: 0.48125
```

Tuning

In [ ]:

```
# The next step is to find the optimal value for lambda, number of epochs, learn
ing rate and batch size.
# CHOSE ANY TWO from the above mentioned to tune.
# Create plot and table to show the effect of the hparams.

# initialize the model
model = Single_layer_classifier( 28*28, 10)

lr = [0.1,0.01,1e-3,1e-4]
lambda_ = [0,0.005,0.01]
num_epochs = 5

# create a dataframe to store the results
import pandas as pd
df = pd.DataFrame(columns = ["lr","lambda","epochs","val_loss","val_accuracy"])


for lr_ in lr:
    for reg_val in lambda_:
        print("lr:",lr_,"lambda:",reg_val,"epochs:",num_epochs, "\t--->" ,end =
" ")
        # train the model
        train_losses, train_acc, val_losses, val_acc = train(model,mini_train_se
t,mnist_valset,epochs = num_epochs, batch_size = 64, lr = lr_, reg_coeff = reg_v
al, verbose = False)
        # test the model on the validation set
        val_loss, val_accuracy = test_loss_and_accuracy(model, mnist_valset)
        print("val_loss:",val_loss,"val_accuracy:",val_accuracy)
        df = df.append({"lr":lr_,"lambda":reg_val,"epochs":num_epochs,"val_los
s":val_loss,"val_accuracy":val_accuracy},ignore_index=True)
```

```
lr: 0.1 lambda: 0 epochs: 5     ---> val_loss: 378406.0234815161 val
_accuracy: 0.5584166666666667
lr: 0.1 lambda: 0.005 epochs: 5         ---> val_loss: 113388.203083
07986 val_accuracy: 0.6938333333333333
lr: 0.1 lambda: 0.01 epochs: 5  ---> val_loss: 39345.056983425304 va
l_accuracy: 0.6938333333333333
lr: 0.01 lambda: 0 epochs: 5    ---> val_loss: 39345.056983425304 va
l_accuracy: 0.6938333333333333
lr: 0.01 lambda: 0.005 epochs: 5        ---> val_loss: 37421.1118566
4472 val_accuracy: 0.6938333333333333
lr: 0.01 lambda: 0.01 epochs: 5         ---> val_loss: 33841.6292305
4688 val_accuracy: 0.6938333333333333
lr: 0.001 lambda: 0 epochs: 5   ---> val_loss: 33841.62923054688 val
_accuracy: 0.6938333333333333
lr: 0.001 lambda: 0.005 epochs: 5       ---> val_loss: 33672.7979923
0808 val_accuracy: 0.6938333333333333
lr: 0.001 lambda: 0.01 epochs: 5        ---> val_loss: 33337.5681601
2119 val_accuracy: 0.6938333333333333
lr: 0.0001 lambda: 0 epochs: 5  ---> val_loss: 33337.56816012119 val
_accuracy: 0.6938333333333333
lr: 0.0001 lambda: 0.005 epochs: 5      ---> val_loss: 33320.9030933
5998 val_accuracy: 0.6938333333333333
lr: 0.0001 lambda: 0.01 epochs: 5       ---> val_loss: 33287.5970501
6231 val_accuracy: 0.6938333333333333
```

In [ ]:

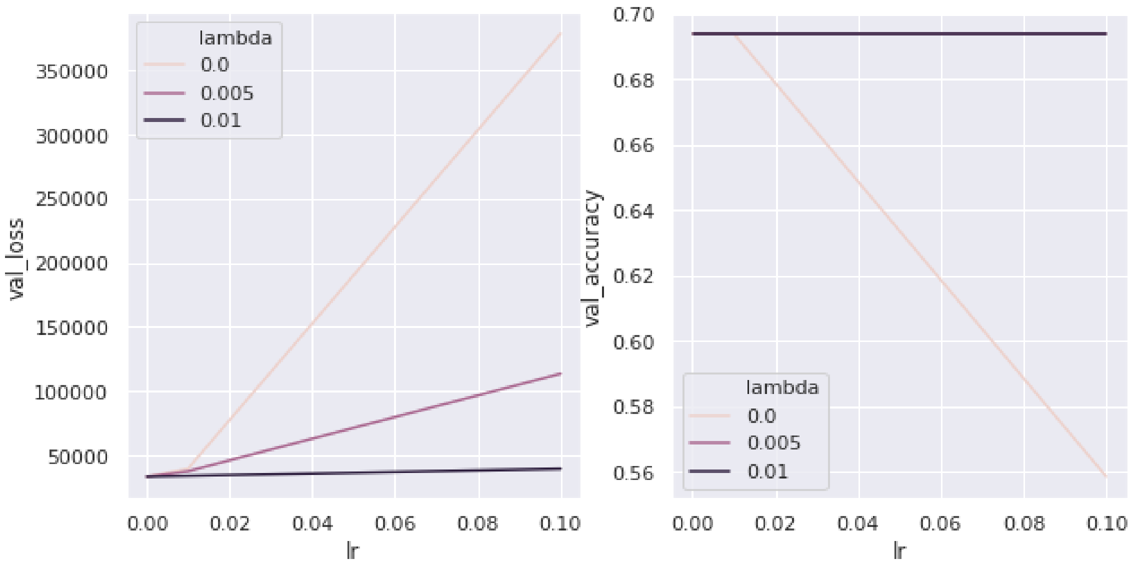
```
df
```

Out[ ]:

	lr	lambda	epochs	val_loss	val_accuracy
0	0.1000	0.000	5.0	378406.023482	0.558417
1	0.1000	0.005	5.0	113388.203083	0.693833
2	0.1000	0.010	5.0	39345.056983	0.693833
3	0.0100	0.000	5.0	39345.056983	0.693833
4	0.0100	0.005	5.0	37421.111857	0.693833
5	0.0100	0.010	5.0	33841.629231	0.693833
6	0.0010	0.000	5.0	33841.629231	0.693833
7	0.0010	0.005	5.0	33672.797992	0.693833
8	0.0010	0.010	5.0	33337.568160	0.693833
9	0.0001	0.000	5.0	33337.568160	0.693833
10	0.0001	0.005	5.0	33320.903093	0.693833
11	0.0001	0.010	5.0	33287.597050	0.693833

In [ ]:

```
# make plots for the results
import seaborn as sns
sns.set()
plt.figure(figsize=(10,5))
plt.subplot(1,2,1)
sns.lineplot(x="lr", y="val_loss", hue="lambda", data=df, legend = 'full')
plt.subplot(1,2,2)
sns.lineplot(x="lr", y="val_accuracy", hue="lambda", data=df, legend = 'full')
plt.show()
```



final training

In [ ]:

```
### Report final performance on MNIST test set
# from above we can see that the best value for lambda is 0.01 and the best valu
e for lr is 0.001

# initialize the model
model = Single_layer_classifier( 28*28, 10)
# train the model
train_losses, train_acc, val_losses, val_acc = train(model,mnist_trainset,mnist_
valset,\
                              epochs = 20, batch_size = 64, lr = 0.001, reg_coeff = 0.
01, verbose = False)

# plot the loss and accuracy curves
# plot_loss_and_accuracy(train_losses, train_acc,val_losses,val_acc)
```

Report final performance on mnist test set

In [ ]:

```
# test on the test set
test_loss, test_accuracy = test_loss_and_accuracy(model, mnist_testset)
print("Test loss:",test_loss)
print("Test accuracy:",test_accuracy)
```

```
Test loss: 1003.3873654058586
Test accuracy: 0.8508
```

Find the best performing class and the worst performing class

In [ ]:

```
# best and worst performing classes on the test set

# seperate the test set into 10 classes
classes = [[] for i in range(10)]
for i in range(len(mnist_testset)):
    # class based on the label
    label = mnist_testset[i][1]
    classes[label].append(mnist_testset[i])


# test the model on each class
class_acc = []
class_loss = []
for i in range(10):
    test_loss, test_accuracy = test_loss_and_accuracy(model, classes[i])
    class_acc.append(test_accuracy)
    class_loss.append(test_loss)

# bar plot for the accuracy
plt.figure(figsize=(10,5))
plt.subplot(1,2,1)
plt.bar(range(10),class_acc)
plt.xlabel("Class")
plt.ylabel("Accuracy")
plt.title("Accuracy for each class")

# bar plot for the loss
plt.subplot(1,2,2)
plt.bar(range(10),class_loss)
plt.xlabel("Class")
plt.ylabel("Loss")
plt.title("Loss for each class")
plt.show()
```

- best performing class : 9
- worst performing class : 7

Training a Linear Classifier on MNIST from scikit-learn

In this section you have to train a linear classifier from the scikit-learn library and compare its results against your implementation. ([https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LinearRegression.html (https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LinearRegression.html)](https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LinearRegression.html))

In [ ]:

```
# Import the necessary libraries
from sklearn.linear_model import LogisticRegression
from sklearn.multiclass import OneVsRestClassifier


def dataset_to_arrays(dataset):
  """ flattens the images, and normalizes.
  """
  X = np.array( [np.array(x).flatten()/255 for x,y in dataset])
  Y = np.array( [np.array(y).flatten() for x,y in dataset])
  return X,Y
```

In [ ]:

```
%%capture

# Define the classifier
classifier = OneVsRestClassifier(LogisticRegression())

X,y = dataset_to_arrays(mnist_trainset)
test_x, test_y = dataset_to_arrays(mnist_testset)

# Fit the model to the training data
classifier.fit(X,y)

# Make predictions on the test data
y_pred = classifier.predict(test_x)
```

In [ ]:

```
# Calculate the accuracy
from sklearn.metrics import accuracy_score
print("Accuracy on the test set: %.2g" % accuracy_score(test_y, y_pred))
```

```
Accuracy on the test set: 0.92
```

Compare the training and test accuracies for the your implementation and linear classifier from scikit-learn

- the linear classifier from sklearn doesn't learn on iterations. so it does not give train accuracy for each iteration.

In [ ]:

```
# trainset
print("sklearn linear model" )
print("--------------------")

for i,dataset in enumerate([mnist_trainset,mnist_valset,mnist_testset]):
  names = ["train", "val", "test"]
  X,y = dataset_to_arrays(dataset)
  y_pred = classifier.predict(X)
  print("Accuracy on the %s set: %.2g" % (names[i], accuracy_score(y, y_pred)))


print("\n my linear model" )
print("--------------------")

for i,dataset in enumerate([mnist_trainset,mnist_valset,mnist_testset]):
  names = ["train", "val", "test"]
  # use model
  test_loss, test_accuracy = test_loss_and_accuracy(model, dataset)
  print("Accuracy on the %s set: %.2g" % (names[i], test_accuracy))
```

```
sklearn linear model
--------------------
Accuracy on the train set: 0.93
Accuracy on the val set: 0.92
Accuracy on the test set: 0.92

 my linear model
--------------------
Accuracy on the train set: 0.85
Accuracy on the val set: 0.85
Accuracy on the test set: 0.85
```

Any additional observations / comments?

BONUS Question

Observe the effect on test set accuracy by changing the number of training samples.

Train on 10%, 20% and 50% training data and plot the percentage of training data v.s. the test accuracy.

In [ ]:

```
# train on 10%,20%,50% of the training set and test on the test set
for percent in [0.1,0.2,0.5]:
  print("Training on %d%% of the training set" % (percent*100))
  # get the training set
  train_set = mnist_trainset[:int(len(mnist_trainset)*percent)]
  # initialize the model
  model = Single_layer_classifier( 28*28, 10)
  # train the model
  train_losses, train_acc, val_losses, val_acc = train(model,train_set,None,\
                           epochs = 20, batch_size = 64, lr = 0.001, reg_coeff =
0.01, verbose = False)
  # test on the test set
  test_loss, test_accuracy = test_loss_and_accuracy(model, mnist_testset)
  print("Test loss:",test_loss)
  print("Test accuracy:",test_accuracy)
  print("")
```

```
Training on 10% of the training set
Test loss: 955.2665598715907
Test accuracy: 0.8687

Training on 20% of the training set
Test loss: 970.3732014170986
Test accuracy: 0.8637

Training on 50% of the training set
Test loss: 809.0257723878053
Test accuracy: 0.8719
```

Question 2

Implementing Backpropagation

Now that you have had some experience with single layer networks, we can proceed to more complex architectures. But first we need to completely understand and implement backpropagation.

Backpropagation:

Simply put, a way of computing gradients of expressions through repeated application of chain rule. If

$$L = f(g(h(\mathbf{x})))$$

then, by the chain rule we have:

$$\frac{\partial L}{\partial \mathbf{x}} = \frac{\partial f}{\partial g} \cdot \frac{\partial g}{\partial h} \cdot \frac{\partial h}{\partial \mathbf{x}}$$

Look into the class Lecture for more detail

Question 2 : Scalar Backpropagation

Evaluate the gradient of the following function w.r.t. the input:

$$f(x, y, z) = log(\sigma(\frac{cos(\pi \times x) + sin(\pi \times y/2)}{tanh(z^2)}))$$

where σ is the sigmoid function. Find gradient for the following inputs:

- (x, y, z) = (2,4,1)
- (x, y, z) = (9,14,3)
- (x, y, z) = (128,42,666)
- (x, y, z) = (52,14,28)

Main idea :

1. make graph
2. forward should give a number
3. backward should take upstream grad
4. seperate class for each function

define classes

```
In [ ]:
```

```python
import numpy as np

class Scalar :
  def __init__(self, value):
    self.value = value
    self.grad = 0
  def forward(self):
    return self.value
  def backward(self,upstream_grad):
    self.grad = upstream_grad
  def __repr__(self):
    """For printing.
      only two decimal places"""
    return "%.2f" % self.value

class Mult:
  def __init__(self,a,x):
    self.a = a
    self.x = x
  def forward(self):
    return self.a.forward() * self.x.forward()
  def backward(self, upstream_grad):
    self.a.backward(upstream_grad * self.x.forward())
    self.x.backward(upstream_grad * self.a.forward())
  def __repr__(self):
    return "%s*%s" % (self.a, self.x)

class Add:
  def __init__(self,a,b):
    self.a = a
    self.b = b
  def forward(self):
    return self.a.forward() + self.b.forward()
  def backward(self, upstream_grad):
    self.a.backward(upstream_grad)
    self.b.backward(upstream_grad)
  def __repr__(self):
    return "%s + %s" % (self.a, self.b)

class div:
  def __init__(self,a,b):
    self.a = a
    self.b = b
  def forward(self):
    return self.a.forward() / self.b.forward()
  def backward(self, upstream_grad):
    self.a.backward(upstream_grad / self.b.forward())
    self.b.backward(- upstream_grad * self.a.forward() / (self.b.forward() **
2))
  def __repr__(self):
    return "{%s}/{%s}" % (self.a, self.b)

class Square:
  def __init__(self,x):
    self.x = x
  def forward(self):
    return self.x.forward() ** 2
  def backward(self, upstream_grad):
    self.x.backward(2 * self.x.forward() * upstream_grad)
  def __repr__(self):
    return "(%s)^2" % self.x
```

```python
class Sigmoid:
  def __init__(self,x):
    self.x = x
  def forward(self):
    return 1 / (1 + np.exp(-self.x.forward()))
  def backward(self, upstream_grad):
    self.x.backward(upstream_grad * (1 - self.forward()) * self.forward())
  def __repr__(self):
    return "σ(%s)" % self.x


class Tanh:
  def __init__(self,x):
    self.x = x
  def forward(self):
    return np.tanh(self.x.forward())
  def backward(self, upstream_grad):
    self.x.backward(upstream_grad * (1 - self.forward() ** 2))
  def __repr__(self):
    return "Tanh(%s)" % self.x

class sin:
  def __init__(self,x):
    self.x = x
  def forward(self):
    return np.sin(self.x.forward())
  def backward(self, upstream_grad):
    self.x.backward(upstream_grad * np.cos(self.x.forward()))
  def __repr__(self):
    return "sin(%s)" % self.x

class cos:
  def __init__(self,x):
    self.x = x
  def forward(self):
    return np.cos(self.x.forward())
  def backward(self, upstream_grad):
    self.x.backward(- upstream_grad * np.sin(self.x.forward()))
  def __repr__(self):
    return "cos(%s)" % self.x

class log:
  def __init__(self,x):
    self.x = x
  def forward(self):
    return np.log(self.x.forward())
  def backward(self, upstream_grad):
    self.x.backward(upstream_grad / self.x.forward())
  def __repr__(self):
    return "log(%s)" % self.x
```

In [ ]:

```
# Now write the class func
# which constructs the graph (all operators), forward and backward functions.

class Func:
    def __init__(self):
        pass

    def forward(self,x,y,z):
      """x,y,z are the input variables and float values not Scalar objects
      """
      # input variables
      self.x = Scalar(x)
      self.y = Scalar(y)
      self.z = Scalar(z)

      # constants
      pi = Scalar(np.pi)
      half_pi = Scalar(np.pi/2)

      # function graph
      a = cos(Mult(pi,self.x))
      b = sin(Mult(half_pi,self.y))
      c = Tanh(Square(self.z))
      d = div(Add(a,b),c)
      e = Sigmoid(d)
      f = log(e)

      self.f = f
      print(f)

      # forward pass
      return self.f.forward()


    def backward(self):
        # backward pass
        self.f.backward(1)
        grad_x = self.x.grad
        grad_y = self.y.grad
        grad_z = self.z.grad

        return [grad_x,grad_y,grad_z]
```

testing

In [ ]:

```
def test(x,y,z):
    f = Func()
    print("f(%.5g,%.5g,%.5g) = %.5g" % (x,y,z,f.forward(x,y,z)))
    print("df/dx = %.5g" % f.backward()[0])
    print("df/dy = %.5g" % f.backward()[1])
    print("df/dz = %.5g" % f.backward()[2])
    print()


test(2,4,1)
test(9,14,3)
test(128,42,666)
test(52,14,28)
```

```
log(σ({cos(3.14*2.00) + sin(1.57*4.00)}/{Tanh((1.00)^2)}))
f(2,4,1) = -0.23823
df/dx = 2.1417e-16
df/dy = 0.43721
df/dz = -0.30697

log(σ({cos(3.14*9.00) + sin(1.57*14.00)}/{Tanh((3.00)^2)}))
f(9,14,3) = -1.3133
df/dx = -2.5314e-15
df/dy = -1.1483
df/dz = 2.6722e-07

log(σ({cos(3.14*128.00) + sin(1.57*42.00)}/{Tanh((666.00)^2)}))
f(128,42,666) = -0.31326
df/dx = 1.3244e-14
df/dy = -0.42245
df/dz = -0

log(σ({cos(3.14*52.00) + sin(1.57*14.00)}/{Tanh((28.00)^2)}))
f(52,14,28) = -0.31326
df/dx = -6.6263e-15
df/dy = -0.42245
df/dz = -0
```

Question 3 : Modular Vector Backpropagation

- Construct a Linear Layer module, implementing the forward and backward functions for arbitrary sizes.
- Construct a ReLU module, implementing the forward and backward functions for arbitrary sizes.
- Create a 2 layer MLP using the constructed modules.
- Modifying the functions built in Question 1 , train this two layer MLP for the same data set, MNIST, with the same train-val split.

define classes

In [ ]:

```
# Class for Linear Layer (If you're stuck, you can refer to code of PyTorch/Tens
orflow packages)

class Linear:
    """ this works for batch input
    it has bais also
    input is batch of inputs not one image"""

    def __init__(self, input_size, output_size,SIGMA = 10**(-3)):
        self.input_size = input_size
        self.output_size = output_size

        # self.W = np.random.randn(input_size, output_size)
        # normal distribution with mean 0 and std  = 10^(-3)

        self.W = np.random.normal(0, SIGMA, (input_size, output_size))
        self.b = np.zeros(output_size)

        self.grad_W = np.zeros((input_size, output_size))
        self.grad_b = np.zeros(output_size)


    def forward(self, x):
        self.x = x
        self.y = np.dot(x, self.W) + self.b
        return self.y

    def backward(self, grad_y):
        """ grad_y is the gradient of the loss w.r.t. the output of the layer fo
r a batch of inputs

            so grad_y has shape (batch_size, output_size)
        """
        self.grad_W = self.x.T @ grad_y             # this is sum of all the gra
dients of the loss w.r.t. the output of the layer for a batch of inputs
        self.grad_b = np.sum(grad_y, axis=0)
        grad_x = np.dot(grad_y, self.W.T)
        return grad_x

    def step(self, learning_rate):
        self.W -= learning_rate * self.grad_W
        self.b -= learning_rate * self.grad_b
# check: how it  changes with zero centring the images
```

In [ ]:

```
# Class for ReLU
class ReLU:
    def __init__(self):
        pass

    def forward(self, x):
        self.x = x
        self.y = np.maximum(0, x)
        return self.y

    def backward(self, grad_y):
        """ grad_y is the gradient of the loss w.r.t. the output of the layer fo
r a batch of inputs
            so grad_y has shape (batch_size, output_size)
        """
        grad_x = grad_y * (self.x > 0)
        return grad_x
```

In [ ]:

```
# Your 2 layer MLP
class MLP:
    """ input_size is the size of the flattened image
    x : batch of images
    image is flattened in forward pass


    """

    def __init__(self, input_size, hidden_size, output_size):
        self.linear1 = Linear(input_size, hidden_size)
        self.relu = ReLU()
        self.linear2 = Linear(hidden_size, output_size)

    def forward(self, x):
        #flatten the image
        x = x.reshape(x.shape[0], -1)
        #normalize the image
        x = x/255

        x = self.linear1.forward(x)
        x = self.relu.forward(x)
        x = self.linear2.forward(x)
        return x

    def backward(self, grad_y):
        """ grad_y is the gradient of the loss w.r.t. the output of the layer
(i.e scores) for a batch of inputs
            so grad_y has shape (batch_size, output_size)
        """
        grad_x = self.linear2.backward(grad_y)
        grad_x = self.relu.backward(grad_x)
        grad_x = self.linear1.backward(grad_x)
        return grad_x

    def step(self, learning_rate):
        self.linear1.step(learning_rate)
        self.linear2.step(learning_rate)

    def parameters(self):
        return [self.linear1.W, self.linear1.b, self.linear2.W, self.linear2.b]

    def gradients(self):
        return [self.linear1.grad_W, self.linear1.grad_b, self.linear2.grad_W, s
elf.linear2.grad_b]
```

In [ ]:

```
# softmax class for the loss function
class Softmax_loss:
    def __init__(self):
        pass
    def forward(self,scores,y_labels):
        """ scores shape : (batch_size, output_size)
            y_labels shape : (batch_size, )


            return loss
            loss is mean of the loss for each batch. so keep the count of the nu
mber of batches. finally, divide the loss by it.
            loss shape : (1, )
        """
        self.scores = scores
        self.y_labels = y_labels

        # subtract max for numerical stability
        scores -= np.max(scores, axis=1, keepdims=True)


        first_term = -scores[range(len(y_labels)), y_labels]
        second_term = np.log(np.sum(np.exp(scores), axis=1))
        loss = first_term + second_term

        return np.mean(loss)

    def backward(self):
        """ return grad_scores
            grad_scores shape : (batch_size, output_size)
        """
        scores = self.scores.copy()
        # subtract max for numerical stability
        scores -= np.max(self.scores, axis=1, keepdims=True)
        # fix the error


        grad_scores = np.exp(scores) / np.sum(np.exp(scores), axis=1, keepdims=T
rue)
        grad_scores[range(len(self.y_labels)), self.y_labels] -= 1
        return grad_scores
```

utils for batched data

In [ ]:

```
def accuracy(scores, y_labels):
    """ scores shape : (batch_size, output_size)
        y_labels shape : (batch_size, )

        return corrects, total
    """
    predictions = np.argmax(scores, axis=1)
    corrects = np.sum(predictions == y_labels)
    total = len(y_labels)
    return corrects, total
```

In [ ]:

```
 from tqdm import tqdm

def train(model, train_set, N_EPOCHS= 10 , BATCH_SIZE=16, LEARNING_RATE=0.001,te
sting = False,print_to_console = True):
  """
  dataset : list of tuples (x,y) where x is the image and y is the label
  model: model.forward(batch_x) => batch_scores

  loss = softmax_loss

  Return:
  training_loss , training_accuracy : lists
  ----------------
  for testing or validation : set testing = True
  """

  if testing:
    N_EPOCHS = 1


  loss_fn = Softmax_loss()


  # store loss and accuracy
  train_loss = []
  train_acc = []

  # Train for t epochs:
  for epoch in tqdm(range(N_EPOCHS)):
      if print_to_console:
        print("Epoch: ",epoch)



      #metrics
      epoch_loss = 0
      batch_count = 0

      epoch_corrects = 0
      epoch_total = 0



      generator = get_minibatch(train_set, batch_size= BATCH_SIZE)

      for mini_batch in generator:

        X = np.array([np.array(x) for x,y in mini_batch])
        Y = np.array([y for x,y in mini_batch])

        # Forward pass
        scores = model.forward(X)
        batch_mean_loss = loss_fn.forward(scores,Y)
        batch_mean_acc = accuracy(scores,Y)

        # update metrics
        epoch_loss += batch_mean_loss
        epoch_corrects += batch_mean_acc[0]
        epoch_total += batch_mean_acc[1]

        batch_count += 1
```

```
        if not testing:
          # Backward pass
          grad_scores = loss_fn.backward()
          model.backward(grad_scores)

          # Update parameters
          model.step(LEARNING_RATE)

      # update metrics for epoch
      epoch_loss /= batch_count
      epoch_acc = epoch_corrects / epoch_total

      # store metrics
      train_loss.append(epoch_loss)
      train_acc.append(epoch_acc)

      if print_to_console:
        print( f"loss: {epoch_loss:.4f}  acc: {epoch_acc:.4f}" )

  return train_loss, train_acc
```

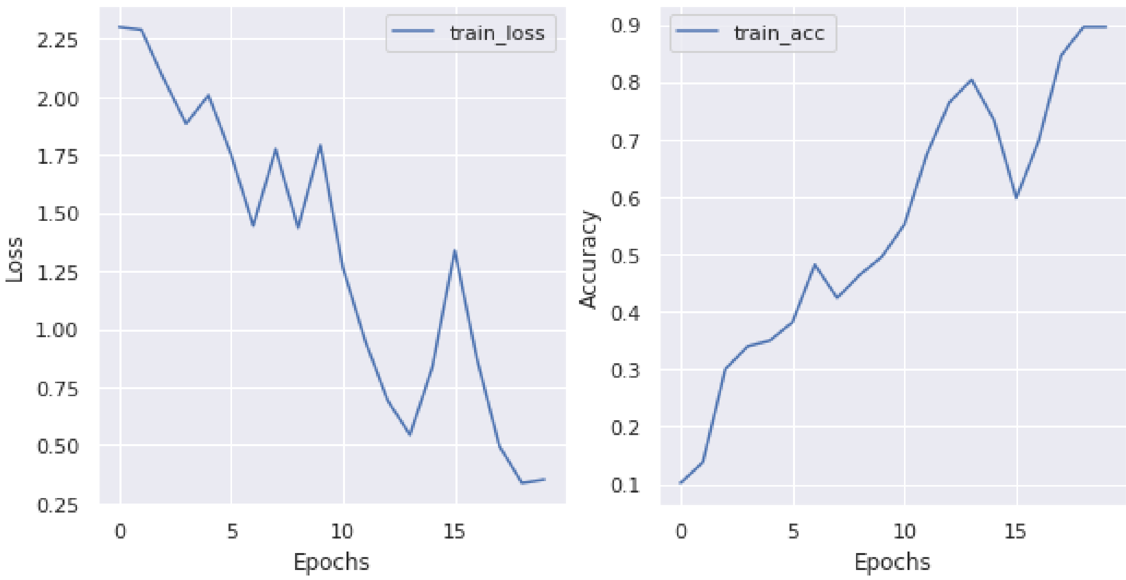
sanity check

In [ ]:

```
%%capture
#  overfit to a small dataset
size = 500        # got 99.55 with 50 epochs
mini_train_set = mnist_trainset[:size]


model = MLP(784, 100, 10)
train_loss, train_acc = train(model, mini_train_set, N_EPOCHS=20, BATCH_SIZE=10
0, LEARNING_RATE=0.01, testing = False, print_to_console = True)
```

In [ ]:

```
plot_loss_and_accuracy(train_loss, train_acc)
```



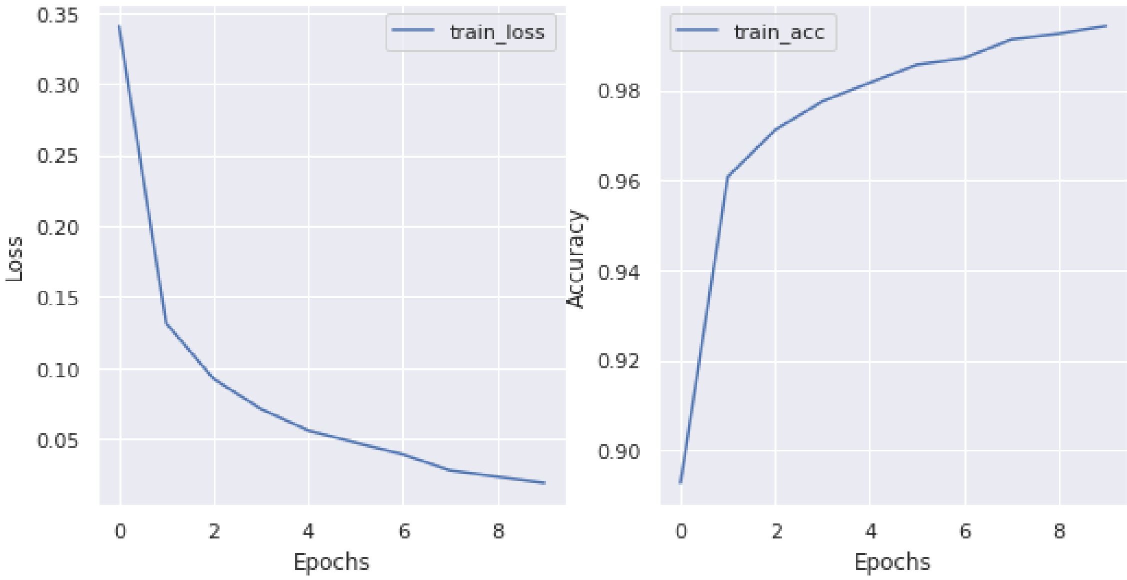
model Training

In [ ]:

```
%%capture
model = MLP(28*28, 100, 10)
train_loss, train_acc = train(model, mnist_trainset, N_EPOCHS= 10 , BATCH_SIZE=1
6, LEARNING_RATE=0.01,testing = False,print_to_console = True)
```

In [ ]:

```
plot_loss_and_accuracy(train_loss, train_acc)
```



In [ ]:

```
# Find the accuracy on the validation set

val_loss, val_acc = train(model, mnist_valset, N_EPOCHS= 1 , BATCH_SIZE=16, LEAR
NING_RATE=0.01,testing = True,print_to_console = True)
```

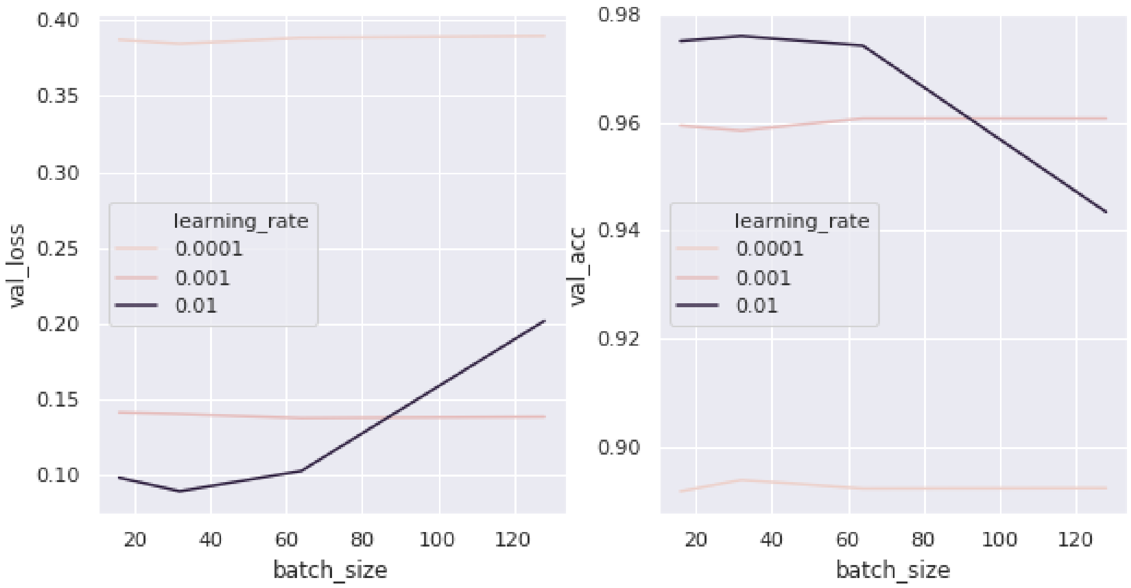
```
100%|██████████| 1/1 [00:00<00:00,  5.01it/s]

Epoch:  0
loss: 0.0964  acc: 0.9732
```

In [ ]:

```
# Find the optimal value of learning rate and batch size.
# Use the same tuning strategy as the previous question
# Create plot and table to show the effect of the hparams.

learning_rates = [0.01, 0.001, 0.0001]
batch_sizes = [16, 32, 64, 128]

# store the results in pandas dataframe
df = pd.DataFrame(columns = ["learning_rate", "batch_size", "train_loss", "train
_acc", "val_loss", "val_acc"])

for lr in learning_rates:
    for bs in batch_sizes:
        model = MLP(28*28, 100, 10)
        train_loss, train_acc = train(model, mnist_trainset, N_EPOCHS= 10 , BATC
H_SIZE=bs, LEARNING_RATE=lr,testing = False,print_to_console = False)
        val_loss, val_acc = train(model, mnist_valset, N_EPOCHS= 1 , BATCH_SIZE=
bs, LEARNING_RATE=lr,testing = True,print_to_console = False)

        df = df.append({"learning_rate": lr, "batch_size": bs, "train_loss": tra
in_loss[-1], "train_acc": train_acc[-1], "val_loss": val_loss[-1], "val_acc": va
l_acc[-1]}, ignore_index=True)
```

In [ ]:

```python
# make plots for the results
import seaborn as sns
sns.set()
plt.figure(figsize=(10,5))
plt.subplot(1,2,1)
# sns.lineplot(x="lr", y="val_loss", hue="lambda", data=df, legend = 'full')
sns.lineplot(x="batch_size", y="val_loss", hue="learning_rate", data=df, legend = 'full')
plt.subplot(1,2,2)
# sns.lineplot(x="lr", y="val_accuracy", hue="lambda", data=df, legend = 'full')
sns.lineplot(x="batch_size", y="val_acc", hue="learning_rate", data=df, legend = 'full')
plt.show()
```



final performance on mnist test set

In [ ]:

```python
### Report final performance on MNIST test set
# best hprams : lr = 0.01, bs = 64

model = MLP(28*28, 100, 10)
train_loss, train_acc = train(model, mnist_trainset, N_EPOCHS= 10 , BATCH_SIZE=64, LEARNING_RATE=0.01,testing = False,print_to_console = False)
test_loss, test_acc = train(model, list(mnist_testset), N_EPOCHS= 1 , BATCH_SIZE=64, LEARNING_RATE=0.01,testing = True,print_to_console = False)

print( f"test loss: {test_loss[-1]:.4f}  test acc: {test_acc[-1]:.4f}" )
```

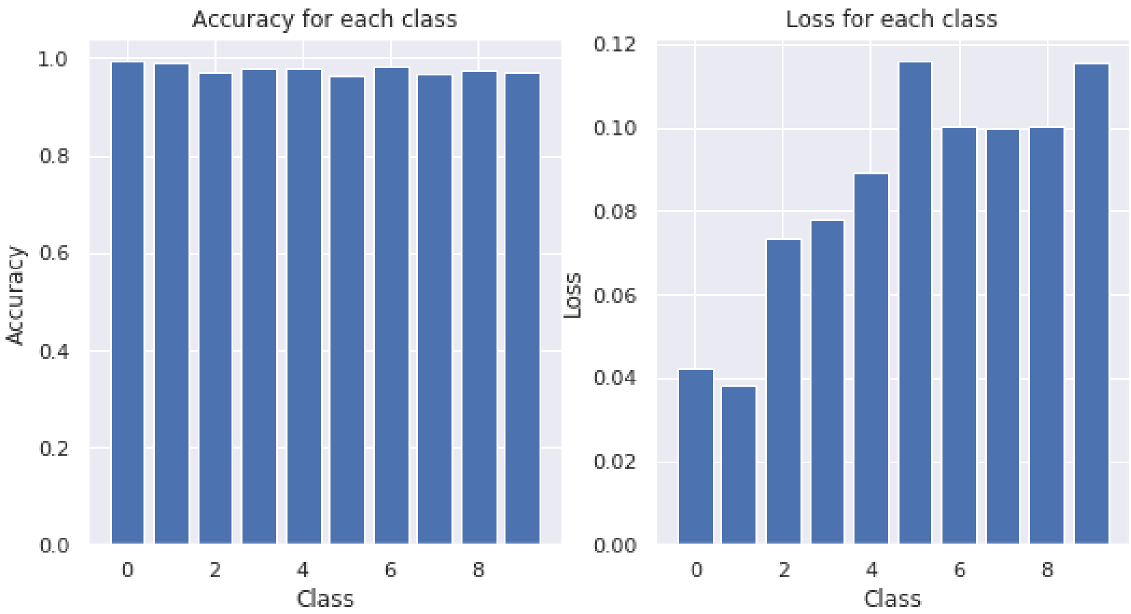
```
100%|██████████| 10/10 [00:08<00:00,  1.18it/s]
100%|██████████| 1/1 [00:00<00:00,  7.87it/s]

test loss: 0.0836  test acc: 0.9767
```

Find the best performing class and the worst performing class

In [ ]:

```
# class wise performance

# best and worst performing classes on the test set

# seperate the test set into 10 classes
classes = [[] for i in range(10)]
for i in range(len(mnist_testset)):
    # class based on the label
    label = mnist_testset[i][1]
    classes[label].append(mnist_testset[i])


# test the model on each class
class_acc = []
class_loss = []
for i in range(10):
    test_loss, test_accuracy = train(model, classes[i] ,testing = True,print_to_
console = False)
    class_acc.append(test_accuracy[0] )
    class_loss.append(test_loss[0] )

# bar plot for the accuracy
plt.figure(figsize=(10,5))
plt.subplot(1,2,1)
plt.bar(range(10),class_acc)
plt.xlabel("Class")
plt.ylabel("Accuracy")
plt.title("Accuracy for each class")

# bar plot for the loss
plt.subplot(1,2,2)
plt.bar(range(10),class_loss)
plt.xlabel("Class")
plt.ylabel("Loss")
plt.title("Loss for each class")
plt.show()
```

```
100%|██████████| 1/1 [00:00<00:00, 43.89it/s]
100%|██████████| 1/1 [00:00<00:00, 42.21it/s]
100%|██████████| 1/1 [00:00<00:00, 38.13it/s]
100%|██████████| 1/1 [00:00<00:00, 38.30it/s]
100%|██████████| 1/1 [00:00<00:00, 42.58it/s]
100%|██████████| 1/1 [00:00<00:00, 53.41it/s]
100%|██████████| 1/1 [00:00<00:00, 64.00it/s]
100%|██████████| 1/1 [00:00<00:00, 56.93it/s]
100%|██████████| 1/1 [00:00<00:00, 54.11it/s]
100%|██████████| 1/1 [00:00<00:00, 54.87it/s]
```



- best performing class: 0
- worst perming class : 9

Any additional observations / comments?

OBSERVATION:

- without normalising the image (divide by 255), the accuracy is not going above 20.
- with very large batch size, it is always outputting one single prediction for all images. but at different runs that prediction is different.
- it may be stuck at local minima

BONUS Question

Observe the effect on test set accuracy by changing the number of training samples.

Train on 10%, 20% and 50% training data and plot the percentage of training data v.s. the test accuracy.

In [ ]:

```
# WRITE CODE HERE
# import time

# train on 10%,20%,50% of the training set and test on the test set
for percent in [0.1,0.2,0.5]:

    print("Training on %d%% of the training set" % (percent*100))
    # time.sleep(0.1)
    size = int(len(mnist_trainset)*percent)
    mini_train_set = mnist_trainset[:size]
    model = MLP(28*28, 100, 10)
    train_loss, train_acc = train(model, mini_train_set, N_EPOCHS= 10 , BATCH_SIZE=64, LEARNING_RATE=0.01,testing = False,print_to_console = False)
    test_loss, test_acc = train(model, list(mnist_testset), N_EPOCHS= 1 , BATCH_SIZE=64, LEARNING_RATE=0.01,testing = True,print_to_console = False)
    print( f"test loss: {test_loss[-1]:.4f}  test acc: {test_acc[-1]:.4f}" )
    print("")
```

```
 20%|██        | 2/10 [00:00<00:00, 11.57it/s]

Training on 10% of the training set

100%|██████████| 10/10 [00:00<00:00, 11.53it/s]
100%|██████████| 1/1 [00:00<00:00,  9.16it/s]
 10%|█         | 1/10 [00:00<00:01,  5.83it/s]

test loss: 0.2026  test acc: 0.9425

Training on 20% of the training set

100%|██████████| 10/10 [00:01<00:00,  5.62it/s]
100%|██████████| 1/1 [00:00<00:00,  9.10it/s]
  0%|          | 0/10 [00:00<?, ?it/s]

test loss: 0.1494  test acc: 0.9574

Training on 50% of the training set

100%|██████████| 10/10 [00:04<00:00,  2.36it/s]
100%|██████████| 1/1 [00:00<00:00,  9.17it/s]

test loss: 0.1162  test acc: 0.9693
```

Accuracy increasing with increasing training dataset size

Question 4

Implementing a Convolution Module with Numpy

- This topic will require you to implement the Convolution operation using Numpy.
- We will use the Module for tasks like Blurring.
- Finally, we implement Backpropagation for the convolution module.

- Implement a naive Convolution module, with basic functionalities: kernel_size, padding, stride and dilation
- Test out the convolution layer by using it to do gaussian blurring on 10 random images of CIFAR-10 dataset

shape checks etc.

reference for size of output :(dialation): https://pytorch.org/docs/stable/generated/torch.nn.Conv2d.html (https://pytorch.org/docs/stable/generated/torch.nn.Conv2d.html)

In [ ]:

```
# a of shape (32,3,5,5)
# b of shape (10,3,5,5)
a = np.random.randn(32,3,5,5)
print(a.shape)
# make extra dimension at dim 1 in a
a = np.expand_dims(a,1)
print(a.shape)
b = np.random.randn(10,3,5,5)

c = a*b
print(c.shape)

# sum last 3 dimensions in c
d = np.sum(c,axis = (2,3,4))
d.shape
```

```
(32, 3, 5, 5)
(32, 1, 3, 5, 5)
(32, 10, 3, 5, 5)
```

Out[ ]:

```
(32, 10)
```

conv class

```
In [ ]:
```

```python
## Define a class Convolution Layer, which is initialized with the various requi
red params:
class Convolution_Layer():

    def __init__(self,in_channels,out_channels , filter_size, bias=False, stride
=1, padding = 0, dilation=1):
        """
        filter_size : int    : square filter
        padding, stride, dilation : int : same for both height and width
        """
        # For an untrained layer, set random initial filter weights
        self.in_channels = in_channels
        self.out_channels = out_channels
        self.filter_size = filter_size
        self.bias_enabled = bias
        self.stride = stride
        self.padding = padding
        self.dilation = dilation

        # Initialize weights
        self.weights = np.random.randn(out_channels, in_channels, filter_size, f
ilter_size)
        self.bias = np.random.randn(out_channels) if bias else None


    def forward(self,input_batch, check_output_shape = False):
        """ main idea for vectorizable implementation:
        use views;

        psuedo code:
        ----------------
        for one output pixel:
            get input patch  (B,in_C,k,k)
            expand dim= 1   (B,1,in_C,k,k)
            multiply (element vise) with kernal    (B,out_C,in_C,k,k)
            sum over in_C and k and k   (B,out_C)
            add bias (if enabled)   (B,out_C)
        ----------------



        """

        # Input Proprocess(According to pad etc.) Input will be of size (Batch_s
ize, in_channels, inp_height, inp_width)
        input_batch = np.pad(input_batch, ((0,0), (0,0), (self.padding, self.pad
ding), (self.padding, self.padding)), 'constant', constant_values=0)
        # normalize input
        input_batch = input_batch / 255.0
        # Reminder: Save Input for backward-prop
        self.input_batch = input_batch
        # since, the padding already applied to the input, removing it from the
formula

        # print("input_batch shape" , input_batch.shape)
        h_out = (input_batch.shape[2] +  - self.dilation * (self.filter_size -
1) - 1) // self.stride + 1
        w_out = (input_batch.shape[3] +  - self.dilation * (self.filter_size -
1) - 1) // self.stride + 1

        self.h_out = h_out
        self.w_out = w_out
```

```
        # Check if output shape is correct
        # if check_output_shape:
        #     print("h_out: ", h_out)
        #     print("w_out: ", w_out)
            # return None

        output = np.zeros((input_batch.shape[0], self.out_channels, h_out, w_ou
t))

        # making storage
        #-----------------------------------------------------
        # _shape = ( *output.shape, *self.weights.shape)

        # self.storage_for_backward = np.zeros(_shape)
        #-----------------------------------------------------


        s = self.stride
        k = self.filter_size
        d = self.dilation


        # Vectorized Conv operation:
        for i in range(h_out):
            for j in range(w_out):
                # get input patch  (B,in_C,k,k)
                input_patch = input_batch[:, :, i*s : i*s + k*d : d ,  j*s : j*s
+ k*d : d]
                # expand dim= 1   (B,1,in_C,k,k)
                input_patch = np.expand_dims(input_patch, 1)

                # check input_patch shape if it is not (B,1,in_C,k,k) then conti
nue
                # if input_patch.shape != (input_batch.shape[0],1,input_batch.sh
ape[1],k,k):
                #     print("input_patch_size mismatch")
                #     print("input_patch_size:",input_patch.shape)
                #     print("i,j:",i,j)
                #     continue

                # multiply (element vise) with kernal    (B,out_C,in_C,k,k)
                output[:, :, i, j] = np.sum(input_patch * self.weights, axis=(2,
3,4))

                # ---------------------------------------------------------
                # # sum the input patch for dim 0
                # # store it in storage
                # # self.storage_for_backward[:, :, i, j] = np.sum(input_patch,
axis=0)                                                          # leave dim1, ch
eck if it broadcasts in backward
                # temp = np.sum(input_patch, axis=0)
# leave dim1, check if it broadcasts in backward
                # #repeat in dim1 to match the shape of output channels
                # temp = np.repeat(temp, self.out_channels, axis=0)
                # # print(temp.shape)
                # self.storage_for_backward[:, :, i, j] = temp
                # ---------------------------------------------------------
```

```
            # sum over in_C and k and k   (B,out_C)
            # add bias (if enabled)   (B,out_C)

            if self.bias_enabled:
                output[:, :, i, j] += self.bias


        # if check_output_shape:
        #     print("output_size:",output.shape)

        # Output will be of the size (Batch_size, out_channels, out_height, out_
width)
        return output


    def backward(self, grad_of_output_size):



        """

        ***************************** Speed is  "A CONSERN" ********************
****

        grad_of_output_size : (B, out_C, out_H, out_W)
        main idea:
        ----------------
        for each output pixel:
            store corresponding input patch { that is converted to shape of kern
el  by summing over broadcasted dimensions }: in forward * operation.

            {:: check rough work 4th paper
            mearge --> B
            make copies --> out_C
            }



        finally multiply (*) with grad_of_output_size (by broadcasting)
        sum over the dimensions without broadcasting ( = summing all branches )
# this is creating out of memory error

so loop over kernel size

            finally, we get patch size = kernel size;

        so,storage size = (output_size , input_patch_size)
        ----------------
        """

        # normalize the grad_of_output_size
        grad_of_output_size = grad_of_output_size / np.max(grad_of_output_size)


        h_out = self.h_out
        w_out = self.w_out
        input_batch = self.input_batch

        s = self.stride
        k = self.filter_size
```

```
        d = self.dilation

        # intialize grad_kernel with all zeros
        grad_kernel = np.zeros(self.weights.shape)


        for i in range(h_out):
            for j in range(w_out):
                # get input patch  (B,in_C,k,k)
                input_patch = input_batch[:, :, i*s : i*s + k*d : d ,  j*s : j*s
+ k*d : d].copy()
                # expand dim= 1   (B,1,in_C,k,k)
                input_patch = np.expand_dims(input_patch, 1)

                grad_patch = grad_of_output_size[:, :, i, j].copy()
                grad_patch = np.expand_dims(grad_patch, 2)

                temp_shape = grad_patch.shape
                # add 2 extra dimensions to grad_patch
                grad_patch = grad_patch.reshape( *temp_shape, 1, 1)

                grad_kernel += np.sum(input_patch * grad_patch, axis=0)

        grad_kernel =  grad_kernel / np.max(grad_kernel)

        self.grad_weights = grad_kernel
        return grad_kernel

                # check input_patch shape if it is not (B,1,in_C,k,k) then conti
nue
                # if input_patch.shape != (input_batch.shape[0],1,input_batch.sh
ape[1],k,k):
                #     print("input_patch_size mismatch")
                #     print("input_patch_size:",input_patch.shape)
                #     print("i,j:",i,j)
                #     continue

                # multiply (element vise) with kernal    (B,out_C,in_C,k,k)
                # output[:, :, i, j] = np.sum(input_patch * self.weights, axis=
(2,3,4))




        # storage = self.storage_for_backward
        # print("storage shape", storage.shape)
        # make extra 4 dimensions to match the shape of grad_of_output_size
        # old_shape = storage.shape
        # new_shape = ( *old_shape, 1,1,1,1)
        # storage = np.reshape(storage, new_shape)
        # grad = grad_of_output_size * storage

        # sum over the dimensions without broadcasting ( = summing all branches
)
        # grad = np.sum(grad, axis=(0,1,2,3))
#  out of memory error
        # print("grad shape", grad.shape)

        # -----------------------------------------------------

        # grad = np.zeros_like(self.weights )
```

```
        # q = grad_of_output_size.shape
        # # for each ouput pixel
        # for i in range(q[0]):
        #     for j in range(q[1]):
        #         for k in range(q[2]):
        #             for l in range(q[3]):
        #                 storage[i,j,k,l] *= grad_of_output_size[i,j,k,l]
# check paper 5. before stating agian { loop over theta_1. then, sum over theta_
1}




        # grad = np.sum(storage, axis=(0,1,2,3))




        # Hint: gradients from each independant operation can be summed





        #  return gradient of the size of the weight kernel
        # self.grad_weights = grad
        # self.grad_bias = np.sum(grad_of_output_size, axis=(0,2,3))

        # return grad
    def step(self, learning_rate):
        """ update the weights using the gradient and the learning rate
        then, make the gradient zero
        """
        # Update the weights using the gradient and the learning rate

        # normalize the gradient by dividing by max value
        self.grad_weights /= np.max(self.grad_weights) * np.max(self.weights)
        # update weights
        self.weights -= learning_rate * self.grad_weights
        # make gradient zero
        self.grad_weights = np.zeros_like(self.weights)

        if self.bias_enabled:
            self.grad_bias /= np.max(self.grad_bias)
            self.bias -= learning_rate * self.grad_bias
            self.grad_bias = np.zeros_like(self.bias)




    def set_weights(self, new_weights):
        """ kernal shape : (out_channels, in_channels, filter_size, filter_size)
        bais shape : (out_channels)

        """
        kernal, bias = new_weights
        self.weights = kernal
        self.bias = bias
```

```
        ## Replace the set of weights with the given 'new_weights'
        ## use this for setting weights for blurring, bilateral filtering etc.
```

sanity check

In [ ]:

```
### Download CIFAR-10 images and load it in a numpy array (https://www.cs.toront
o.edu/~kriz/cifar.html)

# load cifa-10 dataset from torch
trainset = datasets.CIFAR10(root='./data', train=True,
                                        download=True, transform=None)
testset = datasets.CIFAR10(root='./data', train=False,
                                        download=True, transform=None)
# convert to numpy array
train_data = np.array(trainset.data)
test_data = np.array(testset.data)

train_data.shape, test_data.shape
```

```
Files already downloaded and verified
Files already downloaded and verified
```

Out[ ]:

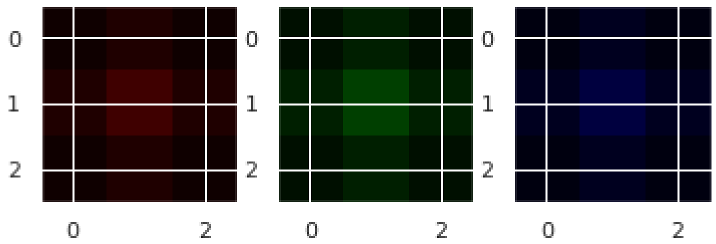
```
((50000, 32, 32, 3), (10000, 32, 32, 3))
```

Initialize a conv layer. Set weights for gaussian blurring (do not train the filter for this part). Visualise the filters using matplotlib

In [ ]:

```
# kernal should blur each color channel seperately
grey_kernal = np.array([[1,2,1],[2,4,2],[1,2,1]])/ 16

kernel = np.zeros((3,3,3,3))
kernel[0,0] = grey_kernal
kernel[1,1] = grey_kernal
kernel[2,2] = grey_kernal



# kernel = kernel/np.sum(kernel)
print(" kernal shape " ,kernel.shape)

# using subplots
fig, ax = plt.subplots(1,3)
for i in range(3):
    ax[i].imshow(kernel[i].transpose(1,2,0))
plt.show()
```

```
 kernal shape  (3, 3, 3, 3)
```



test on images

In [ ]:

```
# 10 images from train_data
size = 10
mini_data = train_data[:size]
# change color channel to first
mini_data = np.transpose(mini_data, (0,3,1,2))
print("mini_data shape", mini_data.shape)
# channel first
```

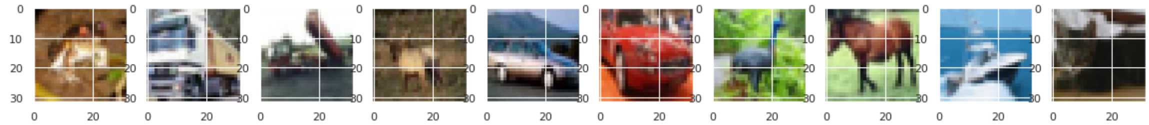
```
mini_data shape (10, 3, 32, 32)
```

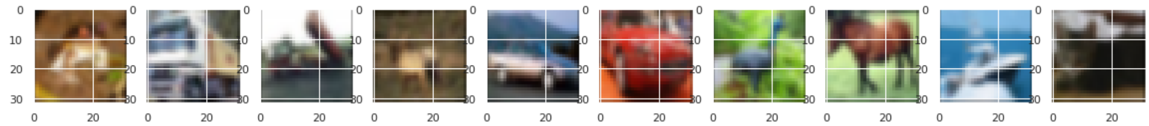
In [ ]:

```
# conv_model = (in_channels=3, out_channels=3, filter_size=3, stride=1, padding=1, dilation=1, bias_enabled=True)
conv_model = Convolution_Layer(in_channels=3, out_channels=3, filter_size=3, stride=1, padding=1, dilation=1, bias=True)
conv_model.weights = kernel
conv_model.bias = np.zeros(3)

blurred_images = conv_model.forward(mini_data)

# delete the model
del conv_model
```

In [ ]:

```
def show_blurring_effect(original_images, blurred_images,size):

    # set fig size
    plt.figure(figsize=(20,10))
    # show original images
    for i in range(size):
        plt.subplot(2,size,i+1)
        plt.imshow(np.transpose(original_images[i], (1,2,0)))
    plt.show()

    plt.figure(figsize=(20,10))
    # show blurred images
    for i in range(size):
        plt.subplot(2,size,i+1+size)
        plt.imshow(np.transpose(blurred_images[i], (1,2,0)))
    plt.show()

show_blurring_effect(mini_data, blurred_images,size)
```





Question 5

Now we will use this module for training a simple Convolution Layer using CIFAR-10 images.

- The goal is to learn a set of weights, by using the backpropagation function created. To test the backpropagation, instead of training a whole network, we will train only a single layer.
  - Instantiate a Convolution layer C_0 with 20 filters, each with size 5$\times$5 (RGB image, so 3 input channels). Load the given numpy array of size (20,3,5,5), which represents the weights of a convolution layer. Set the given values as the filter weights for C_0. Take 100 CIFAR-10 images. Save the output of these 100 images generated from this Convolution layer C_0.
  - Now, initialize a new convolution layer C with weight values sampled from uniform distribution [-1,1]. Use the L_2 loss between the output of this layer C and the output generated in the previous step to learn the filter weights of C_0.

In [ ]:

```
## Load filter weights from given numpy array "C0_weights.npy".
filter_weights = np.load("C0_weights.npy")


BATCH_SIZE = 20
N_BATCHES = 5


C_0 = Convolution_Layer(in_channels=3, out_channels=20, filter_size=5, stride=1,
padding=2, dilation=1, bias=True)
C_0.set_weights((filter_weights, np.zeros(20)))

def cifar_batch_generator(batch_size=BATCH_SIZE, n_batches=N_BATCHES):
    for i in range(n_batches):
        mini_data =  train_data[i*batch_size:(i+1)*batch_size]
        mini_data = np.transpose(mini_data, (0,3,1,2))
        yield mini_data
```

In [ ]:

```
output_C0 = []
generator = cifar_batch_generator()
for i in range(N_BATCHES):
    batch = next(generator)
    output_C0.append(C_0.forward(batch))

del C_0
```

In [ ]:

```
# for part 2 we need to write a class for the  L2 loss
class L2_loss():
    def ___init__(self):
        pass

    def forward(self, C0_output,C_output):
        # Conv. output is of dimension (batchsize,channels,height,width)
        # calculate the L2 norm of (C0_output - C_output)
        loss = np.sum((C0_output - C_output)**2)
        self.C0_output = C0_output
        self.C_output = C_output

        return loss

    def backward(self,output_grad=1):
        # from the loss, and the conv. output, get the grad at each location
        # The grad is of the shape (batchsize,channels,height,width)
        # gradient wrt C_output

        grad = 2*(self.C_output - self.C0_output)
        return grad
```

sanity check

In [ ]:

```
from tqdm import tqdm
def new_train(conv_layer, loss_layer, C0_output,learning_rate= 0.001, batch_size
=BATCH_SIZE, n_batches=N_BATCHES,epochs = 1):


    loss = L2_loss()
    loss_list = []
    # iterate over the batches
    for epoch in range(epochs):
        print("epoch : ",epoch)
        generator = cifar_batch_generator(batch_size,n_batches)

        for i in tqdm(range(n_batches)):

            batch = next(generator)
            # forward pass
            output = conv_layer.forward(batch)
            # calculate the loss
            loss_value = loss.forward(C0_output[i],output)
            # store the loss
            loss_list.append(loss_value)
            # backward pass
            grad = loss.backward()
            conv_layer.backward(grad)
            # update the weights
            conv_layer.step(learning_rate)
            # check if grad is zero or not
            # print( (conv_layer.grad_weights == 0).sum())
            # print( "weights" ,np.linalg.norm(conv_layer.weights))


            print(" iteration : ",i," loss : %.3g"%loss_value)

        # print("grad shape",conv_layer.grad
    return loss_list, conv_layer
```

In [ ]:

```
%%capture

# Now Init a new conv layer C and a L2 loss layer
C = Convolution_Layer(in_channels=3, out_channels=20, filter_size=5, stride=1, p
adding=2, dilation=1, bias=False)
C.weights = np.random.uniform(-1,1,size=(20,3,5,5))

# Train the new conv-layer C using the L2 loss to learn C_0, i.e., the set of gi
ven weights.
# Use mini-batches if required

# Initialize the L2 loss layer
l2_loss = L2_loss()

loss_list,C = new_train(C, l2_loss, output_C0,learning_rate= 0.01,epochs=50)



#Print L2 dist between output from the new trained convolution layer C and the o
utputs generated from C_0.
```

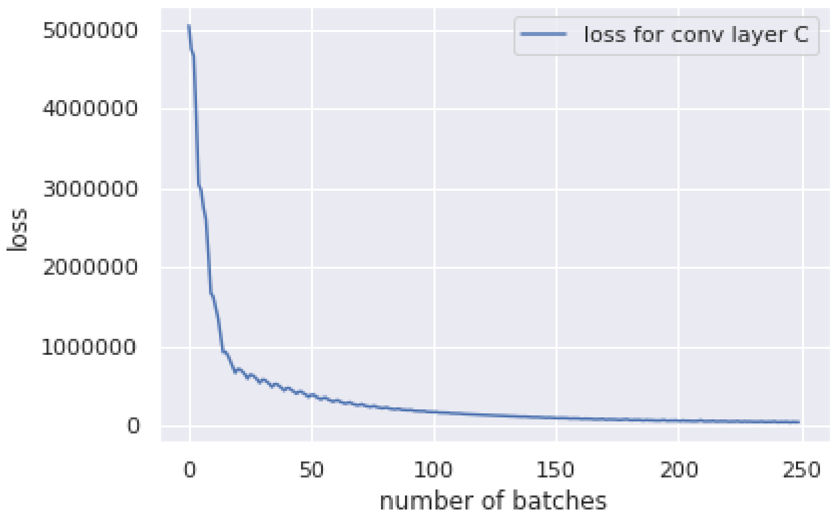
In [ ]:

```
# bais of C
print("bias of C",C.bias)
```

```
bias of C None
```

In [ ]:

```
def plot_loss(loss_list):
    #plot loss_list
    plt.plot(loss_list,label="loss for conv layer C")
    plt.legend()
    # x axis is the number of batches
    plt.xlabel("number of batches")
    plt.ylabel("loss")
    plt.show()

plot_loss(loss_list)
```



In [ ]:

```
# output from trained C
output_C = []
generator = cifar_batch_generator()
for i in range(N_BATCHES):
    batch = next(generator)
    output_C.append(C.forward(batch))
```

In [ ]:

```
# calculate the L2 loss
sum([l2_loss.forward(output_C0[i],output_C[i]) for i in range(N_BATCHES)])
```

Out[ ]:

```
224038.3337523802
```

In [ ]:

```
C.weights = np.random.uniform(-1,1,size=(20,3,5,5))


generator = cifar_batch_generator()
loss_function = L2_loss()
```

In [ ]:

```
batch = next(generator)
output_C = C.forward(batch)
loss = loss_function.forward(output_C0[0],output_C)
grad = loss_function.backward()
a = C.backward(grad)
```

In [ ]:

```
np.max(C.weights - filter_weights)
```

Out[ ]:

```
1.9339866296782997
```