

Matrix chain Multiplication:-

(ex-1)

$A_{3 \times 4}$

$B_{4 \times 2}$

~~$C_{3 \times 2}$~~

~~AB~~

$$C = A \times B$$

$$3 \times 2 \quad 3 \times 4 \quad 4 \times 2$$

\downarrow

3×2 - ele

\downarrow

6 - ele

\downarrow

6×4 - mul

\downarrow

24 - mul.

(ex-2)

$A_{5 \times 4}$

$B_{4 \times 6}$

$C_{6 \times 3}$

$$D = ABC$$

ABC

$(AB)C$

\downarrow

$$120 + (AB)_{5 \times 6} \times 6 \times 3$$

\downarrow

$$120 + 180$$

\downarrow

$$300$$

$A(BC)$

\downarrow

$$A_{5 \times 4} (BC)_{4 \times 3} \times 72$$

\downarrow

$$60 + 72$$

\downarrow

$$132$$

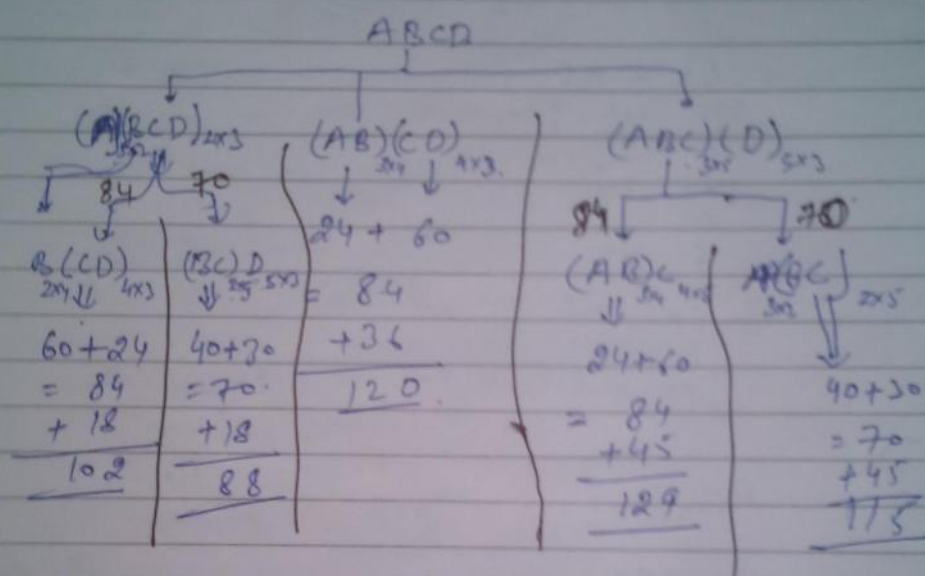
\Rightarrow optimal way
of keeping
Parankthesis.

total no. of Multiplication = 132

Ex-2

$A_{3 \times 2}, B_{2 \times 4}, C_{4 \times 5}, D_{5 \times 3}$

$E = ABCD$



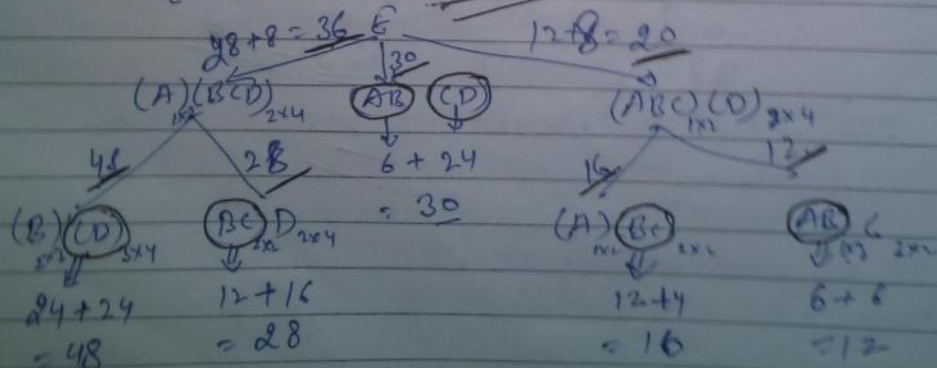
optimal way = $\{A[(BC)D]\}$

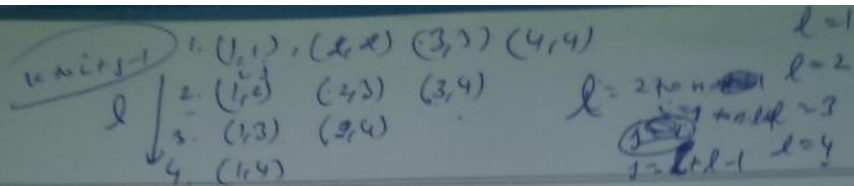
Total no. of Multiplication = 88.

Ex-4

$A_{1 \times 2}, B_{2 \times 3}, C_{3 \times 2}, D_{2 \times 4}$

$E = ABCD$



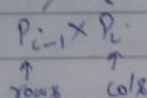


Let $mcm[i,j]$ = The min. no. of multiplications required to multiply i to j matrices.

Assumption

$m_i \rightarrow i$ th matrix

eg. m_5, m_6, \dots, m_{100}



$P_4 \times P_5$

ex.

$mcm(1,4) = \min \left\{ \begin{array}{l} \begin{array}{l} \Rightarrow P_0 \times P_1 \quad P_0 \times P_4 \\ mcm(1,1) + mcm(2,4) + P_0 \times P_4 \times P_1 \end{array} \\ \begin{array}{l} \Rightarrow P_0 \times P_2 \quad P_2 \times P_4 \\ mcm(1,2) + mcm(3,4) + P_0 \times P_4 \times P_2 \end{array} \\ \begin{array}{l} \Rightarrow P_0 \times P_3 \quad P_3 \times P_4 \\ mcm(1,3) + mcm(4,4) + P_0 \times P_4 \times P_3 \end{array} \end{array} \right.$

R.R.

$mcm(i,j) = \min \left\{ \begin{array}{l} 0 \quad \text{if } i=j \\ \begin{array}{l} \Rightarrow P_{i-1} \times P_k \quad P_k \times P_j \\ mcm(i,k) + mcm(k+1,j) + P_{i-1} \times P_k \times P_j \\ i \leq k < j \end{array} \end{array} \right.$

$\rightarrow mcm(1,n)$ will generate n -level, ~~(n-1) array tree~~ [approximately]

$$\text{Total no. of function calls} = (n-1)(n-2)(n-3) \dots (2)(1) \\ = (n-1)!$$

$$\text{Time Complexity} = \# \text{ function calls} \times \text{each function cost} \\ = (n-1)! \times n \rightarrow \text{min. \& max.} \\ = n! \\ = O(n^n)$$

$$\boxed{\text{T.C.} = O(n^n)}$$

$$\text{Space Complexity} = \text{i/p} + \text{extra} \\ \downarrow \quad \quad \quad \swarrow \quad \searrow \\ 4B \times n \cdot \text{stack} \quad \text{table} \\ \downarrow \quad \quad \quad \downarrow \\ n \quad \quad \quad n$$

$$= O(n)$$

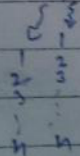
$$\boxed{\text{S.C.} = O(n)}$$

→ In the above recursive tree, some function calls are repeating. So, we will go to DP. which will solve only distinct function calls.

→ How many distinct function calls are there in $\text{mem}(1, n)$?

Soln:-

$\text{mem}(1, n)$



$\text{mem}(1, 4)$

or $\text{arr} \downarrow \text{map}$

(1,1)	(2,2)	3,3	4,4
(1,2)	(2,3)	3,4	
(1,3)	(2,4)		
(1,4)			

$$\text{So, Total distinct function calls} = n + n-1 + n-2 + \dots + 2 + 1 \\ = \frac{n(n+1)}{2} \\ = \underline{\underline{O(n^2)}}$$

Time Complexity = $n^2 * o(n)$
 $= o(n^3)$

G.C. = $O(n^3)$

Space Complexity = i/p + extra

\downarrow

$4n$

Stack $\downarrow n$

Table $\downarrow n^2$

$= O(n^2)$

S.C. = $O(n^2)$

[illegible]

Split:

	0	1	2	3	4
0	0	0	0	0	0
1	0	0	1	1	1
2	0	0	0	2	3
3	0	0	0	0	3
4	0	0	0	0	0