

Assignment-1

EE:1205 (*Signals Systems*)

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Question 10.5.3.12:

Find the sum of the first 40 positive integers divisible by 6.

Solution:

Parameter	Description	Value
x(0)	First Term	6
d	Common Difference	6

TABLE 0
PARAMETER TABLE

We can observe that there is only a 3 times repeated pole at $z=1$,

$$\Rightarrow R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (11)$$

$$= \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{6z^{41}}{(z-1)^3} \right) \quad (12)$$

$$= 3 \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (z^{41}) \quad (13)$$

$$= 4920 \quad (14)$$

$$\therefore \boxed{y(39) = 4920} \quad (15)$$

$$x(n) = (6 + 6n)(u(n)) \quad (1)$$

$$X(z) = \frac{x(0)}{(1-z^{-1})} + \frac{dz^{-1}}{(1-z^{-1})^2} \quad (2)$$

$$= \frac{6}{1-z^{-1}} + \frac{6z^{-1}}{(1-z^{-1})^2} \quad (3)$$

$$X(z) = \frac{6}{(1-z^{-1})^2}, |z| > 1 \quad (4)$$

$$y(n) = x(n) * u(n) \quad (5)$$

$$Y(z) = X(z)U(z) \quad (6)$$

$$= \frac{6}{(1-z^{-1})^2} \frac{1}{(1-z^{-1})} \quad (7)$$

$$= \frac{6}{(1-z^{-1})^3}, |z| > 1 \quad (8)$$

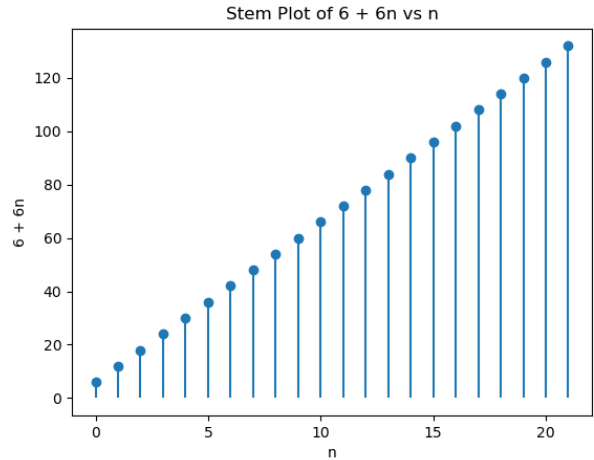


Fig. 0. Plot of $x(n)$ vs n

Using contour integration to find the inverse Z-transform:

$$\Rightarrow y(39) = \frac{1}{2\pi j} \oint_C Y(z) z^{38} dz \quad (9)$$

$$= \frac{1}{2\pi j} \oint_C \frac{6z^{38}}{(1-z^{-1})^3} dz \quad (10)$$