

Assignment-11.14.7

EE:1205-Signals and Systems
Indian Institute of Technology, Hyderabad

Md Ayaan Ashraf
EE23BTECH11041

Question

The motion of a particle executing simple harmonic motion is described by the displacement function, $x(t) = A \cos(\omega t + \phi)$. If the initial ($t = 0$) position of the particle is 1cm and its initial velocity is $\omega \text{ cm/s}$, what are its amplitude and initial phase angle? The angular frequency of the particle is $\pi \text{ s}^{-1}$. If instead of the cosine function, we choose the sine function to describe the SHM : $x = B \sin(\omega t + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions.

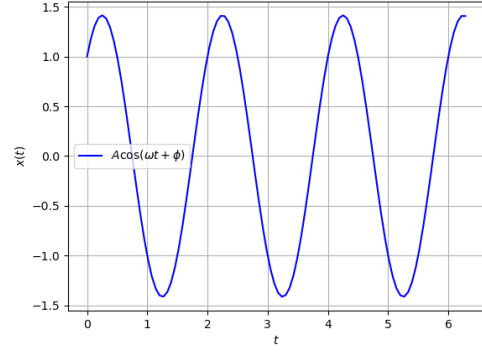


Fig. 1: $x(t) = \sqrt{2} \cos(\pi t - \frac{\pi}{4})$

Solution

Parameter	Description	Value
$x(0)$	Initial position of particle	1cm
f	Frequency of particle	$\frac{\omega}{2\pi}$
$x'(0)$	Initial velocity of particle	ω
ϕ	Initial Phase Angle	?
α	New Phase Angle	?
A	Initial Amplitude	?
B	New Amplitude	?

TABLE 1: Parameter Table 11.14.7

1) In Fig. 1, the equation is

$$x(t) = A \cos(2\pi f t + \phi) \quad (1)$$

Given:

$$x(0) = A \cos(\phi) = 1 \text{ cm} \quad (2)$$

$$x'(0) = -A 2\pi f \sin(\phi) = 2\pi f \text{ cm/s} \quad (3)$$

Solving for ϕ and A :

$$\tan(\phi) = -1 \quad (4)$$

$$\Rightarrow \phi = -\frac{\pi}{4} \quad (5)$$

$$\Rightarrow A = \sqrt{2} \text{ cm} \quad (6)$$

2) In Fig. 2, the equation is

$$x(t) = B \sin(2\pi f t + \alpha) \quad (7)$$

Given:

$$B \sin(\alpha) = 1 \text{ cm} \quad (8)$$

$$B 2\pi f \cos(\alpha) = 2\pi f \text{ cm/s} \quad (9)$$

Solving for α and B :

$$\tan(\alpha) = 1 \quad (10)$$

$$\Rightarrow \alpha = \frac{\pi}{4} \quad (11)$$

$$\Rightarrow B = \sqrt{2} \text{ cm} \quad (12)$$

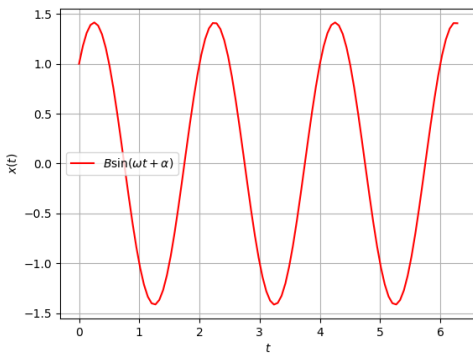


Fig. 2: $x(t) = \sqrt{2} \sin(\pi t + \frac{\pi}{4})$