Experiment No. 05

1. Problem Statement: A multiple-input multiple-output (MIMO) channel is equipped with n_T transmitting antennas, n_R receiving antennas, and n_I interfering antennas

(a) Derive the expressions of the channel capacity

(b) Write the programs of channel capacity for MIMO interference channel using MATLAB

(c) Explain the numerical results of the channel capacity for MIMO interference channel

2. Derivation of the channel capacity for MIMO channel

Let,

Received signal: $\boldsymbol{y = Hx + Gs + z = Hx + w}$

Where,

$\boldsymbol{y} \in C^{n_R \times 1}$ = Received Signal Vector
$\boldsymbol{H} \in C^{n_R \times n_T}$ = Channel Coefficient between the Transmitter and Receiver
$\boldsymbol{G} \in C^{n_R \times n_i}$ = Channel Coefficient between the Interferer and Receiver
$\boldsymbol{x} \in C^{n_R \times 1}$ = Transmitted Signal Vector from the Transmitter
$\boldsymbol{s} \in C^{n_T \times 1}$ =Transmitted Signal Vector from the Interferer
$\boldsymbol{z} \in C^{n_i \times 1}$ = Noise Vector

w = Interference plus Noise Vector

(i) Covariance of Received Signal:

$$\begin{aligned}
\boldsymbol{R}_y = E[yy^\dagger] &= E[(\boldsymbol{Hx}+\boldsymbol{z})\,(Hx + z)^\dagger] \\
&= E[(\boldsymbol{Hx}+\boldsymbol{z})\{\,(Hx)^\dagger + (z)^\dagger\}] \\
&= E[(\boldsymbol{Hx}+z)\{\,(x^\dagger H^\dagger + z^\dagger\}] \\
&= E[(Hxx^\dagger H^\dagger + Hxz^\dagger + zx^\dagger H^\dagger + zz^\dagger)] \\
&= E[Hxx^\dagger H^\dagger] + E[Hxz^\dagger] + E[zx^\dagger H^\dagger] + E[zz^\dagger] \\
&= E[Hxx^\dagger H^\dagger] + E[zz^\dagger] \\
&= HE[xx^\dagger H^\dagger] + E[zz^\dagger] \\
&= H\,\{\tfrac{P_T}{n_T} I_{nT}\}H^\dagger + N_0 I_{nR} + G\,\{\tfrac{P_l}{n_l} I_{nl}\}G^\dagger \\
&= N_0[\,I_{nR} + MQM^\dagger\,]
\end{aligned}$$

Where, $\boldsymbol{M = [\,HG\,] \in C^{n_R(n_T+n_l)}}$ *and* $\boldsymbol{Q = \frac{P_T}{N_0 n_T} I_{nR} \oplus \frac{P_l}{N_0 n_l} I_{nl}}$

(ii) Covariance of Interference plus Noise signals

$$\begin{aligned}
\boldsymbol{R}_w = E[ww^\dagger] &= E[(Gs + z)(Gs + z)^\dagger] \\
&= E[Gss^\dagger G^\dagger] + E[zz^\dagger] \\
&= GE[ss^\dagger]G^\dagger + E[zz^\dagger] \\
&= N_0 I_{nR} + G\,\{\tfrac{P_l}{n_l} I_{nl}\}G^\dagger
\end{aligned}$$

(iii) Entropy of Received Signal

$$\boldsymbol{H}(\boldsymbol{y}) = log_2 det(\pi e \boldsymbol{R}_y) = log_2 det\{\pi e N_0[\,I_{nR} + MQM^\dagger\,]\}$$

(iv) Entropy of interference plus noise signal

$$\boldsymbol{H}(\mathbf{w}) = log_2 det(\pi e \boldsymbol{R}_w) = log_2 det\{\pi e N_0 [\, I_{nR} + G\, \{\frac{P_l}{n_l} I_{nl}\} G^{\dagger}\,]\}$$

v) Mutual information between x and y

$$I(\boldsymbol{x};\boldsymbol{y}) = \boldsymbol{H}(\boldsymbol{y}) - \boldsymbol{H}(\mathrm{w}) = log_2 det(\pi e \boldsymbol{Ry}) - log_2 det(\pi e \boldsymbol{R}\mathrm{w})$$

$$= log_2 \frac{det\,(I_{nR} + MQM^{\dagger})}{det\,(I_{nR} + G\,\{\frac{P_l}{n_l} I_{nl}\} G^{\dagger})}$$, which is the Mutual Information.

(vii) Capacity at the Receiver:

$$C = \max_{Q} I(x;y) = \max_{Q} log_2 \frac{det\,(I_{nR} + MQM^{\dagger})}{det\,(I_{nR} + G\,\{\frac{P_l}{n_l} I_{nl}\} G^{\dagger})},$$

$$= log_2 [\frac{\det(I_{nR} + MQM^{\dagger})}{\det\left(I_{nR} + G\left\{\frac{P_l}{n_l} I_{nl}\right\} G^{\dagger}\right)}]$$,which is the channel capacity.

3. Program for the channel capacity for MIMO interference channel

- **MATLAB Code for plotting the capacity**

```
clc
clear all;
%%%%%%%% Initialization %%%%%%%%
N=30000;       % Number of Iterations for H
No=1;          % Noise Variance
%%%%%%%% Number of antennas %%%%%%%%
nt=2;          % Number of Transmit antennas
ni=2;       % Number of interference antenna
nt=Nt+ni;
nr=6;          % Number of Receive antennas
%%%%%%%% Correlation of antennas %%%%%%%%
rhot=0;        % Correlation coefficient (Transmitter)
rhor=0;        % Correlation coefficient (Receiver)
ExponencorrMrtx=ones(nt);
%%%%%%%% Creation of SNR from 1 to L %%%%%%%%
for i=1:nt
   for j=1:nt
    if i==j
 ExponencorrMrtx(i,j)=1;
    end
    if i>j
   ExponencorrMrtx(i,j)=rhot^(i-j);
  else
   ExponencorrMrtx(i,j)=rhot^(j-i);
  end
 end
end
Rt=ExponencorrMrtx;
% generate an nxn exponential correlation matrix for Rr
ExponencorrMrtx=ones(nt);
for i=1:nr
 for j=1:nr
   if i==j
   ExponencorrMrtx(i,j)=1;
   end
   if i>j
   ExponencorrMrtx(i,j)=rhor^(i-j);
```

```
      else
      ExponencorrMrtx(i,j)=rhor^(j-i);
      end
     end
   end
   Rr=ExponencorrMrtx;
   SNRdB=[0:1:35];
   l=length (SNRdB);
   SNR=zeros (1,l);
   for i=1:l
       SNR (i)=10^(0.1*SNRdB (i));
       P(i)=SNR(i)*No;
   end
   Erg_Cap_unknown=zeros (1,l);
   for i=1:l
       C_unknown=zeros (1,N);
   for j=1:N
       H=CGM (nr,nt,Rr,Rt);
       C_unknown (j)=log (det (eye (nr)+(SNR (i)/nt)*(H*H')));
   end
    Erg_Cap_unknown (i)=mean (C_unknown);
   fprintf ('% e\t% e \n',SNRdB (i),Erg_Cap_unknown (i));
   end
   Erg_Cap_unknown=zeros (1,l);
   for i=1:l
   C_unknown=zeros (1,N);
   for j=1:N
       G=CGM (nr,nt,Rr,Rt);
       C_unknown (j)=log (det (eye (nr)+(SNR (i)/nt)*(G*G')));
   end
    Erg_Cap_unknown (i)=mean (C_unknown);
   fprintf ('% e\t% e \n',SNRdB (i),Erg_Cap_unknown (i));
   end
   plot (SNRdB,Erg_Cap_unknown)
   xlabel ('SNR (dB)');
   ylabel ('Ergodic capacity (bits/sec/Hz)');
   title ('Ergodic capacity versus SNR');
   grid on
   hold on
```

4. Numerical results of the MIMO channel

(a) Numerical data for measuring capacity at different SNR (dB)

SNR (dB)	Capacity(bits/sec/Hz)	SNR (dB)	Capacity(bits/sec/Hz)
0	3.21022	16	14.7718
1	3.68846	17	15.6565
2	4.21794	18	16.5476
3	4.77797	19	17.4404
4	5.38851	20	18.3512
5	6.02162	21	19.2530
6	6.69693	22	20.1611
7	7.40800	23	21.0785
8	8.14525	24	21.9772
9	8.90590	25	22.8973
10	9.69533	26	23.8161
11	10.4940	27	24.7259
12	11.3237	28	25.6441
13	12.1671	29	26.5609

14	13.0249	30	27.4844
15	13.8944		

- **Graphical representation**

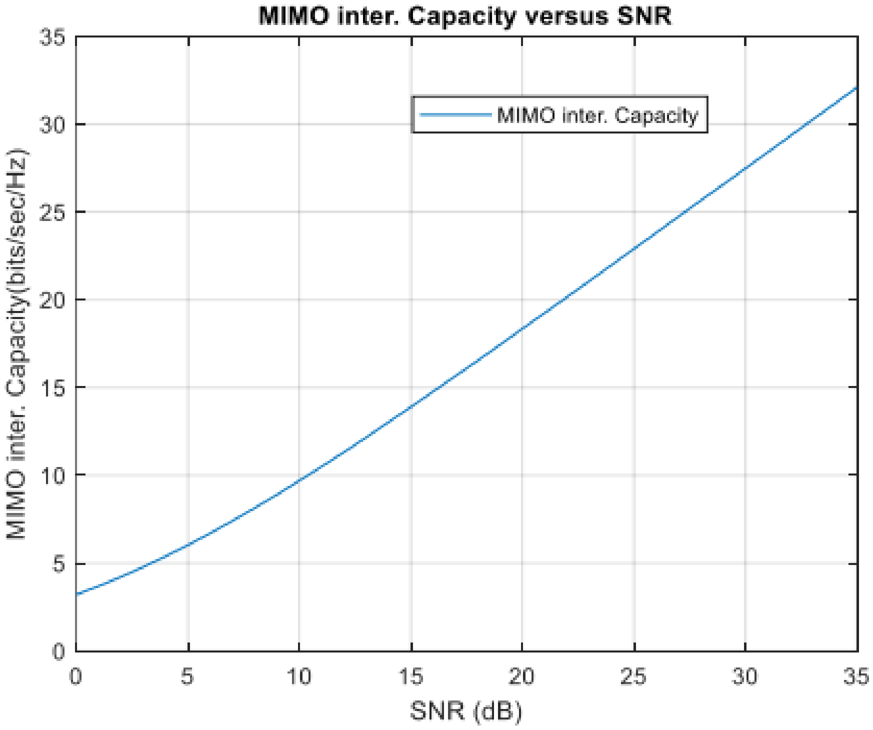

Figure 5.1: Capacity of MIMO interference channel with respect to Signal to noise ratio.

8. Discussion and Conclusion

a) Discussion: The picture and data table depict the capacity variation of a MIMO interference channel. It It has been shown that as SNR increases, capacity increases. As we all know, interference occurs when the primary signal from one transmitter collides with another signal from another transmitter of the same frequency. Interference distorts the transmission, and the power of the signal can be raised or diminished. To solve this problem, many multiplexing techniques like as CDM, TDM, SDM, and FDM are used.

b) Conclusion: From our investigation, we can draw the following conclusions:

- **Capacity of MIMO interference channel increase with SNR.** MIMO communication systems increase channel capacity with signal-to-noise ratio (SNR), enabling higher reliability and spatial diversity. This improves robustness, modulation schemes, and resistance to noise and interference, resulting in a larger MIMO system capacity.
- **There is an effect of interfering antenna on the capacity of the channel.** Interfering antennas in communication environments impact channel capacity by introducing unwanted signals and noise, reducing signal quality and data rate. Factors like interference strength and proximity contribute to channel capacity reduction.
- **For better capacity interference should be avoided.** Minimizing interference is crucial for improving communication channel capacity, as it introduces errors and reduces SINR. Planning frequency allocations, antenna placements, and transmission strategies helps achieve higher SINR and improved channel capacity. Techniques like smart antenna design and interference cancellation are employed.