Assignment-11.14.7

EE:1205-Signals and Systems
Indian Institute of Technology, Hyderabad

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Question

The motion of a particle executing simple harmonic motion is described by the displacement function, $x(t) = A\ cos\ (\omega t + \phi)$. If the initial ($t = 0$) position of the particle is $1cm$ and its initial velocity is $\omega \quad cm/s$, what are its amplitude and initial phase angle ? The angular frequency of the particle is $\pi \quad s^{-1}$. If instead of the cosine function, we choose the sine function to describe the SHM : $x = B\ sin\ (\omega t + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions.

Solution

Parameter	Description	Value
$x(0)$	Initial position of particle	1cm
ω	Angular frequency of particle	πs^{-1}
$x'(0)$	Initial velocity of particle	ω
ϕ	Initial Phase Angle	?
α	New Phase Angle	?
A	Initial Amplitude	?
B	New Amplitude	?

TABLE 1: Parameter Table 11.14.7

The displacement function for simple harmonic motion (SHM) is given by:

$$x(t) = A\cos(\omega t + \phi) \tag{1}$$

Given:

$$x(0) = A\cos(\phi) = 1 \text{ cm} \tag{2}$$
$$x'(0) = -A\omega\sin(\phi) = \omega \text{ cm/s} \tag{3}$$

Solving for ϕ and A:

$$tan(\phi) = -1 \tag{4}$$
$$\implies \phi = -\frac{\pi}{4} \tag{5}$$
$$\implies A = \sqrt{2}cm \tag{6}$$

If we choose the sine function instead, the displacement function becomes:

$$x(t) = B\sin(\omega t + \alpha) \tag{7}$$

Given:

$$x(0) = B\sin(\alpha) = 1 \text{ cm} \tag{8}$$
$$x'(0) = B\omega\cos(\alpha) = \omega \text{ cm/s} \tag{9}$$

Solving for α and B:

$$tan(\alpha) = 1 \tag{10}$$
$$\implies \alpha = \frac{\pi}{4} \tag{11}$$
$$\implies B = \sqrt{2}cm \tag{12}$$

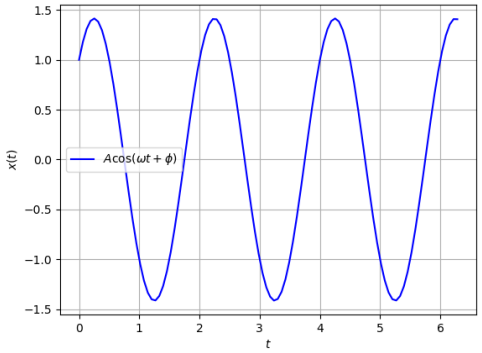

Fig. 1: $x(t) = \sqrt{2}\cos(\pi t - \frac{\pi}{4})$

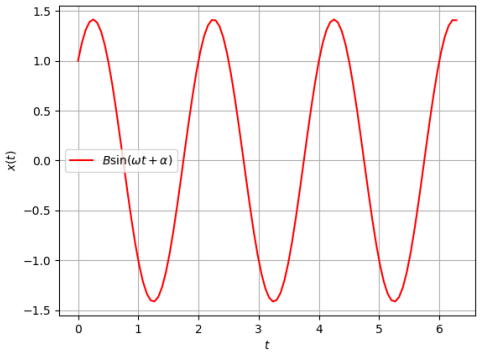

Fig. 2: $x(t) = \sqrt{2}\sin(\pi t + \frac{\pi}{4})$