Assignment-11.14.7

EE:1205-Signals and Systems
Indian Institute of Technology, Hyderabad

Md Ayaan Ashraf
EE23BTECH11041

Question

The motion of a particle executing simple harmonic motion is described by the displacement function, $x(t) = A\ cos\ (\omega t + \phi)$. If the initial ($t = 0$) position of the particle is $1cm$ and its initial velocity is $\omega \quad cm/s$, what are its amplitude and initial phase angle ? The angular frequency of the particle is $\pi \quad s^{-1}$. If instead of the cosine function, we choose the sine function to describe the SHM : $x = B\ sin\ (\omega t + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions.

Solution

Parameter	Description	Value
$x_1(0)$	Initial position of particle in 1)	1cm
$x_2(0)$	Initial position of particle in 2)	1cm
f	Frequency of particle	$\frac{\omega}{2\pi}$
$x_1'(0)$	Initial velocity of particle in 1)	ω
$x_2'(0)$	Initial velocity of particle in 2)	ω
ϕ	Initial Phase Angle	?
α	New Phase Angle	?
A	Initial Amplitude	?
B	New Amplitude	?

TABLE 1: Parameter Table 11.14.7

1) In Fig. 1, the equation is

$$x_1(t) = A\cos(2\pi f t + \phi) \tag{1}$$

Given:

$$x_1(0) = A\cos(\phi) = 1 \text{ cm} \tag{2}$$
$$x_1'(0) = -A2\pi f \sin(\phi) = 2\pi f \text{ cm/s} \tag{3}$$

Solving for ϕ and A:

$$\tan(\phi) = -1 \tag{4}$$
$$\implies \phi = -\frac{\pi}{4} \tag{5}$$
$$\implies A = \sqrt{2}cm \tag{6}$$

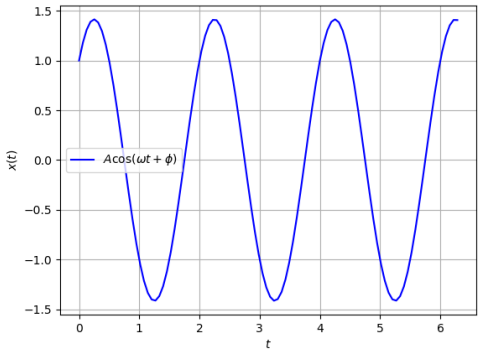

Fig. 1: $x_1(t) = \sqrt{2}\cos(\pi t - \frac{\pi}{4})$

2) In Fig. 2, the equation is

$$x_2(t) = B\sin(2\pi f t + \alpha) \tag{7}$$

Given:

$$x_2(0) = B\sin(\alpha) = 1 \text{ cm} \tag{8}$$
$$x_2'(0) = B2\pi f\cos(\alpha) = 2\pi f \text{ cm/s} \tag{9}$$

Solving for α and B:

$$\tan(\alpha) = 1 \tag{10}$$
$$\implies \alpha = \frac{\pi}{4} \tag{11}$$
$$\implies B = \sqrt{2}cm \tag{12}$$

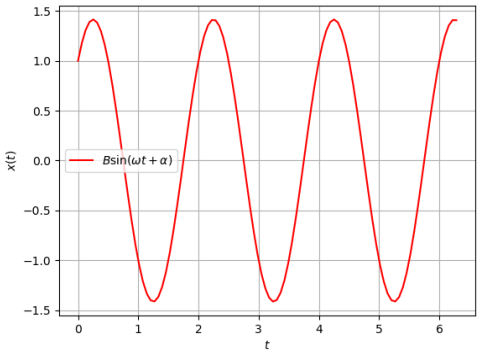

Fig. 2: $x_2(t) = \sqrt{2}\sin(\pi t + \frac{\pi}{4})$