Quantum Galton Board Circuit: Summary

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2025 Quantum Projects

Introduction

Simulating quantum systems on classical computers has always been a challenging task for scientists and researchers due to the exponential growth of computational resources required (Universal Quantum Simulators). Statistical techniques like Monte Carlo, which use random sampling to estimate numerical results (Monte Carlo Methods in Quantum Problems) (Lattice Gauge Theories and Monte Carlo Simulations), have shown remarkable efficiency compared to conventional numerical methods because they can approximate the statistics without needing to explore all configurations, instead relying on randomness to guide the sampling. Monte Carlo is used in several areas, whether it is solving integrals in high-dimensional spaces, sampling states from a Boltzmann distribution (An Introduction to Monte Carlo Methods for the Boltzmann Equation), lattice gauge theory and quantum chemistry for energy estimation (Lattice Gauge Theories and Monte Carlo Simulations), or simply rendering 3D software animations.

The Galton board simulation is one of the key concepts in our study, used for modeling various statistical distributions, and can be considered a Monte Carlo experiment, as it simulates the probability of the ball falling at each peg with a 50% chance of going left or right, in a fully random manner. The quantum Monte Carlo approach for simulating the Galton board—the quantum Galton board—is a hybrid or full adaptation of the classical one that uses superposition to sample multiple states in parallel. In other words, the board tests all possible peg interactions in parallel.

In the classical Galton board, the random left-or-right movement naturally leads to a binomial distribution, which approaches a Gaussian distribution when the number of layers becomes large. However, in the quantum Galton board, the use of adjustable rotation gates, biased coin operations, and controlled interference enables the generation of a variety of output distributions such as exponential and Hadamard quantum walks, which is our main goal in this challenge, along with an optimized implementation on a noisy model and NISQ devices so that we can analyze the distance between our ideal distribution and the obtained one.

Background

Gaussian Distribution: Continuous probability with probability density function

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where μ is the mean and σ is the standard deviation.

Binomial Distribution: Discrete distribution describing number of successes in n independent trials,

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

De Moivre–Laplace Theorem: The binomial distribution converges to a Gaussian distribution as n increases, with

$$\mu = np, \quad \sigma = \sqrt{np(1-p)}.$$

Quantum Galton Board Circuit

The circuit with n_{layers} produces a discrete distribution over $n_{\text{layers}}+1$ bins. Key properties include:

- Number of qubits: $2 \times (n_{\text{layers}} + 1)$
- Ball qubit located at position $n_{\text{layers}} + 1$
- Measurement wires spaced at even offsets around the ball qubit

The control qubit is initialized with an `RX` rotation gate, creating a superposition to represent the probability p of the ball moving up or down at each peg.

After each peg, the control qubit is reset to avoid interference effects and maintain consistent probabilities across layers.

Design of One Peg

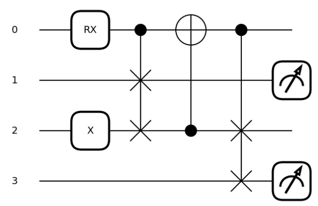

Figure 1: Design of one peg in the Quantum Galton Board

Results

Increasing the number of layers n_{layers} leads to discrete distributions that increasingly resemble Gaussian distributions. This convergence validates the quantum Galton board's ability to simulate classical probabilistic behavior, with the added flexibility of biasing and interference to generate a variety of distributions.