2. BOOLEAN ALGEBRA (or) SWITCHING ALGEBRA

Boolean algebra is used to simplify boolean equations, to make simple logic circuits.

Representation of Boolean function (or) Switching function (or) logical function :-

Ex:- $y = ab + bc$

→ a, b, c are called variables (or) literals. which are inputs for the logic gates.

→ To implement a logic function with less no. of gates we have to minimise variables & the number of terms.

Boolean algebra rules :-

(i) commutative law

$$A + B = B + A$$
$$A \cdot B = B \cdot A$$

(ii) Associative law

$$A + (B + C) = (A + B) + C$$
$$(AB)C = A(BC).$$

(iii) Distributive law

$$A(B + C) = AB + AC$$
$$A + BC = (A + B)(A + C)$$

(iv) Additive law

$$A + 0 = A$$
$$A + A = A$$
$$A + 1 = 1$$
$$A + \overline{A} = 1$$

(v) multiplication property.

$$A \cdot 0 = 0$$
$$A \cdot 1 = A$$
$$A \cdot A = A$$
$$A \cdot \bar{A} = 0$$

(vi) Double negation.

$$\overline{(\bar{A})} = A$$

(vii) Absorption law

$$A(A+B) = A$$
$$A + AB = A$$
$$A(\bar{A}+B) = AB$$
$$AB + \bar{B} = A + \bar{B}$$
$$A\bar{B} + B = A + B$$

(viii) Duality property. $AB + \bar{A}\bar{B} = (A+B)(\bar{A}+\bar{B})$

(ix) Demorgan's theorem. $\overline{A \cdot B} = \bar{A} + \bar{B}$

$\overline{A+B} = \bar{A} \cdot \bar{B}$

(x) consensus theorem. $AB + \bar{A}C + BC = AB + \bar{A}C$

① Prove that $(A+B)(A+C) = A + BC$.

Proof:- L.H.S $= (A+B)(A+C)$

$= AA + AC + BA + BC$ $\quad (\because A.A = A)$

$= A + AC + BA + BC$

$= BC + (1 + C + B)A$ $\quad (\because 1 + A = 1)$

$= A + BC$ $\quad$ (1+ Anything = 1 in logical gates).

$=$ R.H.S.

$$\therefore \boxed{A + BC = (A+B)(A+C)}$$

Ex:- Simplify following boolean expressions to a min.
no. of variables or literals. ($\bar{B}$ (or) B' both are same)

① $(A+B)(A+\bar{B})$

Sol:- $(A+B)(A+\bar{B}) = A\cdot A + A\cdot\bar{B} + B\cdot A + B\cdot\bar{B}$

$= A + A\cdot\bar{B} + B\cdot A + 0$

$= A(1 + B + \bar{B})$

$= A(1+1)$ $\quad (\because 1+\text{anything} = 1)$

$= \underline{\underline{A}}$

② $xy + xyz + xy\bar{z} + \bar{x}yz$

Sol:- $xy + xyz + xy\bar{z} + \bar{x}yz$

$= xy[1 + z + \bar{z}] + \bar{x}yz$ $\quad (\because 1+\text{anything} = 1)$

$= xy\cdot 1 + \bar{x}yz$

$= \bar{x}yz + xy$ $\quad (\because A+BC = (A+B)(A+C))$

$= y[\bar{x}z + x] = y[(x+\bar{x})\cdot(x+z)]$

$= y[x+z]$

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③ $x'y + xy' + xy + x'y'$

Sol:- $x'[y+y'] + x[y+y']$

$= x'[1] + x[1] = x + x' = \underline{\underline{1}}$

④ $\bar{A} + AB + A\bar{C} + A\bar{B}\bar{C}$

Sol:- $\bar{A} + AB + A\bar{C}(A + A\bar{B})$

$= \bar{A} + AB + A\bar{C}(1)$

$= \bar{A} + A(B+\bar{C})$

$= 1 - A + AB + A\bar{C}(1-C)$

$= 1 - AC$

$= 1 - A + AB + A - AC$

$\bar{A} + AB + A\bar{C}(1+\bar{B})$

$= \bar{A} + AB + A\bar{C}$

$= (A+\bar{A})(\bar{A}+B) + A\bar{C}$

$= \bar{A} + B + A\bar{C}$ $\quad (\because \text{distributive law})$

$= \bar{A} + A\bar{C} + B$

$= (\bar{A}+A)(\bar{A}+\bar{C}) + B$

⑤ $ABC + \bar{A}B + AB\bar{C}$

Sol:- $ABC + AB\bar{C} + \bar{A}B$

$= AB(C+\bar{C}) + \bar{A}B$

$= AB + \bar{A}B$

$= B(A+\bar{A}) = B$

⑥ $x'y' + xy + x'y$

Sol:- $x'(y'+y) + xy$

$= x' + xy$

$= (x'+x)(x'+y)$ $\quad$ ($\because$ distributive law)

$= x'+y$

⑦ $a + a'b + a'b'c + a'b'c'd + \cdots$

Sol:- $a + a'[b + b'c + b'c'd + \cdots]$ $\quad$ (absorption law) ($\because A + \bar{A}B = A+B$)

$= a + b + b'c + b'c'd + \cdots$

$= a + b + b'[c + c'd + c'd'e + \cdots]$ $\quad$ ($\because$ absorption law) $A+\bar{A}B = A+B$

$= a + b + c + c'd + c'd'e + \cdots$

$\vdots$

$= a + b + c + d + e + \cdots$

⑧ prove that $A(\bar{A}+C)(\bar{A}B+\bar{C}) = 0$

Sol:- $(A.\bar{A} + AC)(\bar{A}B + \bar{C})$

$= AC(\bar{A}B + \bar{C})$

$= AC.\bar{A}B + A(C.\bar{C})$ $\quad$ ($\because A.\bar{A} = 0$)

$= (A.\bar{A})BC + A(0)$

$= 0 + 0 = 0.$

⑨ P.T $(A+C)(A+D)(B+C)(B+D) = AB+CD$

Sol:- ${}^{Y}C$ $[A\cdot A + AD + CA + CD][B\cdot B + BD + C\cdot B + C\cdot D]$

$= [A + AD + CA + CD][B + BD + CB + CD]_{Jx}$

$(A+C)(A+D)(B+C)(B+C)$ $\quad [\because A+BC = (A+B)(A+C)]$

$= (A+CD)(B+CD) \quad = A$

$= CD + AB$

⑩ P.T $AB + \overline{AC} + A\overline{B}C(AB+C) = 1$

Sol:- $AB + \overline{A} + \overline{C} + \underbrace{A\overline{B}C\cdot AB}_{0} + A\overline{B}C\cdot C$

${}^{Y}C = \underline{AB + \overline{AC}}\,\overline{C} + A\overline{B}C$ $\quad (\because B\cdot\overline{B} = 0)$

$= (A+B)(\overline{A}+\overline{B}) + A\overline{B}C_{Jx}$

$= AB + \overline{AC} + A\overline{B}C$

$= A(B + \overline{B}C) + \overline{AC}$ $\quad (\because A+BC = (A+B)(A+C))$

$= A[(B+\overline{B}) + (B+C)] + \overline{AC}$ $\quad (\because B+\overline{B} = 1)$

$= AB + \underbrace{AC + \overline{AC}}_{1}$

$= 1 + AB$ $\quad (\because 1 + \text{any thing} = 1)$

$= 1$

⑪ Prove that $\overline{A}\overline{B}\overline{C} + \overline{A}B\overline{C} + AB\overline{C} + A\overline{B}\overline{C} = \overline{C}$

Sol:- $\overline{A}\overline{C}[B+\overline{B}] + A\overline{C}[B+\overline{B}]$

$= \overline{A}\overline{C} + A\overline{C}$ $\quad (\because A+\overline{A} = 1)$

$= \overline{C}[A+\overline{A}] = \overline{C}$

Ex:- Reduce the following Boolean expression to indicated no. of variables or literals.

(12) $\bar{A}\bar{C} + ABC + A\bar{C}$ to 3 literals.

Sol:- $(\bar{A}+A)\bar{C} + ABC$

$= \bar{C} + ABC.$

$= \bar{C} + C.AB$ $\quad (\because A+BC = (A+B)(A+C))$

$= (\bar{C}+C)(\bar{C}+AB)$ $\quad (\bar{A}+A = 1)$

$= \bar{C} + AB$

(13) $(\overline{\bar{x}\bar{y}+z}) + z + xy + wz$ to 3 variables.

Sol:- $(\overline{\bar{x}\bar{y}}) \cdot \bar{z} + z + xy + wz$ $\quad (\because \overline{A+B} = \bar{A}\cdot\bar{B})$

$xy\bar{z} + z + xy + wz$

$= z + \bar{z}.xy + xy + wz$ $\quad (A+BC = (A+B)(A+C))$

$= (z+\bar{z})(z+xy) + xy + wz$

$= z + xy + xy + wz$

$= z(1+w) + xy(1+1)$

$= z + xy.$

(or)

$= xy(1+\bar{z}) + z(1+w)$

$= xy + z$ $\quad (\because 1+\text{anything}=1)$

(13)(i) $(\overline{\bar{x}\,\bar{y}+z}) + z + xy + wz$ to 3 variables.

Sol:- $\overline{\bar{x}\cdot\bar{y}}\cdot\bar{z} + z + xy + wz$

$(\bar{\bar{x}} + \bar{\bar{y}})\bar{z} + z(1+w) + xy$

$= xy\bar{z} + xy +$

$= x\bar{z} + y\bar{z} + z + xy.$

$= z + x\bar{z} + y\bar{z} + xy$

$= (z+x)(z+\bar{z}) + y\bar{z} + xy$

$= x + z + xy + y\bar{z}$ $\quad = x(1+y) + (z+y)(z+\bar{z})$

$= x + y + z$ $\quad$ ($\because$ $1 +$ anything $= 1$, $A + \bar{A} = 1$).

(14) $A'B(C'D + D') + B(A + A'CD)$ to 1 literal

Sol:- $A'BC'D + A'BD' + AB + A'BCD$

$= A'BD(C + C') + AB + A'BD'$ $\quad$ ($\because$ $A + BC = (A+B)(A+C)$, $(A + \bar{A} = 1)$)

$= A'BD + AB + A'BD'$

$= A'B(D + D') + AB$

$= A'B + AB \quad = B(A + A') = \underline{\underline{B}}$

Ex: Find the compliment of the Boolean function and reducing to a minimum number of variables.

Sol: (15) $(B\bar{C} + \bar{A}D)(A\bar{B} + C\bar{D})$

Sol: after taking complement

$\overline{(B\bar{C} + \bar{A}D)(A\bar{B} + C\bar{D})}$

$= \overline{(AB\bar{B}\bar{C} + B\bar{C}C\bar{D} + A\bar{A}D\bar{B} + \bar{A}DC\bar{D})}$ $\quad$ ($\because$ $A \cdot \bar{A} = 0$)

$= \bar{0} = 1$

(16) $x'(y' + z')(x + y + z')$

Sol:- $\overline{x'(y' + z')(x + y + z')}$

$= \overline{(x'y' + x'z')(x + y + z')}$

$= \overline{(x'y'x + x'z'x + x'y'y + x'y'z' + x'z'y + x' \cdot z' \cdot z')}$

$= \overline{x'y'z' + x'z'y + x' \cdot z'}$

$= \overline{x'z'(y + y') + x' \cdot z'}$ $\quad = \overline{x'z' + x'z'}$ $\quad$ $C = \overline{x'(\cancel{y'} + z)}$

$= \overline{x'z'(1+1)}$ $\quad = \overline{x'} = (\bar{\bar{x}})$

$= \overline{\bar{x}\bar{z}}$ $\quad = x$

$= (\bar{\bar{x}}) + (\bar{\bar{z}}) = x + z$

(17) Find dual of $F = \bar{A}B + BC + A\bar{C}$

Sol:- Duality of $F = (\bar{A}+B)(B+C)(A+\bar{C})$

Sub. x's → +'s
+'s → x's
0's → 1's
1's → 0's

(18) Find dual of $xy + x'z = 0$

Sol:- Duality ⇒ $(x+y)(x'+z) = 1$

LOGIC GATES :-

1. Inverter (or) NOT gate :-

NOT Gate performs inversion operation. It has '1' input and one output.

Logic Symbol :-

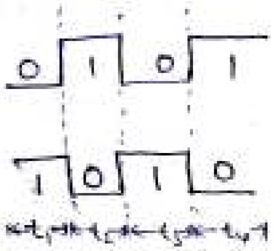
Truth Table :- Truth table indicates outputs for different possibilities of input.

i/p	o/p
A	$Y = \bar{A}$
0	1
1	0

Timing diagram :- The input & output wave forms showing time relationship is called Timing diagram.

(i/p) A

(o/p) Y



2. Buffer gate :-

output is Same as the i/p

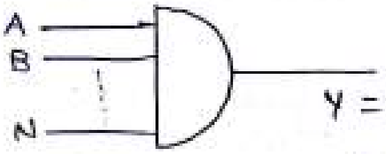
i/p = o/p.

It used for time delay.

3. AND gate :-

'AND' gate performs logical multiplication. It has two or more i/p's and one output.

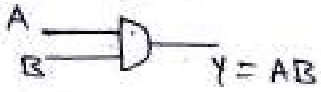
logic Symbol :-



Y = A and B and ....N
= A · B. .....N
= AB.....N.

2 i/p AND gate :-

logic symbol :-

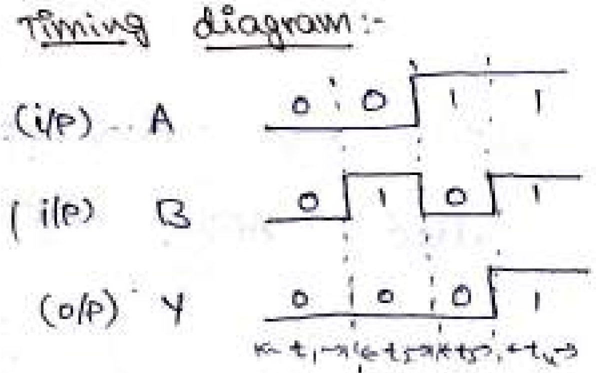


Truth table :-

i/p's		o/p
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

AND gate o/p is high when all i/p's are high.

Timing diagram :-

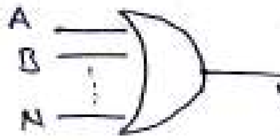


1. i/p AND gate :-

④ OR gate :-

OR gate performs logical Addition. It has two or more inputs & one output.

logic symbol :-

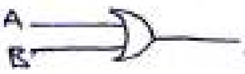


$Y = A$ or B or $\ldots N$
$= A + B + \ldots N$

1 i/p OR gate :-

2 i/p OR gate :-

logic symbol :-



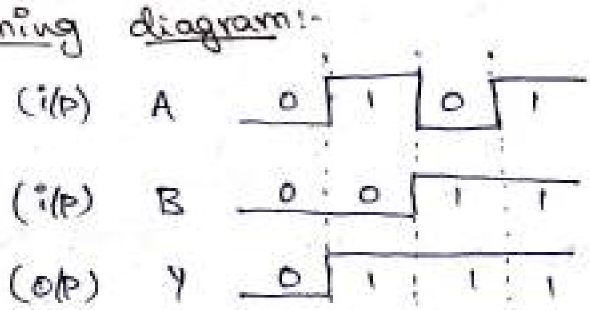
$Y = A+B$

Truth table :-

i/p's		o/p
A	B	Y
0	0	0
1	0	1
0	1	1
1	1	1

OR gate o/p is low when all the inputs are low.

Timing diagram :-



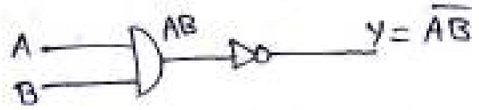
Note :- Logic circuits which use AND, OR, NOT gates only are called AOI logic circuits

Universal gates :-

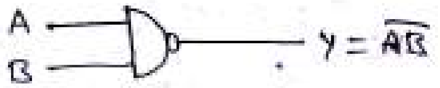
NAND & NOR are called universal gates because by using either NAND or NOR gates we can implement any logic circuit.

NAND gate :- The Term NAND is a combination of AND & NOT gates.

logic symbol :-



2 i/p NAND gate :-



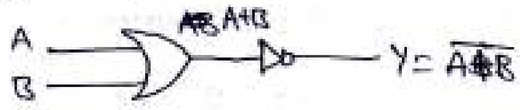
Truth table :-

i/p's		o/p
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

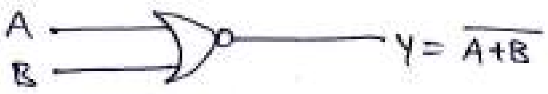
NAND gate o/p is low when the all the i/p's are high.

NOR gate :- The Term NOR is a combination of OR & NOT gates.

logic symbol :-



2 i/p's NOR gate :-



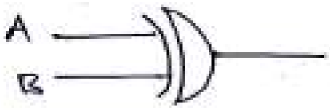
Truth table :-

i/p's		o/p
A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

NOR gate o/p is high when all i/p's are low.

EXOR (or) XOR (or) EXCLUSIVE-OR gate :-

logic symbol :-



$Y = A \text{ XOR } B$

$= A \oplus B$

$= A\bar{B} + \bar{A}B$.

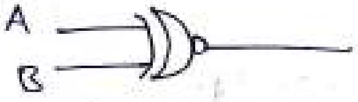
Truth table :-

i/p's		o/p
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

XOR o/p is low when all i/p's are same.

EXNOR (or) XNOR (or) EXCLUSIVE NOR gate :-

logic symbol :-



$Y = A \text{ XNOR } B$

$= A \odot B \quad = \overline{A \oplus B}$

$= \bar{A}\bar{B} + AB \quad = \overline{\bar{A}B + A\bar{B}}$

Truth table:-

i/p's		o/p
A	B	y
0	0	1
0	1	0
1	0	0
1	1	1

EXNOR o/p is high when all i/p's are same.

Note:- dual of XOR gate is XNOR.

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* Dual of XOR gate is XNOR.

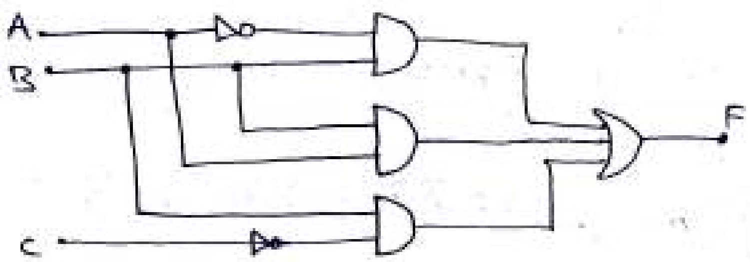
dual of $\bar{A}B + A\bar{B} = (\bar{A}+B)(A+\bar{B})$

$= \bar{A}\cdot A + \bar{A}\cdot\bar{B} + B\cdot A + B\cdot\bar{B}$

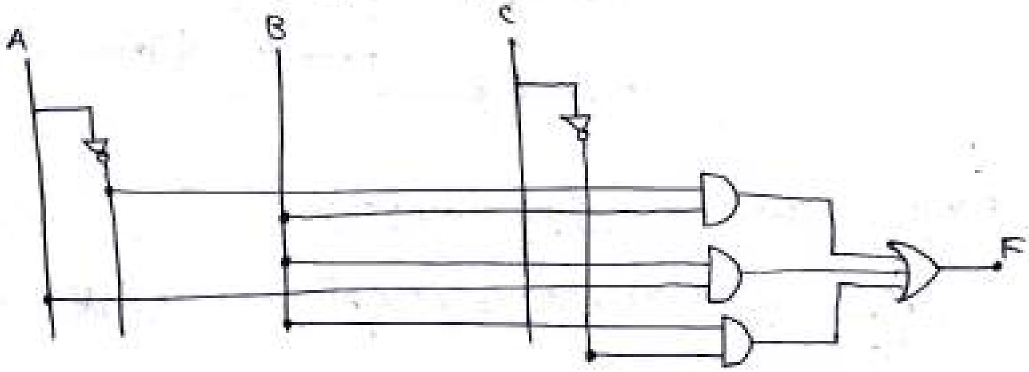
$= \bar{A}\bar{B} + AB$

Ex: Implement $F = \bar{A}B + AB + B\bar{C}$ using basic logic gates.

Sol:-

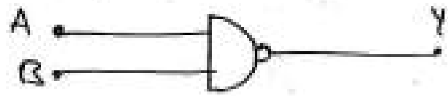


(or)



Ex: Implement NOT, AND, OR, NOR, EXNOR, EXOR gates using NAND gate.

Sol:- NAND gate:-

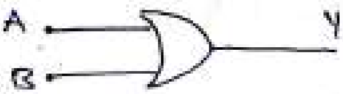


$Y = \overline{AB}$

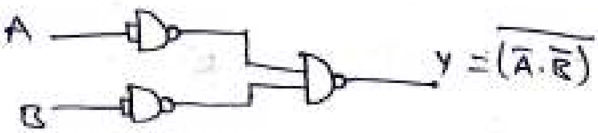
NOT gate using NAND gate:-

AND gate using NAND:-

OR gate using NAND:-

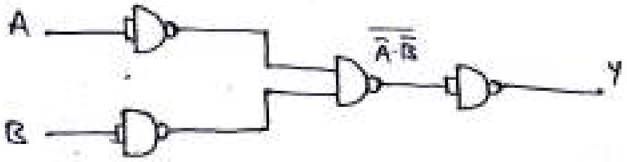


$Y = A+B = \overline{\overline{A+B}} = \overline{\bar{A}\cdot\bar{B}}$ (∵ demorgans)

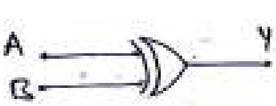


NOR gate using NAND:-

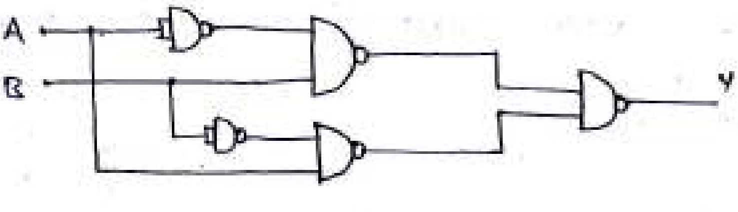
$Y = \overline{A+B} = \bar{A}\cdot\bar{B} = \overline{\overline{\bar{A}\cdot\bar{B}}}$



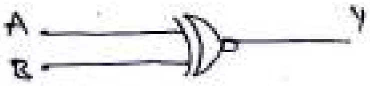
EXOR gate using NAND:-



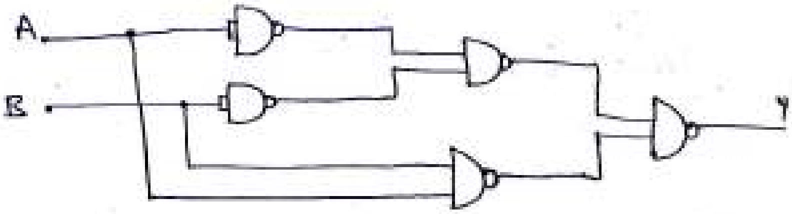
$Y = A\oplus B = \bar{A}B + A\bar{B} = \overline{\overline{\bar{A}B + A\bar{B}}} = \overline{\overline{\bar{A}B}\cdot\overline{A\bar{B}}}$



EXNOR gate using NAND:-



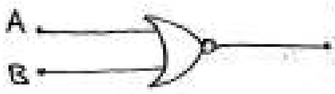
$Y = A \odot B = \bar{A}\bar{B} + AB = \overline{\overline{\bar{A}\bar{B} + AB}} = \overline{\overline{\bar{A}\bar{B}} \cdot \overline{AB}}$



$Y = \overline{\overline{\bar{A}\bar{B}} \cdot \overline{AB}}$

Ex: Implement all logic gates expect NOR gate using NOR gate.

Sol:- NOR gate :-

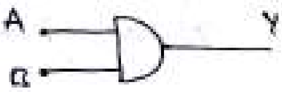


$Y = \overline{A+B}$

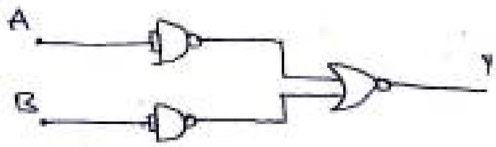
NOT gate using NOR gate:-

$Y = \bar{A}$ $\equiv$ $Y = \bar{A}$

AND gate using NOR gate:-

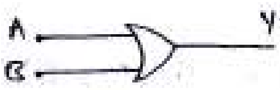


$Y = AB = \overline{\overline{AB}} = \overline{\bar{A} + \bar{B}}$

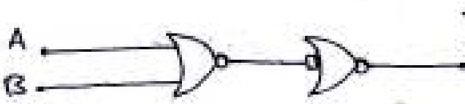


$Y = \overline{\bar{A}+\bar{B}}$

OR gate using NOR gate:-

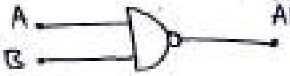


$Y = A+B = \overline{\overline{A+B}}$

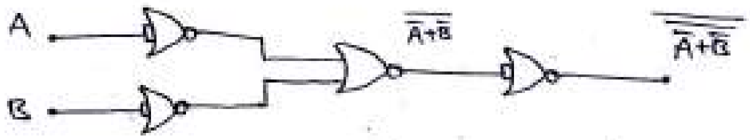


$\overline{\overline{(A+B)}}$

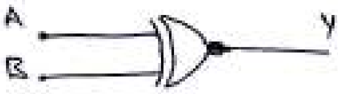
NAND gate using NOR gate :-



$\overline{AB} = \overline{A}+\overline{B} = \overline{\overline{\overline{A}+\overline{B}}}$

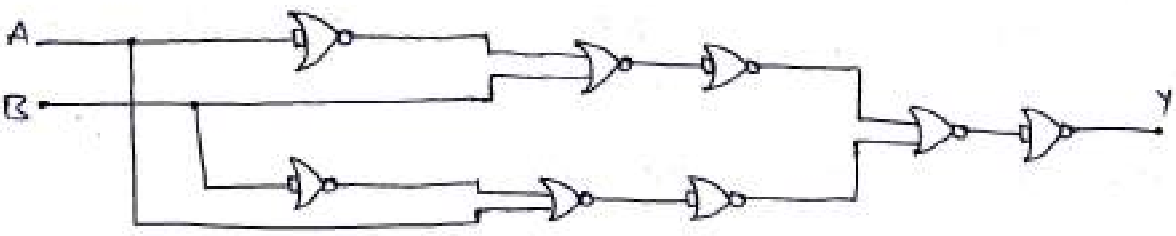


EXOR gate using NOR gate :-

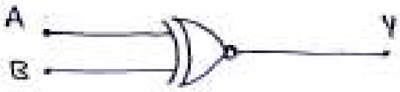


$Y = A\oplus B = \overline{A}B + A\overline{B} = \overline{\overline{\overline{A}B}} + \overline{\overline{A\overline{B}}}$

$= \overline{\overline{\overline{\overline{A}B} + \overline{A\overline{B}}}}$



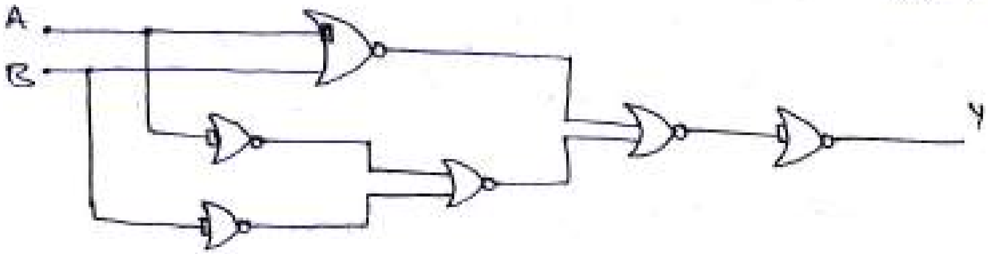
EXNOR gate using NOR gate :-



$Y = A\odot B = \overline{A}\overline{B} + AB = \overline{\overline{\overline{A}\overline{B}+AB}} = \overline{\overline{\overline{A}\overline{B}}} + \overline{\overline{AB}}$

$= \overline{\overline{A}+\overline{B}} + \overline{A+B} = \overline{\overline{\overline{A}\overline{B}}\cdot\overline{AB}} \quad \overline{A+B} + \overline{\overline{A}+\overline{B}}$

$= \overline{\overline{\overline{A+B} + \overline{\overline{A}+\overline{B}}}}$



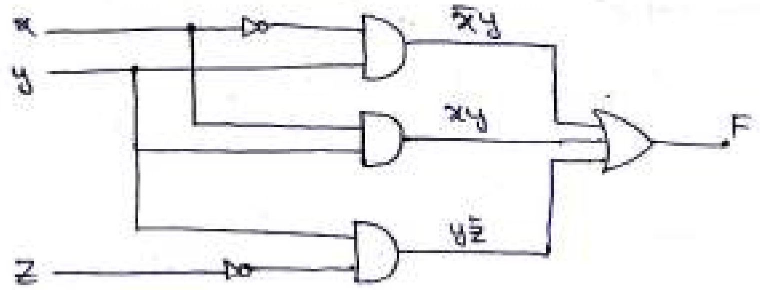
Ex:- Implement $F = \bar{x}y + xy + y\bar{z}$ using (i) Basic gates

(ii) AND & NOT gates (iii) OR & NOT gates

(iv) only NAND gate (iv) only NOR gate.

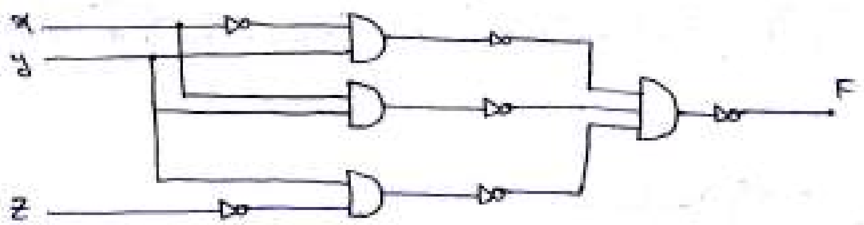
Sol:- $F = \bar{x}y + xy + y\bar{z}$

(i) using Basic gates.



(ii) using AND & NOT gates

$$F = \bar{x}y + xy + y\bar{z} = \overline{\overline{\bar{x}y + xy + y\bar{z}}} = \overline{\overline{\bar{x}y} \cdot \overline{xy} \cdot \overline{y\bar{z}}}$$



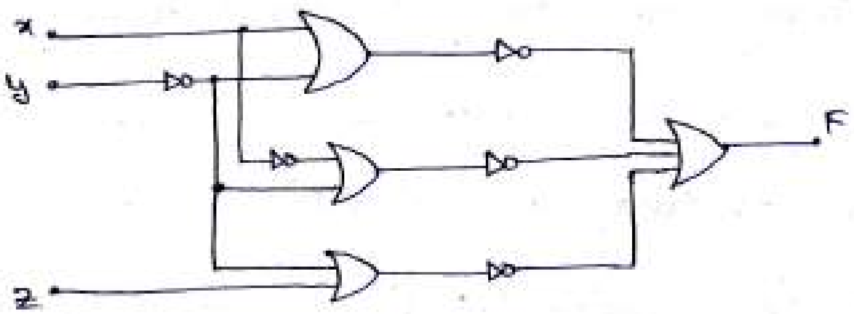
(iii) using OR & NOT gates.

$$F = \bar{x}y + xy + y\bar{z}$$

$$= \overline{\overline{\bar{x}y}} + \overline{\overline{xy}} + \overline{\overline{y\bar{z}}}$$

$$= \overline{\bar{\bar{x}} + \bar{y}} + \overline{\bar{x} + \bar{y}} + \overline{(\bar{y} + \bar{\bar{z}})}$$

$$= \overline{x + \bar{y}} + \overline{\bar{x} + \bar{y}} + \overline{\bar{y} + z}$$

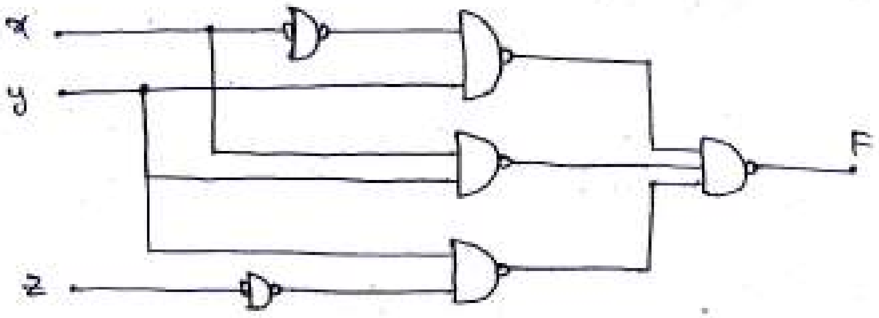


(iv) using NAND gate

$$F = \bar{x}y + xy + y\bar{z}$$

$$= \overline{\overline{\bar{x}y + xy + y\bar{z}}} \quad (\because \text{demorgans})$$

$$= \overline{\overline{\bar{x}y} \cdot \overline{xy} \cdot \overline{y\bar{z}}}$$



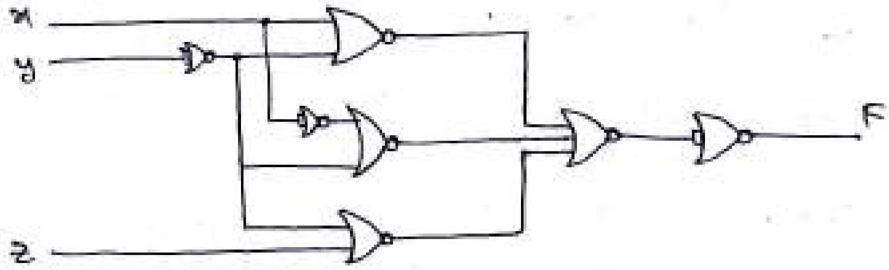
(v) using NOR gates.

$$F = \bar{x}y + xy + y\bar{z}$$

$$= \overline{\overline{\bar{x}y}} + \overline{\overline{xy}} + \overline{\overline{y\bar{z}}}$$

$$= \overline{\bar{\bar{x}}+\bar{y}} + \overline{\bar{x}+\bar{y}} + \overline{\bar{y}+\bar{\bar{z}}}$$

$$= \overline{x+\bar{y}} + \overline{\bar{x}+\bar{y}} + \overline{\bar{y}+z} = \overline{\overline{\overline{x+\bar{y}} + \overline{\bar{x}+\bar{y}} + \overline{\bar{y}+z}}}$$



* Representation of switching functions (or) logic functions (or) Boolean function :-

there are two standard forms in which logic functions can be expressed.

(i) Sum of products form (SOP)

(ii) Product of Sum's (POS).

(i) SOP :- SOP is a group of product terms ORed together.

Ex: $F = \bar{x}y + xy + yz$

(ii) <u>POS</u>:- POS is a group of Sum terms ANDed together.

<u>Ex</u>:- $F = (\bar{x}+\bar{y})(x+y)(y+z)$

<u>Standard</u> ~~cos~~ and <u>canonical</u> <u>SOP</u> & <u>POS</u> forms :-

In standard the SOP & POS forms all the individual terms do not contain all variables. If each term is SOP & POS form contains all variables then it is canonical ~~standard~~ SOP & POS forms.

Ex. of CSOP is,

$$F(A,B,C) = \bar{A}BC + ABC + A\bar{B}C$$

Ex. of CPOS is,

$$F(A,B,C) = (\bar{A}+B+C)(A+B+C)(A+\bar{B}+C)$$

<u>Ex</u>:- convert the given expression is into canonical ~~standard~~ SOP form.

(i) $Y(A,B,C) = \bar{A}B + A\bar{B}C$

<u>Sol</u>:-
$$= \bar{A}B(C+\bar{C}) + A\bar{B}C \quad (\because C+\bar{C}=1)$$
$$= \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}C$$

(ii) $Y(A,B,C) = AB + BC + CA$

<u>Sol</u>:-
$$= AB(C+\bar{C}) + BC(A+\bar{A}) + CA(B+\bar{B})$$
$$= ABC + AB\bar{C} + ABC + \bar{A}BC + ABC + A\bar{B}C$$
$$= ABC(1+1+1) + AB\bar{C} + A\bar{B}C + \bar{A}BC \quad (\because 1+\text{anything}=1)$$
$$= ABC + AB\bar{C} + A\bar{B}C + \bar{A}BC.$$

(iii) $A(X,Y,Z) = X + XY + XYZ$

<u>Sol</u>:-
$$= X(Y+\bar{Y})(Z+\bar{Z}) + XY(Z+\bar{Z}) + XYZ$$
$$= X(YZ + Y\bar{Z} + \bar{Y}Z + \bar{Y}\bar{Z}) + XY(Z+\bar{Z}) + XYZ$$
$$= XYZ + XY\bar{Z} + X\bar{Y}Z + X\bar{Y}\bar{Z} + XYZ + XY\bar{Z} + XYZ$$
$$= XYZ(1+1+1) + XY\bar{Z} + X\bar{Y}Z + X\bar{Y}\bar{Z} + XY\bar{Z}$$
$$= XYZ + XY\bar{Z} + X\bar{Y}Z + X\bar{Y}\bar{Z} + XY\bar{Z}$$
$$= XYZ + XY\bar{Z} + X\bar{Y}Z + X\bar{Y}\bar{Z}$$

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Ex:- Convert the given expression into canonical POS form.

(i) $F(A,B,C) = (A+B)(B+C)(A+C)$

Sol:-

$$= (A+B+C\bar{C})(B+C+A\bar{A})(A+C+B\bar{B}) \quad (\because A+BC = (A+B)(A+C)) \; (\because \text{distributive})$$

$$= (A+B+C)(A+B+\bar{C})(B+C+A)(B+C+\bar{A})(A+C+B)(A+\bar{B}+C)$$

$$= (A+B+C)(\bar{A}+B+C)(A+\bar{B}+C)(A+B+\bar{C})$$

(ii) $F(x,y,z) = x(x+y)(x+y+\bar{z})$

Sol:-

$$= (x+y\bar{y}+z\bar{z})(x+y+z\bar{z})(x+y+\bar{z})$$

$$= (x+y\bar{y}+z)(x+y\bar{y}+\bar{z})(x+y+z)(x+y+\bar{z})(x+y+\bar{z})$$

$$= (x+y+z)(x+\bar{y}+z)(x+y+\bar{z})(x+\bar{y}+\bar{z})(x+y+z)(x+y+\bar{z})$$

$$= (x+y+z)(x+\bar{y}+z)(x+y+\bar{z})(x+\bar{y}+\bar{z})$$

* Minterms and Maxterms :-

→ Each individual term in CSOP is called Minterm.

→ Each individual term in CPOS is called Maxterm.

* For 'n' variable logic function there are 2^n minterms and 2^n maxterms.

Minterms and Maxterms for 3 Binary variables :-

Variables			Minterms (m_i)		Maxterm (M_i)	
A	B	C	Term	designation	Term	designation
0	0	0	$\bar{A}\bar{B}\bar{C}$	m_0	$A+B+C$	M_0
0	0	1	$\bar{A}\bar{B}C$	m_1	$A+B+\bar{C}$	M_1
0	1	0	$\bar{A}B\bar{C}$	m_2	$A+\bar{B}+C$	M_2
0	1	1	$\bar{A}BC$	m_3	$A+\bar{B}+\bar{C}$	M_3
1	0	0	$A\bar{B}\bar{C}$	m_4	$\bar{A}+B+C$	M_4
1	0	1	$A\bar{B}C$	m_5	$\bar{A}+B+\bar{C}$	M_5
1	1	0	$AB\bar{C}$	m_6	$\bar{A}+\bar{B}+C$	M_6
1	1	1	ABC	m_7	$\bar{A}+\bar{B}+\bar{C}$	M_7

→ The minterm is represented by m, Maxterm by M

→ The subscript 'i' is the decimal number equivalent of binary input.

* With these short hand notations, logic function can be represented as follows.

Ex:- (i) $Y = \overset{1}{A}\overset{1}{B}\overset{1}{C} + \overset{0}{\bar{A}}\overset{1}{B}\overset{1}{C} + \overset{1}{A}\overset{0}{\bar{B}}\overset{1}{C} + \overset{1}{A}\overset{0}{\bar{B}}\overset{0}{\bar{C}}$

$= m_7 + m_3 + m_5 + m_4$

$= \Sigma m(3,4,5,7)$

$= \Sigma(3,4,5,7)$ $\boxed{\Sigma \text{ denotes SOP}}$

(ii) $Y = (\overset{0}{A}+\overset{0}{B}+\overset{0}{C})(\overset{1}{\bar{A}}+\overset{0}{B}+\overset{1}{\bar{C}})(\overset{1}{\bar{A}}+\overset{1}{\bar{B}}+\overset{1}{\bar{C}})$

$= M_0 \cdot M_5 \cdot M_7$

$= \Pi M(0,5,7)$

$= \Pi(0,5,7)$ $\boxed{\Pi \text{ denotes POS}}$

Note:- If o/p func is '1' then it corresponds to Minterm.
If o/p func is '0' then it corresponds to Maxterm.

Ex:- Find SOP and POS expressions from given truthtable

i/p's			o/p
A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Note:-

o/p is '1' → Minterm

o/p is '0' → Maxterm.

Sol:- SOP

$Y(A,B,C) = \bar{A}\bar{B}\bar{C} + A\bar{B}C + \bar{A}BC + ABC$

$= m_0 + m_2 + m_3 + m_7 = \Sigma m(0,2,3,7)$

POS

$Y(A,B,C) = (A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$

$= M_1 \cdot M_4 \cdot M_5 \cdot M_6 = \Pi M(1,4,5,6)$

* From the above expressions, there is a complementary

Ex:- Simplify the following 3 variable expressions using boolean algebra.

(i) $Y = \Sigma m(1,3,5,7)$ considering A, B, C as 3 i/p variables.

Sol:- $Y = m_1 + m_3 + m_5 + m_7$

$= \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$

$= \bar{A}C(B+\bar{B}) + AC(B+\bar{B})$

$= \bar{A}C + AC$ $\quad (\because A+\bar{A} = 1)$

$= C(A+\bar{A}) = C$

(ii) $Y = \Pi M(3,7)$

Sol:- $Y = M_3 \cdot M_7$

$= (A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+\bar{C})$

$= A\cdot\bar{A} + A\cdot\bar{B} + A\cdot\bar{C} + \bar{A}\cdot\bar{B} + \bar{B} + \bar{B}\cdot\bar{C} + \bar{C}\bar{A} + \bar{B}\bar{C} + \bar{C}$

$= A\bar{B} + A\bar{C} + \bar{A}\cdot\bar{B} + \bar{B} + \bar{B}\cdot\bar{C} + \bar{C}\cdot\bar{A} + \bar{C}$

$= \bar{B}(A+\bar{A}) + \bar{C}(A+\bar{A}) + \bar{B}(1+\bar{C}) + \bar{C}$

$= \bar{B} + \bar{C} + \bar{B} + \bar{C}$ $\quad (\because 1 + \text{anything} = 1)$

$= \bar{B} + \bar{C}$.

Ex:- Convert the given expression into Minterms using complementary property and simplify the expression.

(i) $F = \Pi M(3,5,7)$

Sol:- $F = M_3 \cdot M_5 \cdot M_7 \longrightarrow$ This is POS form.

$\therefore$ SOP form is,

$F(A,B,C) = \Sigma m(0,1,2,4,6)$

$= m_0 + m_1 + m_2 + m_4 + m_6$

$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

$= \bar{A}\bar{B}(C+\bar{C}) + \bar{A}B\bar{C} + A\bar{C}(B+\bar{B})$

$= \bar{A}\bar{B} + \bar{A}B\bar{C} + A\bar{C}$

$= \bar{A}\bar{B} + \bar{C}(\bar{A}B + A)$ (∵ distributive law)

$= (\bar{A}\bar{B} + \bar{C})(\bar{A}\bar{B} + \bar{A}B + A)$

$= (\bar{A}\bar{B} + \bar{C})(\bar{A}(B + \bar{B}) + A)$

$= (\bar{A}\bar{B} + \bar{C})(A + \bar{A}) = \bar{A}\bar{B} + \bar{C}.$

Ex:- Express the following functions in sum of minterms (canonical SOP) and product of max. terms (canonical POS) forms.

(i) $F(w, x, y, z) = w'y + xy' + yz + xyz.$

Sol:- $w'y$

w'	x	y	z	
0	0	1	0	— m_2
0	0	1	1	— m_3
0	1	1	0	— m_6
0	1	1	1	— m_7

xy'

w	x	y'	z	
0	1	0	0	— m_4
0	1	0	1	— m_5
1	1	0	0	— m_{12}
1	1	0	1	— m_{13}

yz

w	x	y	z	
0	0	1	1	— m_3
0	1	1	1	— m_7
1	0	1	1	— m_{11}
1	1	1	1	— m_{15}

xyz

w	x	y	z	
0	1	1	1	— m_7
1	1	1	1	— m_{15}

CSOP (or) Sum of minterms form,

$$F(w, x, y, z) = m_2 + m_3 + m_4 + m_5 + m_6 + m_7 + m_{11} + m_{12} + m_{13} + m_{15}$$

$$= \sum m(2, 3, 4, 5, 6, 7, 11, 12, 13, 15)$$

Remaining terms are maxterms.

CPOS (or) Product of Maxterms form.

$$F(w, x, y, z) = M_0 \cdot M_1 \cdot M_8 \cdot M_9 \cdot M_{10} \cdot M_{14}$$

$$= \prod M(0, 1, 8, 9, 10, 14)$$

② $F(A,B,C) = (AB+C)(AC+B)$

Sol:- $F(A,B,C) = ABC + AB + AC + BC$.

ABC: A B C — 1 1 1 → m_7

AB: A B C — 1 1 0 — m_6; 1 1 1 — m_7

BC: A B C — 0 1 1 — m_3; 1 1 1 — m_7

AC: A B C — 1 0 1 — m_5; 1 1 1 — m_7

Sum of minterms $F = \sum m(3,5,6,7)$.

Product of maxterms $F = \prod M(0,1,2,4)$

06/08/2014

* Karnaugh Map (K-Map) :-

K-Map gives a systematic approch for simplifying a boolean expression. The basis of this method is a graphical chart known as "K-Map". It contains boxes called cells. Each of the cell represents one of the 2^n possible products that can be formed from 'n' variables.

2-variable K-Map

(i) Representation using minterms

A \ B	$\bar{B}$	B
$\bar{A}$	$\bar{A}\bar{B}$	$\bar{A}B$
A	$A\bar{B}$	AB

~~2-variable K-Map representation using minterms~~

A \ B	0	1
0	m_0 00	m_1 01
1	m_2 10	m_3 11

2-variable K-map representation using Maxterms

(ii) Representation using maxterms.

A \ B	B	$\bar{B}$
A	$A+B$	$A+\bar{B}$
$\bar{A}$	$\bar{A}+B$	$\bar{A}+\bar{B}$

A \ B	0	1
0	M_0 00	M_1 01
1	M_2 10	M_3 11

Ex:- (i) Simplify $F(A,B) = \sum m(0,1)$ using K-Map.

Sol:-

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	1	1
A 1		

$F(A,B) = \sum m(0,1)$

$= m_0 + m_1$

Fill minterms with '1'.

$\therefore \boxed{F(A,B) = \bar{A}}$

$(\because F(A,B) = \bar{A}\bar{B} + \bar{A}B$

$= \bar{A}(B+\bar{B}) = \bar{A})$

(ii) Simplify $F(A,B) = \sum m(0,2)$ using K-map.

Sol:-

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	1	
A 1	1	

$F(A,B) = \sum(m(0,2))$

$= m_0 + m_2$

$\therefore \boxed{F(A,B) = \bar{B}}$

(iii) Simplify $F(A,B) = \sum(0,3)$ using K-map.

Sol:-

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	1	
A 1		1

$F(A,B) = \sum(0,3)$

$= \sum m(0,3)$

$= m_0 + m_3$

$\therefore \boxed{F(A,B) = \bar{A}\bar{B} + AB}$

(iv) Simplify $F(A,B) = \pi M(1,3)$ using K-map.

Sol:-

A \ B	B 0	$\bar{B}$ 1
A 0		0
$\bar{A}$ 1		0

$F(A,B) = \pi M(1,3)$

$= M_1 \cdot M_3$

Fill maxterms with '0's.

$\therefore \boxed{F(A,B) = \bar{B}}$

(v) Simplify $F(x,y) = \pi(1,2)$ using K-map.

Sol:-

x \ y	y 0	$\bar{y}$ 1
x 0		0
$\bar{x}$ 1	0	

$F(x,y) = \pi M(1,2)$

$= M_1 \cdot M_2$

Fill maxterms with '0's.

$\therefore \boxed{F(x,y) = (x+\bar{y})(\bar{x}+y)}$

3- Variable K- Map :-

Representation using Minterms

A \ BC	$\bar B\bar C$	$\bar B C$	BC	$B\bar C$
$\bar A$	$\bar A\bar B\bar C$	$\bar A\bar B C$	$\bar A BC$	$\bar A B\bar C$
A	$A\bar B\bar C$	$A\bar B C$	ABC	$AB\bar C$

A \ BC	00	01	11	10
0	m_0 0	m_1 1	m_3 3	m_2 2
1	m_4 4	m_5 5	m_7 7	m_6 6

Representation using Maxterms

A \ BC.	$B+C$	$B+\bar C$	$\bar B+\bar C$	$\bar B+C$
A	$A+B+C$	$A+B+\bar C$	$A+\bar B+\bar C$	$A+\bar B+C$
$\bar A$	$\bar A+B+C$	$\bar A+B+\bar C$	$\bar A+\bar B+\bar C$	$\bar A+\bar B+C$

A \ BC	00	01	11	10
0	M_0 0	M_1 1	M_3 3	M_2 2
1	M_4 4	M_5 5	M_7 7	M_6 6

Ex: ① simplify $F(A,B,C) = \sum m(1,5,2,6)$ using K-Map.

Sol:

A \ BC	$\bar B\bar C$ 00	$\bar B C$ 01	BC 11	$B\bar C$ 10
$\bar A$ 0	0	1 1	3	1 2
A 1	4	1 5	7	1 6

$F(A,B,C) = \sum m(1,5,2,6)$

$= m_1+m_2+m_5+m_6.$

Fill minterms with 1's.

$\therefore F(A,B,C) = \bar B C + B\bar C$

$\therefore \boxed{F(A,B,C) = B\oplus C}$

$(\because F(A,B,C) = \bar A\bar B C+\bar A B\bar C+A\bar B C+AB\bar C$

$= \bar B C(A+\bar A)+B\bar C(A+\bar A)$

$= \bar B C+B\bar C$

$= B\oplus C.)$

(ii) Simplify $F(A,B,C) = \sum(1,3,5,7,6)$ using K-Map.

Sol:

A \ BC	$\bar B\bar C$ 00	$\bar B C$ 01	BC 11	$B\bar C$ 10
$\bar A$ 0	0	1 1	1 3	2
A 1	4	1 5	1 7	1 6

$F(A,B,C) = \sum(1,3,5,7,6)$

$= m_1+m_3+m_5+m_6+m_7$

Fill minterms with 1's.

$\therefore \boxed{F(A,B,C) = C+BA}$

(iii) $F(A,B,C) = \Sigma(0,1,4,5,2,6)$.

Sol:- $F(A,B,C) = m_0+m_1+m_2+m_4+m_5+m_6$.

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1 (0)	1 (1)	(3)	1 (2)
A 1	1 (4)	1 (5)	(7)	1 (6)

$\therefore F(A,B,C) = \bar{B} + \bar{C}$

(iv) $F(x,y,z) = \Sigma m(0,1,2,3,5)$.

Sol:-

x \ yz	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
0	1 (0)	1 (1)	1 (3)	1 (2)
1	(4)	1 (5)	(7)	(6)

$\therefore F(x,y,z) = \bar{x} + \bar{y}z$

(v) $F(x,y,z) = \Sigma(1,4,5,6)$.

Sol:-

x \ yz	00 $\bar{y}\bar{z}$	01 $\bar{y}z$	11 yz	10 $y\bar{z}$
0 $\bar{x}$	(0)	1 (1)	(3)	(2)
1 x	1 (4)	1 (5)	(7)	1 (6)

$\therefore F(x,y,z) = \bar{y}z + x\bar{z}$

don't care condition

X $\rightarrow$ don't care symbol, it can be 0 or 1.

(vi) $Y(A,B,C) = \Sigma m(1,3,5) + \Sigma d(6,7)$.

Sol:- $Y(A,B,C) = m_1+m_3+m_5+d_6+d_7$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	(0)	1 (1)	1 (3)	(2)
A 1	(4)	1 (5)	X (7)	X (6)

Fill mixterms with '1'.
Fill don't terms with 'X'.
It may be '0' or '1'.
consider $d_7 = 1$.

$\therefore Y(A,B,C) = C$

Ex:- Simplify the following boolean expression and draw logic circuit.

(vii) $Y(A,B,C) = \Sigma(0,1,2,6)$.

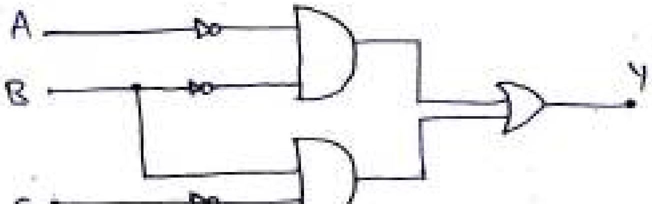
Sol:-

$A \backslash BC$	$\bar B \bar C$ 00	$\bar B C$ 01	BC 11	$B\bar C$ 10
$\bar A$ 0	1 (0)	1 (1)	(3)	1 (2)
A 1	(4)	(5)	(7)	1 (6)

$Y(A,B,C) = \Sigma(0,1,2,6)$
$= m_0 + m_1 + m_2 + m_6$

$\therefore Y(A,B,C) = \bar A \bar B + B \bar C$.

logic circuit:-



Ex:- Simplify the following boolean expression and draw logic circuits using NAND gate only.

(viii) $F = \Sigma(0,1,5,6) + \Sigma d(4,3)$

Sol:-

$A \backslash BC$	$\bar B \bar C$ 00	$\bar B C$ 01	BC 11	$B\bar C$ 10
$\bar A$ 0	1 (0)	1 (1)	X (3)	(2)
A 1	X (4)	1 (5)	(7)	1 (6)

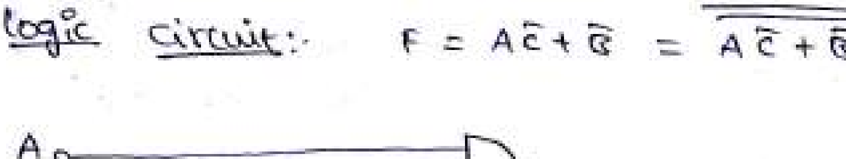
$F = m_0 + m_1 + m_5 + m_6 + d_3 + d_4$

Fill minterms with 1's

Fill don't terms with x's.

$F = \bar B + \quad + A\bar C \qquad = A\bar C + \bar B$

logic circuit:- $F = A\bar C + \bar B = \overline{\overline{A\bar C + \bar B}} = \overline{\overline{\bar B} \cdot \overline{A \bar C}}$
$= \overline{B \cdot \overline{A\bar C}}$



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(ix) $F(A,B,C) = \pi M(0,1,3,4,7)$

Sol:

A \ BC	00 $B+C$	01 $B+\bar{C}$	11 $\bar{B}+\bar{C}$	10 $\bar{B}+C$
0 A	0	0	0	
1 $\bar{A}$	0		0	

$F(A,B,C) = \pi M(0,1,3,4,7)$

$= M_0 \cdot M_1 \cdot M_3 \cdot M_4 \cdot M_7$

Fill maxterms with 0's.

$\therefore F(A,B,C) = (B+C)(\bar{B}+\bar{C})(A+\bar{C})$

(or)

$(B+C)(\bar{B}+\bar{C})(A+B)$

(x) $F = \pi M(0,1,3,7) + \pi d(2,5)$

Sol:

A \ BC	$B+C$ 00	$B+\bar{C}$ 01	$\bar{B}+\bar{C}$ 11	$\bar{B}+C$ 10
A 0	0	0	0	X
$\bar{A}$ 1		X	0	

$F = \pi M(0,1,3,7) + \pi d(2,5)$

$= M_0 \cdot M_1 \cdot M_3 \cdot M_7 + d_2 d_5$.

Fill maxterms with 0's.

Fill don't terms with X's

consider d_2, d_5 as 0's.

$\therefore F(A,B,C) = A\bar{C}$.

(xi) Given $F = \Sigma(0,3,4,7)$. Find (a) F (b) dual of F (c) F'.

Sol:

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1		1	
A 1	1		1	

$F = \Sigma m(0,3,4,7)$

$= m_0 + m_3 + m_4 + m_7$

Fill minterms with 1's.

$F \rightarrow$ ~~POS~~ SOP

(i) $F(A,B,C) = \bar{B}\bar{C} + BC = B \odot C$

(ii) Dual of F is $(\bar{B}+\bar{C})(B+C)$

$= B \cdot \bar{B} + C \cdot \bar{B} + \bar{C} \cdot B + C \cdot \bar{C}$

$= B\bar{C} + \bar{B}C = B \oplus C$.

$\therefore \boxed{\text{Dual of } F = B\bar{C} + \bar{B}C = B \oplus C}$

(iii) $F' \rightarrow$ SOP.

$\therefore F' = \pi(1,2,5,6)$

$\therefore F' = (\bar{B}+C)(B+C')$

$(\because F' = \text{dual of } F)$

4-variable K-Map :-

Representation using Minterms

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}C\bar{D}$
$\bar{A}B$	$\bar{A}B\bar{C}\bar{D}$	$\bar{A}B\bar{C}D$	$\bar{A}BCD$	$\bar{A}BC\bar{D}$
AB	$AB\bar{C}\bar{D}$	$AB\bar{C}D$	$ABCD$	$ABC\bar{D}$
$A\bar{B}$	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}\bar{C}D$	$A\bar{B}CD$	$A\bar{B}C\bar{D}$

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

Representation using maxterms

AB \ CD	$C+D$	$C+\bar{D}$	$\bar{C}+\bar{D}$	$\bar{C}+D$
$A+B$	$A+B+C+D$	$A+B+C+\bar{D}$	$A+B+\bar{C}+\bar{D}$	$A+B+\bar{C}+D$
$A+\bar{B}$	$A+\bar{B}+C+D$	$A+\bar{B}+C+\bar{D}$	$A+\bar{B}+\bar{C}+\bar{D}$	$A+\bar{B}+\bar{C}+D$
$\bar{A}+\bar{B}$	$\bar{A}+\bar{B}+C+D$	$\bar{A}+\bar{B}+C+\bar{D}$	$\bar{A}+\bar{B}+\bar{C}+\bar{D}$	$\bar{A}+\bar{B}+\bar{C}+D$
$\bar{A}+B$	$\bar{A}+B+C+D$	$\bar{A}+B+C+\bar{D}$	$\bar{A}+B+\bar{C}+\bar{D}$	$\bar{A}+B+\bar{C}+D$

AB \ CD	00	01	11	10
00	M_0	M_1	M_3	M_2
01	M_4	M_5	M_7	M_6
11	M_{12}	M_{13}	M_{15}	M_{14}
10	M_8	M_9	M_{11}	M_{10}

Ex:- (i) $F(A,B,C,D) = \Sigma m(2,4,5,9,10,12,13)$

Sol:-

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	$_{0}$	$_{1}$	$_{3}$	1_{2}
$\bar{A}B$ 01	1_{4}	1_{5}	$_{7}$	$_{6}$
AB 11	1_{12}	1_{13}	$_{15}$	$_{14}$
$A\bar{B}$ 10	$_{8}$	1_{9}	$_{11}$	1_{10}

$F(A,B,C,D) = \Sigma m(2,4,5,9,10,12,13)$

$= m_2 + m_4 + m_5 + m_9 + m_{10} + m_{12} + m_{13}$

Fill Minterms with 1s.

$\therefore \; F(A,B,C,D) = B\bar{C} + A\bar{C}D + \bar{B}C\bar{D}$

(ii) $F(w,x,y,z) = \Sigma m(1,3,5,7,8,9,10,11,12,14)$.

Sol:-

$wx \backslash yz$	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{w}\bar{x}$ 00	${}_{0}$	1_{1}	1_{3}	${}_{2}$
$\bar{w}x$ 01	${}_{4}$	1_{5}	1_{7}	${}_{6}$
wx 11	1_{12}	${}_{13}$	${}_{15}$	1_{14}
$w\bar{x}$ 10	1_{8}	1_{9}	1_{11}	1_{10}

$F(w,x,y,z) = m_1+m_3+m_5+m_7+m_8+m_9+m_{10}+m_{11}+m_{12}+m_{14}$

Fill minterms with 1's.

$$F(w,x,y,z) = z\bar{w} + w\bar{z} + w\bar{x}$$
$$= w\bar{x} + w\bar{z} + \bar{w}z$$

(iii) $F(A,B,C,D) = \Sigma(0,1,2,3,5,7,8,9,10,11,13,15)$.

Sol:-

$AB \backslash CD$	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1_{0}	1_{1}	1_{3}	1_{2}
$\bar{A}B$ 01	${}_{4}$	1_{5}	1_{7}	${}_{6}$
AB 11	${}_{12}$	1_{13}	1_{15}	${}_{14}$
$A\bar{B}$ 10	1_{8}	1_{9}	1_{11}	1_{10}

$F(A,B,C,D) = m_0+m_1+m_2+m_3+m_5+m_7+m_8+m_9+m_{10}+m_{11}+m_{13}+m_{15}$

Fill minterms with 1's.

$$F(A,B,C,D) = D + \bar{B}$$

(iv) $F = \Sigma(0,5,7,8,10,15) + \Sigma d(2,6,13)$.

Sol:-

$AB \backslash CD$	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1_{0}	${}_{1}$	${}_{3}$	X_{2}
$\bar{A}B$ 01	${}_{4}$	1_{5}	1_{7}	X_{6}
AB 11	${}_{12}$	X_{13}	1_{15}	${}_{14}$
$A\bar{B}$ 10	1_{8}	${}_{9}$	${}_{11}$	1_{10}

$F = \Sigma(0,5,7,8,10,15) + \Sigma d(2,6,13)$
$= m_0+m_5+m_7+m_8+m_{10}+m_{15}+d_2+d_6+d_{13}$.

Fill minterms with 1's.
Fill don't terms with x's.
consider d_{13}, d_2 as 1's.

$$\therefore F(A,B,C,D) = BD + \bar{B}\bar{D} = B \odot D.$$

(v) $F = \Sigma(0,1,2,4,5,6,7,13,15) + \Sigma\phi(3,12,14,9,11)$.

Sol:- $F = m_0+m_1+m_2+m_4+m_5+m_6, m_7, m_{13}+m_{15}+\phi_3+\phi_{12}+\phi_{14}+\phi_9+\phi_{11}$

Fill minterms with 1's.

... x's ... consider ϕ_3, ϕ_{11} as 1's.

$AB \backslash CD$	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1_{0}	1_{1}	X_{3}	1_{2}
$\bar{A}B$ 01	1_{4}	1_{5}	1_{7}	1_{6}
AB 11	X_{12}	1_{13}	1_{15}	X_{14}
$A\bar{B}$ 10	$_{8}$	X_{9}	X_{11}	$_{10}$

$\therefore F(A,B,C,D) = \bar{A} + D.$

(or)

$\bar{A} + B.$

(vi) $F = \Sigma m(1,5,8,15,13,14) + \Sigma d(6,7).$

Sol:-

$AB \backslash CD$	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	$_{0}$	1_{1}	$_{3}$	$_{2}$
$\bar{A}B$ 01	$_{4}$	1_{5}	X_{7}	X_{6}
AB 11	$_{12}$	1_{13}	1_{15}	1_{14}
$A\bar{B}$ 10	1_{8}	$_{9}$	$_{11}$	$_{10}$

$F = m_1 + m_5 + m_8 + m_{15} + m_{13} + m_{14} + d_6 + d_7$

Fill 'd' terms with x's

consider d_6, d_7 as is.

$\therefore F(A,B,C,D) = BD + BC + \bar{A}\bar{C}D + A\bar{B}\bar{C}\bar{D}$

(vii) $F = \Pi M(0,4,7,10) + \Pi d(3,13)$

Sol:-

$AB \backslash CD$	$C+D$ 00	$C+\bar{D}$ 01	$\bar{C}+\bar{D}$ 11	$\bar{C}+D$ 10
$A+B$ 00	0_{0}	$_{1}$	X_{3}	$_{2}$
$A+\bar{B}$ 01	0_{4}	$_{5}$	0_{7}	$_{6}$
$\bar{A}+\bar{B}$ 11	$_{12}$	X_{13}	$_{15}$	$_{14}$
$\bar{A}+B$ 10	$_{8}$	$_{9}$	$_{11}$	0_{10}

$F = M_0 \cdot M_4 \cdot M_7 \cdot M_{10} + d_3 \cdot d_{13}$

consider d_3 as 0.

$\therefore F(A,B,C,D) = (A+C+D)(A+\bar{C}+\bar{D})(\bar{A}+B+\bar{C}+D)$

(viii) $F(w,x,y,z) = \pi(0,3,4,7,8,11,12,14) + \pi\phi(2,6)$.

Sol:-

wx \ yz	$y+z$ 00	$y+\bar{z}$ 01	$\bar{y}+\bar{z}$ 11	$\bar{y}+z$ 10
$w+x$ 00	0 ₀	₁	0 ₃	X ₂
$w+\bar{x}$ 01	0 ₄	₅	0 ₇	X ₆
$\bar{w}+\bar{x}$ 11	0 ₁₂	₁₃	₁₅	0 ₁₄
$\bar{w}+x$ 10	0 ₈	₉	0 ₁₁	₁₀

$F(w,x,y,z) = M_0 \cdot M_3 \cdot M_4 \cdot M_7 \cdot M_8 M_{11} \cdot M_{12} M_{14} \cdot \phi_2 \cdot \phi_6$

Fill maxterms with 0's.
Fill ϕ terms with x's.
consider ϕ_2, ϕ_6 as 0's.

$$\therefore F(w,x,y,z) = (y+z)(\bar{x}+z)(x+\bar{y}+\bar{z})$$

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5-variable K-Map :-

Representing using minterms & Maxterms

AB \ CDE	000	001	011	010	110	111	101	100
00	0	1	3	2	6	7	5	4
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	16	17	19	18	22	23	21	20

Ex:- Simplify the following boolean expressions using K-map.

(i) $F = \pi M(0,1,4,5,9,11,13,15,16,17,25,27,28,29,31) + \pi d(20,21,22,30)$

Sol:-

AB \ CDE	$C+D+E$ 000	$C+D+\bar{E}$ 001	$C+\bar{D}+\bar{E}$ 011	$C+\bar{D}+E$ 010	$\bar{C}+\bar{D}+E$ 110	$\bar{C}+\bar{D}+\bar{E}$ 111	$\bar{C}+D+\bar{E}$ 101	$\bar{C}+D+E$ 100
$A+B$ 00	0 ₀	0 ₁	₃	₂	₆	₇	0 ₅	0 ₄
$A+\bar{B}$ 01	₈	0 ₉	0 ₁₁	₁₀	₁₄	0 ₁₅	0 ₁₃	₁₂
$\bar{A}+\bar{B}$ 11	₂₄	0 ₂₅	0 ₂₇	₂₆	X ₃₀	0 ₃₁	0 ₂₉	0 ₂₈
$\bar{A}+B$ 10	0 ₁₆	0 ₁₇	₁₉	₁₈	X ₂₂	₂₃	X ₂₁	X ₂₀

$$\therefore F = (\bar{B}+\bar{E})(B+D)(\bar{A}+\bar{C}+D)$$

(ii) $F = \Sigma m(0,2,4,6,9,11,13,15,17,21,25,27,29,31)$

Sol:-

AB \ CDE	$\bar C\bar D\bar E$ 000	$\bar C\bar D E$ 001	$\bar C DE$ 011	$\bar C D\bar E$ 010	$CD\bar E$ 110	CDE 111	$C\bar D E$ 101	$C\bar D\bar E$ 100
$\bar A\bar B$ 00	1 (0)	(1)	(3)	1 (2)	1 (6)	(7)	(5)	1 (4)
$\bar A B$ 01	(8)	1 (9)	1 (11)	(10)	(14)	1 (15)	1 (13)	(12)
AB 11	(24)	1 (25)	1 (27)	(26)	(30)	1 (31)	1 (29)	(28)
$A\bar B$ 10	(16)	1 (17)	(19)	(18)	(22)	(23)	1 (21)	(20)

$$\therefore F = BE + A\bar D E + \bar A\bar B\bar E$$

<u>Note</u>:- (i) In K-Map pair is a group of two adjecent cells which cancels one variable.

(ii) Quad is a group of 4 adjecent cells which cancels two variables.

(iii) octet is a group of 8 adjecent cells which cancels three variables.

<u>Ex</u>:- ① Using demorgan's law simplify the following.

(i) $Y_1 = ((a+b')(c'+d))'$

<u>Sol</u>:- $Y_1 = \overline{((a+\bar b)(\bar c+d))}$ (∵ demorgan's law $\overline{X+Y} = \bar X\cdot\bar Y$, $\overline{X\cdot Y} = \bar X+\bar Y$)

$= (\overline{a+\bar b})+(\overline{\bar c+d})$

$= (\bar a\cdot\bar{\bar b})+(\bar{\bar c}\cdot\bar d)$

$= (\bar a\cdot b)+(c\cdot\bar d)$

$\therefore \boxed{Y_1 = \bar a\cdot b + c\cdot\bar d}$

(ii) $Y_2 = (a+b')'$

<u>Sol</u>:- $Y_2 = (a+b')'$

$= \bar a\cdot\bar{\bar b} = \bar a\cdot b$

$\boxed{Y_2 = \bar a\cdot b}$

Ex: ② simplify the following expressions and implement using 2-level NAND gate

(i) $AB' + ABD + ABD' + A'C'D' + A'BC'$

Sol. $AB' + AB(D+D') + A'C'D' + A'BC'$ $\quad (\because A+BC = (A+B)(A+C))$

$= AB' + AB + A'C'D' + A'BC'$

$= A + A'C'D' + A'BC'$ $\quad = (A+A')(A+C'D') + A'BC'$

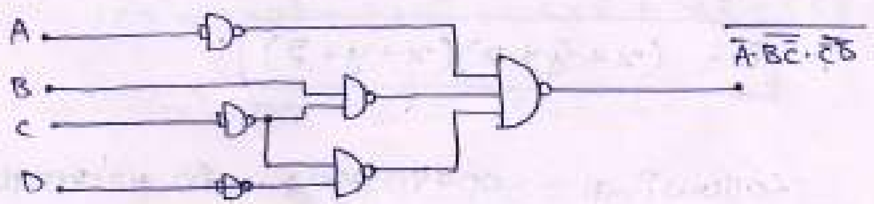
$= (A + A'C'D' + A'BC')$ $\quad = A + C'D' + A'BC'$

$= (A+A')(A+BC') + C'D'$

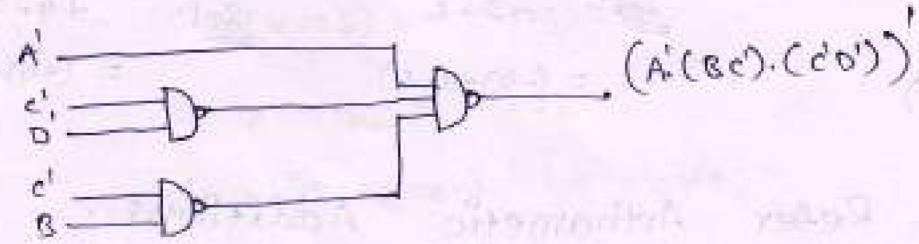
$= A + BC' + C'D'$

$= ((A + BC' + C'D')')'$

$= \overline{\bar{A} \cdot \overline{B\bar{C}} \cdot \overline{\bar{C}\bar{D}}}$



(or)



Note:-

i/p variables — n
∴ Total number of Maxterm $= 2^n$
" " " Minterms $= 2^n$
" " " Boolean expressions $= 2^{2^n}$.

Ex:- ③ The minimum number of NAND gates required to implement $A + AB' + ABC'$.

Sol:- $A(1 + \bar{B} + B\bar{C})$ $\quad$ ($\because$ 1 + anything = 1)

$= A$ $\qquad$ A

$\therefore$ the no. of NAND gates required = 0.

④ Find complement of $F = \bar{x}y\bar{z} + \bar{x}\bar{y}z$

Sol:- $F = \bar{x}y\bar{z} + \bar{x}\bar{y}z$

complement of F is $\bar{F} = \overline{\bar{x}y\bar{z} + \bar{x}\bar{y}z}$

$= \overline{\bar{x}y\bar{z}} \cdot \overline{\bar{x}\bar{y}z}$

$= (\bar{\bar{x}} + \bar{y} + \bar{\bar{z}})(\bar{\bar{x}} + \bar{\bar{y}} + \bar{z})$

$= (x + \bar{y} + z)(x + y + \bar{z})$

$\therefore \boxed{\bar{F} = (x + \bar{y} + z)(x + y + \bar{z})}$

⑤ Perform the following operations in 2's compliment form. 8-bit Binary arthametic.

(i) $+6 - 3$ $\qquad$ (ii) $-2 - 6$ $\qquad$ (iii) $+4 - 7$

Sol:- $+6 - 3$

$= (+6) + (-3)$

Sol:- $-2 - 6$

$= (-2) + (-6)$

Sol:- $+4 - 7$

$= (+4) + (-7)$

Refer Arthametic Additions.

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⑥ Determine the value of Base x.

(i) $(211)_x = (152)_8$ convert both numbers into decimal.

Sol:- $2\times x^2 + 1\times x^1 + 1\times x^0 = 1\times 8^2 + 5\times 8 + 2\times 8^0$

$\Rightarrow 2x^2 + x + 1 = 64 + 40 + 2$

$\therefore 2x^2 + x - 105 = 0$

$2x^2 + 15x - 14x - 105 = 0$

$2x(x-7) + 15(x-7) = 0$

$(x-7)(2x+15) = 0$

$\therefore x = 7$ (or) $-\frac{15}{2}$

It can't be '-ve'

$\therefore \boxed{x = 7}$

(ii) $(193)_x = (623)_8$

Sol:- $1\times x^2 + 9\times x^1 + 3\times x^0 = 6\times 8^2 + 2\times 8^1 + 3\times 8^0$

$\Rightarrow x^2 + 9x + \cancel{3} = 384 + 16 + \cancel{3}$

$\therefore x^2 + 9x - 400 = 0$

$x^2 + 25x - 16x - 400 = 0$

$x(x+25) - 16(x+25) = 0$

$\therefore x = 16$ (or) -25

It can't be '-ve'

$\therefore \boxed{x = 16}$

(iii) $(\sqrt{41})_x = 5_{10}$

Sol:- $(\sqrt{41})_x = 5$

$(\sqrt{41})^2 = 5^2$

$\therefore (41)_x = 25$

$4\times x + 1\times x^0 = 25$

$4x = 24$

$\therefore \boxed{x = 6}$

EXOR function :-

$Y = A \oplus B = \bar{A}B + A\bar{B}$

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

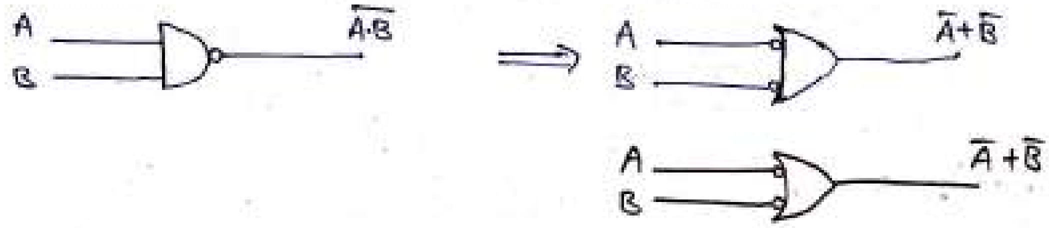
EXOR properties :-

1. $A \oplus A = 0$
2. $A \oplus \bar{A} = 1$
3. $A \oplus 1 = \bar{A}$ (Inverter)
4. $A \oplus 0 = A$ (Non - Inverter)

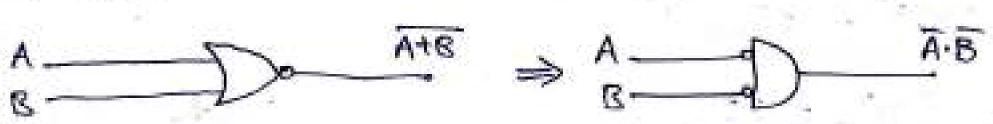
Alternative gate representation :-

1. NOT Gate :-

2. NAND Gate



3. NOR Gate



4. AND Gate

5. OR gate

Conversion of AND, OR, NOT logic to NAND/NOR logic using Graphical procedure :-

Rules:-

(i) Draw logic circuit using ~~logic circuit~~ ~~usi~~ basic gates.

(ii) If it is an AND/OR gate place a bubble at output side.

(iii) If it is an OR/AND gate place a bubble at input side.

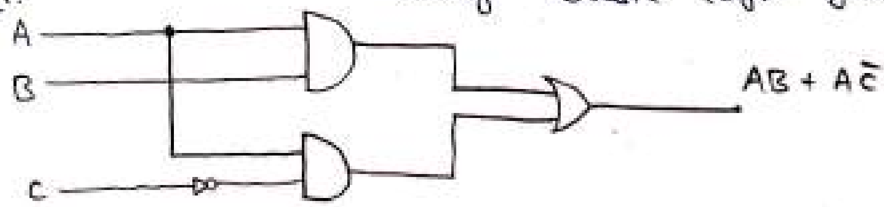
(iv) Place a NOT gate where we placed the bubble.

(v) Replace bubbled (OR)/AND by using NAND/NOR.

(vi) Eliminate double inverters.

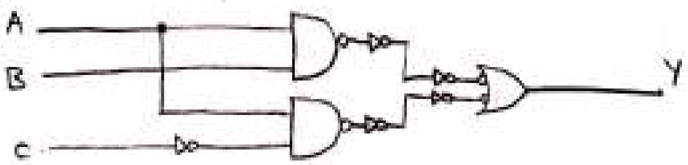
Ex:- $Y = AB + A\bar{C}$ using NAND logic.

Sol:- Step 1:- using basic logic gates.

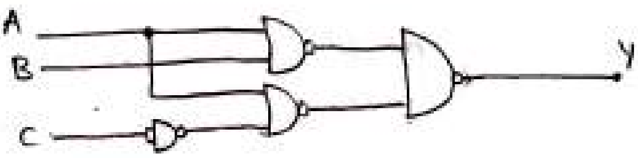


Graphical method.

Step 2:-

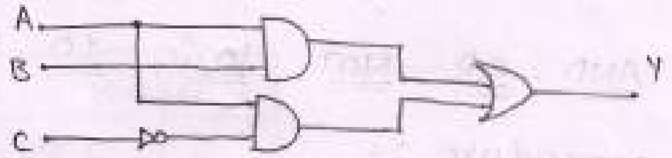


Step 3:-

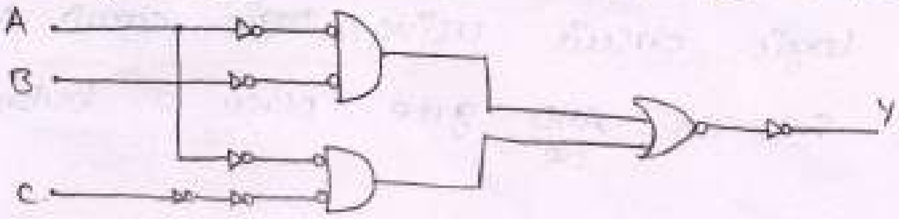


Ex:- $Y = AB + A\bar{C}$ using NOR gate.

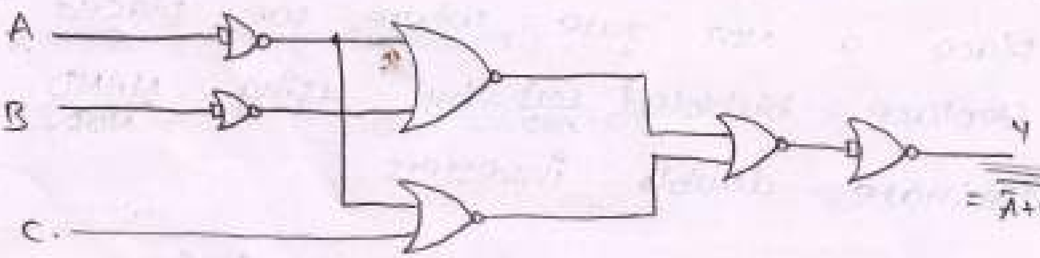
Sol:- Step 1:-



Step 2:- In OR gate * place a bubble at o/p side & place a NOT gate.
In AND gate place a bubble at i/p side & place a NOT gate.



Step 3:- Replace bubble AND ~~with~~ by using NOR.



$= \overline{\overline{\bar{A}+\bar{B}} + \overline{\bar{A}+C}}$