

Assignment-11.14.7

EE:1205-Signals and Systems
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Question

The motion of a particle executing simple harmonic motion is described by the displacement function, $x(t) = A \cos(\omega t + \phi)$. If the initial ($t = 0$) position of the particle is 1 cm and its initial velocity is $\omega \text{ cm/s}$, what are its amplitude and initial phase angle? The angular frequency of the particle is $\pi \text{ s}^{-1}$. If instead of the cosine function, we choose the sine function to describe the SHM : $x = B \sin(\omega t + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions.

Solution

Parameter	Description	Value
$x(0)$	Initial position of particle	1 cm
ω	Angular frequency of particle	$\pi \text{ s}^{-1}$
$x'(0)$	Initial velocity of particle	ω
ϕ	Initial Phase Angle	?
α	New Phase Angle	?
A	Initial Amplitude	?
B	New Amplitude	?

TABLE 1: Parameter Table 11.14.7

The displacement function for simple harmonic motion (SHM) is given by:

$$x(t) = A \cos(\omega t + \phi) \quad (1)$$

Given:

$$x(0) = A \cos(\phi) = 1 \text{ cm} \quad (2)$$

$$x'(0) = -A\omega \sin(\phi) = \omega \text{ cm/s} \quad (3)$$

Solving for ϕ and A :

$$\tan(\phi) = -1 \quad (4)$$

$$\Rightarrow \phi = -\frac{\pi}{4} \quad (5)$$

$$\Rightarrow A = \sqrt{2}\text{ cm} \quad (6)$$

If we choose the sine function instead, the displacement function becomes:

$$x(t) = B \sin(\omega t + \alpha) \quad (7)$$

Given:

$$B \sin(\alpha) = 1 \text{ cm} \quad (8)$$

$$B\omega \cos(\alpha) = \omega \text{ cm/s} \quad (9)$$

Solving for α and B :

$$\tan(\alpha) = 1 \quad (10)$$

$$\Rightarrow \alpha = \frac{\pi}{4} \quad (11)$$

$$\Rightarrow B = \sqrt{2}\text{ cm} \quad (12)$$

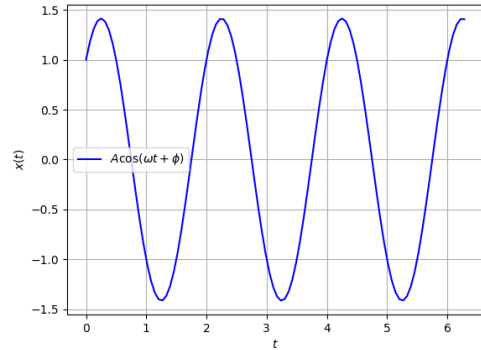


Fig. 1: $x(t) = \sqrt{2} \cos(\pi t - \frac{\pi}{4})$

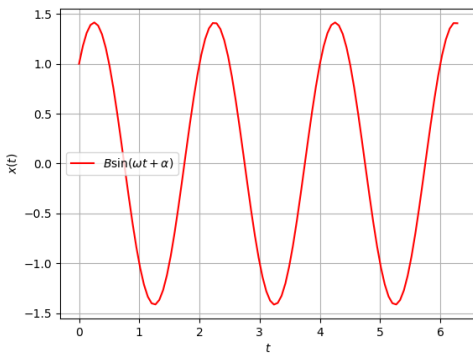


Fig. 2: $x(t) = \sqrt{2} \sin(\pi t + \frac{\pi}{4})$