Revision: Test Paper 2

Statistics - Mixed Topics

Jenny and John are each allowed two attempts to pass an examination.

(i) Jenny estimates that her chances of success are as follows.

- The probability that she will pass on her first attempt is $\frac{2}{3}$.
- If she fails on her first attempt, the probability that she will pass on her second attempt is $\frac{3}{4}$.

Calculate the probability that Jenny will pass. [3]

(ii) John estimates that his chances of success are as follows.

- The probability that he will pass on his first attempt is $\frac{2}{3}$.
- Overall, the probability that he will pass is $\frac{5}{6}$.

Calculate the probability that if John fails on his first attempt, he will pass on his second attempt. [3]

For each of 50 plants, the height, h cm, was measured and the value of $(h - 100)$ was recorded. The mean and standard deviation of $(h - 100)$ were found to be 24.5 and 4.8 respectively.

(i) Write down the mean and standard deviation of h. [2]

The mean and standard deviation of the heights of another 100 plants were found to be 123.0 cm and 5.1 cm respectively.

(ii) Describe briefly how the heights of the second group of plants compare with the first. [2]

(iii) Calculate the mean height of all 150 plants. [2]

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Statistics - Mixed Topics

Past experience has shown that when seeds of a certain type are planted, on average 90% will germinate. A gardener plants 10 of these seeds in a tray and waits to see how many will germinate.

(i) Name an appropriate distribution with which to model the number of seeds that germinate, giving the value(s) of any parameters. State any assumption(s) needed for the model to be valid. [4]

(ii) Use your model to find the probability that fewer than 8 seeds germinate. [2]

Later the gardener plants 20 trays of seeds, with 10 seeds in each tray.

(iii) Calculate the probability that there are at least 19 trays in each of which at least 8 seeds germinate. [4]

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Statistics - Mixed Topics

In a supermarket the proportion of shoppers who buy washing powder is denoted by p. 16 shoppers are selected at random.

(i) Given that $p = 0.35$, use tables to find the probability that the number of shoppers who buy washing powder is

(a) at least 8, [3]

(b) between 4 and 9 inclusive. [2]

(ii) Given instead that $p = 0.38$, find the probability that the number of shoppers who buy washing powder is exactly 6. [3]

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The table shows the latitude, x (in degrees correct to 3 significant figures), and the average rainfall y (in cm correct to 3 significant figures) of five European cities.

City	x	y
Berlin	52.5	58.2
Bucharest	44.4	58.7
Moscow	55.8	53.3
St Petersburg	60.0	47.8
Warsaw	52.3	56.6

$[n = 5,\ \Sigma x = 265.0,\ \Sigma y = 274.6,\ \Sigma x^2 = 14\,176.54,\ \Sigma y^2 = 15\,162.22,\ \Sigma xy = 14\,464.10.]$

(i) Calculate the product moment correlation coefficient. [3]

(ii) The values of y in the table were in fact obtained from measurements in inches and converted into centimetres by multiplying by 2.54. State what effect it would have had on the value of the product moment correlation coefficient if it had been calculated using inches instead of centimetres. [1]

(iii) It is required to estimate the annual rainfall at Bergen, where $x = 60.4$. Calculate the equation of an appropriate line of regression, giving your answer in simplified form, and use it to find the required estimate. [5]

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Statistics - Mixed Topics

A factory makes chocolates of different types. The proportion of milk chocolates made on any day is denoted by p. It is desired to test the null hypothesis $H_0 : p = 0.8$ against the alternative hypothesis $H_1 : p < 0.8$. The test consists of choosing a random sample of 25 chocolates. H_0 is rejected if the number of milk chocolates is k or fewer. The test is carried out at a significance level as close to 5% as possible.

(i) Use tables to find the value of k, giving the values of any relevant probabilities. [3]

(ii) The test is carried out 20 times, and each time the value of p is 0.8. Each of the tests is independent of all the others. State the expected number of times that the test will result in rejection of the null hypothesis. [2]

(iii) The test is carried out once. If in fact the value of p is 0.6, find the probability of rejecting H_0. [2]

(iv) The test is carried out twice. Each time the value of p is equally likely to be 0.8 or 0.6. Find the probability that exactly one of the two tests results in rejection of the null hypothesis. [4]

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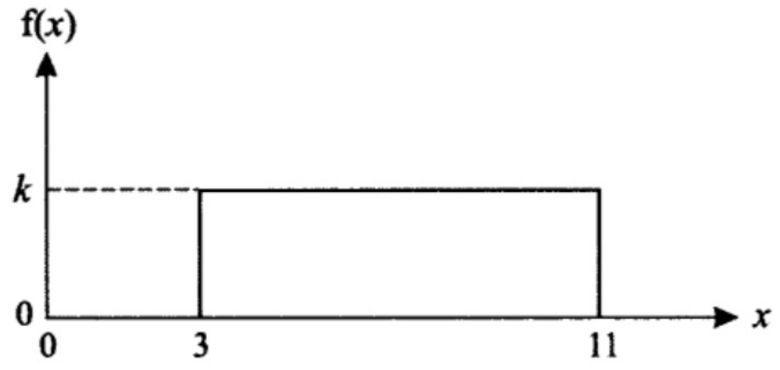
Statistics - Mixed Topics

The continuous random variable X has the probability density function shown in the diagram.



(i) Find the value of the constant k. [2]

(ii) Write down the mean of X, and use integration to find the variance of X. [5]

(iii) Three observations of X are made. Find the probability that $X < 9$ for all three observations. [3]

(iv) The mean of 32 observations of X is denoted by $\bar{X}$. State the approximate distribution of $\bar{X}$, giving its mean and variance. [3]

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Statistics - Mixed Topics

A continuous random variable X has probability density function given by

$$f(x) = \begin{cases} kx^n & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where n and k are positive constants.

(i) Find k in terms of n. [3]

(ii) Show that $\mathrm{E}(X) = \dfrac{n+1}{n+2}$. [3]

It is given that $n = 3$.

(iii) Find the variance of X. [3]

(iv) One hundred observations of X are taken, and the mean of the observations is denoted by $\overline{X}$. Write down the approximate distribution of $\overline{X}$, giving the values of any parameters. [3]

(v) Write down the mean and the variance of the random variable Y with probability density function given by

$$g(y) = \begin{cases} 4\left(y + \frac{4}{5}\right)^3 & -\frac{4}{5} \leqslant y \leqslant \frac{1}{5}, \\ 0 & \text{otherwise.} \end{cases}$$ [3]

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Statistics - Mixed Topics

(i) A clock is designed to chime once each hour, on the hour. The clock has a fault so that each time it is supposed to chime there is a constant probability of $\frac{1}{10}$ that it will not chime. It may be assumed that the clock never stops and that faults occur independently. The clock is started at 5 minutes past midnight on a certain day. Find the probability that the first time it does not chime is

(a) at 0600 on that day, **[3]**

(b) before 0600 on that day. **[3]**

(ii) Another clock is designed to chime twice each hour: on the hour and at 30 minutes past the hour. This clock has a fault so that each time it is supposed to chime there is a constant probability of $\frac{1}{20}$ that it will not chime. It may be assumed that the clock never stops and that faults occur independently. The clock is started at 5 minutes past midnight on a certain day.

(a) Find the probability that the first time it does not chime is at either 0030 or 0130 on that day. **[2]**

(b) Use the formula for the sum to infinity of a geometric progression to find the probability that the first time it does not chime is at 30 minutes past some hour. **[3]**