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UNIT-I

I. NUMBER SYSTEMS AND BOOLEAN ALGEBRA

1. Decimal number system:-

0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (10 digits)

Ex:- 73_{10}, 48.92_{10} → Base (or) Radix (or) Index.

731.23

Most significant Bit (or) Digit (MSB (or) MSD)

Integer Part

Fractional Part

Decimal point.

least significant Bit (or) digit (LSB (or) LSD)

$10^2\ 10^1\ 10^0\ 10^{-1}\ 10^{-2}$ → positional values (or) weights.
7 3 1 . 2 3

2. Binary number system:-

0, 1 (2 digits)

Ex:- 101_2, 110010_2

$2^2\ 2^1\ 2^0\ 2^{-1}\ 2^{-2}$ → positional values (or) weights
1 0 1 . 0 1

Binary point

3. Octal number system:-

0, 1, 2, 3, 4, 5, 6, 7 (8 digits)

Ex:- 73_8, 4072_8, 123.65_8

$8^2\ 8^1\ 8^0\ 8^{-1}\ 8^{-2}$ → weights (or) positional values.
1 2 3 . 6 5

Octal point.

4. Hexa decimal number system :-

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F (16 digits)

Ex:- $B73_{16}$, $57A.C3_{16}$

$\overset{16^2}{1}\ \overset{16^1}{2}\ \overset{16^0}{A}\ .\ \overset{16^{-1}}{1}\ \overset{16^{-2}}{3}$ ⟶ weights (or) positional values.

↓

Hexa decimal point.

* Relation b/w number systems :-

Decimal	Octal	Hexa decimal	Binary
0	0	0	0
1	1	1	1
2	2	2	10
3	3	3	11
4	4	4	100
5	5	5	101
6	6	6	110
7	7	7	111
8	10	8	1000
9	11	9	1001
10	12	A	1010
11	13	B	1011
12	14	C	1100
13	15	D	1101
14	16	E	1110
15	17	F	1111
16	20	10	10000
17	21	11	10001

Number Base conversion :-

1. Binary to Decimal conversion :-

Ex:- ① $(101.11)_2 = (\ ?\ )_{10}$

Sol:- $\overset{2^2}{1}\ \overset{2^1}{0}\ \overset{2^0}{1}\ .\ \overset{2^{-1}}{1}\ \overset{2^{-2}}{1}$

$= 1\times2^2 + 0\times2^1 + 1\times2^0 + 1\times2^{-1} + 1\times2^{-2}$

$= 4 + 0 + 1 + 0.5 + 0.25 = 5.75$

$$\boxed{(101.11)_2 = (5.75)_{10}}$$

② $(1011.101)_2 = (\ ?\ )_{10}$

Sol:- $1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0 + 1\times2^{-1} + 0\times2^{-2} + 1\times2^{-3}$

$= 8 + 2 + 1 + \frac{1}{2} + \frac{1}{8}$

$= (11.625)_{10}$

$$\boxed{(1011.101)_2 = (11.625)_{10}}$$

2. Octal to Decimal conversion :-

Ex:- ① $(75.3)_8 = (\ ?\ )_{10}$

Sol:- $= 7\times8^1 + 5\times8^0 + 3\times8^{-1}$

$= 56 + 5 + \frac{3}{8} = (61.375)_{10}$

$$\boxed{(75.3)_8 = (61.375)_{10}}$$

② $(624.712)_8 = (\ ?\ )_{10}$

Sol:- $6\times8^2 + 2\times8^1 + 4\times8^0 + 7\times8^{-1} + 1\times8^{-2} + 2\times8^{-3}$

$= 6\times64 + 16 + 4 + \frac{7}{8} + \frac{1}{64} + \frac{2}{64\times8}$

$= 384 + 20 + 0.875 + 0.015625 + 0.00390625$

$= (404.894)_{10}$

$$\boxed{(624.712)_8 = (404.894)_{10}}$$

Ex:- ③ $(482.31)_8 = (\ ?\ )_{10}$

It is not an octal number.

So, conversion is not possible.

③ Hexadecimal to Decimal conversion :-

Ex:- ① $(7A2.C9)_{16} = (\ ?\ )_{10}$

Sol:- $7\times16^2 + A\times16^1 + 2\times16^0 + C\times16^{-1} + 9\times16^{-2}$

$= 7\times256 + 10\times16 + 2\times1 + \frac{12}{16} + \frac{9}{256}$

$= 1792 + 160 + 2 + \frac{3}{4} + 0.3515625$

$= (1954.78)_{10}$

$\boxed{(7A2.C9)_{16} = (1954.78)_{10}}$

② $(CD3.B7)_{16} = (\ ?\ )_{10}$

Sol:- $C\times16^2 + D\times16^1 + 3\times16^0 + B\times16^{-1} + 7\times16^{-2}$

$= 12\times256 + 13\times16 + 3\times\frac{11}{16} + \frac{7}{256}$

$= (3283.715)_{10}$

$\boxed{(CD3.B7)_{16} = (3283.715)_{10}}$

④ Base 5 no. to Decimal conversion :-

Ex:- ① $(431.23)_5 = (116.52)_{10}$

Sol:- $4\times5^2 + 3\times5^1 + 1\times5^0 + 2\times5^{-1} + 3\times5^{-2}$

$= 100 + 15 + 1 + \frac{2}{5} + \frac{3}{25}$

$= (116.52)_{10}$

$\boxed{(431.23)_5 = (116.52)_{10}}$

⑤ $(231.12)_3 = (?)_{10}$

Sol:- It is not a Base 3 no.

Base 3 no. digits are 0, 1, 2.

⑥ Decimal to Binary conversion :-

Ex:- ① $(13.25)_{10} = (?)_2$

Sol:- Integer part conversion

$$\begin{array}{r|l} 2 & 13 \\ 2 & 6 - 1 \\ 2 & 3 - 0 \\ 2 & 1 - 1 \\ & 0 - 1 \end{array}$$

↑ all remainders from bottom to top

$(13)_{10} = 1101_2$

Fractional part conversion

$0.25 \times 2 = 0.5$

$0.5 \times 2 = 1.0$

↓ Integer parts of all product terms from top to bottom.

$0.25_{10} = 0.01_2$

$$\boxed{(13.25)_{10} = (1101.01)_2}$$

② $(5.6)_{10} = (?)_2$

Sol:- I.P.C.

$$\begin{array}{r|l} 2 & 5 \\ 2 & 2 - 1 \\ 2 & 0 - 0 \\ & 0 - 1 \end{array}$$

↑

$(5)_{10} = (101)_2$

F.P.C

$0.6 \times 2 = 1.2$

$0.2 \times 2 = 0.4$

$0.4 \times 2 = 0.8$

$0.8 \times 2 = 1.6$

$0.6 \times 2 = 1.2$

$0.2 \times 2 = 0.4$

$0.4 \times 2 = 0.8$

$0.8 \times 2 = 1.6$.

↓ $0.\overline{1001}$

$(5.6)_{10} = (101.\overline{1001})_2$

Note :- If there is no repetation in that case the process of multiplication is to be stopped after 4 or 5 places.

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7. **Decimal to octal conversion :-**

Ex:- (i) $(73.625)_{10} = (\ ?\ )_8$

Sol:- Integer conversion

$$\begin{array}{r|l} 8 & 73 \\ 8 & 9 - 1 \\ 8 & 1 - 1 \\ & 0 - 1 \end{array} \uparrow$$

$(73)_{10} = (111)_8$

Fractional part convertion

$0.625 \times 8 = 5.000 \downarrow$

$(0.625)_{10} = (0.5)_8$

$$\therefore \boxed{(73.625)_{10} = (111.5)_8}$$

(ii) $(296.198)_{10} = (\ ?\ )_8$

Sol:- Integer part conversion

$$\begin{array}{r|l} 8 & 296 \\ 8 & 37 - 0 \\ 8 & 4 - 5 \\ & 0 - 4 \end{array} \uparrow$$

$(296)_{10} = (450)_8$

Fractional part convertion

$0.198 \times 8 = 1.584$
$0.584 \times 8 = 4.672$
$0.672 \times 8 = 5.376$
$0.376 \times 8 = 3.008$
$0.008 \times 8 = 0.064$
$0.064 \times 8 = 0.512$

$(0.198)_{10} = (0.145300)_8$

$$\boxed{(296.198)_{10} = (450.145300)_8}$$

8. **Decimal to Hexa Decimal :-**

Ex:- (i) $(1954.785)_{10} = (\ ?\ )_{16}$

Sol:- Integer part conversion

$$\begin{array}{r|l} 16 & 1954 \\ 16 & 122 - 2 \\ 16 & 7 - 10\ A \\ & 0 - 7 \end{array} \uparrow$$

$(1954)_{10} = (7A2)_{16}$

Fractional part conversion

$0.785 \times 16 = 12.56$ C
$0.56 \times 16 = 8.96$ 8
$0.96 \times 16 = 15.36$ F
$0.36 \times 16 = 5.76$ 5
$0.76 \times 16 = 12.16$ C

$(0.785)_{10} = (0.C8F5C)_{16}$

$$\therefore \boxed{(1954.785)_{10} = (7A2.C8F5C)_{16}}$$

(ii) $(3283.715)_{10} = (\ ?\ )_{16}$

Sol:- Integer part conversion

$$\begin{array}{r|l} 16 & 3283 \\ 16 & 205 - 3 \\ 16 & 12 - 13\ D \\ & 0 - 12\ C \end{array}$$

$(3283)_{10} = (CD3)_{16}$

Fractional part conversion

$0.715 \times 16 = 11.44$ — B
$0.44 \times 16 = 7.04$ — 7
$0.04 \times 16 = 0.64$ — 0
$0.64 \times 16 = 10.24$ — A

$(0.715)_{10} = (0.B70A)_{16}$.

$\therefore \boxed{(3283.715)_{10} = (CD3.B70A)_{16}}$

9. Base 3 to Base 5 :-

Ex:- (i) $(21.1)_3 = (\ ?\ )_5$

Sol:- Base 3 $\longrightarrow$ decimal $\longrightarrow$ Base 5.

$2\times3^1 + 1\times3^0 + 1\times3^{-1} = 7.333$

$\therefore (21.1)_3 = (7.333)_{10}$.

$(7.333)_{10} = (\ ?\ )_5$.

$$\begin{array}{r|l} 5 & 7 \\ 5 & 1 - 2 \\ & 0 - 1 \end{array}$$

$(7)_{10} = (12)_5$

$0.333\times5 = 1.665$
$1.665\times5 = 3.325$
$0.325\times5 = 1.625$
$0.625\times5 = 3.125$
$0.125\times5 = 0.725$
$0.725\times5 = 3.625$

$(0.333)_{10} = (0.13\overline{130})_5$.

$\therefore \boxed{(7.333)_{10} = (12.13\overline{130})_5 = (21.1)_3}$

10. Octal to Binary conversion :-

Ex:- (i) $(73.2)_8 = (\ ?\ )_2$

Sol:-

$$\begin{array}{ccccc} 7 & 3 & . & 2 \\ \downarrow & \downarrow & & \downarrow \\ 111 & 011 & & 010 \end{array}$$

$\therefore \boxed{(73.2)_8 = (111011.010)_2}$

(ii) $(23.46)_8 = (\ ?\ )_2$

Sol:-
$$\begin{array}{cccc} 2 & 3 & . & 4 & 6 \\ \downarrow & \downarrow & & \downarrow & \downarrow \\ 010 & 011 & & 100 & 110 \end{array}$$

$\therefore \boxed{(23.46)_8 = (10011.10011)_2}$

11. **Hexadecimal to Binary conversion**:-

Ex:- (i) $(7A.2C)_{16} = (\ ?\ )_2$

Sol:-
$$\begin{array}{ccccc} 7 & A & . & 2 & C \\ \downarrow & \downarrow & & \downarrow & \downarrow \\ 0111 & 1010 & & 0010 & 1100 \end{array}$$

$\boxed{(7A.2C)_{16} = (1111010.00101100)_2}$

(ii) $(D2.E9)_{16} = (\ ?\ )_2$

Sol:-
$$\begin{array}{ccccc} D & 2 & . & E & 9 \\ \downarrow & \downarrow & & \downarrow & \downarrow \\ 1101 & 0010 & & 1110 & 1001 \end{array}$$

$\boxed{(D2.E9)_{16} = (11010010.11101001)_2}$

12. **Binary to octal conversion**:-

16 8 4 2 1

Ex:- (i) $(10111.101)_2 = (\ ?\ )_8$

Sol:- $\underbrace{010}_{2}\underbrace{111}_{7}.\underbrace{101}_{5} = (27.5)_8$

$\therefore \boxed{(10111.101)_2 = (27.5)_8}$

(ii) $(1011.1)_2 = (\ ?\ )_8$

Sol:- $\underbrace{001}_{1}\underbrace{011}_{3}.\underbrace{100}_{4} = (13.4)_8$

$\therefore \boxed{(011011.100)_2 = (13.4)_8}$

13. Binary to Hexadecimal :-

Ex:- (i) $(01110111.1010)_2 = (\ ?\ )_{16}$

Sol:- $\underbrace{0111}_{7}\underbrace{0111}_{7}.\underbrace{1010}_{A} = (77.A)_{16}$

$\therefore \boxed{(01110111.1010)_2 = (77.A)_{16}}$

(ii) $(10111.1000)_2 = (\ ?\ )_{16}$

Sol:- $\underbrace{0001}_{1}\underbrace{0111}_{7}.\underbrace{1000}_{8} = (17.8)_{16}$

$\therefore \boxed{(10111.1)_2 = (17.8)_{16}}$

14. Octal to Hexadecimal :-

Ex: (i) $(35.7)_8 = (\ ?\ )_{16}$

Sol: Octal $\longrightarrow$ Binary $\longrightarrow$ Hexa decimal.

$$\begin{array}{ccc} 3 & 5 & .\ 7 \\ \downarrow & \downarrow & \downarrow \\ 011 & 101 & 111 \end{array}$$

$(35.7)_8 = (\underbrace{01}_{1}\underbrace{1101}_{D}.\underbrace{111}_{E})_2$

$\therefore \boxed{(35.7)_8 = (1D.E)_{16}}$

(ii) $(73.2)_8 = (\ ?\ )_{16}$

Sol: 7 3 . 2

111 011 010

$(73.2)_8 = (\underbrace{11}_{3}\underbrace{1011}_{B}.\underbrace{010}_{4})_2$

$\therefore \boxed{(73.2)_8 = (3B.4)_{16}}$

15. Hexadecimal to octal :-

Hexadecimal $\longrightarrow$ Binary $\longrightarrow$ octal

Ex:- ① $(1D.E)_{16} = (\ ?\ )_8$

Sol:-

$$\begin{array}{ccc} 1 & D & . & E \\ 0001 & 1101 & & 1110 \end{array}$$

$$\therefore (1D.E)_{16} = (\underbrace{011}_{3}\underbrace{101}_{5}.\underbrace{111}_{7})_2$$

$$= (35.7)_8$$

$$\therefore \boxed{(1D.E)_{16} = (35.7)_8}$$

② $(3B.4)_{16} = (\ ?\ )_8$

Sol:-

$$\begin{array}{ccc} 3 & B & . & 4 \\ 0011 & 1011 & & 0100 \end{array}$$

$$= (\underbrace{111}_{7}\underbrace{011}_{3}.\underbrace{010}_{2})_2$$

$$\therefore \boxed{(3B.4)_{16} = (73.2)_8}$$

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* Binary addition

Rules

	Sum	Carry	Result
0+0	0	0	0
0+1	1	0	1
1+0	1	0	1
1+1	0	1	10

$$\begin{array}{r} 1 \\ +1 \\ \hline 10 \\ +1 \\ \hline 11 \\ +1 \\ \hline 100 \\ +1 \\ \hline 101 \end{array}$$

Ex:- (i) perform $1011_2 + 1111_2$

Sol:-

$$\begin{array}{r} 1011 \\ +1111 \\ \hline \underbrace{11}_{\text{Carry}}\underbrace{010}_{\text{Sum}} \end{array}$$

$1011_2 + 1111_2 = 11010_2$

11 + 15 = 26

(ii) Perform $1011_2 + 1101_2 + 1110_2$

Sol:

$$\begin{array}{r} 1011 \\ 1101 \\ +\ 1110 \\ \hline \underbrace{10}_{\text{carry}}\underbrace{0110}_{\text{sum}} \end{array}$$

$\therefore \boxed{1011_2 + 1101_2 + 1110_2 = 100110_2}$

(iii) $1001.1_2 + 1101.01_2$

Sol:

$$\begin{array}{r} 1001.10 \\ 1101.01 \\ \hline \underbrace{1}_{\text{carry}}\underbrace{0110.11}_{\text{sum}} \end{array}$$

$\therefore \boxed{1001.1_2 + 1101.01_2 = 10110.11_2}$

* Binary multiplication :-

Ex: (i) $101_2 \times 111_2$

Sol:

$$\begin{array}{r} 101 \\ \times 111 \\ \hline 101 \\ 101 \\ 101 \\ \hline 100011 \end{array}$$

$\therefore \boxed{101_2 \times 111_2 = 100011_2}$

(ii) $1010.1_2 \times 1011.1_2$

Sol:

$$\begin{array}{r} 1010.1 \times 1011.1 \\ \hline 10101 \\ 10101 \\ 10101 \\ 00000 \\ 10101 \\ \hline 1111000.11 \end{array}$$

$\therefore \boxed{1010.1_2 \times 1011.1_2 = 1111000.11}$

* Binary division :-

Ex: (i) $11101_2 \div 100_2$

Sol:

$$\begin{array}{l} 100)\,11101\,(\,111 \\ \underline{100} \\ \ 111 \\ \ \underline{100} \\ \ \ 110 \\ \ \ \underline{100} \\ \ \ \ 101 \\ \ \ \ \underline{100} \\ \ \ \ \ \ 1 \end{array}$$

Q R

$\therefore \boxed{11101 \div 100 = 111, 1}$

Ex: (ii) $1010.1 \div 101.01$

Sol:

$$\frac{1010.1}{101.01} = \frac{10101/10}{10101/100}$$

$$= \frac{101010}{10101}$$

$$= 10$$

* ## Binary subtraction & complements :-

Note:-

$A - B = A + r$'s complement of B (Subtraction using radix complement)

$A - B = A + (r-1)$'s complement of $B + 1$ (subtraction using diminished radix complement)

$r \longrightarrow$ radix (or) Base of A & B no.'s

If A & B are Binary no.'s then

$A - B = A + 1$'s comp. of $B + 1$

$= A + 2$'s comp. of B.

where
$A \rightarrow$ Minuend
$B \rightarrow$ subtrahend.

1's complement:-

1's complement of any number can be obtained by changing each '0' to '1' and '1' to '0'.

∴ 1's complement of 1101001 is 0010110.

1's complement subtraction:-

rules:-

(i) Add minuend and 1's complement of subtrahend.
(ii) If there is carry after addition, then the result is '+ve'. else result is '+ve'. else result is '-ve'
(iii) If result is '+ve', add carry to sum to get the actual value.
(iv) If result is '-ve' take 1's complement of sum to get the actual value.

Ex:- (1) Perform unsigned binary subtraction using 1's complement.

(i) $1101_2 - 1001_2$

Sol:-
1101 $\longrightarrow$ minuend
1001 $\longrightarrow$ Subtrahend
0110 $\longrightarrow$ 1's complement of Subtrahend.

```
 1101  → minuend
 0110  → 1's comp. of Sub
10011
```

$\underbrace{1}_{carry}\underbrace{0011}_{sum}$

∴ There is a carry. ∴ Result is '+ve'.

To get the actual value (difference) add carry to sum

```
 0011
    1
 0100  → difference.
```

∴ $\boxed{1101_2 - 1001_2 = 0100_2}$

(ii) $1001_2 - 1101_2$

Sol:

```
 1001  → minuend
 1101  → Subtrahend.

 1001  → minuend
 0010  → 1's complement of Subtrahend
 1011
```

$\underbrace{1011}_{sum}$

There is no carry. ∴ Result is '-ve'.

To get the actual value take 1's complement of sum.

1's complement of 1011 is 0100

∴ $\boxed{1001_2 - 1101_2 = -0100_2}$

2's complement :-

2's complement of A = 1's complement of A + 1

Ex:- Find 2's complement of 1010.

Sol:

= 1's complement of 1010 + 1

= 0101 + 1

= 0110

```
 0101
    1
 0110
```

∴ $\boxed{\text{2's complement of } 1010 = 0110}$

2's comp of 101100 is 010100 (write no. as it is from right to left until u found 1. if u found 1 then write that first 1 as it is. then change the remaining no.s from 0's to 1's and 1's to 0's)

2's complement subtraction rules:-

(i) Add minuend and 2's complement of subtrahend.

(ii) After addition if there is carry then the result is '+ve', else '-ve'.

(iii) If the result is +ve to get the actual value discard the carry.

(iv) If the result is -ve to get the actual values take 2's complement of sum.

Ex:- 1. perform unsign binary subtraction using 2's complement.

& (i) $1101_2 - 1010_2$

Sol:

1101 → minuend
1010 → subtrahend.

$$\begin{array}{r} 0101 \\ +1 \\ \hline 0110 \end{array}$$

2's comp. of subtrahend (1010) = 0101 + 1 = 0110.

$$\begin{array}{rl} 1101 & \rightarrow \text{minuend} \\ 0110 & \rightarrow \text{2's comp. of sub.} \\ \hline \underbrace{1}_{\text{carry}}\underbrace{0011}_{\text{sum}} & \end{array}$$

∴ It has carry. ∴ Result is '+ve'.

∴ To get actual value discard the carry.

$= 0011_2$

∴ $\boxed{1101_2 - 1010_2 = 0011_2}$

(ii) $1010_2 - 1101_2$

Sol:

1010 → min.
1101 → sub.

$$\begin{array}{rl} 0011 & \rightarrow \text{2's comp. of sub.} \\ 1010 & \rightarrow \text{min} \\ \hline \underbrace{1101}_{\text{sum}} & \end{array}$$

∴ It has no carry. ∴ Result is '-ve'.

To get actual value take 2's comp. of sum.

2's comp. of $1101 = 0010 + 1 = 0011_2$

∴ $\boxed{1010_2 - 1101_2 = -0011_2}$

9's complement:-

9's complement of a decimal number can be obtained by the sub.tracting each digit from 9.

Ex:- 9's comp. of 73_{10} is

$$\begin{array}{r} 99 \\ -73 \\ \hline 26 \end{array}$$

$\therefore$ $\boxed{\text{9's comp. of } 73_{10} \text{ is } 26}$

9's complement subtraction:-

Rules same as is complement but instead of 1's complement take 9's complement.

Ex:- (i) $73_{10} - 26_{10}$

Sol:- 73 → min

26 → Sub.

$$\begin{array}{r} 99 \\ -26 \\ \hline 73 \end{array}$$

73 → min.

73 → 9's comp. of sub.

$$\begin{array}{r} 73 \\ 73 \\ \hline 146 \end{array}$$

carry sum

$\therefore$ It has carry. $\therefore$ Result is '+ve'.

To get actual value add carry to sum.

$$\begin{array}{r} 46 \\ 1 \\ \hline 47_{10} \end{array}$$

$\therefore$ $\boxed{73_{10} - 26_{10} = 47_{10}}$

(ii) $26_{10} - 73_{10}$

Sol:- 26 → min.

73 → sub.

$$\begin{array}{r} 99 \\ 73 \\ \hline 26 \end{array}$$ → 9's comp. of sub.

$$\begin{array}{r} 26 \\ 26 \\ \hline 52 \end{array}$$

It has no carry. Result is '-ve'.

To get actual values Take 9's comp. of sum.

$$\begin{array}{r} 99 \\ -52 \\ \hline 47 \end{array} \longrightarrow \text{9's comp. of 52.}$$

$\therefore$ $\boxed{26_{10} - 73_{10} = -47_{10}}$

10's complement :-

10's comp. of A = 9's comp. of A + 1

Ex:- Find 10's comp. of 23_{10}.

Sol:- 10's comp. of 23 = 9's comp. of 23 + 1

= 76 + 1

= 77

$$\begin{array}{r} 99 \\ -23 \\ \hline 76. \end{array}$$

$\therefore$ $\boxed{\text{10's comp. of } 23 = 77}$

10's complement subtraction :-

Rules same as 2's subtraction complement but take 10's complement instead of 2's comp.

Ex:- $45_{10} - 16_{10}$

Sol:- 45 → min.

16 → sub.

10's comp. of 16 = 9's comp of 16 + 1

= 83 + 1 = 84

$$\begin{array}{r} 99 \\ -16 \\ \hline 83 \end{array}$$

$\therefore$ 84 → 10's comp. of sub.

45 → min.

$$\underbrace{1}_{\text{carry}}\ \underbrace{29}_{\text{sum}}$$

It has carry. $\therefore$ Result is '+ve'.

To get actual value discard the carry.

$\therefore$ $\boxed{45_{10} - 16_{10} = 29_{10}}$

Ex:- (ii) $16_{10} - 45_{10}$.

Sol:
16 → min.
45 → sub.

10's comp. of 45 = 9's comp of 45 + 1 $\begin{array}{r} 99 \\ -45 \\ \hline 54 \end{array}$
= 54 + 1 = 55.

$$\begin{array}{rl} 55 & \rightarrow \text{10's comp. of 45} \\ 16 & \rightarrow \text{min} \\ \hline \underbrace{71}_{\text{sum}} & \end{array}$$

It has no carry. ∴ Result is '-ve'.

To get actual value take 10's comp. of sum.

10's comp. of 71 is = 9's comp. of 71 + 1 $\begin{array}{r} 99 \\ -71 \\ \hline 28 \end{array}$
= 28 + 1 = 29

∴ $\boxed{16_{10} - 45_{10} = -29_{10}}$

Octal subtraction :-

If A & B are octal no.'s then

A − B = A + 8's comp. of B
= A + 7's comp. of B + 1

7's comp. of 23_8 is $\begin{array}{r} 77 \\ -23 \\ \hline 54 \end{array}$

Hexadecimal Subtraction :-

If A & B are octal no.'s then,

A − B = A + 16's comp. of B
= A + 15's comp. of B + 1

15's comp. of $A2_{16}$ is

$$\begin{array}{r} FF \\ -A2 \\ \hline 5D \end{array}$$

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Signed Binary numbers :-

MSB LSB

B_7	B_6	B_5	B_4	B_3	B_2	B_1	B_0

$\underbrace{B_7}_{\text{Sign Bit}}\ \underbrace{B_6\ B_5\ B_4\ B_3\ B_2\ B_1\ B_0}_{\text{Magnitude}}$

sign bit = 0 $\longrightarrow$ +ve binary number

sign bit = 1 $\longrightarrow$ −ve binary number.

Ex:- Represent -5_{10}, -9_{10}, $+17_{10}$, in signed magnitude form.

Sol:- $5 \longrightarrow 101$

$-5_{10} = (\underbrace{1}_{\text{Sign bit}}\underbrace{101}_{\text{Magn.}})_{\text{Signed binary}}$ (∵ add '1' in MSB place)

$9 \longrightarrow 1001$

$-9_{10} = (\underline{1}\,\underline{1001})_{\text{signed binary}}$

$+17_{10} = \underline{0}\,\underline{10001}$

$+9_{10} = (\underline{0}\,\underline{1001})_{\text{signed binary}}$.

Ex:- Represent -5_{10}, -9_{10}, $+17_{10}$ in 8 bit signed magnitude form.

Sol:- -5_{10} =

1	0	0	0	0	1	0	1

$-5_{10} = (10000\underline{101})_{\text{signed Binary (2)}}$

$-9_{10} = (1000\underline{1001})_{\text{signed binary}}$

$+17_{10} = (000\underline{10001})_{\text{Signed binary}}$.

Ex:- Represent +7, −7 in 8bit signed binary, signed 1's complement, signed 2's complement form.

Sol: (i) +7 = 0111 (signed binary representation)

+7 = 0111 (signed 1's complement ")

+7 = 0111 (signed 2's complement ")

+7 = 0000111 (8-bit signed binary ")

+7 = 00000111 (signed 1's comp. ")

+7 = 00000111 (signed 2's " ").

(ii) −7 = 1111 (signed magnitude Representation)

−7 = 1000 (signed 1's comp. ") (+7 1's complement)

−7 = 1001 (" 2's " ") (+7 2's complement = 1's comp + 1.)

−7 = 10000111 (8-Bit Signed magnitude rep)

−7 = 11111000 (signed 1's comp. rep)

−7 = 11111001 (" 2's comp. ")

Note: In signed 1's complement '−7' is obtained by complementing all the bits of +7, including the sign bit.

In signed 2's complement '−7' is obtained by taking 2's complement of +7.

* Arthametic Addition :-

Ex:- Perform the following using 8-Bit Binary arthametic.

(i) (+6) + (+13) (ii) (−6) + (+13) (iii) (+6) + (−13)

(iv) (−6) + (−13).

(i) sol:- +6 → 00000110

+13 → 00001101

$$\begin{array}{r} 00000110 \\ 00001101 \\ \hline \underline{0}\,\underbrace{0010011}_{19} \end{array}$$

Signed

Signed bit is '0'.

∴ Result is '+ve' number.

$\boxed{(+6) + (+13) \Rightarrow +19}$

(ii) sol:- +6 → 00000110

(−6 → 10000110)×

Represent '-ve' no's in signed 2's comp. form

−6 = (2's comp. of +6)

−6 = 11111010

+13 = 00001101

$$\begin{array}{r} 11111010 \\ 00001101 \\ \hline \underline{1}00\,\underbrace{000111}_{7} \end{array}$$

Carry.

Sign bit = 0 , ∴ Result is '+ve' no.

If carry is generated, discard that carry.

$\boxed{(-6) + (+13) = +7}$

(iii) $+6 \rightarrow 00000110$

$-13 \rightarrow 11110011$ (2's comp. of +13)

$$
\begin{array}{r}
00000110 \\
11110011 \\
\hline
\underline{1}1111001
\end{array}
$$

Sign bit

sign bit = 1, ∴ Result is '-ve' no.

'-ve' results are already in 2's comp. form.

To get result take 2's comp. of sum including sign bit.

2's comp. of 11111001 is $\underbrace{00000111}_{7}$.

$$\therefore \boxed{(+6) + (-13) = -7}$$

(iv) $-6 \rightarrow 11111010$

$-13 \rightarrow 11110011$

$$
\begin{array}{rl}
11111010 & \text{(2's comp of -6)} \\
11110011 & \text{(2's comp of -13)} \\
\hline
\underline{1}\,\underline{1}1101101 &
\end{array}
$$

carry sign bit

carry is discard

sign bit = 1 ∴ Result is '-ve' no.

'-ve' results are already in 2's comp. form.

To get result take 2's comp. of sum including sign bit.

2's comp. of 11101101 is $\underbrace{00010011}_{19}$

$$\therefore \boxed{(-6) + (-13) = -19}$$

Binary codes :-

Binary codes

- weighted codes
  - Binary
  - BCD codes
    - 8421
    - 2421
    - 3321
    - 4221
    - 5211
    - 5311
    - 5421
    - 6311
    - 7421
    - $74\bar{2}\bar{1}$ (or) 74-2-1
    - $84\bar{2}\bar{1}$ (or) 84-2-1
- non weighted codes
  1. Excess-3
  2. Gray
  3. 5-bit BCD codes
- Reflective codes
  1. 2421
  2. 5211
  3. Excess-3
- Sequential codes
  1. 8421
  2. Excess-3
- Alphanumeric codes
  1. ASCII
  2. EBCDIC
  3. Hollerith
- Error detecting & correcting codes
  1. Parity
  2. Hamming code.

Weighted codes :-

In weighted codes each digit position of the number representing a specific weight.

1. Straight Binary code.

-------- 2^4 2^3 2^2 2^1 2^0

MSB ↳ depends upon no. of digits. LSB

2. Binary coded decimal (BCD)

BCD is a numeric code in which each digit of a decimal number is represented by a seperate group of bits.

The most common BCD code is 8421

Decimal	BCD codes 8421	4221	7421̄ (or) 7 4 -2 -1
0	0000	0000	0000
1	0001	0001	0111
2	0010	0100 (or) 0010	0110
3	0011	0011 (or) 0101	0101
4	0100	1000 (or) 0110	1011
5	0101	1001 (or) 0111	1010
6	0110	1100 (or) 1010	1001
7	0111	1101 (or) 1011	1000
8	1000	1110	1100
9	1001	1111	1110

Ex:- Represent 65_{10} in straight binary & BCD code.

Sol:- S.B $\quad 65_{10} = (1000001)_2$

$$\begin{array}{r|l} 2 & 65 \\ 2 & 32 - 1 \\ 2 & 16 - 0 \\ 2 & 8 - 0 \\ 2 & 4 - 0 \\ 2 & 2 - 0 \\ 2 & 1 - 0 \\ & 0 - 1 \end{array} \uparrow$$

BCD code

$$\begin{array}{cc} 6 & 5 \\ \downarrow & \downarrow \\ 0110 & 0101 \end{array}$$

$65_{10} = (01100101)_{\substack{BCD \\ (8421)}}$

Ex:- ① convert $2A_{16}$ to BCD code.

Sol:- Hexa decimal $\longrightarrow$ decimal $\longrightarrow$ BCD.

$= 2 \times 16^1 + A \times 16^0$

$= 32 + 10 = 42$

$(2A)_{16} = (42)_{10}$

$$\begin{array}{cc} 4 & 2 \\ \downarrow & \downarrow \\ 0100 & 0010 \end{array}$$

$(2A)_{16} = (42)_{10} = (01000010)_{\substack{BCD \\ 8421}}$

② convert $(01000010)_{BCD} = (\ ?\ )_{16}$

BCD $\rightarrow$ decimal $\rightarrow$ Hexadeci

Sol:- $\underbrace{0100}_{4}\underbrace{0010}_{2}$

$(01000010)_{BCD} = (42)_{10}$

$$\begin{array}{r|l} 16 & 42 \\ 16 & 2 - 10\ A \\ & 0 - 2 \end{array} \uparrow$$

$(42)_{10} = (2A)_{16}$

$\therefore (01000010)_{BCD} = (42)_{10} = (2A)_{16}$

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BCD Addition rules :-

(i) Add two BCD numbers using Binary addition rules

(ii) If a 4 bit sum is greater than 9 or if a carry is generated from a 4 bit sum, the sum is Invalid

(iii) Add 6 to the 4 bit sum inorder to skip the invalid states.

Ex:- perform the following using BCD arthametic.

(i) $1273_{10} + 9587_{10}$

Sol:- $1273_{10} \longrightarrow \overline{0001}^{1}\,\overline{0010}^{2}\,\overline{0111}^{7}\,\overline{0011}^{3}{}_{BCD}$

$9587_{10} \longrightarrow \underbrace{1001}_{9}\,\underbrace{0101}_{5}\,\underbrace{1000}_{8}\,\underbrace{0111}_{7}{}_{BCD}$

```
  0001001001110011
+ 1001010110000111
  ----------------
  1010 0111 1111 1010
   >9   >9   >9   >9

  1010  1111  1111  1010
  0110        0110  0110
  ----------------------
 10000  1000  0110  0000
  1 0    8     6     0
```

in the first level of addition add 0110(6) to the groups which are >9(1001) (or) carry generated out of 4-bit group

From second level onwards add 6 to the groups which are >9

$$\boxed{1273_{10} + 9587_{10} = 10860_{10}}$$

(ii) $999_{10} + 989_{10}$

Sol:- $999_{10} \longrightarrow 1001\ 1001\ 1001_{BCD}$

$989_{10} \longrightarrow 1001\ 1000\ 1001_{BCD}$

```
          1001  1001  1001
          1001  1000  1001
          ----------------
         10011  0010  0010
carry      0110  0110  0110
          ----------------
         11001  1000  1000
         1  9    8     8
```

→ In the first level of addition carry is generated so, add 6 (0110) to all 4-bit groups.

999 + 9587 = 1988_{10}

(iii) $7762_{10} + 3838_{10}$

Sol:- $7762_{10} \rightarrow$ 0111 0111 0110 0010_{BCD}

$3838_{10} \rightarrow$ 0011 1000 0011 1000_{BCD}.

```
  0111   0111  0110   0010
  0011   1000  0011   1000
 -------------------------
  1010   1111  1000   1010
  0110   0110         0110
 -------------------------
 10001   0101  1000   0000
               0110
 -------------------------
 10001   0110  0000   0000
  1  1    6     0      0
```

In the first level of addition add 0110 to the groups which are > 1001 (or) carry generated out of 4-bit group

from 2nd level onwards add 6 to the groups which are > 9 only

$$\boxed{7762_{10} + 3838_{10} = 11600_{10}}$$

* Non-weighted (or) unweighted codes :-

Un weighted codes are not assigned with any weight to each digit position.

Ex:- EXCESS-3, GRAY codes.

EXCESS-3 code :- It can be derived from the natural BCD code by adding '3' to each coded number.

Decimal	BCD 8421	Excess-3 (BCD+0011)
0	0000	0011
1	0001	0100
2	0010	0101
3	0011	0110
4	0100	0111
5	0101	1000
6	0110	1001
7	0111	1010
8	1000	1011
9	1001	1100

Ex:- Find EXCESS-3 code for 592_{10}

Sol:- 5 9 2_{10}

1000 1100 0101 → EXCESS-3 form.

$592_{10} = 100011000101_{EXCESS-3}$.

→ Binary to Gray code conversion :-

rules :-

B_1 B_2 B_3 B_4 ----- Binary no.

G_1 G_2 G_3 G_4 ----- Gray code no.

$G_1 = B_1$

$G_2 = B_1 \oplus B_2 = B_1\bar{B}_2 + \bar{B}_1 B_2$

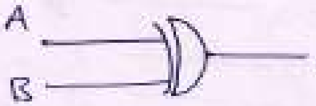
$G_3 = B_2 \oplus B_3$

$G_4 = B_3 \oplus B_4$

Ex:- $(1100)_2 = (\ ?\ )_{Gray}$

Sol:- MSB is same. Remaining digits are replaced with opposite digits 1's → 0's, 0's → 1's. Remaining are as below table.

EXOR (or) XOR (or) Exclusive or gate.



$C = \bar{A}B + A\bar{B} = A \oplus B$.

A	B	C=A⊕B
0	0	0
0	1	1
1	0	1
1	1	0

1 1 0 1 1 0 0

Ex:- (i) $(1111)_2 = (\ ?\ )_{Gray}$

Sol:-

1 1 1 1

1 0 0 0

$(1111)_2 = (1000)_{Gray}$.

(ii) $(101010)_2 = (\ ?\ )_{Gray}$.

Sol:- $(101010)_2 = (111111)_{Gray}$.

Gray to Binary :-

rules :-

$G_1 \quad G_2 \quad G_3$ ---- Gray code no.

$B_1 \quad B_2 \quad B_3$ ---- Binary code no.

$B_1 = G_1$

$B_2 = B_1 \oplus G_2$

$B_3 = B_2 \oplus G_3$

$B_4 = B_3 \oplus B_4$

⋮

Ex:- (i) $(1000)_{Gray} = (\; ? \;)_2$

Sol:-

$$\begin{array}{cccc} 1 & 0 & 0 & 0 \\ \downarrow & \oplus\downarrow & \oplus\downarrow & \oplus\downarrow \\ 1 & 1 & 1 & 1 \end{array}$$

$$\boxed{(1000)_{Gray} = (1111)_2}$$

Reflective codes :-

A code is said to be replective when the code for '9' is the complement for the code for '0', '8' for '1', '7' for '2', '6' for '3', '5' for '4'.

Ex:- 2421, 5211, EXCESS-3.

Sequential codes :-

In sequential codes each succeeding code is one binary no. greater than it's preceeding code.

Alphanumeric codes :-

The codes which represent the alphabetic characters, numbers are called alpha numeric codes.

Ex:- ASCII. (American standard code for Information Interchange).

EBCDIC (Extended Binary coded Decimal Interchange code)

→ Binary to Gray code:-

unit distance code is gray code.

decimal	Binary	Gray
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000