Tutorial Sheet	**:**	**2**
Course Code	**:**	**BMAT101L**
Course Title	**:**	**Calculus**

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Module 2: Multivariable Calculus

EASY

1. Evaluate the limits of the following

(i)

$$\lim_{(x,y)\to(0,0)} \frac{xy}{y^2 - x^2}$$

(ii)

$$\lim_{(x,y)\to(\infty,3)} \frac{2xy - 3}{x^3 + 4y^3}$$

(iii)

$$\lim_{(x,y)\to(1,2)} \frac{2x^2 y}{x^2 + y^2 + 1}$$

Solution: (i) limit does not exist, (ii) 0, (iii) 2/3

2. If $f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

Then discuss the continuity of $f(x,y)$ at the origin.

Solution: Discontinuous at the origin

3. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the following equation

$$x^2 \sin y^3 + xe^{3z} - \cos z^2 = 3y - 6z + 8$$

Solution: $\frac{\partial z}{\partial x} = \frac{2x\sin y^3 + e^{3z}}{-6-3xe^{3z}-2z\sin z^2}$ $\quad \frac{\partial z}{\partial y} = \frac{3x^2y^2\cos y^3 - 3}{-6-3xe^{3z}-2z\sin z^2}$

4. Let $x = r\cos\theta$, $y = r\sin\theta$ for all $r \geq 0$ and $0 \leq \theta \leq 2\pi$ and $z = x^2+y^2$, $\forall(x,y) \in R^2$. Express z_r and z_θ in terms of r and θ.

> **Solution:** Hint: $z_r = \frac{\partial z}{\partial r}$ and $z_\theta = \frac{\partial z}{\partial \theta}$. Use chain rule

5. If $z = f(x,y)$, $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, then show that

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$$

> **Solution:** Use chain rule

MEDIUM

1. If $u = f(e^{y-z},\ e^{z-x},\ e^{x-y})$, then find the value of

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$$

> **Solution:** 0

2. Find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ of following the transformation:

$$\begin{aligned} u &= x^3 - 3xy^2 \\ v &= 3x^2y - y^3 \end{aligned}$$

> **Solution:** $\frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{9(x^2+y^2)^2}$

3. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then find $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u$

Solution: $\frac{-9}{(x+y+z)^2}$

4. Find the Jacobian $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ of the following transformation:

$$\begin{aligned} u &= xyz \\ v &= x^2+y^2+z^2 \\ w &= x+y+z \end{aligned}$$

Solution: $\frac{\partial(x,y,z)}{\partial(u,v,w)} = \frac{-1}{2(x-y)(y-z)(z-x)}$

5. Find limit of the following function at $(0,0)$ if it exists

$$f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Solution: Limit does not exist.

HARD

1. Find limit of the following function at $(0,0)$ if it exists

$$f(x,y) = \begin{cases} (x^2+y^2)\cos\left(\frac{1}{x^2+y^2}\right) & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Solution: Limit : 0 (Hint: $\delta = \sqrt{\epsilon}$)

2. If $u = f(r)$ and $x = r\cos\theta$, $y = r\sin\theta$ then find

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

Solution: $\frac{d^2f}{dr^2} + \frac{1}{r}\frac{df}{dr}$

3. Prove that $x+y+z=u,\ xy+yz+zx=v,\ x^2+y^2+z^2=w$ are functionally dependent. Also, find the relation between them.

Solution: $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0,\ u^2 = w + 2v$

4. Show that

$$\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{(x+y)}$$

does not exist.

Solution: Hint: Check the limit along the path $y = -\sin(x)$.

5. Let

$$f(x,y) = \begin{cases} \frac{\cos y \sin x}{x} & x \neq 0 \\ \cos y & x = 0 \end{cases}$$

Is f continuous at (0,0) ? Is f continuous everywhere ?

Solution: f is continuous at (0,0). f is continuous everywhere.

******************END******************