Assignment-1

EE:1205 (*SignalsSystems*)
Indian Institute of Technology, Hyderabad

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Question 10.5.3.12:

Find the sum of the first 40 positive integers divisible by 6.

Solution:

Parameter	Description	Value
x(0)	First Term	6
d	Common Difference	6

TABLE 0
PARAMETER TABLE

$$x(n) = (6+6n)\,u(n) \tag{1}$$

$$\implies X(z) = \frac{x(0)}{(1-z^{-1})} + \frac{dz^{-1}}{(1-z^{-1})^2} \tag{2}$$

$$= \frac{6}{1-z^{-1}} + \frac{6z^{-1}}{(1-z^{-1})^2} \tag{3}$$

$$\implies X(z) = \frac{6}{(1-z^{-1})^2}, \quad |z| > 1 \tag{4}$$

$$y(n) = x(n) * u(n) \tag{5}$$

$$\implies Y(z) = X(z)\,U(z) \tag{6}$$

$$= \frac{6}{(1-z^{-1})^2}\frac{1}{(1-z^{-1})} \tag{7}$$

$$= \frac{6}{(1-z^{-1})^3}, \quad |z| > 1 \tag{8}$$

Using contour integration to find the inverse Z-transform:

$$\implies y(39) = \frac{1}{2\pi j}\oint_C Y(z)\; z^{38}\, dz \tag{9}$$

$$= \frac{1}{2\pi j}\oint_C \frac{6z^{38}}{(1-z^{-1})^3}\, dz \tag{10}$$

We can observe that there is only a three times repeated pole at z=1,

$$\implies R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{11}$$

$$= \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\frac{6z^{41}}{(z-1)^3}\right) \tag{12}$$

$$= 3\lim_{z\to 1}\frac{d^2}{dz^2}\left(z^{41}\right) \tag{13}$$

$$= 4920 \tag{14}$$

$$\therefore \boxed{y(39) = 4920} \tag{15}$$

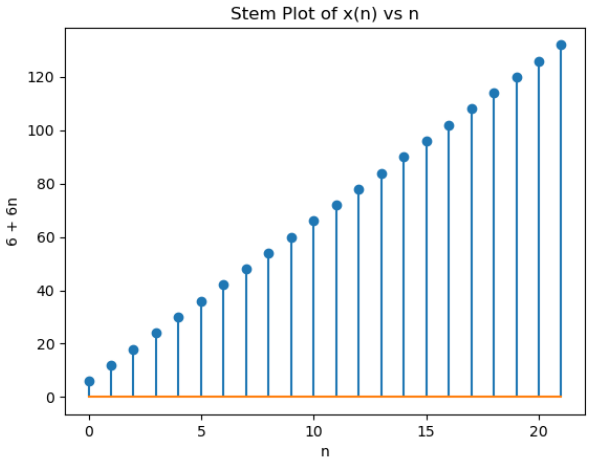

Fig. 0. Plot of x(n) *vs* n