*Heaven's light is our guide.*

# Rajshahi University of Engineering and Technology

## Department of Electrical & Electronic Engineering

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|---|---|
| **Course no.** | EEE 4184 |
| **Course title:** | Digital Communication Sessional |
| **Experiment no.** | 3 |
| **Date of experiment:** | July 29, 2023 |
| **Date of submission:** | August 8, 2023 |



| Submitted to: | Submitted by: |
|---|---|
| **Dr. Md. Zahurul Islam Sarkar**<br>**Professor**<br>Dept. of Electrical & Electronic Engineering,<br>Rajshahi University of Engineering and Technology. | **Ashraf Al- Khalique**<br>**Roll:** 1801171<br>**Session:** 2018-2019<br>Dept. of Electrical & Electronic Engineering,<br>Rajshahi University of Engineering and Technology. |

## Experiment No. 03

**1. Problem Statement:** A multiple-input multiple-output (MIMO) channel is equipped with $n_T$ transmitting antennas and $n_R$ receiving antennas.

**(a) Derive the expressions of** the channel capacity.

**(b) Write the programs of** channel capacity for MIMO channel using MATLAB.

(**c) Explain the numerical results of** the channel capacity for MIMO channel.

**2. Derivation of the channel capacity for MIMO channel**

Let,

$\boldsymbol{P_T}$ **=** Transmit power
$\boldsymbol{N_0}$ **=** Noise power
$\boldsymbol{n_T}$ **=** Number of Antennas at the transmitter
$\boldsymbol{n_R}$ **=** Number of Antennas at the Receiver
$\boldsymbol{I_{nR}}$ **=** Identity Matrix

Required formula for the derivation of the capacity are -

$$(\boldsymbol{A}+\boldsymbol{B})^{\dagger} = \boldsymbol{A}^{\dagger} + \boldsymbol{B}^{\dagger} \quad \ldots\ldots (1)$$

$$(\boldsymbol{AB})^{\dagger} = \boldsymbol{B}^{\dagger}\boldsymbol{A}^{\dagger}$$

$$E[\boldsymbol{x}z^{\dagger}] = 0 \quad \ldots\ldots (2)$$

Since x and z are independent,

$$E(\boldsymbol{A}+\boldsymbol{B}) = E(\boldsymbol{A})+E(\boldsymbol{B}) \quad \ldots\ldots (3)$$

$$det(\alpha\boldsymbol{A}) = \alpha^{n} det(\boldsymbol{A}),\ n = matrix\ order$$

**(i) Covariance of Transmit Signal:**

$$\boldsymbol{R_x} = \boldsymbol{E[xx^{\dagger}]} = \frac{\boldsymbol{P_T}}{\boldsymbol{n_T}}\boldsymbol{I_{nT}}$$

**(ii) Covariance of Noise Signal:**

$$\boldsymbol{R_z} = \boldsymbol{E[zz^{\dagger}]} = \boldsymbol{N_0 I_{nR}}$$

**(iii) Covariance of Received Signal:**

$$\begin{aligned}
\boldsymbol{R}_y = E[yy^{\dagger}] &= E[(\boldsymbol{Hx}+\boldsymbol{z})\,(Hx+z)^{\dagger}] \\
&= E[(\boldsymbol{Hx}+\boldsymbol{z})\{\,(Hx)^{\dagger}+(z)^{\dagger}\}] \\
&= E[(\boldsymbol{Hx}+z)\{\,(x^{\dagger}H^{\dagger}+z^{\dagger}\}] \\
&= E[(Hxx^{\dagger}H^{\dagger}+Hxz^{\dagger}+zx^{\dagger}H^{\dagger}+zz^{\dagger})] \\
&= E[Hxx^{\dagger}H^{\dagger}] + E[Hxz^{\dagger}] + E[zx^{\dagger}H^{\dagger}] + E[zz^{\dagger}] \\
&= E[Hxx^{\dagger}H^{\dagger}] + E[zz^{\dagger}] \\
&= HE[xx^{\dagger}H^{\dagger}] + E[zz^{\dagger}] \\
&= H\,\{\frac{P_T}{n_T}I_{nT}\}H^{\dagger} + N_0 I_{nR}
\end{aligned}$$

$\boldsymbol{R}_y = \frac{P_T}{n_T}\{HH^{\dagger}\} + N_0 I_{nR}$ ,which is the received signal covariance.

**(iv) Entropy of Noise Signal:**

$$\boldsymbol{H}(\boldsymbol{z}) = log_2 det(\pi e\boldsymbol{R}_z) = log_2 det(\pi e N_0 I_{nR})$$



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**(v) Entropy of Received Signal:**

$$\boldsymbol{H}(\boldsymbol{y}) = log_2 det(\pi e \boldsymbol{R}_y) = log_2 det\{\pi e(\frac{P_T}{n_T}\{HH^{\dagger}\} + N_0 I_{nR})\}$$

**(vi) Mutual Information:**

$$I(\boldsymbol{x};\boldsymbol{y}) = \boldsymbol{H}(\boldsymbol{y}) - \boldsymbol{H}(\boldsymbol{z}) = log_2 det(\pi e \boldsymbol{Ry}) - log_2 det(\pi e \boldsymbol{Rz})$$

$$= log_2 \frac{det(\pi e Ry)}{det(\pi e Rz)}$$

$$= log_2 \frac{\det\{\pi e(\frac{P_T}{n_T}\{\mathrm{H}H^{\dagger}\} + N_0 I_{nR})\}}{det(\pi e N_0 I_{nR})}$$

$$= log_2 \frac{(\pi e N_0)^{n_R} \det\{(\frac{P_T}{n_T}\{\mathrm{H}H^{\dagger}\} + I_{nR})\}}{(\pi e N_0)^{n_R} det(I_{n_R})}$$

$$= log_2 \frac{\det\{(\frac{P_T}{n_T N_0}\{\mathrm{H}H^{\dagger}\} + N_0 I_{nR})\}}{1}$$

$I(\boldsymbol{x};\boldsymbol{y}) = log_2 \det\{(\frac{P_T}{n_T N_0}\{\mathrm{H}H^{\dagger}\} + N_0 I_{nR})\}$, which is the Mutual Information.

**(vii) Capacity at the Receiver:**

$$C = {}^{max}_{Q} I(x;y) = {}^{max}_{Q} log_2 det\{(\frac{P_T}{n_T N_0}\{HH^{\dagger}\} + N_0 I_{nR})\}$$

$C = log_2 det\{(\frac{P_T}{n_T N_0}\{HH^{\dagger}\} + N_0 I_{nR})\}$, which is the channel capacity.

**3. Program for the channel capacity for MIMO channel**

- **MATLAB Code for plotting the capacity**

```
% Ergodic Capacity of Nr x Nt fading MIMO Channel with
% no channel knowledge at the transmitter.
clc
clear all;
%%%%%%%% Initialization %%%%%%%%
N=30000;        % Number of Iterations for H
No=1;           % Noise Variance
%%%%%%%% Number of antennas %%%%%%%%
nt=2;           % Number of Transmit antennas
nr=2;           % Number of Receive antennas
%%%%%%%% Correlation of antennas %%%%%%%%
rhot=0;         % Correlation coefficient (Transmitter)
rhor=0;         % Correlation coefficient (Receiver)
Rt=ExponencorrMrtx(nt,rhot);    % Correlation Matrix Rt
Rr=ExponencorrMrtx(nr,rhot);    % Correlation Matrix Rr
%%%%%%%% SNR of Channel %%%%%%%%
SNRdB=[0:1:15];         % Signal-to-Noise Ratio
l=length (SNRdB);
SNR=zeros(1,l);
%%%%%%%% Creation of SNR from 1 to L %%%%%%%%
for i=1:l
    SNR(i)=10^(0.1*SNRdB(i)); %Conversion of SNR in magnitude form
    P(i)=SNR(i)*No;
end
```

```
%%%%%%%% Calculation of Ergodic capacity %%%%%%%%
Erg_Cap_unknown=zeros(1,l);
for i=1:l                    % Loop of SNR
    C_unknown=zeros(1,N);
    %%%%%%%% Calculation of capacity %%%%%%%%
    for j=1:N
        H=CGM (nr,nt,Rr,Rt);     % Generate Rayleigh Distributed fading
        C_unknown(j)=log(det(eye(nr)+(SNR(i)/nt)*(H*H')));
end
%%%%%%%% Ergodic Capacity %%%%%%%%
    Erg_Cap_unknown(i)=mean(C_unknown); %Ergodic capacity with no CSIT
%%%%%%%% Printing Data %%%%%%%%
fprintf ('% e\t% e \n',SNRdB(i),Erg_Cap_unknown(i));
end
%%%% Plotting of Ergodic capacity versus SNR  %%%%
plot (SNRdB,Erg_Cap_unknown,':r*')

xlabel ('SNR (dB)');
ylabel ('Ergodic capacity (bits/sec/Hz)');
title ('Ergodic capacity versus SNR');
grid on
hold on
```

- **To build n*n identity matrix the user defined function**

```
function out=ExponencorrMrtx(n,rho);
% generate an nxn exponential correlation matrix
A=ones(n);
for i=1:n
    for j=1:n
        if i==j
            A(i,j)=1;
        end
        if i>j
            A(i,j)=rho^(i-j);
        else A(i,j)=rho^(j-i);
        end
    end
end
out=A;
```

- **To generate relay distributed fading user defined function**

```
function out=CGM(nr,nt,Rr,Rt)
% Generate Complex Gaussian matrix
X = sqrt(0.5)*randn(nr,nt)+i*sqrt(0.5)*randn(nr,nt);
out=Rr^(1/2)*X*Rt^(1/2);
```

### 4. Numerical results of the channel capacity for MIMO channel

**(a) Numerical data for measuring capacity at different SNR (dB)**

| SNR (dB) | Capacity(bits/sec/Hz) | SNR (dB) | Capacity(bits/sec/Hz) |
|---|---|---|---|
| 0 | 1.169220 | 8 | 3.178715 |
| 1 | 1.353969 | 9 | 3.518419 |
| 2 | 1.558950 | 10 | 3.840310 |
| 3 | 1.781360 | 11 | 4.206453 |
| 4 | 2.034536 | 12 | 4.573855 |
| 5 | 2.297278 | 13 | 4.936794 |

| | | | |
|---|---|---|---|
| 6 | 2.571197 | 14 | 5.333823 |
| 7 | 2.861399 | 15 | 5.720873 |

**(b) Graphical representation**

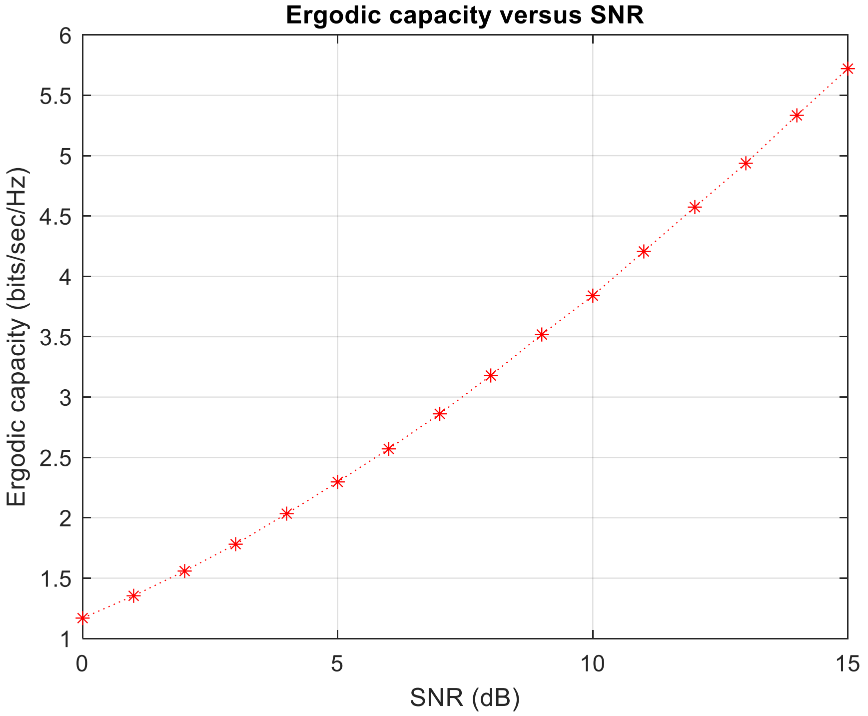


**Figure 3.1.** Ergodic capacity vs Signal to Noise Ratio plot for MIMO channel.

**(c) Description of Figure 3.1:** This is a plot of the capacity vs signal to noise ratio for various values of the average SNR (SNRADB) for MIMO) channel is equipped with $n_T$ transmitting antennas and $n_R$ receiving antennas. This graph depicts the impact of SNR on channel capacity. The graph above demonstrates that as the SNR value increases, the capacity rises.

## 6. Discussion and Conclusion

**a) Discussion:** The linearity property of channel ergodic capacity in response to the signal-to-noise ratio is depicted in Figure 3.1. Capacity, as we know, is the conditional data transmission rate at which the data transmission rate is maximal and the error probability is minimum. It varies depending on the case and the SNR capacity. The graph and data table illustrate that increasing the SNR increases capacity.

**b) Conclusion:** From our investigation, we can draw the following conclusions:

- Capacity varies depending on the channel.
- Increasing the SNR increases channel capacity.
- Noise power has an adverse influence on channel capacity.
- The number of antennas on the transmitter and receiving sides determines channel capacity.

Finally, comprehending the statistical features of the MIMO channel as revealed by the ergodic capacity analyses is critical for building robust and efficient digital communication systems.

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| **Dr. Md. Zahurul Islam Sarkar**<br>**Professor**<br>Dept. of Electrical & Electronic Engineering,<br>Rajshahi University of Engineering and Technology. | **Ashraf Al- Khalique**<br>**Roll:** 1801171<br>**Session:** 2018-2019<br>Dept. of Electrical & Electronic Engineering,<br>Rajshahi University of Engineering and Technology. |

# Experiment No. 04

**1. Problem Statement:** A multiple-input multiple-output (MIMO) channel is equipped with $n_T$ transmitting antennas and $n_R$ receiving antennas.

**(a) Investigate the effect** of transmitting antennas on the channel capacity of MIMO channel.

**(b) Investigate the effect** of receiving antennas on the channel capacity of MIMO channel.

**2. Derivation of the channel capacity for MIMO channel**

Let,

$\boldsymbol{P_T}$ = Transmit power
$\boldsymbol{N_0}$ = Noise power
$\boldsymbol{n_T}$ = Number of Antennas at the transmitter
$\boldsymbol{n_R}$ = Number of Antennas at the Receiver
$\boldsymbol{I_{nR}}$ = Identity Matrix

Required formula for the derivation of the capacity are -

$$(\boldsymbol{A}+\boldsymbol{B})^\dagger = \boldsymbol{A}^\dagger + \boldsymbol{B}^\dagger \;\ldots\ldots (1)$$

$$(\boldsymbol{AB})^\dagger = \boldsymbol{B}^\dagger \boldsymbol{A}^\dagger$$

$$E[\boldsymbol{x}z^\dagger] = 0 \;\ldots\ldots (2)$$

Since x and z are independent,

$$E(\boldsymbol{A}+\boldsymbol{B}) = E(\boldsymbol{A})+E(\boldsymbol{B}) \;\ldots\ldots (3)$$

$$det(\alpha\boldsymbol{A}) = \alpha^n det(\boldsymbol{A}),\; n = matrix\ order$$

**(i) Covariance of Transmit Signal:**

$$\boldsymbol{R_x} = \boldsymbol{E[xx^\dagger]} = \frac{\boldsymbol{P_T}}{\boldsymbol{n_T}}\boldsymbol{I_{nT}}$$

**(ii) Covariance of Noise Signal:**

$$\boldsymbol{R_z} = \boldsymbol{E[zz^\dagger]} = \boldsymbol{N_0 I_{nR}}$$

**(iii) Covariance of Received Signal:**

$$\begin{aligned}
\boldsymbol{R}_y = E[yy^\dagger] &= E[(\boldsymbol{Hx}+\boldsymbol{z})\,(Hx+z)^\dagger] \\
&= E[(\boldsymbol{Hx}+\boldsymbol{z})\{\,(Hx)^\dagger + (z)^\dagger\}] \\
&= E[(\boldsymbol{Hx}+z)\{\,(x^\dagger H^\dagger + z^\dagger\}] \\
&= E[(Hxx^\dagger H^\dagger + Hxz^\dagger + zx^\dagger H^\dagger + zz^\dagger)] \\
&= E[Hxx^\dagger H^\dagger] + E[Hxz^\dagger] + E[zx^\dagger H^\dagger] + E[zz^\dagger] \\
&= E[Hxx^\dagger H^\dagger] + E[zz^\dagger] \\
&= HE[xx^\dagger H^\dagger] + E[zz^\dagger] \\
&= H\,\{\frac{P_T}{n_T} I_{nT}\} H^\dagger + N_0 I_{nR}
\end{aligned}$$

$\boldsymbol{R}_y = \frac{P_T}{n_T}\{HH^\dagger\} + N_0 I_{nR}$ ,which is the received signal covariance.

**(iv) Entropy of Noise Signal:**

$$\boldsymbol{H}(\boldsymbol{z}) = log_2 det(\pi e \boldsymbol{R}_z) = log_2 det(\pi e N_0 I_{nR})$$

**(v) Entropy of Received Signal:**

$$\boldsymbol{H}(\boldsymbol{y}) = log_2 det(\pi e \boldsymbol{R}_y) = log_2 det\{\pi e(\frac{P_T}{n_T}\{HH^\dagger\} + N_0 I_{nR})\}$$

**(vi) Mutual Information:**

$$I(\boldsymbol{x};\boldsymbol{y}) = \boldsymbol{H}(\boldsymbol{y}) - \boldsymbol{H}(\boldsymbol{z}) = log_2 det(\pi e \boldsymbol{Ry}) - log_2 det(\pi e \boldsymbol{Rz})$$

$$= log_2 \frac{det(\pi e Ry)}{det(\pi e Rz)}$$

$$= log_2 \frac{\det\{\pi e(\frac{P_T}{n_T}\{\mathrm{H}H^\dagger\} + N_0 I_{nR})\}}{det(\pi e N_0 I_{nR})}$$

$$= log_2 \frac{(\pi e N_0)^{n_R} \det\{(\frac{P_T}{n_T}\{\mathrm{H}H^\dagger\} + I_{nR})\}}{(\pi e N_0)^{n_R} det(I_{n_R})}$$

$$= log_2 \frac{\det\{(\frac{P_T}{n_T N_0}\{\mathrm{H}H^\dagger\} + N_0 I_{nR})\}}{1}$$

$I(\boldsymbol{x};\boldsymbol{y}) = log_2 \det\{(\frac{P_T}{n_T N_0}\{\mathrm{H}H^\dagger\} + N_0 I_{nR})\}$, which is the Mutual Information.

**(vii) Capacity at the Receiver:**

$$C = {}^{max}_{Q}\, I(x;y) = {}^{max}_{Q}\, log_2 det\{(\frac{P_T}{n_T N_0}\{HH^\dagger\} + N_0 I_{nR})\}$$

$C = log_2 det\{(\frac{P_T}{n_T N_0}\{HH^\dagger\} + N_0 I_{nR})\}$, which is the channel capacity.

## 3. Program for the channel capacity for MIMO channel

- **MATLAB Code for plotting the capacity**

```
% Ergodic Capacity of Nr x Nt fading MIMO Channel with
% no channel knowledge at the transmitter.
clc
clear all;
%%%%%%%% Initialization %%%%%%%%
N=30000;        % Number of Iterations for H
No=1;           % Noise Variance
%%%%%%%% Number of antennas %%%%%%%%
nt=2;           % Number of Transmit antennas
nr=2;           % Number of Receive antennas
%%%%%%%% Correlation of antennas %%%%%%%%
rhot=0;         % Correlation coefficient (Transmitter)
rhor=0;         % Correlation coefficient (Receiver)
Rt=ExponencorrMrtx(nt,rhot);     % Correlation Matrix Rt
Rr=ExponencorrMrtx(nr,rhot);     % Correlation Matrix Rr
%%%%%%%% SNR of Channel %%%%%%%%
SNRdB=[0:1:15];          % Signal-to-Noise Ratio
l=length (SNRdB);
SNR=zeros(1,l);
%%%%%%%% Creation of SNR from 1 to L %%%%%%%%
for i=1:l
    SNR(i)=10^(0.1*SNRdB(i)); %Conversion of SNR in magnitude form
    P(i)=SNR(i)*No;
end
%%%%%%%% Calculation of Ergodic capacity %%%%%%%%
```

```
Erg_Cap_unknown=zeros(1,l);
for i=1:l                    % Loop of SNR
    C_unknown=zeros(1,N);
%%%%%%%% Calculation of capacity %%%%%%%%
    for j=1:N
        H=CGM (nr,nt,Rr,Rt);        % Generate Rayleigh Distributed fading
        C_unknown(j)=log(det(eye(nr)+(SNR(i)/nt)*(H*H')));
end
%%%%%%%% Ergodic Capacity %%%%%%%%
    Erg_Cap_unknown(i)=mean(C_unknown); %Ergodic multicast capacity with no CSIT
%%%%%%%% Printing Data %%%%%%%%
fprintf ('% e\t% e \n',SNRdB(i),Erg_Cap_unknown(i));
end
%%%% Plotting of Ergodic capacity versus SNR  %%%%
plot (SNRdB,Erg_Cap_unknown,':r*')
xlabel ('SNR (dB)');
ylabel ('Ergodic capacity (bits/sec/Hz)');
title ('Ergodic capacity versus SNR');
grid on
hold on
```

- **To build n*n identity matrix the user defined function**

```
function out=ExponencorrMrtx(n,rho);
% generate an nxn exponential correlation matrix
A=ones(n);
for i=1:n
    for j=1:n
        if i==j
            A(i,j)=1;
        end
        if i>j
            A(i,j)=rho^(i-j);
        else A(i,j)=rho^(j-i);
        end
    end
end
out=A;
```

- **To generate relay distributed fading user defined function**

```
function out=CGM(nr,nt,Rr,Rt)
% Generate Complex Gaussian matrix
X = sqrt(0.5)*randn(nr,nt)+i*sqrt(0.5)*randn(nr,nt);
out=Rr^(1/2)*X*Rt^(1/2);
```

## 4. Numerical results of the MIMO channel

### (a) Numerical data for measuring capacity at different SNR (dB)

| SNR (dB) | Capacity(bits/sec/Hz) | SNR (dB) | Capacity(bits/sec/Hz) |
|---|---|---|---|
| 0 | 1.169220 | 8 | 3.178715 |
| 1 | 1.353969 | 9 | 3.518419 |
| 2 | 1.558950 | 10 | 3.840310 |
| 3 | 1.781360 | 11 | 4.206453 |
| 4 | 2.034536 | 12 | 4.573855 |
| 5 | 2.297278 | 13 | 4.936794 |
| 6 | 2.571197 | 14 | 5.333823 |
| 7 | 2.861399 | 15 | 5.720873 |

## 5. Effects of transmitting antennas

- **Graphical representation**

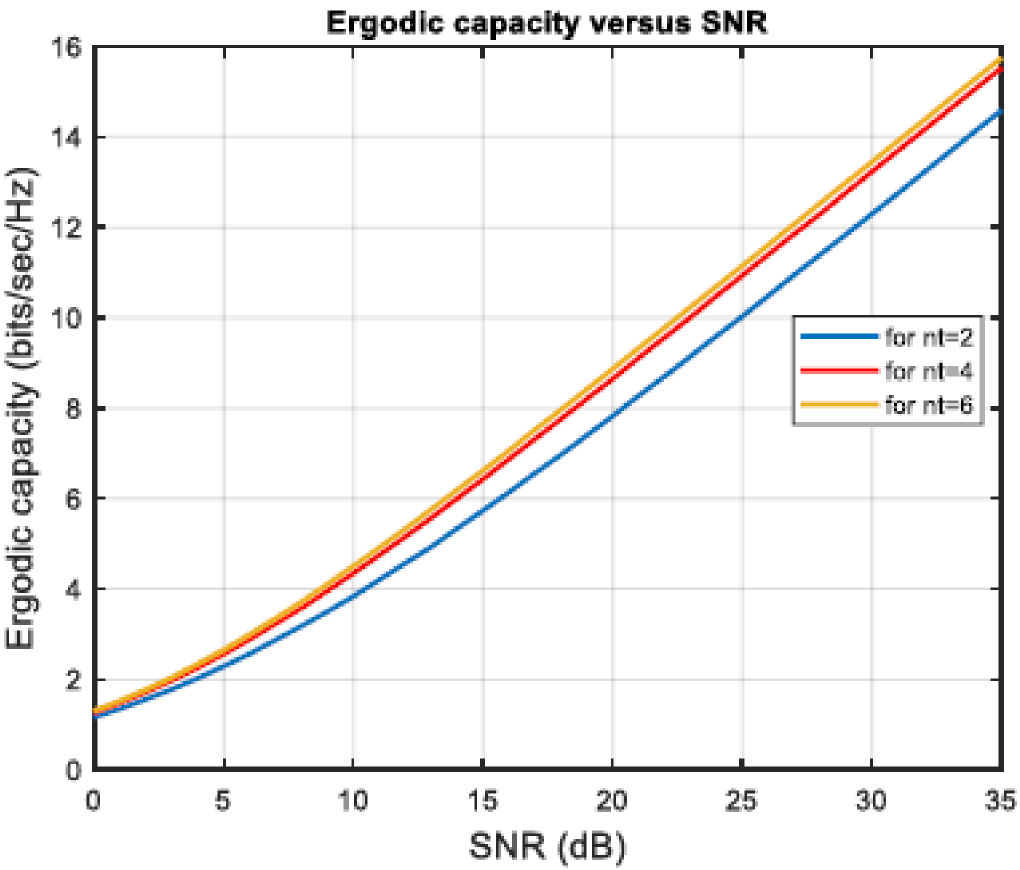


**Figure 4.1:** Variation of capacity with respect to SNR for $N_t = 2$, $n_t = 4$, $N_t = 6$ taking $n_r = 4$.

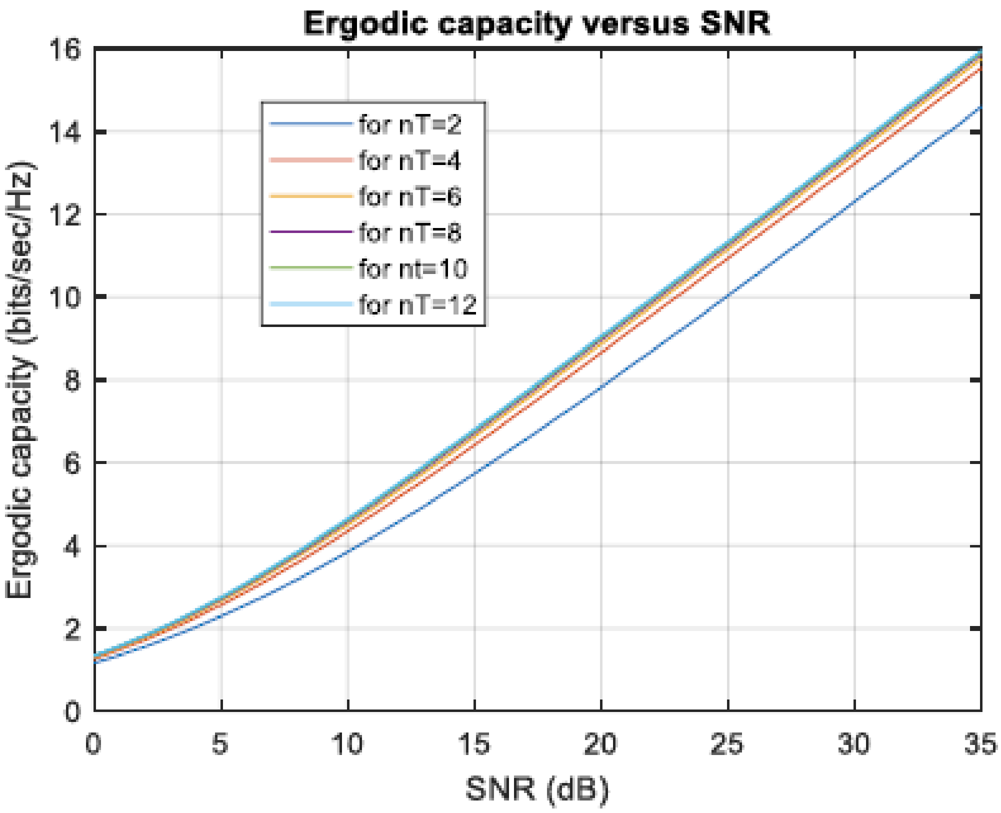


**Figure 4.2:** Effect on capacity by increasing the number of antennas at transmitter side taking number of receiving antenna constant at $n_R$=2.

## 6. Effects of receiving antennas

- **Graphical representation**

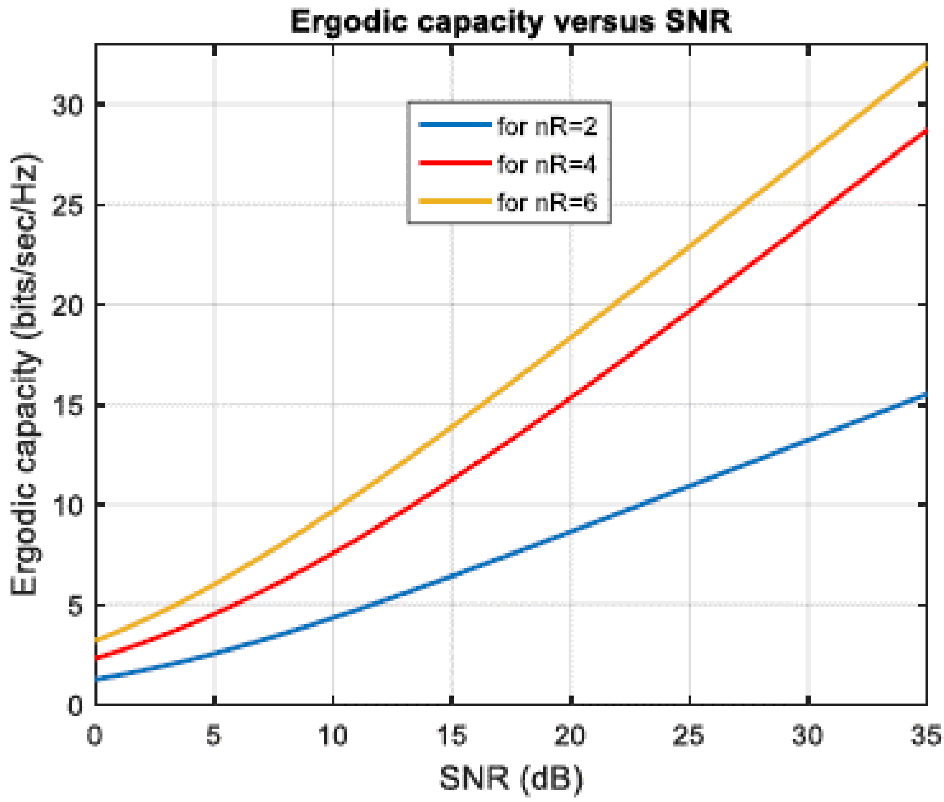


**Figure 4.3:** Variation of capacity with respect to SNR for $n_R$=2, $n_R$=4, $n_R$=6 taking $n_T$=4.

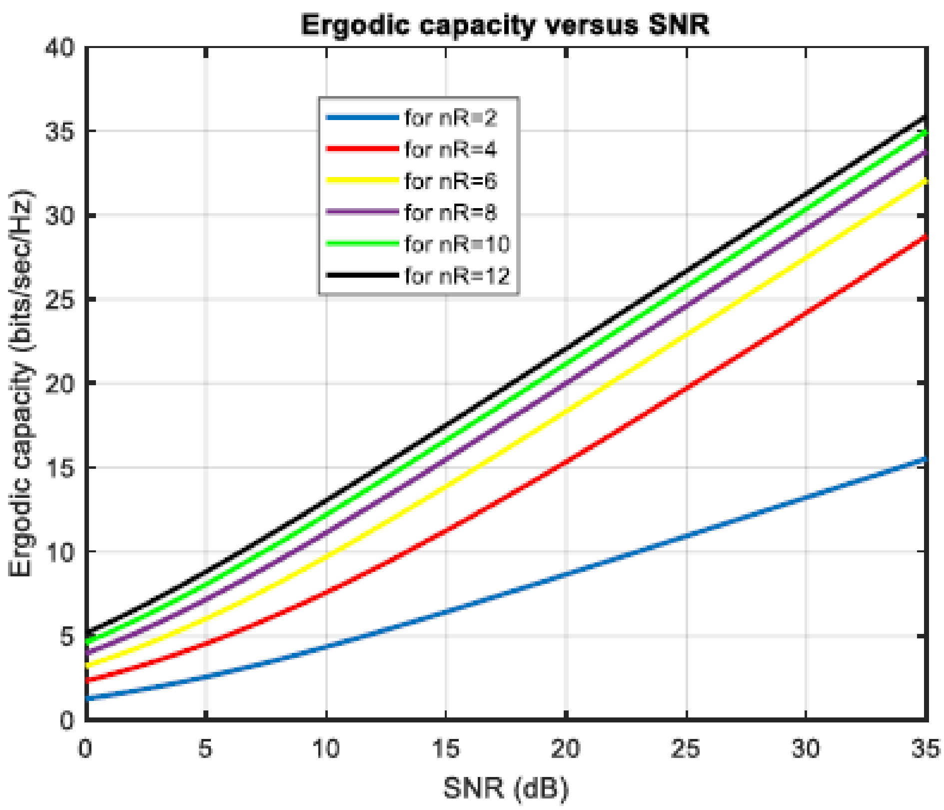


**Figure 4.4:** Effect on capacity by increasing the number of antennas at the receiver side.

## 7. Combined effect

- **Graphical representation**

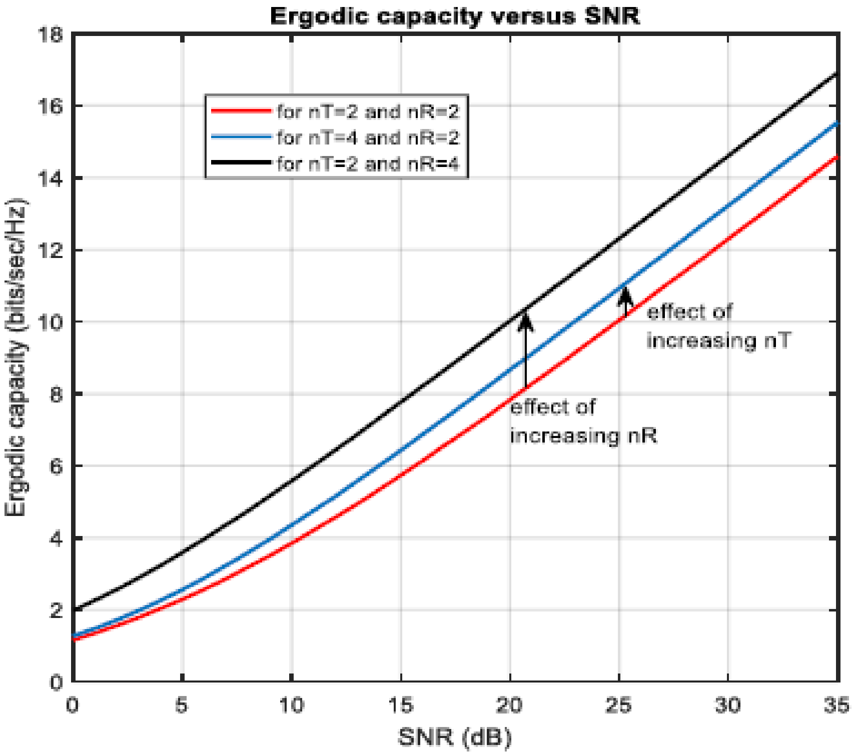


**Figure 4.5:** Variation of capacity by separately the receiving antenna and transmitting antenna.

## 8. Discussion and Conclusion

**a) Discussion:** All of the figures depict the variation in capacity caused by adjusting the number of antennas on both the receiver side and the receiver side. Some graphs are drawn with the number of receiving antennas fixed and the number of broadcasting antennas variable, or vice versa. Figure 4.5 depicts the effect of increasing the sending antenna while keeping the receiving antenna the same size on capacity. It demonstrates that receiving antennas have a bigger influence than emitting antennas.

**b) Conclusion:** From our investigation, we can draw the following conclusions:

- We have generated a graph of capacity versus SNR, which is dependent on noise. Power. Because there is no noise power at the transmitter end, SNR is quite high. The number of antennas at the transmitter end can be changed to alter the noise effect but not the capacity. As a result, the number of the transmitting antenna has a minor impact on capacity.

- As we all know, increasing the number of receiving antennas creates multiple pathways for the signal to reach the receiver, and there is a variation in encountering different SNR values. However, the receiver produces an output with the highest SNR or the lowest error. As a result, capacity grows. As a result, the number of receiving antennas has a considerable impact on any channel's capacity.

Finally, comprehending the statistical features of the MIMO channel as revealed by the ergodic capacity analyses is critical for building robust and efficient digital communication systems.

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# Rajshahi University of Engineering and Technology

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## Department of Electrical & Electronic Engineering

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| **Course no.** | EEE 4184 |
| **Course title:** | Digital Communication Sessional |
| **Experiment no.** | 5 |
| **Date of experiment:** | July 29, 2023 |
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# Experiment No. 05

**1. Problem Statement:** A multiple-input multiple-output (MIMO) channel is equipped with $n_T$ transmitting antennas, $n_R$ receiving antennas, and $n_I$ interfering antennas

**(a) Derive the expressions of** the channel capacity

**(b) Write the programs of** channel capacity for MIMO interference channel using MATLAB

**(c) Explain the numerical results of** the channel capacity for MIMO interference channel

**2. Derivation of the channel capacity for MIMO channel**

Let,

Received signal: $\boldsymbol{y = Hx + Gs + z = Hx + w}$

Where,

$\boldsymbol{y} \in C^{n_R \times 1}$ = Received Signal Vector
$\boldsymbol{H} \in C^{n_R \times n_T}$ = Channel Coefficient between the Transmitter and Receiver
$\boldsymbol{G} \in C^{n_R \times n_i}$ = Channel Coefficient between the Interferer and Receiver
$\boldsymbol{x} \in C^{n_R \times 1}$ = Transmitted Signal Vector from the Transmitter
$\boldsymbol{s} \in C^{n_T \times 1}$ =Transmitted Signal Vector from the Interferer
$\boldsymbol{z} \in C^{n_i \times 1}$ = Noise Vector

w = Interference plus Noise Vector

**(i) Covariance of Received Signal:**

$$\begin{aligned}
\boldsymbol{R}_y = E[yy^\dagger] &= E[(\boldsymbol{Hx}+\boldsymbol{z})\,(Hx+z)^\dagger] \\
&= E[(\boldsymbol{Hx}+\boldsymbol{z})\{\,(Hx)^\dagger + (z)^\dagger\}] \\
&= E[(\boldsymbol{Hx}+z)\{\,(x^\dagger H^\dagger + z^\dagger\}] \\
&= E[(Hxx^\dagger H^\dagger + Hxz^\dagger + zx^\dagger H^\dagger + zz^\dagger)] \\
&= E[Hxx^\dagger H^\dagger] + E[Hxz^\dagger] + E[zx^\dagger H^\dagger] + E[zz^\dagger] \\
&= E[Hxx^\dagger H^\dagger] + E[zz^\dagger] \\
&= HE[xx^\dagger H^\dagger] + E[zz^\dagger] \\
&= H\,\{\frac{P_T}{n_T} I_{nT}\}H^\dagger + N_0 I_{nR} + G\,\{\frac{P_l}{n_l} I_{nl}\}G^\dagger \\
&= N_0[\,I_{nR} + MQM^\dagger\,]
\end{aligned}$$

Where, $\boldsymbol{M = [\,HG\,] \in C^{n_R(n_T+n_l)}}$ *and* $\boldsymbol{Q = \frac{P_T}{N_0 n_T} I_{nR} \oplus \frac{P_l}{N_0 n_l} I_{nl}}$

**(ii) Covariance of Interference plus Noise signals**

$$\begin{aligned}
\boldsymbol{R}_w = E[ww^\dagger] &= E[(Gs+z)(Gs+z)^\dagger] \\
&= E[Gss^\dagger G^\dagger] + E[zz^\dagger] \\
&= GE[ss^\dagger]G^\dagger + E[zz^\dagger] \\
&= N_0 I_{nR} + G\,\{\frac{P_l}{n_l} I_{nl}\}G^\dagger
\end{aligned}$$

**(iii) Entropy of Received Signal**

$$\boldsymbol{H}(\boldsymbol{y}) = log_2 det(\pi e \boldsymbol{R}_y) = log_2 det\{\pi e N_0[\,I_{nR} + MQM^\dagger\,]\}$$

**(iv) Entropy of interference plus noise signal**

$$\boldsymbol{H}(\mathbf{w}) = log_2 det(\pi e \boldsymbol{R}_w) = log_2 det\{\pi e N_0 [\, I_{nR} + G\, \{\frac{P_I}{n_I} I_{nI}\} G^{\dagger}\, ]\}$$

**v) Mutual information between x and y**

$$I(\boldsymbol{x};\boldsymbol{y}) = \boldsymbol{H}(\boldsymbol{y}) - \boldsymbol{H}(\mathrm{w}) = log_2 det(\pi e \boldsymbol{Ry}) - log_2 det(\pi e \boldsymbol{R}\mathrm{w})$$

$= log_2 \frac{det\,(I_{nR} + MQM^{\dagger})}{det\,(I_{nR} + G\,\{\frac{P_I}{n_I} I_{nI}\} G^{\dagger})}$, which is the Mutual Information.

**(vii) Capacity at the Receiver:**

$$C = \max_Q I(x;y) = \max_Q log_2 \frac{det\,(I_{nR} + MQM^{\dagger})}{det\,(I_{nR} + G\,\{\frac{P_I}{n_I} I_{nI}\} G^{\dagger})},$$

$= log_2 [\frac{\det(I_{nR} + MQM^{\dagger})}{\det\left(I_{nR} + G\left\{\frac{P_I}{n_I} I_{nI}\right\} G^{\dagger}\right)}]$ ,which is the channel capacity.

**3. Program for the channel capacity for MIMO interference channel**

- **MATLAB Code for plotting the capacity**

```
clc
clear all;
%%%%%%%% Initialization %%%%%%%%
N=30000;       % Number of Iterations for H
No=1;          % Noise Variance
%%%%%%%% Number of antennas %%%%%%%%
nt=2;          % Number of Transmit antennas
ni=2;       % Number of interference antenna
nt=Nt+ni;
nr=6;          % Number of Receive antennas
%%%%%%%% Correlation of antennas %%%%%%%%
rhot=0;        % Correlation coefficient (Transmitter)
rhor=0;        % Correlation coefficient (Receiver)
ExponencorrMrtx=ones(nt);
%%%%%%%% Creation of SNR from 1 to L %%%%%%%%
for i=1:nt
   for j=1:nt
    if i==j
 ExponencorrMrtx(i,j)=1;
    end
    if i>j
   ExponencorrMrtx(i,j)=rhot^(i-j);
  else
   ExponencorrMrtx(i,j)=rhot^(j-i);
  end
 end
end
Rt=ExponencorrMrtx;
% generate an nxn exponential correlation matrix for Rr
ExponencorrMrtx=ones(nt);
for i=1:nr
 for j=1:nr
   if i==j
   ExponencorrMrtx(i,j)=1;
   end
   if i>j
   ExponencorrMrtx(i,j)=rhor^(i-j);
```

```
        else
        ExponencorrMrtx(i,j)=rhor^(j-i);
        end
      end
    end
    Rr=ExponencorrMrtx;
    SNRdB=[0:1:35];
    l=length (SNRdB);
    SNR=zeros (1,l);
    for i=1:l
        SNR (i)=10^(0.1*SNRdB (i));
        P(i)=SNR(i)*No;
    end
    Erg_Cap_unknown=zeros (1,l);
    for i=1:l
        C_unknown=zeros (1,N);
    for j=1:N
        H=CGM (nr,nt,Rr,Rt);
        C_unknown (j)=log (det (eye (nr)+(SNR (i)/nt)*(H*H')));
    end
     Erg_Cap_unknown (i)=mean (C_unknown);
    fprintf ('% e\t% e \n',SNRdB (i),Erg_Cap_unknown (i));
    end
    Erg_Cap_unknown=zeros (1,l);
    for i=1:l
    C_unknown=zeros (1,N);
    for j=1:N
        G=CGM (nr,nt,Rr,Rt);
        C_unknown (j)=log (det (eye (nr)+(SNR (i)/nt)*(G*G')));
    end
     Erg_Cap_unknown (i)=mean (C_unknown);
    fprintf ('% e\t% e \n',SNRdB (i),Erg_Cap_unknown (i));
    end
    plot (SNRdB,Erg_Cap_unknown)
    xlabel ('SNR (dB)');
    ylabel ('Ergodic capacity (bits/sec/Hz)');
    title ('Ergodic capacity versus SNR');
    grid on
    hold on
```

## 4. Numerical results of the MIMO channel

### (a) Numerical data for measuring capacity at different SNR (dB)

| SNR (dB) | Capacity(bits/sec/Hz) | SNR (dB) | Capacity(bits/sec/Hz) |
|---|---|---|---|
| 0 | 3.21022 | 16 | 14.7718 |
| 1 | 3.68846 | 17 | 15.6565 |
| 2 | 4.21794 | 18 | 16.5476 |
| 3 | 4.77797 | 19 | 17.4404 |
| 4 | 5.38851 | 20 | 18.3512 |
| 5 | 6.02162 | 21 | 19.2530 |
| 6 | 6.69693 | 22 | 20.1611 |
| 7 | 7.40800 | 23 | 21.0785 |
| 8 | 8.14525 | 24 | 21.9772 |
| 9 | 8.90590 | 25 | 22.8973 |
| 10 | 9.69533 | 26 | 23.8161 |
| 11 | 10.4940 | 27 | 24.7259 |
| 12 | 11.3237 | 28 | 25.6441 |
| 13 | 12.1671 | 29 | 26.5609 |

| | | | |
|---|---|---|---|
| 14 | 13.0249 | 30 | 27.4844 |
| 15 | 13.8944 | | |

- **Graphical representation**

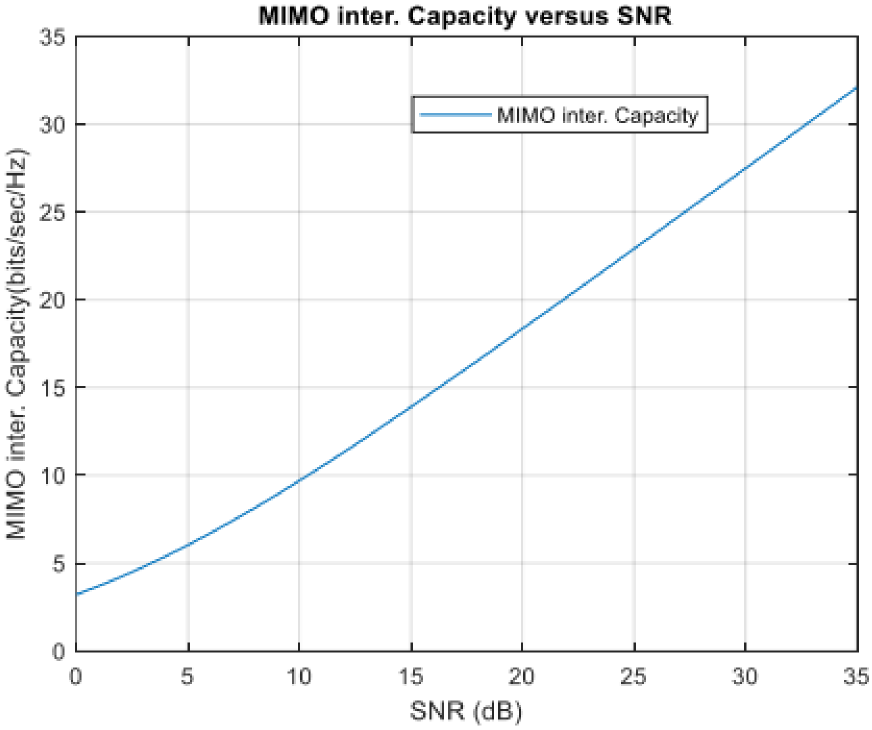


**Figure 5.1:** Capacity of MIMO interference channel with respect to Signal to noise ratio.

**8. Discussion and Conclusion**

**a) Discussion:** The picture and data table depict the capacity variation of a MIMO interference channel. It It has been shown that as SNR increases, capacity increases. As we all know, interference occurs when the primary signal from one transmitter collides with another signal from another transmitter of the same frequency. Interference distorts the transmission, and the power of the signal can be raised or diminished. To solve this problem, many multiplexing techniques like as CDM, TDM, SDM, and FDM are used.

**b) Conclusion:** From our investigation, we can draw the following conclusions:

- **Capacity of MIMO interference channel increase with SNR.** MIMO communication systems increase channel capacity with signal-to-noise ratio (SNR), enabling higher reliability and spatial diversity. This improves robustness, modulation schemes, and resistance to noise and interference, resulting in a larger MIMO system capacity.
- **There is an effect of interfering antenna on the capacity of the channel.** Interfering antennas in communication environments impact channel capacity by introducing unwanted signals and noise, reducing signal quality and data rate. Factors like interference strength and proximity contribute to channel capacity reduction.
- **For better capacity interference should be avoided.** Minimizing interference is crucial for improving communication channel capacity, as it introduces errors and reduces SINR. Planning frequency allocations, antenna placements, and transmission strategies helps achieve higher SINR and improved channel capacity. Techniques like smart antenna design and interference cancellation are employed.

*Heaven's light is our guide.*

# Rajshahi University of Engineering and Technology

RUET

## Department of Electrical & Electronic Engineering

**Course no.** EEE 4184

**Course title:** Digital Communication Sessional

**Experiment no.** 6

**Date of experiment:** July 29, 2023

**Date of submission:** August 8, 2023



| Submitted to: | Submitted by: |
|---|---|
| **Dr. Md. Zahurul Islam Sarkar**<br>**Professor**<br>Dept. of Electrical & Electronic Engineering,<br>Rajshahi University of Engineering and Technology. | **Ashraf Al- Khalique**<br>**Roll:** 1801171<br>**Session:** 2018-2019<br>Dept. of Electrical & Electronic Engineering,<br>Rajshahi University of Engineering and Technology. |

# Experiment No. 06

**1. Problem Statement:** A multiple-input multiple-output (MIMO) channel is equipped with $n_T$ transmitting antennas, $n_R$ receiving antennas, and $n_I$ interfering antennas

**(a) Investigate the effect of** transmitting antennas on the channel capacity of MIMO interference channel
**(b) Investigate the effect of** receiving antennas on the channel capacity of MIMO interference channel
**(c) Investigate the effect of** interfering antennas on the channel capacity of MIMO interference channel

## 2. Derivation of the channel capacity for MIMO channel

Let,

Received signal: $$\boldsymbol{y = Hx + Gs + z = Hx + w}$$

Where,

$\boldsymbol{y} \in C^{n_R \times 1}$ = Received Signal Vector
$\boldsymbol{H} \in C^{n_R \times n_T}$ = Channel Coefficient between the Transmitter and Receiver
$\boldsymbol{G} \in C^{n_R \times n_i}$ = Channel Coefficient between the Interferer and Receiver
$\boldsymbol{x} \in C^{n_R \times 1}$ = Transmitted Signal Vector from the Transmitter
$\boldsymbol{s} \in C^{n_T \times 1}$ =Transmitted Signal Vector from the Interferer
$\boldsymbol{z} \in C^{n_i \times 1}$ = Noise Vector

w = Interference plus Noise Vector

### (i) Covariance of Received Signal:

$$\begin{aligned}
\boldsymbol{R}_y = E[yy^\dagger] &= E[(\boldsymbol{Hx}+\boldsymbol{z})\,(Hx+z)^\dagger] \\
&= E[(\boldsymbol{Hx}+\boldsymbol{z})\{\,(Hx)^\dagger + (z)^\dagger\}] \\
&= E[(\boldsymbol{Hx}+z)\{\,(x^\dagger H^\dagger + z^\dagger\}] \\
&= E[(Hxx^\dagger H^\dagger + Hxz^\dagger + zx^\dagger H^\dagger + zz^\dagger)] \\
&= E[Hxx^\dagger H^\dagger] + E[Hxz^\dagger] + E[zx^\dagger H^\dagger] + E[zz^\dagger] \\
&= E[Hxx^\dagger H^\dagger] + E[zz^\dagger] \\
&= HE[xx^\dagger H^\dagger] + E[zz^\dagger] \\
&= H\{\frac{P_T}{n_T} I_{nT}\}H^\dagger + N_0 I_{nR} + G\{\frac{P_I}{n_I} I_{nl}\}G^\dagger \\
&= N_0[\,I_{nR} + MQM^\dagger\,]
\end{aligned}$$

Where, $\boldsymbol{M = [\,HG\,] \in C^{n_R(n_T+n_I)}}$ and $\boldsymbol{Q = \frac{P_T}{N_0 n_T} I_{nR} \oplus \frac{P_I}{N_0 n_I} I_{nl}}$

### (ii) Covariance of Interference plus Noise signals

$$\begin{aligned}
\boldsymbol{R}_w = E[ww^\dagger] &= E[(Gs+z)(Gs+z)^\dagger] \\
&= E[Gss^\dagger G^\dagger] + E[zz^\dagger] \\
&= GE[ss^\dagger]G^\dagger + E[zz^\dagger] \\
&= N_0 I_{nR} + G\{\frac{P_I}{n_I} I_{nl}\}G^\dagger
\end{aligned}$$

### (iii) Entropy of Received Signal

$$\boldsymbol{H}(\boldsymbol{y}) = log_2 det(\pi e \boldsymbol{R}_y) = log_2 det\{\pi e N_0[\,I_{nR} + MQM^\dagger\,]\}$$

**(iv) Entropy of interference plus noise signal**

$$\boldsymbol{H}(\mathbf{w})= log_2 det(\pi e \boldsymbol{R}_w) = log_2 det\{\pi e N_0[\, I_{nR} + G\, \{\frac{P_I}{n_I} I_{nI}\} G^{\dagger}\,]\}$$

**v) Mutual information between x and y**

$$I(\boldsymbol{x};\boldsymbol{y})=\boldsymbol{H}(\boldsymbol{y})-\boldsymbol{H}(\mathrm{w}) = log_2 det(\pi e \boldsymbol{Ry}) - log_2 det(\pi e \boldsymbol{R}\mathrm{w})$$

$$= log_2 \frac{det\,(\,I_{nR}+MQM^{\dagger}\,)}{det\,(I_{nR}+G\,\{\frac{P_I}{n_I}I_{nI}\}G^{\dagger}\,)}$$, which is the Mutual Information.

**(vii) Capacity at the Receiver:**

$$C = \max_Q I(x;y) = \max_Q log_2 \frac{det\,(\,I_{nR}+MQM^{\dagger}\,)}{det\,(I_{nR}+G\,\{\frac{P_I}{n_I}I_{nI}\}G^{\dagger}\,)},$$

$$= log_2[\frac{\det(\,I_{nR}+MQM^{\dagger}\,)}{\det\left(I_{nR}+G\left\{\frac{P_I}{n_I}I_{nI}\right\}G^{\dagger}\right)}]$$ ,which is the channel capacity.

**3. Program for the channel capacity for MIMO interference channel**

- **MATLAB Code for plotting the capacity**

```
% Ergodic Capacity of Nr x Nt fading MIMO Channel with
% no channel knowledge at the transmitter.
clc
clear all;
%%%%%%%% Initialization %%%%%%%%
N=30000;        % Number of Iterations for H
No=1;           % Noise Variance
%%%%%%%% Number of antennas %%%%%%%%
nt=2;           % Number of Transmit antennas
ni=2;           % Number of interference antenna
nt=Nt+ni;
nr=6;           % Number of Receive antennas
%%%%%%%% Correlation of antennas %%%%%%%%
rhot=0;         % Correlation coefficient (Transmitter)
rhor=0;         % Correlation coefficient (Receiver)
ExponencorrMrtx=ones(nt);
%%%%%%%% Creation of SNR from 1 to L %%%%%%%%
for i=1:nt
   for j=1:nt
    if i==j
       ExponencorrMrtx(i,j)=1;
    end
    if i>j
      ExponencorrMrtx(i,j)=rhot^(i-j);
  else
      ExponencorrMrtx(i,j)=rhot^(j-i);
  end
 end
end
Rt=ExponencorrMrtx;
% generate an nxn exponential correlation matrix for Rr
ExponencorrMrtx=ones(nt);
for i=1:nr
 for j=1:nr
   if i==j
      ExponencorrMrtx(i,j)=1;
   end
```

```
    if i>j
       ExponencorrMrtx(i,j)=rhor^(i-j);
    else
       ExponencorrMrtx(i,j)=rhor^(j-i);
    end
  end
end
Rr=ExponencorrMrtx;
SNRdB=[0:1:35];
l=length (SNRdB);
SNR=zeros (1,l);
for i=1:l
    SNR (i)=10^(0.1*SNRdB (i));
    P(i)=SNR(i)*No;
end
Erg_Cap_unknown=zeros (1,l);
for i=1:l
    C_unknown=zeros (1,N);

for j=1:N
    H=CGM (nr,nt,Rr,Rt);
    C_unknown (j)=log (det (eye (nr)+(SNR (i)/nt)*(H*H')));
end
 Erg_Cap_unknown (i)=mean (C_unknown);

fprintf ('% e\t% e \n',SNRdB (i),Erg_Cap_unknown (i));
end
Erg_Cap_unknown=zeros (1,l);
for i=1:l
C_unknown=zeros (1,N);

for j=1:N
    G=CGM (nr,nt,Rr,Rt);
    C_unknown (j)=log (det (eye (nr)+(SNR (i)/nt)*(G*G')));
end
 Erg_Cap_unknown (i)=mean (C_unknown);

fprintf ('% e\t% e \n',SNRdB (i),Erg_Cap_unknown (i));
end
plot (SNRdB,Erg_Cap_unknown)
xlabel ('SNR (dB)');
ylabel ('Ergodic capacity (bits/sec/Hz)');
title ('Ergodic capacity versus SNR');
grid on
hold on
```

## 4. Numerical results of the MIMO channel

### (a) Numerical data for measuring capacity at different SNR (dB)

| SNR (dB) | Capacity(bits/sec/Hz) | SNR (dB) | Capacity(bits/sec/Hz) |
|---|---|---|---|
| 0 | 3.21022 | 16 | 14.7718 |
| 1 | 3.68846 | 17 | 15.6565 |
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| 3 | 4.77797 | 19 | 17.4404 |
| 4 | 5.38851 | 20 | 18.3512 |
| 5 | 6.02162 | 21 | 19.2530 |
| 6 | 6.69693 | 22 | 20.1611 |
| 7 | 7.40800 | 23 | 21.0785 |
| 8 | 8.14525 | 24 | 21.9772 |

| | | | |
|---|---|---|---|
| 9 | 8.90590 | 25 | 22.8973 |
| 10 | 9.69533 | 26 | 23.8161 |
| 11 | 10.4940 | 27 | 24.7259 |
| 12 | 11.3237 | 28 | 25.6441 |
| 13 | 12.1671 | 29 | 26.5609 |
| 14 | 13.0249 | 30 | 27.4844 |
| 15 | 13.8944 | | |

## 5. Effects of transmitting antennas

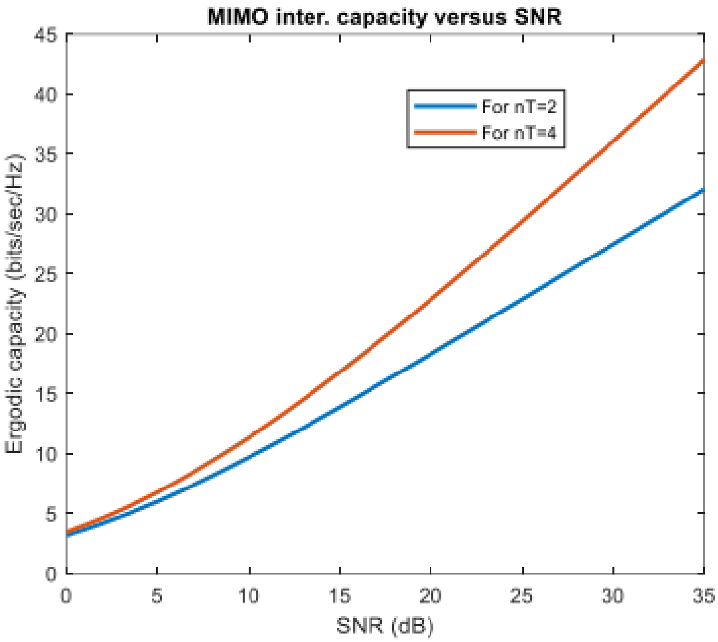


**Figure 6.1:** Increase in MIMO interference Capacity Vs SNR curve with transmitting antennas

## 6. Effects of receiving antennas

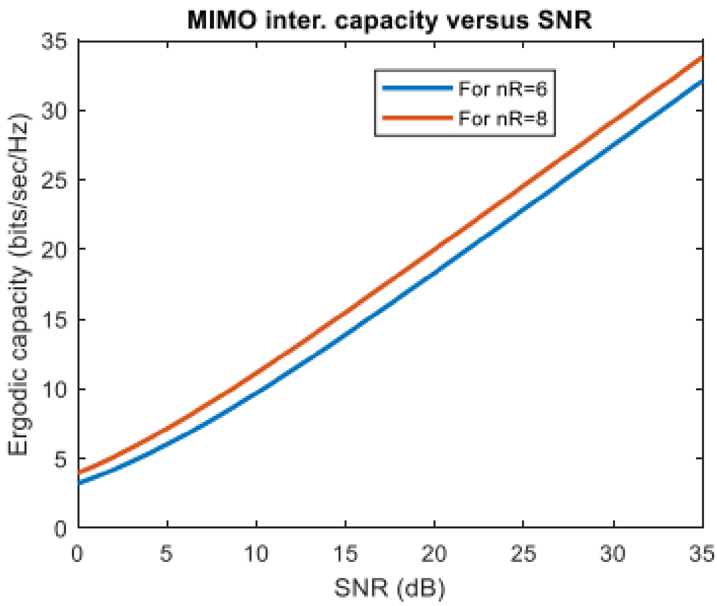


**Figure 6.2:** Increase in MIMO interference Capacity Vs SNR curve with receiving antennas

## 7. Effects of interfering antennas

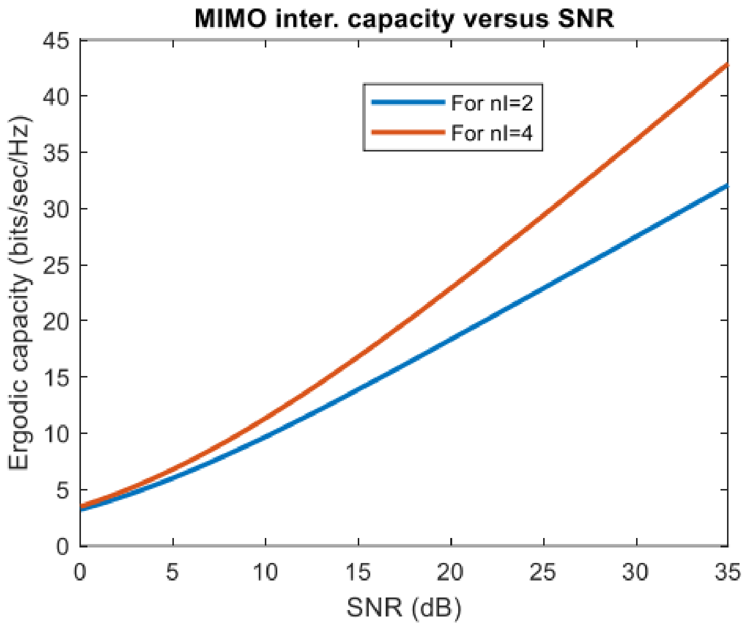


**Figure 6.3:** Increase in MIMO interference Capacity Vs SNR curve with interfering antennas

## 8. Discussion and Conclusion

**a) Discussion:** All of the figures depict the fluctuation in capacity in relation to SNR for various transmitting, receiving, and interfering antenna values. The channel capacity of a MIMO interference channel is determined by all factors, and we can improve the capacity by adding or decreasing the number of antennas. Interference is an adversary of every signal and should be avoided for optimal signal delivery.

**b) Conclusion:** From our investigation, we can draw the following conclusions:

- **The effect of interfering antenna on the capacity is more than another antenna.** An interfering antenna has a bigger impact on capacity than other antennas. When numerous antennas are functioning in a shared communication environment, the presence of interfering antennas can have a major impact on the channel's total capacity. Interfering antennas introduce new signal components into the mix, resulting in increased noise and probable signal deterioration.

- **Number of interfering antennas should be reduced to get high capacity of any channel.** Interference degrades signal quality and increases the risk of mistakes, lowering the possible data rate. As a result, the signal-to-interference-plus-noise ratio (SINR) improves, allowing for higher data rates and increased overall channel capacity.

- **Increased receiving antenna number gives us better SNR as well as the capacity.** Increasing the number of receiving antennas (using multiple-input, multiple-output, or MIMO techniques) can significantly improve both SNR and capacity. As the number of receiving antennas grows, the system can take advantage of spatial diversity and multipath propagation, resulting in higher SNR.

Finally, comprehending the statistical features of the MIMO channel as revealed by the ergodic capacity analyses is critical for building robust and efficient digital communication systems.