greatlearning
Learning for Life

TIME SERIES FORECASTING

greatlearning
Learning for Life

AUTOREGRESSIVE MODELS FOR TIME SERIES

Alternative Approach: ARIMA

Exponential smoothing and ARIMA

Most widely-used approaches to time series forecasting Provide complementary approaches to the problem

Exponential smoothing

Description of trend and seasonality in the data

ARIMA models :

Autocorrelations in the data

Stationary Time Series

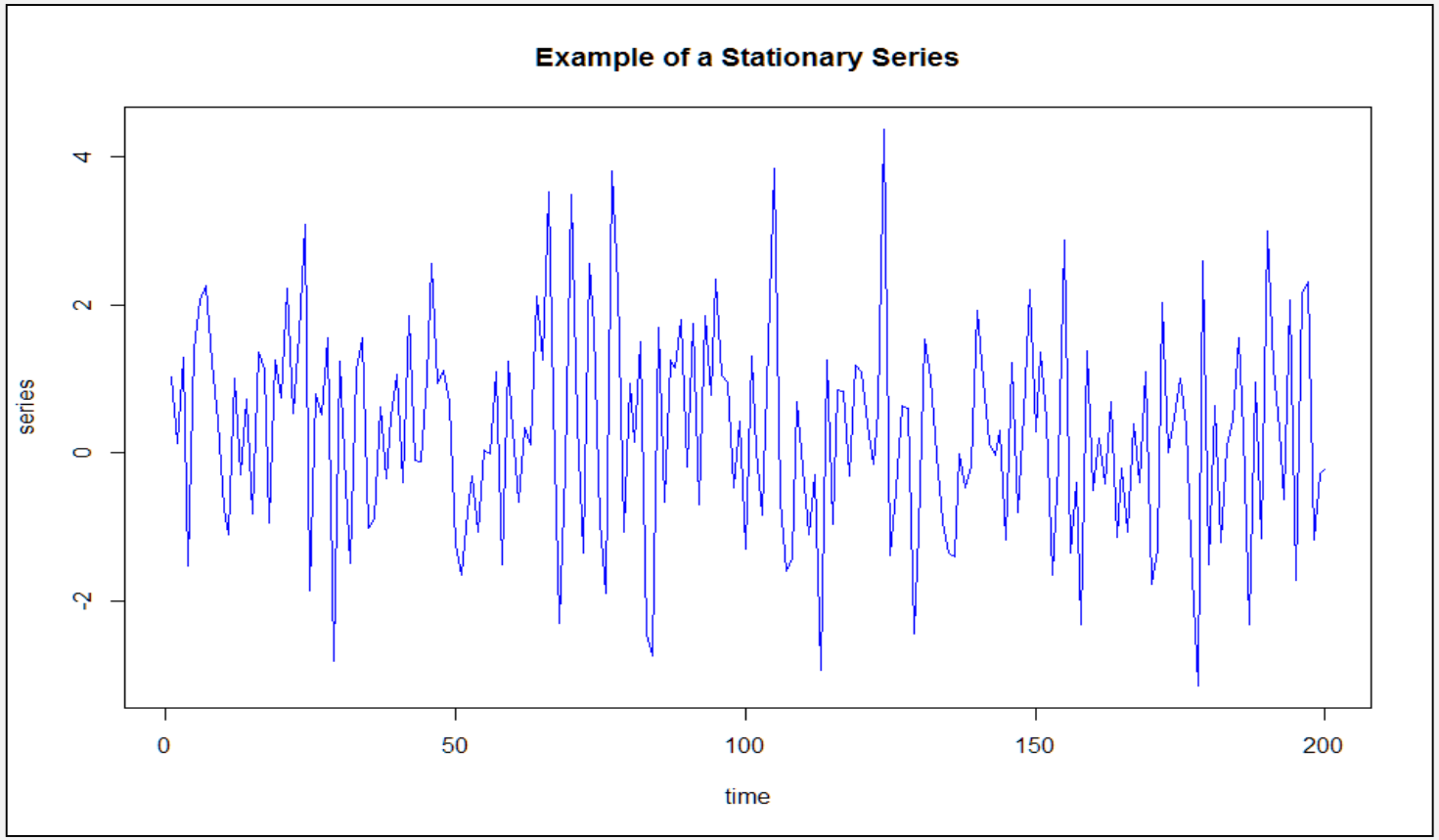
greatlearning
Learning for Life

- Only Stationary Series can be forecasted!!
- If Stationarity condition is violated, the first step is to stationarize the series

A stationary time series is one whose properties do not depend on the time at which the series is observed. The series will not have any predictable pattern

Another name: White Noise

Stationary Time Series

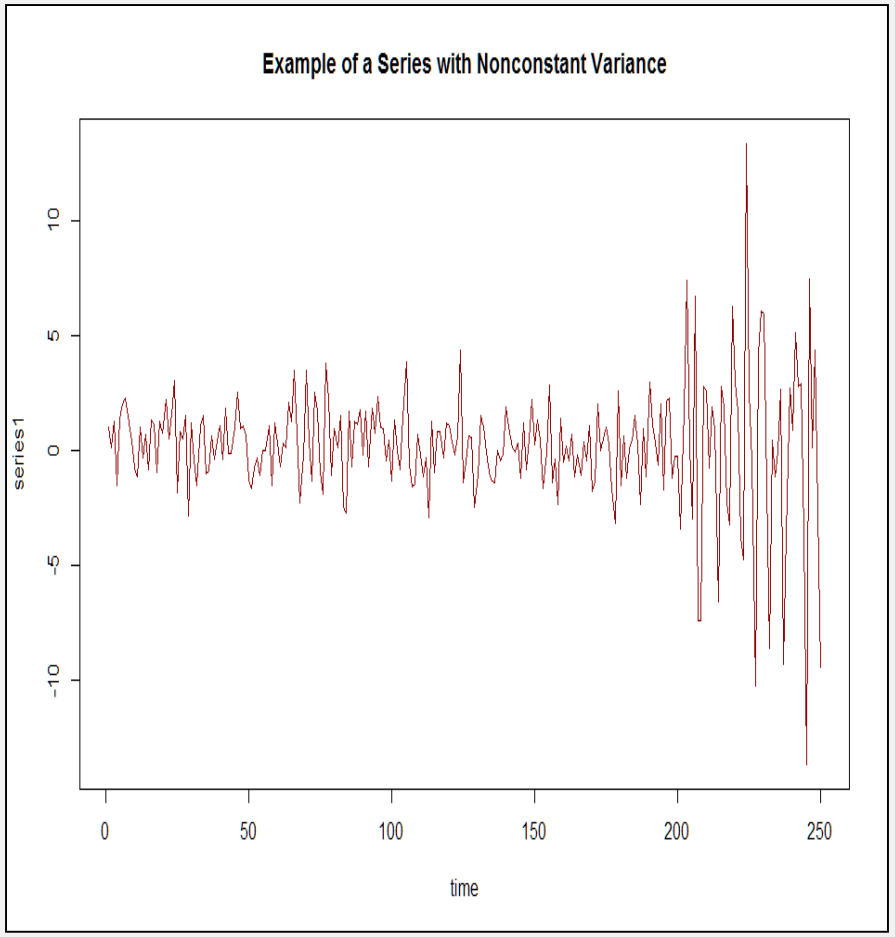


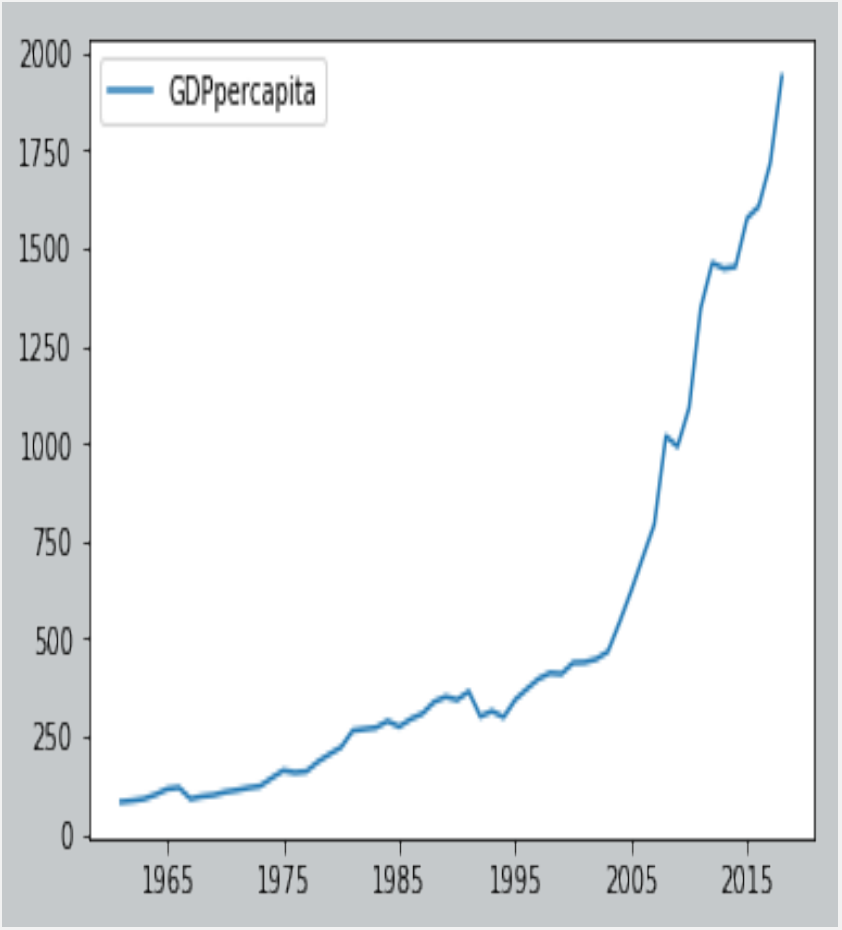
Stationary Time Series

greatlearning
Learning for Life

- Mean of the time series will be a constant.
  - Time series with trends are not stationary
  - Variance of the time series will be a constant
  - The correlation between the t-th term in the series and the t+m-th term in the series is constant for all time periods and for all m

Non-Stationary Time Series





Steps for Analysis

greatlearning
Learning for Life

1. Visualization
2. Stationarization
   - Do a formal test of hypothesis
   - If series non-stationary, stationarize
3. Explore Autocorrelations and Partial Autocorrelations
4. Build ARIMA Model
   - Identify training and test periods
   - Decide on model parameters
   - Compare models using accuracy measures
   - Make prediction

Simple Test for Stationarity

Augmented Dickey-Fuller Test

Tests whether a time series is NON-STATIONARY

Null hypothesis H_0: Time series non-stationary
Alternative hypothesis H_a: Time series is stationary

Rejection of null hypothesis implies that the series is stationary

greatlearning
Learning for Life

Stationary Series

It is possible to make a non-stationary series stationary by taking differences between consecutive observations

Difference Series

greatlearning
Learning for Life

- Simple but effective method to stationarize a time series
- Take difference of consecutive terms in a series
- Known as a Difference Series of Order 1

Autoregression

greatlearning
Learning for Life

- When value of a time series depends on its value at the previous time point
- Various economic and financial series are autoregressive
  - GDP of a country
  - Stock prices
  - Consumption expenditure

Autoregression

greatlearning
Learning for Life

- Current value of the series may depend on only one past observation
  - Current GDP depends only on the past year's GDP
  - AR(1) process

$$Y(t) = \beta_1 Y(t-1) + \varepsilon(t)$$

Autoregression

- It can depend on several past observations
  - AR(p) process
  - p: parameter (to be determined from data)

$$Y(t) = \beta_1 Y(t-1) + \beta_2 Y(t-2) + \beta_3 Y(t-3) + \ldots + \beta_p Y(t-p) + \varepsilon(t)$$

$\beta_1, \beta_2, \ldots \beta_p$: autoregressive parameters of various orders

$\varepsilon(t)$: White noise, iid r.v. with mean 0, variance σ^2

Explore Autocorrelation

greatlearning
Learning for Life

- Autocorrelation: Correlation with self
- Autocorrelation of different orders gives *inside information* regarding time series
- Determines order p of the series

$$-1 \leq ACF \leq 1$$

$$ACF(0) = 1$$

ACF makes sense only if the series is stationary

Explore Autocorrelation

greatlearning
Learning for Life

- Correlation between Original series and Lag(1) series = ACF(1)
- Correlation between Original series and Lag(2) series = ACF(2)
- Correlation between Original series and Lag(3) series = ACF(3)
- And so on ….

```
acf(BASF, lag = 50)
```

Explore Autocorrelation

greatlearning
Learning for Life

- Autocorrelations decreasing as lag increases
- Autocorrelations significant till high order
- Significant autocorrelations imply observations of long past influences current observation

Is there really such a dependency or the dependency is only apparent?

Explore Partial Autocorrelations

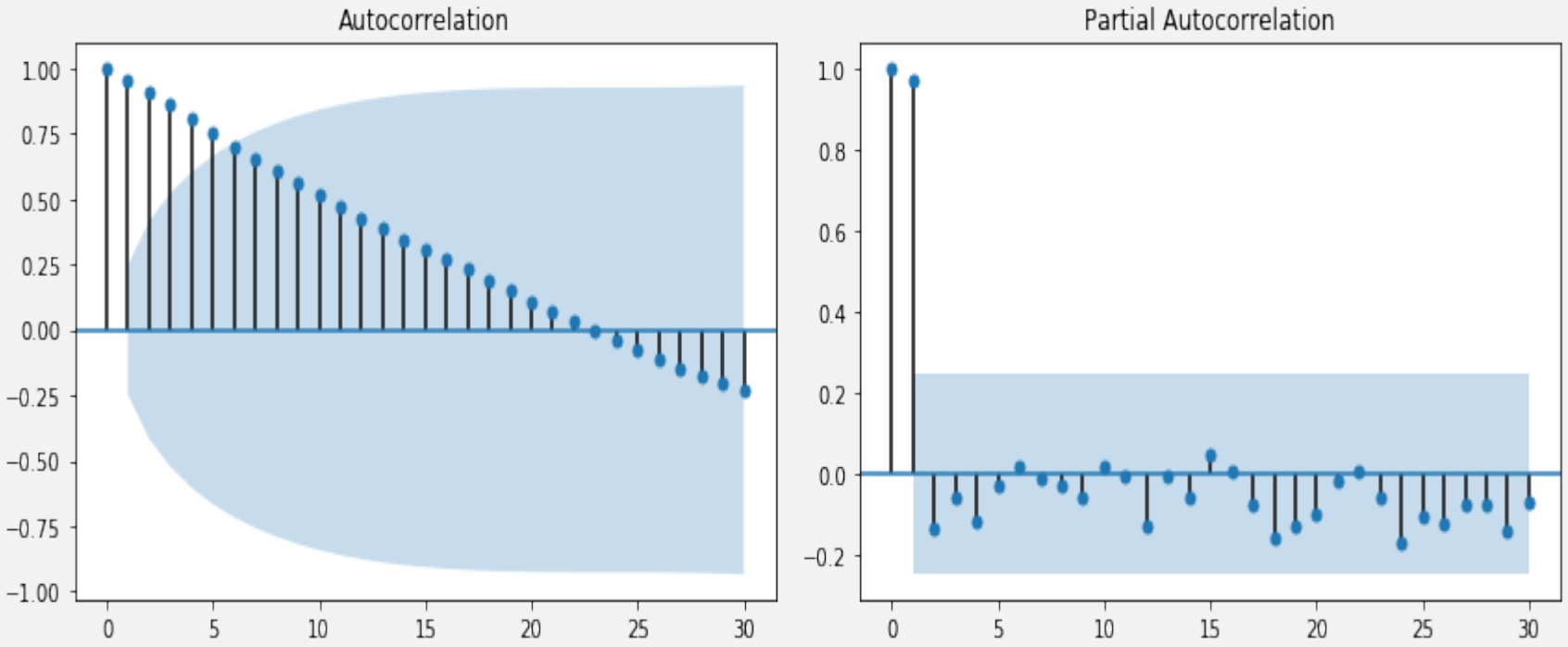
greatlearning
Learning for Life

- Partial autocorrelation adjusts for the intervening periods
- PACF(1) = ACF(1)
- PACF(2) is the correlation between Original and Lag(2) series AFTER the influence of Lag(1) series has been eliminated
- PACF(3) is the correlation between Original and Lag(3) series AFTER the influence of Lag(1) and Lag(2) series has been eliminated
- PACF(50) is the correlation between Original and Lag(50) series AFTER the influence of Lag(1) through Lag(49) series has been eliminated

Explore Partial Autocorrelation

greatlearning
Learning for Life

Dramatic reduction in PACF beyond order 1



Tools for Identification

greatlearning
Learning for Life

- ACF and PACF together to be considered for identification of order of Autoregression
- Seasonal ACF show significant correlation at seasonal points

Generalized Model: ARIMA

AR is a special and simpler form of a general class of models: ARIMA(p, d, q)

AR: Current observation is regressed on past observations

$$Y(t) = \beta_1 Y(t-1) + \beta_2 Y(t-2) + \beta_3 Y(t-3) + \ldots + \beta_p Y(t-p) + \varepsilon(t)$$

MA (Moving Average): Current observation is regressed on past forecast errors

$$Y(t) = \varepsilon(t) + \alpha_1\, \varepsilon(t-1) + \alpha_2\, \varepsilon(t-2) + \ldots + \alpha_q\, \varepsilon(t-q) \qquad |\alpha_1| < 1$$

$\alpha_1, \alpha_2, \ldots, \alpha_q$: Moving average parameters

Generalized Model: ARIMA

ARMA: When Current observation is a linear combination of past observations and past white (random) noises

$$Y(t) = \beta_1 Y(t-1) + \beta_2 Y(t-2) + \beta_3 Y(t-3) + \ldots + \beta_p Y(t-p) + \varepsilon(t) + \alpha_1 \varepsilon(t-1) + \alpha_2 \varepsilon(t-2) + \ldots + \alpha_q \varepsilon(t-q)$$

Theoretically (p, q) may take any value but usually values higher than 2 not preferred in practical situation

Generalized Model: ARIMA

greatlearning
Learning for Life

- If the original series is not stationary, differencing is necessary
- Most often differencing of order 1 makes the series stationary
- But higher order differencing may be needed
- Order = d

ARIMA(p, d, q) identifies a non-seasonal model which needs to be differenced d times to make it stationary and contains p AR terms and q MA terms

Seasonal ARIMA

greatlearning
Learning for Life

ARIMA(p, d, q)x(P, D, Q)[freq]

Seasonal difference = D

Appropriate for seasonal series

Final Model

- Based on accuracy choose the model which works best
- Must have proper interpretability
- Often a simple model works better