

CS 18200 ✦ Spring 2024

🌀 Homework 02 🌀

Due: Friday, January 26, 2024 at 11:00 pm Eastern Time

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- Homework **must be typed** and uploaded as one .pdf file to Gradescope.
 - Every problem has to start on a new page.
 - Follow Gradescope instructions on matching problems to page numbers.
 - Homework not following guidelines will not be graded.
 - Answering to Problem 1 is **mandatory** for everyone. Failure to submit a solution to Problem 1 results in student submission not being graded.
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Problem 1. (MANDATORY) In this question you must write the following sentence with your full name typed in between:

I, [YOUR FULL NAME GOES HERE], affirm that I have not given or received any unauthorized help on this assignment and that this work is my own. What I have submitted is expressed and explained in my own words. I have not used any online websites that provide a solution. I will not post any parts of this problem set to any online platform and doing so is a violation of course policy.

So for example, if "John Doe" is in the class, they will submit the following as the solution to this problem:

I, John Doe, affirm that I have not given or received any unauthorized help on this assignment and that this work is my own. What I have submitted is expressed and explained in my own words. I have not used any online websites that provide a solution. I will not post any parts of this problem set to any online platform and doing so is a violation of course policy.

Problem 2. (8 × 2 points)

In a zoo, animals are classified into two types: mammals and birds. The zookeeper keeps track of which animals share the same enclosure and whether they were born in captivity. Let

- $M(x)$ denote "x is a mammal"
- $B(x)$ denote "x is a bird"
- $S(x, y)$ denote "x and y share the same enclosure"
- $C(x)$ denote "x was born in captivity".

Take the universal set U to be all the animals in the zoo.

- Write a predicate logic statement to express that no mammals share an enclosure with birds.
- Express the statement that all birds that were born in captivity share an enclosure with at least one mammal.

Problem 3. (16 points)

Prove or disprove that the equation $4x^2 + 5y^2 + 20y + 3 = 100$ has no integer solutions.

[Hint: Try narrowing down the range of x or y (Proof by cases). For proof by contradiction, suppose an integer solution exists.]

Problem 4. (12 points)

Prove that if n is an integer and $n^2 + 10n + 5$ is not divisible by 5, then n is not divisible by 5.

Problem 5. (12 points)

Rewrite $\neg\forall x\exists y\exists z(P(x, y) \wedge Q(x, z) \wedge R(z))$ such that the negation symbols immediately precede the predicates. Show your steps to receive full credit.

Problem 6. (14 points)

Prove that $x^2 - 12x + 36$ is odd **if and only if** x is odd. Remember, to prove $p \leftrightarrow q$, you must show $p \rightarrow q$ and $q \rightarrow p$.

Problem 7. (8 × 2 points)

Let a and b be real numbers. Consider the statement "If $0 \leq a < b$, then $a^2 < b^2$."

- Rewrite the statement as a proposition using predicate variables p and q . Be sure to define p and q .
- Given the statement is true, can we conclude that "If $a^2 < b^2$, then $0 \leq a < b$ " is correct? Why or why not?

Problem 8. (14 points)

Which of the following is equivalent to $\exists! x P(x)$ and why is the other one incorrect? The notation $\exists!$ is used for “there exists a **unique** element.”

- $\exists x (P(x) \wedge \forall y (P(y) \rightarrow x = y))$
- $\exists x \forall y (P(y) \rightarrow x = y)$