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# Assignment-11.14.7

EE:1205-Signals and Systems Indian Institute of Technology, Hyderabad

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## **Question**

The motion of a particle executing simple harmonic motion is described by the displacement function,  $x(t) = A \cos(\omega t + \phi)$ . If the initial (t = 0)position of the particle is 1cm and its initial velocity is  $\omega$  cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is  $\pi$  s<sup>-1</sup>. If instead of the cosine function, we choose the sine function to describe the SHM:  $x = B \sin \theta$  $(\omega t + \alpha)$ , what are the amplitude and initial phase of the particle with the above initial conditions.

## Solution

Parameter	Description	Value
x(0)	Initial position of particle	1cm
ω	Angular frequency of particle	$\pi s^{-1}$
x'(0)	Initial velocity of particle	ω
φ	Initial Phase Angle	?
α	New Phase Angle	?
A	Initial Amplitude	?
В	New Amplitude	?

TABLE 1: Parameter Table 11.14.7

The displacement function for simple harmonic motion (SHM) is given by:

$$x(t) = A\cos(\omega t + \phi) \tag{1}$$

Given:

$$x(0) = A\cos(\phi) = 1 \text{ cm}$$
 (2)

$$x'(0) = -A\omega \sin(\phi) = \omega \text{ cm/s}$$
 (3)

Solving for  $\phi$  and A:

$$\tan(\phi) = -1 \tag{4}$$

$$\implies \phi = -\frac{\pi}{4} \tag{5}$$

$$\implies A = \sqrt{2}cm \tag{6}$$

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If we choose the sine function instead, the displacement function becomes:

$$x(t) = B\sin(\omega t + \alpha) \tag{7}$$

Given:

$$B\sin(\alpha) = 1 \text{ cm}$$
 (8)

$$B\omega\cos(\alpha) = \omega \text{ cm/s} \tag{9}$$

Solving for  $\alpha$  and B:

$$\tan(\alpha) = 1 \tag{10}$$

$$\implies \alpha = \frac{\pi}{4} \tag{11}$$

$$\implies B = \sqrt{2}cm$$
 (12)

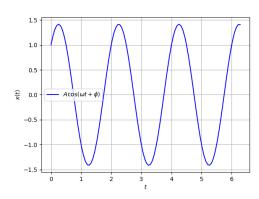


Fig. 1:  $x(t) = \sqrt{2}\cos(\pi t - \frac{\pi}{4})$ 

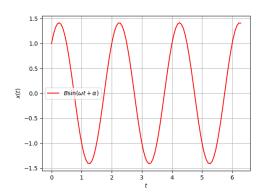


Fig. 2:  $x(t) = \sqrt{2}\sin(\pi t + \frac{\pi}{4})$