

GATE MA-28(2022)

EE:1205-Signals and Systems
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Question

The radius of convergence of the series

$$\sum_{n=0}^{\infty} 3^{n+1} z^{2n}, \quad z \in \mathbb{C}$$

is ?

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For Radius of Convergence,

$$|3z^2| < 1 \quad (10)$$

$$|z| < \frac{1}{\sqrt{3}} \quad (11)$$

Solution:

Parameter	Description	Value
$Y(z)$	General Term	$3^{n+1} z^{2n}$
$x(n)$	Given Sum	$\sum_{n=0}^{\infty} 3^{n+1} z^{2n}, \quad z \in \mathbb{C}$

TABLE 1: GATE MA-28(2022)

$$x(n) = 3 \sum_{n=0}^{\infty} 3^n z^{2n}, \quad z \in \mathbb{C} \quad (1)$$

$$= 3 \sum_{n=0}^{\infty} (3z^2)^n \quad (2)$$

$$= \frac{3}{1 - 3z^2} \quad (3)$$

$$= \frac{3}{(1 - \sqrt{3}z)(1 + \sqrt{3}z)} \quad (4)$$

$$= \frac{3}{2} \left(\frac{1}{1 - \sqrt{3}z} + \frac{1}{1 + \sqrt{3}z} \right) \quad (5)$$

$$= \frac{3}{2} \left(\frac{z^{-1}}{z^{-1} - \sqrt{3}} + \frac{z^{-1}}{z^{-1} + \sqrt{3}} \right) \quad (6)$$

$$= \frac{3}{2} \left(1 - \frac{\sqrt{3}}{\sqrt{3} - z^{-1}} + 1 - \frac{\sqrt{3}}{z^{-1} + \sqrt{3}} \right) \quad (7)$$

$$(8)$$

\therefore Inverse Z transform,

$$\Rightarrow \frac{3}{2} \left(2\delta(n) + \left(\frac{1}{\sqrt{3}} \right)^n - \left(\frac{-1}{\sqrt{3}} \right)^n \right) \quad (9)$$

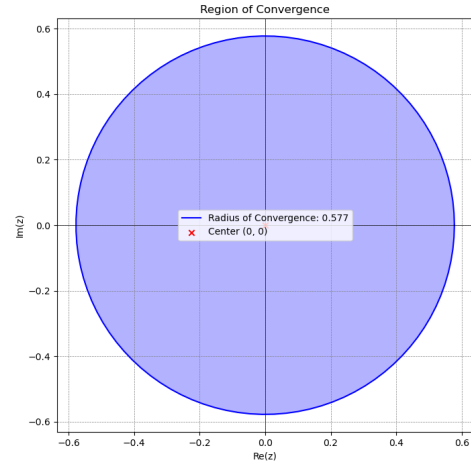


Fig. 1: ROC - $|z| < \frac{1}{\sqrt{3}}$