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TUTORIAL SHEET for Modulue 3

1. Find the relative maxima and minima of the following functions (a) $f(x,y) = xy + \frac{9}{x} + \frac{3}{y}$
(b) $f(x,y) = x^2 + y^2 + xy + \frac{1}{x} + \frac{1}{y}$
(c) $f(x,y) = x^2 + \frac{2}{x^2 y} + y^2$

2. Find the relative and absolute maxima and minima of the following functions in the given domain. (a) $f(x,y) = x^2 - y^2 - 2y, x^2 + y^2 \leq 1$
(b) $f(x,y) = xy, x^2 + y^2 \leq 1$
(c) $f(x,y) = x + y, 4x^2 + 9y^2 \leq 36$
(d) $f(x,y) = 4x^2 + y^2 - 2x + 1, 2x^2 + y^2 \leq 1$
(e) $f(x,y) = x^2 + y^2 - x - y + 1, 0 \leq x \leq 2, 0 \leq y \leq 2$
(f) $f(x,y) = x^3 + y^3 - xy$ over the triangular region bounded by the lines $x = 0, y = 0$, and $y = 2x$

3. Using the Lagrange method of multipliers, solve the following problems.
(a) Find the smallest and the largest values of xy on the line segment $x + 2y = 2, x \geq 0, y \geq 0$
(b) Find the smallest and the largest values of $x + 2y$ on the circle $x^2 + y^2 = 1$
(c) Find the points on the curve $x^2 + xy + y^2 = 16$ which are nearest and farthest form the origin.
(d) Find the triangle whose perimeter is constant and has largest area.
(e) Find the extreme value of xyz when $x + y + z = a, a > 0$.
(f) Find the extreme value of $a^3x^2 + b^3y^2 + c^3z^2$ such that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, where
$$a > 0, b > 0, c > 0$$

4. Expand $f(x,y) = \sin(x + 2y)$ in Taylor's series upto third degree terms about $(1,3)$. Hence, approximate $f(0.99, 3.01)$

5)

Finding Quadratic and Cubic Approximations

In Exercises 1–10, use Taylor's formula for $f(x, y)$ at the origin to find quadratic and cubic approximations of f near the origin.

1. $f(x, y) = xe^y$
2. $f(x, y) = e^x \cos y$
3. $f(x, y) = y \sin x$
4. $f(x, y) = \sin x \cos y$

6)

Use Taylor's formula to find a quadratic approximation of $f(x, y) = \cos x \cos y$ at the origin. Estimate the error in the approximation if $|x| \leq 0.1$ and $|y| \leq 0.1$.

Use Taylor's formula to find a quadratic approximation of $e^x \sin y$ at the origin. Estimate the error in the approximation if $|x| \leq 0.1$ and $|y| \leq 0.1$.