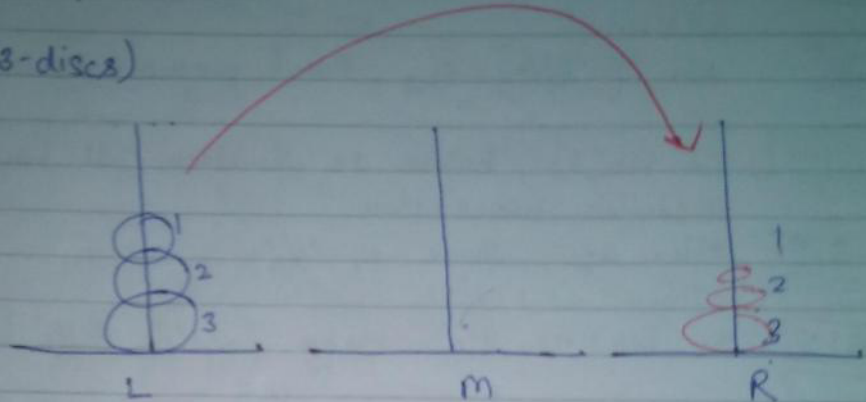


Condition : i) We can't put bigger disk on smaller disk
 ii) We can move one disk at a time.

Towers of Hanoi :-

$n=3$ (3-discs)



1. L-R
2. L-M
3. R-M
4. L-R
5. M-L
6. M-R
7. L-R

TOH (3, L, M, R)

{

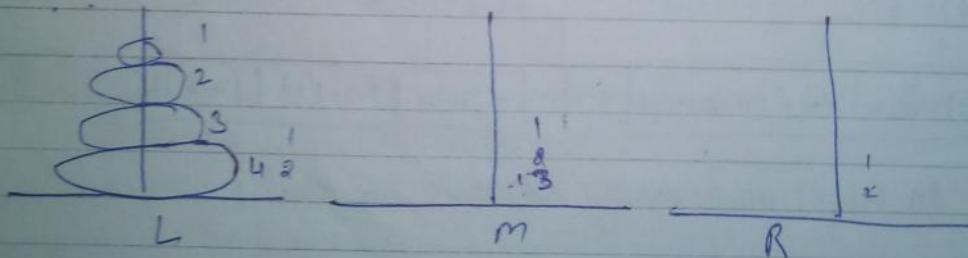
TOH (2, L, R, M)

mov (L, R)

TOH (2, M, L, R)

}

$n=4$.



- | | |
|----------------------|--------------------------|
| 1. L-M | 8. L-R |
| 2. L-R | 9. L-R M-R |
| 3. M-R | 10. L-R M-M-L |
| 4. L-M | 11. R-M R-L |
| 5. R- M L | 12. L-R M-R |
| 6. R-M | 13. M-L L-M |
| 7. L-M | |

TOH (4, L, M, R)

{

TOH (3, L, R, M)

mov (L, R)

TOH (3, M, L, R)

}

Q.5

R.P.: \Rightarrow

TOH(n, L, m, R)

{

if (n == 0)

return;

else

{ TOH(n-1, L, R, m) $\Rightarrow T(n-1)$

mov(L, R) $\Rightarrow C$

TOH(n-1, m, L, R) $\Rightarrow T(n-1)$

}

}

R.R.:-

TOH(n, L, m, R) = ~~if n == 0~~

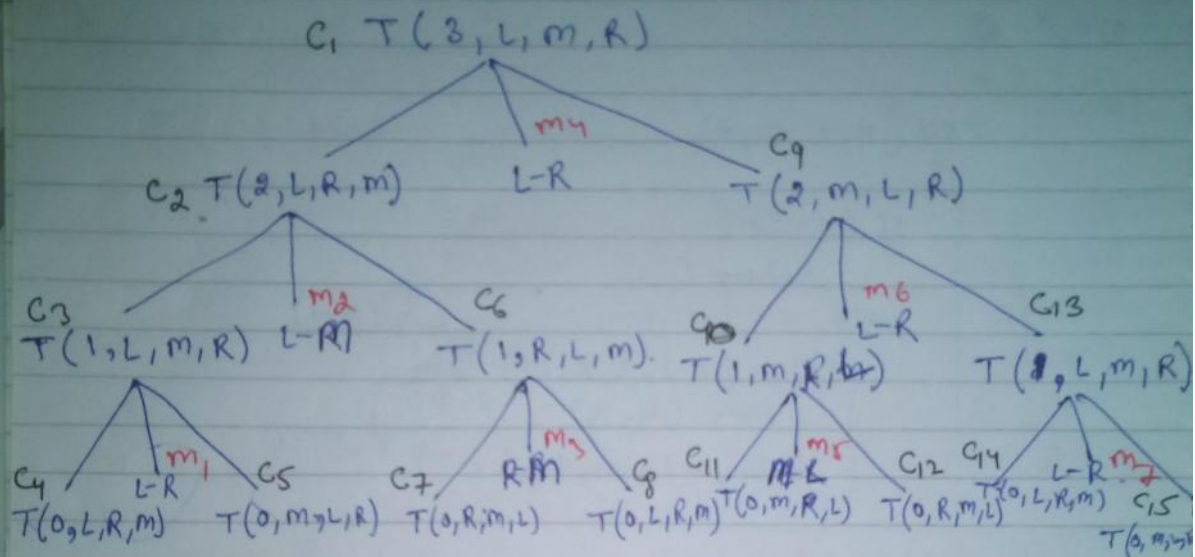
T.C. of TOH(n) \rightarrow

$$T(n) = 2T(n-1) + C$$
$$= O(2^n)$$

S.C. of TOH(n) \rightarrow

n+1 level B.T.

$$= O(\underline{n})$$



Q:-1 In TOH(n), After how many functions call, first move will be taken place?

Soln: $n+1$ (After C_4 there is m_1)

Q:-2 In TOH(n), Is there ~~more~~ any move After last function call?

Soln: No (After C_{15} no ~~more~~ move will there).

Q:-3 In TOH(n), Is any function call after last move?

Soln: Yes (After m_7 there is C_{15})

Q:-4 In TOH(3), After how many function call there is move from M-R.

Soln: 12 (After C_9 , there is m_6)

Q:-5 In TOH(n), how many function calls are there?

Solⁿ

Total function calls

$$T(0) = 0$$

$$T(1) = 3$$

$$T(2) = 7 \quad (2^{2+1} - 1)$$

$$T(3) = 15 \quad (2^{3+1} - 1)$$

$$T(4) = 31 \quad (2^{4+1} - 1)$$

$$T(50) = 2 \times T(49) + 1$$

$$\begin{aligned} T(n) &= 2T(n-1) + 1 \\ &= 2(2T(n-2) + 1) + 1 \\ &= 2^2 T(n-2) + 2 + 1 \\ &= 2^n - 1 \end{aligned}$$

Total moves

$$T(0) = 0$$

$$T(1) = 1$$

$$T(2) = 3$$

$$T(3) = 7$$

$$T(4) = 15$$

$$T(5) = 31$$

$$\begin{aligned} T(n) &= 2T(n-1) + 1 \\ &= 2^n - 1 \end{aligned}$$

Q:-6 In TOH(n), how many moves are there?

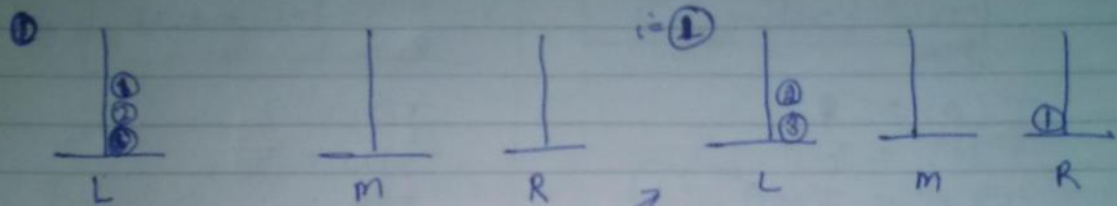
⇒ Iterative Algo:-

- ① Calculate total # moves for n disks = $2^n - 1$
- ② If # disks are even (n = even), interchange (m, R)
- ③ for i = 1 to total # moves:
 - if $(i \% 3 == 0)$
move disk b/w (m, R)
 - if $(i \% 3 == 1)$
move disk b/w (L, R)
 - if $(i \% 3 == 2)$
move disk b/w (L, m).

Continue:- →

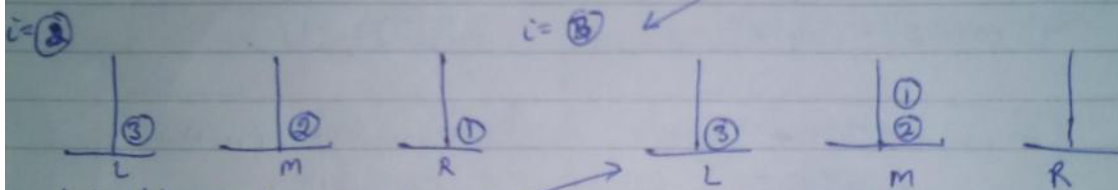
⇒ Iterative Algo for TON :-

$n = 3$ (disks), So total # moves = 7



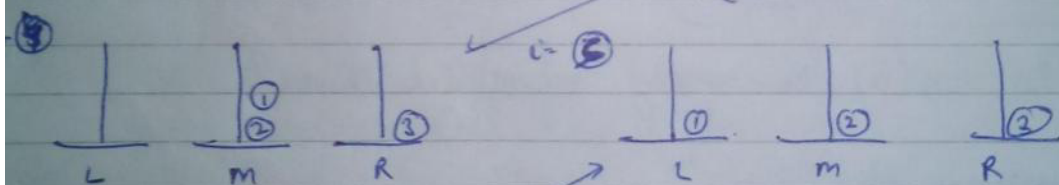
when $i=1$, $(i \% 3 == 1) \Rightarrow \text{mov}(L \rightarrow R)$

$i=2$, $(i \% 3 == 2) \Rightarrow \text{mov}(L \rightarrow M)$



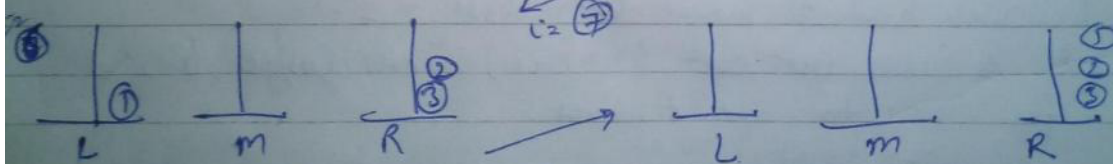
$i=3$, $(i \% 3 == 0) \Rightarrow \text{mov}(R \rightarrow M)$

$i=4$, $(i \% 3 == 1) \Rightarrow \text{mov}(L \rightarrow R)$

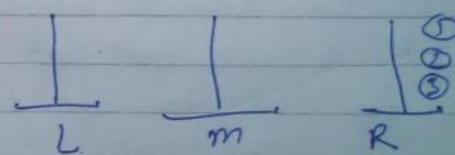


$i=5$, $(i \% 3 == 2) \Rightarrow \text{mov}(M \rightarrow L)$

$i=6$, $(i \% 3 == 0) \Rightarrow \text{mov}(M \rightarrow R)$



$i=7$, $(i \% 3 == 1) \Rightarrow \text{mov}(L \rightarrow R)$



$n=4$

