Matrix chain Multiplication :-

ex-1 $A_{3\times4}$ $B_{4\times2}$

$C = A * B$

3×2 3×4 4×2

⇓

$3*2$ - ele

⇓

6 - ele

⇓

$6*4$ - mul

⇓

24 - mul.

ex-2 $A_{5\times4}$ $B_{4\times6}$ $C_{6\times3}$

$D = ABC$

ABC

(AB)C ⇓ $120 + (AB)_{5\times6}\,C_{6\times3}$ ⇓ 120 + 90 ⇓ 210

A(BC) ⇓ $A_{5\times4}\,(BC)_{4\times3} + 72$ ⇓ 60 + 72 ⇓ 132

A(BC) ⇒ optimal way of keeping Paranthesis.

total no of Multiplication = 132

Ex-3

$A_{3\times2},\ B_{2\times4},\ C_{4\times5},\ D_{5\times3}$

$E = ABCD$

ABCD

$(A)(BCD)_{2\times3}$

84   70

$B(CD)$ 2×4, 4×3

$60+24 = 84$
$+18$
102

$(BC)D$ 2×5, 5×3

$40+30 = 70$
$+18$
$\underline{88}$

$(AB)(CD)$ 3×4, 4×3

$24 + 60 = 84$
$+36$
$\underline{120}$

$(ABC)(D)_{5\times3}$ 3×5

84   70

$(AB)C$ 3×4, 4×5

$24+60 = 84$
$+45$
$\underline{129}$

$A(BC)$ 3×2, 2×5

$40+30 = 70$
$+45$
$\underline{115}$

optimal way $= \{A[(BC)D]\}$

Total no. of Multiplication $= \underline{88}$.

Ex-4

$A_{1\times2}\ B_{2\times3}\ C_{3\times2}\ D_{2\times4}$ $\Rightarrow$ 20

$E = ABCD$

$28+8 = \underline{36}$ E   30 $12+8 = \underline{20}$

$(A)(BCD)_{2\times4}$ 1×2

48   28

$(B)(CD)$ 2×3, 3×4

$24+24 = 48$

$(BC)D_{2\times4}$ 2×2

$12+16 = 28$

$(AB)(CD)$

$6 + 24 = \underline{30}$

$(ABC)(D)_{2\times4}$ 1×2

16   12

$(A)(BC)$ 1×2, 2×2

$12+4 = 16$

$(AB)C$ 1×3, 3×2

$6+6 = 12$

$k = i$ to $j-1$

l ↓

1. (1,1), (2,2) (3,3) (4,4) $l=1$
2. (1,2) (2,3) (3,4) $l=2$
3. (1,3) (2,4) $l=3$
4. (1,4) $l=4$

$l = 2$ to n

$i = 1$ to $n-l+1$

$j = i+l-1$

Let mcm[i,j] = The min. no. of multiplications required to multiply i to j matrices.

Assumption

$m_i \rightarrow i^{th}$ matrix

$P_{i-1} \times P_i$

↑ rows ↑ cols

e.g. $m_5 m_6 \ldots m_{100}$

⇓

$P_4 * P_{100}$

ex.

$$mcm(1,4) = \min \begin{cases} \underbrace{mcm(1,1)}_{P_0*P_1} + \underbrace{mcm(2,4)}_{P_1*P_4} + P_0*P_4*P_1 \\ \underbrace{mcm(1,2)}_{P_0*P_2} + \underbrace{mcm(3,4)}_{P_2*P_4} + P_0*P_4*P_2 \\ \underbrace{mcm(1,3)}_{P_0*P_3} + \underbrace{mcm(4,4)}_{P_3*P_4} + P_0*P_4*P_3. \end{cases}$$

R.R.

$$mcm(i,j) = \begin{cases} 0 & \text{if } i==j \\ \min\limits_{i \le k < j} \left\{ \underbrace{mcm(i,k)}_{P_{i-1}*P_k} + \underbrace{mcm(k+1,j)}_{P_k*P_j} + P_{i-1}*P_j*P_k \right\} & \text{if } i<j \end{cases}$$

→ mcm(1,n) will generate n level (n-1) array tree [approximately]

Total no. of function Calls $= (n-1)(n-2)(n-3)\ldots(2)(1)$
$= (n-1)!$

Time Complexity = # function Calls * each function cost
$= (n-1)! * n \rightarrow$ min 1 may
$= n!$
$= O(n^n)$

$\boxed{T.C. = O(n^n)}$

Space Complexity = i/P + extra
$\Downarrow$
4B*n · stack
$\Downarrow$
n

$= O(n)$

$\boxed{S.C. = O(n)}$

→ In the above recursive tree, some function Calls are repeating. So, we will going to DP. which will Solve only distinct function Calls.

→ How many distinct function Calls are there in mcm(1,n)?

Soln.

mcm (1, n)

mcm (1,4)

(1,1)	(2,2)	3,3	4,4.
(1,2)	(2,3)	3,4	
(1,3)	(2,4)		
(1,4)			

So, Total distinct function Calls $= n + n-1 + n-2 + \ldots + 2+1$
$= \frac{n(n+1)}{2}$
$= O(n^2)$

Time Complexity = $n^2 * O(n)$

$= O(n^3)$

$\boxed{T.C. = O(n^3)}$

Space Complexity = i/p + extra

i/p ⇓ $4n$

extra → Stack ⇓ n, Table ⇓ n^2

$= O(n^2)$

$\boxed{S.C. = O(n^2)}$

	0	1	2	3	4	5
0						
1		0	0	24	70	88
2			0	0	40	70
3				0	0	60
4					0	0

mcm

	0	1	2	3	4
0	0	0	0	0	0
1	0	0	24	70	88
2	0	0	0	40	70
3	0	0	0	0	60
4	0	0	0		0

Split.

	0	1	2	3	4
0	0	0	0	0	0
1	0	0	1	1	1
2	0	0	0	2	3
3	0	0	0	0	3
4	0	0	0	0	0