Solve the following differential equations and display x values, computed y values, exact y values, error.

Problem 1

$$\frac{\mathrm{d}^4 y}{\mathrm{d}x^4} + 4y = 1;$$

subject to the boundary conditions $y(\pm 1) = y''(\pm 1) = 0$. Taking the step length $\Delta x = 0.05$. The exact solution to the above problem is given by

$$y(x) = \frac{1}{4} [1 - 2(\sin 1 \sinh 1 \sin x \sinh x + \cos 1 \cosh 1 \cos x \cosh x) / (\cos 2 + \cosh 2)].$$

Problem 2

$$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} - xy = (x^3 - 2x^2 - 5x - 3)\mathrm{e}^x; \quad y(0) = 0, y'(0) = 1, y'(1) = -\mathrm{e}.$$

Taking the step length $\Delta x = 0.02$. The exact solution to the above problem is given by

$$y(x) = x(1-x)e^x.$$