# 0/1 Knapsack :-

$n=3$, $m=15$

| | | | |
|---|---|---|---|
| object : | obj 1 | obj 2 | obj 3 |
| Profit : | 60 | 40 | 45 |
| weight : | 10 | 7 | 8 |

**Manually**

$(\underset{x_1}{0} \quad \underset{x_2}{1} \quad \underset{x_3}{1})$

0 0 0 ✗
1 0 0 ⇒ 60
0 1 0 ⇒ 40
0 0 1 ⇒ 45
0 1 1 ⇒ 85 ⇒ optimal

**Greedy Technique:**

$\frac{P_1}{w_1} = 6$,

$\frac{P_2}{w_2} = \frac{40}{7} = 5.7$

$\frac{P_3}{w_3} = \frac{45}{8} = 5.6$

$(\underset{x_1}{1} \quad \underset{x_2}{0} \quad \underset{x_3}{0}) \Rightarrow 60$

→ For 0/1 knapsack Problem, Greedy Algo will give wrong Answer. ∵ of this reason we are going to DP which will cover all possibilities & give always Correct Ans.

## Using DP

(ex) $n=5$ , $m=20$

| Obj : | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Profit : | 75 | 50 | 80 | 90 | 20 |
| weight : | 13 | 5 | 30 | 2 | 30 |
| | 130 | 5 | 30 | 200 | 30 |

140
75
210

245

$0/1KS(n,m)$ = The max. Profit we get where no. of object are 'n' & Capacity of knapsack is 'm'

(1) $0/1KS(5,0) = 0$
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(2) $0/1KS(n,m) = 0/1KS(n-1,m)$, if $w_n > m$

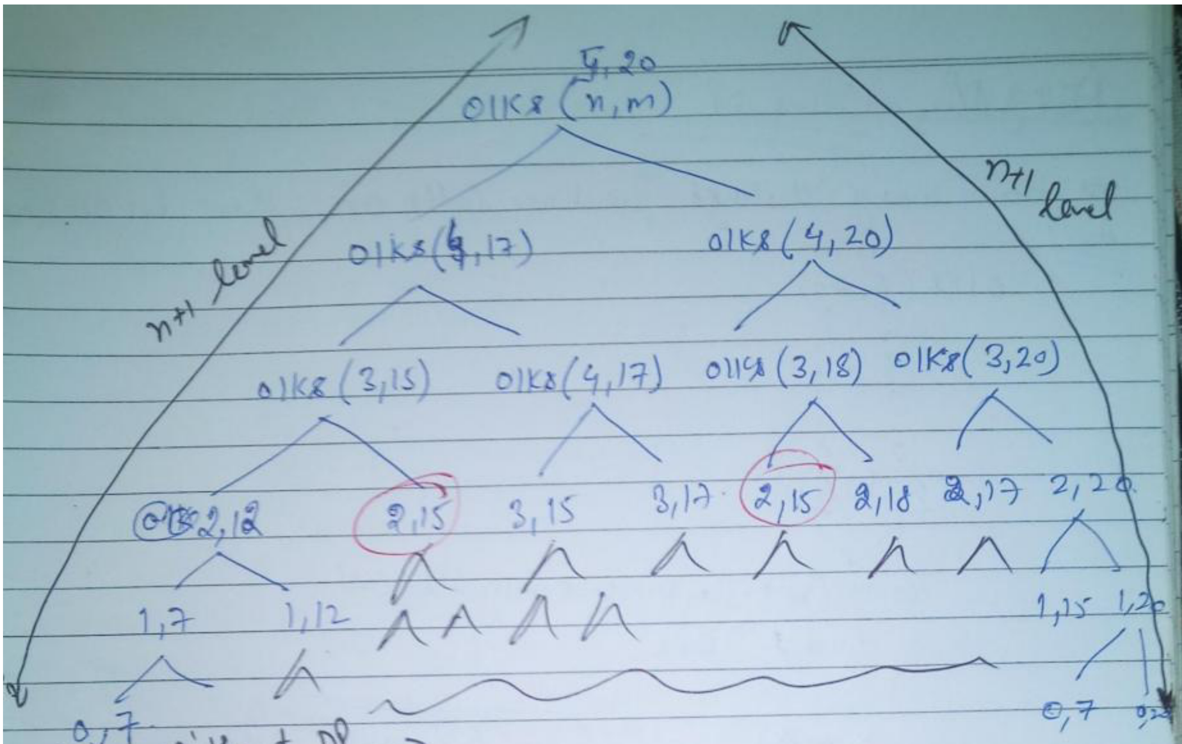
5, 20
4, 20

(3) $0/1KS(n,m) = \max \begin{cases} 0/1KS(n-1, m-w_n) + P_n \\ 0/1KS(n-1, m) \end{cases}$ if $w_n \le m$

5, 20

**R.R.**

$$0/1KS(n,m) = \begin{cases} 0 & \text{if } m=0 \text{ (or) } n=0 \\ 0/1KS(n-1,m) & \text{if } w_n > m \\ \max \begin{cases} 0/1KS(n-1, m-w_n) + P_n \\ 0/1KS(n-1, m) \end{cases} & \text{if } w_n \le m. \end{cases}$$




**Using without DP** →

$01KS(n, m)$

$\Downarrow$

$n+1$ level CBT [UB]

$\Downarrow$

$2^{n+1} - 1$ nodes

$\Downarrow$

$2^n$ function calls

$\Downarrow$

$2^n * 1$-comparison

$\Downarrow$

$O(2^n)$.

$\boxed{T.C. = O(2^n)}$

Space Complexity

$= i/p + \text{extra}$

$\Downarrow$ $n$    $\Downarrow$ stack.

$n + (n+1)$

$= O(n)$

$\boxed{S.C. = O(n)}$

→ In above Recursive Tree, some function calls are repeating [very less repetation] ∴ of this reason we are going to DP.

## ~~Using ...~~ Using DP

→ How many distinct function calls are there in 01KS(n,m)?

**Ans:**

$n, m$

01KS(5, 20)

4 19

3 18

⋮ ⋮

0 0

$\underbrace{(5+1)}$ $\underbrace{(20+1)}$

$(n+1)(m+1)$ – Distinct function call

$mn$ – DFC

$\Downarrow$

$O(mn)$. $\boxed{T.C = O(mn)}$

Space Complexity = i/p + extra

i/p $\Downarrow$ $n$

extra → stack $\Downarrow$ $n+1$, table $\Downarrow$ $mn$ - DFC

$= O(mn)$.

$\boxed{S.C = O(mn)}$

**NOTE:-**

① Because of less repetitions, The T.C. of 01KS $O(mn)$ is approximately equal to $O(2^n)$. So, it is consider to be NPC Problem.

② 01KS problem is polynomially reducible to ~~...~~ "Sum of Subsets Problems". So Sum of Subset Problem is also NPC problem.