

Assignment-1

EE:1205 (*Signals Systems*)

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Question 10.5.3.12:

Find the sum of the first 40 positive integers divisible by 6.

Solution:

Parameter	Description	Value
x(0)	First Term	6
d	Common Difference	6

TABLE 0
PARAMETER TABLE 10.5.3.12

$$x(n) = (6 + 6n) u(n) \quad (1)$$

$$\Rightarrow X(z) = \frac{6}{1 - z^{-1}} + \frac{6z^{-1}}{(1 - z^{-1})^2} \quad (??) \quad (2)$$

$$\Rightarrow X(z) = \frac{6}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (3)$$

$$y(n) = x(n) * u(n) \quad (4)$$

$$\Rightarrow Y(z) = X(z) U(z) \quad (5)$$

$$= \frac{6}{(1 - z^{-1})^3}, \quad |z| > 1 \quad (6)$$

Using contour integration to find the inverse Z-transform:

$$\Rightarrow y(39) = \frac{1}{2\pi j} \oint_C Y(z) z^{38} dz \quad (7)$$

$$= \frac{1}{2\pi j} \oint_C \frac{6z^{41}}{(z - 1)^3} dz \quad (8)$$

We can observe that there is only a three times repeated pole at $z=1$,

$$\Rightarrow R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (9)$$

$$= \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{6z^{41}}{(z-1)^3} \right) \quad (10)$$

$$= 3 \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (z^{41}) \quad (11)$$

$$= 4920 \quad (12)$$

$$\therefore y(39) = 4920 \quad (13)$$

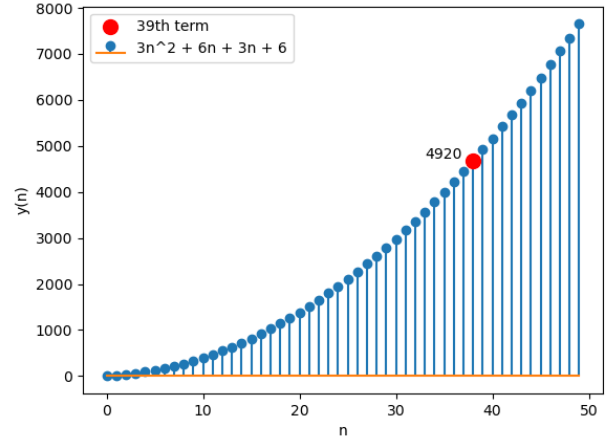


Fig. 0. Plot of $y(n)$ vs n