Assignment-11.14.7

EE:1205-Signals and Systems
Indian Institute of Technology, Hyderabad

Md Ayaan Ashraf
EE23BTECH11041

Question

The motion of a particle executing simple harmonic motion is described by the displacement function, $x(t) = A\ cos\ (\omega t + \phi)$. If the initial $(t = 0)$ position of the particle is $1cm$ and its initial velocity is $\omega \quad cm/s$, what are its amplitude and initial phase angle ? The angular frequency of the particle is $\pi \quad s^{-1}$. If instead of the cosine function, we choose the sine function to describe the SHM : $x = B\ sin\ (\omega t + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions.

Solution

Parameter	Description	Value
$x(0)$	Initial position of particle	1cm
ω	Angular frequency of particle	πs^{-1}
$v(0)$	Initial velocity of particle	ω

TABLE 0
Parameter Table

The displacement function for simple harmonic motion (SHM) is given by:

$$x(t) = A\cos(\omega t + \phi)$$

where A is the amplitude, ω is the angular frequency, and ϕ is the phase angle.
Given:

$$x(0) = A\cos(\phi) = 1 \text{ cm}$$

$$v(0) = -A\omega\sin(\phi) = \omega \text{ cm/s}$$

We are given that $\omega = \pi\ \text{s}^{-1}$. Substituting this into the initial velocity equation:

$$-A\pi\sin(\phi) = \pi$$

Solving for $\sin(\phi)$:

$$\sin(\phi) = -1$$

This implies that $\phi = -\frac{\pi}{2}$ since the sine function is negative in the third quadrant.

Therefore, the amplitude A is 1 cm, and the initial phase angle ϕ is $-\frac{\pi}{2}$.

If we choose the sine function instead, the displacement function becomes:

$$x(t) = B\sin(\omega t + \alpha)$$

Following the same logic, if the initial conditions are the same:

$$B\sin(\alpha) = 1 \text{ cm}$$

$$-B\omega\cos(\alpha) = \omega \text{ cm/s}$$

Solving for $\sin(\alpha)$:

$$\sin(\alpha) = -1$$

This implies that $\alpha = -\frac{\pi}{2}$.

Therefore, for the sine function, the amplitude B is 1 cm, and the initial phase angle α is $-\frac{\pi}{2}$.