GATE MA-28(2022)

EE:1205-Signals and Systems
Indian Institute of Technology, Hyderabad

Md Ayaan Ashraf
EE23BTECH11041

Question

The radius of convergence of the series

$$\sum_{n=0}^{\infty} 3^{n+1} z^{2n}, \quad z \in \mathbb{C}$$

is ?

(GATE MA-28 (2022))

Solution:

Parameter	Description	Value
$Y(z)$	General Term	$3^{n+1}z^{2n}$
$x(n)$	Given Sum	$\sum_{n=0}^{\infty} 3^{n+1}z^{2n}, \quad z \in \mathbb{C}$

TABLE 1: GATE MA-28(2022)

$$x(n) = 3\sum_{n=0}^{\infty} 3^n z^{2n}, \quad z \in \mathbb{C} \tag{1}$$

$$= 3\sum_{n=0}^{\infty} (3z^2)^n \tag{2}$$

$$= \frac{3}{1-3z^2} \tag{3}$$

$$= \frac{3}{(1-\sqrt{3}z)(1+\sqrt{3}z)} \tag{4}$$

$$= \frac{3}{2}\left(\frac{1}{1-\sqrt{3}z} + \frac{1}{1+\sqrt{3}z}\right) \tag{5}$$

$$= \frac{3}{2}\left(\frac{z^{-1}}{z^{-1}-\sqrt{3}} + \frac{z^{-1}}{z^{-1}+\sqrt{3}}\right) \tag{6}$$

$$= \frac{3}{2}\left(1 - \frac{\sqrt{3}}{\sqrt{3}-z^{-1}} + 1 - \frac{\sqrt{3}}{z^{-1}+\sqrt{3}}\right) \tag{7}$$

$$\tag{8}$$

$\therefore$ For inverse Z transform,

$$\implies \frac{3}{2}\left(2\delta(n) + \left(\frac{1}{\sqrt{3}}\right)^n - \left(\frac{-1}{\sqrt{3}}\right)^n\right) \tag{9}$$

For Radius of Convergence,

$$|3z^2| < 1 \tag{10}$$

$$|z| < \frac{1}{\sqrt{3}} \tag{11}$$

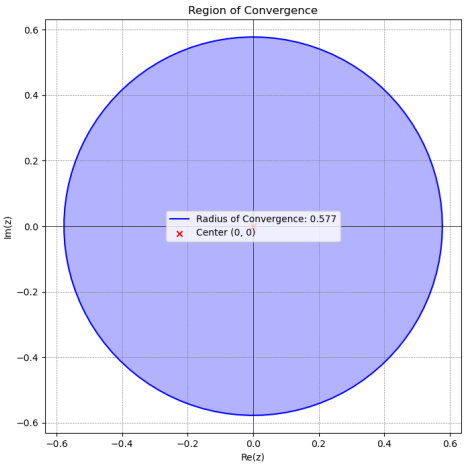

Fig. 1: ROC - $|z| < \frac{1}{\sqrt{3}}$