Experiment No. 06

1. Problem Statement: A multiple-input multiple-output (MIMO) channel is equipped with n_T transmitting antennas, n_R receiving antennas, and n_I interfering antennas

(a) Investigate the effect of transmitting antennas on the channel capacity of MIMO interference channel
(b) Investigate the effect of receiving antennas on the channel capacity of MIMO interference channel
(c) Investigate the effect of interfering antennas on the channel capacity of MIMO interference channel

2. Derivation of the channel capacity for MIMO channel

Let,

Received signal: $\boldsymbol{y = Hx + Gs + z = Hx + w}$

Where,

$\boldsymbol{y} \in C^{n_R \times 1}$ = Received Signal Vector
$\boldsymbol{H} \in C^{n_R \times n_T}$ = Channel Coefficient between the Transmitter and Receiver
$\boldsymbol{G} \in C^{n_R \times n_i}$ = Channel Coefficient between the Interferer and Receiver
$\boldsymbol{x} \in C^{n_R \times 1}$ = Transmitted Signal Vector from the Transmitter
$\boldsymbol{s} \in C^{n_T \times 1}$ =Transmitted Signal Vector from the Interferer
$\boldsymbol{z} \in C^{n_i \times 1}$ = Noise Vector

w = Interference plus Noise Vector

(i) Covariance of Received Signal:

$$
\begin{aligned}
\boldsymbol{R}_y = E[yy^\dagger] &= E[(\boldsymbol{Hx}+\boldsymbol{z})\,(Hx+z)^\dagger] \\
&= E[(\boldsymbol{Hx}+\boldsymbol{z})\{\,(Hx)^\dagger + (z)^\dagger\}] \\
&= E[(\boldsymbol{Hx}+z)\{\,(x^\dagger H^\dagger + z^\dagger\}] \\
&= E[(Hxx^\dagger H^\dagger + Hxz^\dagger + zx^\dagger H^\dagger + zz^\dagger)] \\
&= E[Hxx^\dagger H^\dagger] + E[Hxz^\dagger] + E[zx^\dagger H^\dagger] + E[zz^\dagger] \\
&= E[Hxx^\dagger H^\dagger] + E[zz^\dagger] \\
&= HE[xx^\dagger H^\dagger] + E[zz^\dagger] \\
&= H\,\{\frac{P_T}{n_T} I_{nT}\}H^\dagger + N_0 I_{nR} + G\,\{\frac{P_I}{n_I} I_{nI}\}G^\dagger \\
&= N_0[\, I_{nR} + MQM^\dagger\,]
\end{aligned}
$$

Where, $\boldsymbol{M} = [\,\boldsymbol{HG}\,] \in \boldsymbol{C}^{n_R(n_T+n_I)}$ and $\boldsymbol{Q} = \frac{P_T}{N_0 n_T}\boldsymbol{I}_{nR} \oplus \frac{P_I}{N_0 n_I}\boldsymbol{I}_{nI}$

(ii) Covariance of Interference plus Noise signals

$$
\begin{aligned}
\boldsymbol{R}_w = E[ww^\dagger] &= E[(Gs+z)(Gs+z)^\dagger] \\
&= E[Gss^\dagger G^\dagger] + E[zz^\dagger] \\
&= GE[ss^\dagger]G^\dagger + E[zz^\dagger] \\
&= N_0 I_{nR} + G\,\{\frac{P_I}{n_I} I_{nI}\}G^\dagger
\end{aligned}
$$

(iii) Entropy of Received Signal

$$\boldsymbol{H}(\boldsymbol{y}) = log_2 det(\pi e \boldsymbol{R}_y) = log_2 det\{\pi e N_0[\, I_{nR} + MQM^\dagger\,]\}$$

(iv) Entropy of interference plus noise signal

$$\boldsymbol{H}(\mathbf{w}) = log_2 det(\pi e \boldsymbol{R}_w) = log_2 det\{\pi e N_0 [\, I_{nR} + G\,\{\frac{P_l}{n_l} I_{nl}\} G^{\dagger}\,]\}$$

v) Mutual information between x and y

$$I(\boldsymbol{x};\boldsymbol{y}) = \boldsymbol{H}(\boldsymbol{y}) - \boldsymbol{H}(\mathrm{w}) = log_2 det(\pi e \boldsymbol{Ry}) - log_2 det(\pi e \boldsymbol{R}\mathrm{w})$$

$= log_2 \frac{det\,(\,I_{nR}+MQM^{\dagger}\,)}{det\,(I_{nR}+G\,\{\frac{P_l}{n_l} I_{nl}\} G^{\dagger}\,)}$, which is the Mutual Information.

(vii) Capacity at the Receiver:

$$C = \max_{Q} I(x;y) = \max_{Q} log_2 \frac{det\,(\,I_{nR}+MQM^{\dagger}\,)}{det\,(I_{nR}+G\,\{\frac{P_l}{n_l} I_{nl}\} G^{\dagger}\,)},$$

$= log_2 [\frac{\det(\,I_{nR}+MQM^{\dagger}\,)}{\det\left(I_{nR}+G\left\{\frac{P_l}{n_l} I_{nl}\right\} G^{\dagger}\right)}]$,which is the channel capacity.

3. Program for the channel capacity for MIMO interference channel

- **MATLAB Code for plotting the capacity**

```
% Ergodic Capacity of Nr x Nt fading MIMO Channel with
% no channel knowledge at the transmitter.
clc
clear all;
%%%%%%%% Initialization %%%%%%%%
N=30000;        % Number of Iterations for H
No=1;           % Noise Variance
%%%%%%%% Number of antennas %%%%%%%%
nt=2;           % Number of Transmit antennas
ni=2;           % Number of interference antenna
nt=Nt+ni;
nr=6;           % Number of Receive antennas
%%%%%%%% Correlation of antennas %%%%%%%%
rhot=0;         % Correlation coefficient (Transmitter)
rhor=0;         % Correlation coefficient (Receiver)
ExponencorrMrtx=ones(nt);
%%%%%%%% Creation of SNR from 1 to L %%%%%%%%
for i=1:nt
   for j=1:nt
    if i==j
       ExponencorrMrtx(i,j)=1;
    end
    if i>j
      ExponencorrMrtx(i,j)=rhot^(i-j);
  else
      ExponencorrMrtx(i,j)=rhot^(j-i);
  end
 end
end
Rt=ExponencorrMrtx;
% generate an nxn exponential correlation matrix for Rr
ExponencorrMrtx=ones(nt);
for i=1:nr
 for j=1:nr
   if i==j
      ExponencorrMrtx(i,j)=1;
   end
```

```
    if i>j
       ExponencorrMrtx(i,j)=rhor^(i-j);
    else
       ExponencorrMrtx(i,j)=rhor^(j-i);
    end
  end
end
Rr=ExponencorrMrtx;
SNRdB=[0:1:35];
l=length (SNRdB);
SNR=zeros (1,l);
for i=1:l
    SNR (i)=10^(0.1*SNRdB (i));
    P(i)=SNR(i)*No;
end
Erg_Cap_unknown=zeros (1,l);
for i=1:l
    C_unknown=zeros (1,N);

for j=1:N
    H=CGM (nr,nt,Rr,Rt);
    C_unknown (j)=log (det (eye (nr)+(SNR (i)/nt)*(H*H')));
end
 Erg_Cap_unknown (i)=mean (C_unknown);

fprintf ('% e\t% e \n',SNRdB (i),Erg_Cap_unknown (i));
end
Erg_Cap_unknown=zeros (1,l);
for i=1:l
C_unknown=zeros (1,N);

for j=1:N
    G=CGM (nr,nt,Rr,Rt);
    C_unknown (j)=log (det (eye (nr)+(SNR (i)/nt)*(G*G')));
end
 Erg_Cap_unknown (i)=mean (C_unknown);

fprintf ('% e\t% e \n',SNRdB (i),Erg_Cap_unknown (i));
end
plot (SNRdB,Erg_Cap_unknown)
xlabel ('SNR (dB)');
ylabel ('Ergodic capacity (bits/sec/Hz)');
title ('Ergodic capacity versus SNR');
grid on
hold on
```

4. Numerical results of the MIMO channel

(a) Numerical data for measuring capacity at different SNR (dB)

SNR (dB)	Capacity(bits/sec/Hz)	SNR (dB)	Capacity(bits/sec/Hz)
0	3.21022	16	14.7718
1	3.68846	17	15.6565
2	4.21794	18	16.5476
3	4.77797	19	17.4404
4	5.38851	20	18.3512
5	6.02162	21	19.2530
6	6.69693	22	20.1611
7	7.40800	23	21.0785
8	8.14525	24	21.9772

9	8.90590	25	22.8973
10	9.69533	26	23.8161
11	10.4940	27	24.7259
12	11.3237	28	25.6441
13	12.1671	29	26.5609
14	13.0249	30	27.4844
15	13.8944		

5. Effects of transmitting antennas

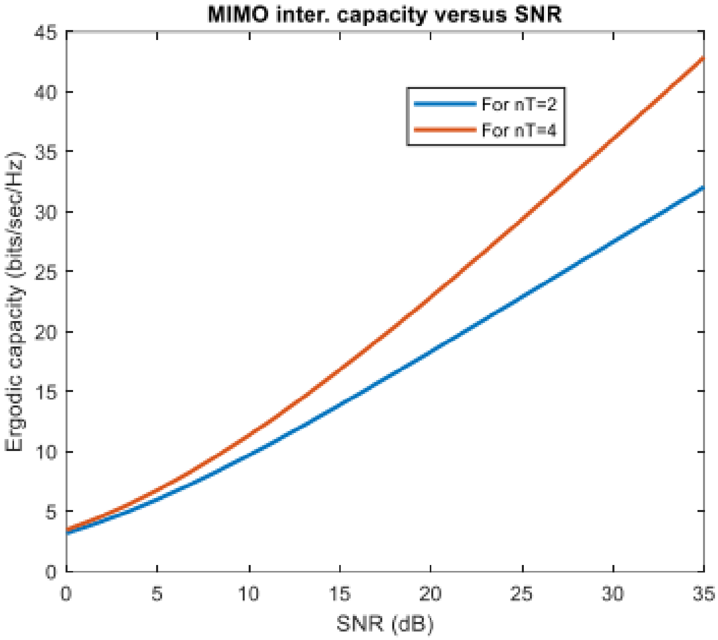

Figure 6.1: Increase in MIMO interference Capacity Vs SNR curve with transmitting antennas

6. Effects of receiving antennas

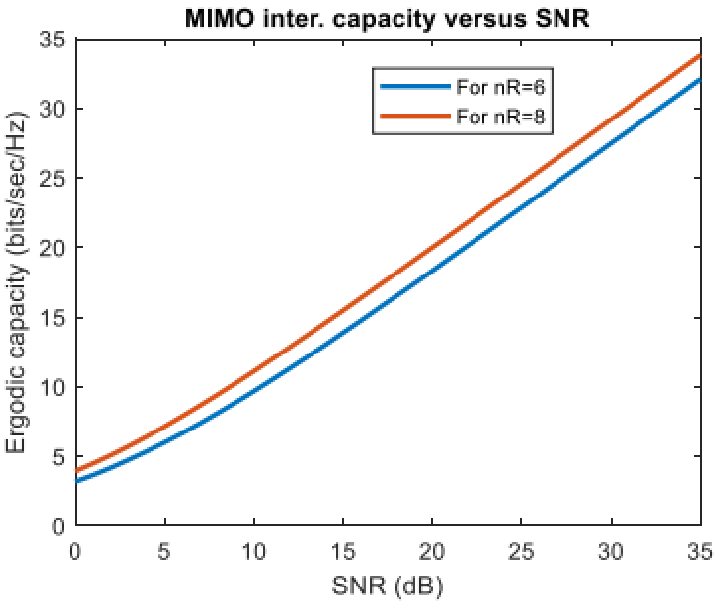

Figure 6.2: Increase in MIMO interference Capacity Vs SNR curve with receiving antennas

7. Effects of interfering antennas

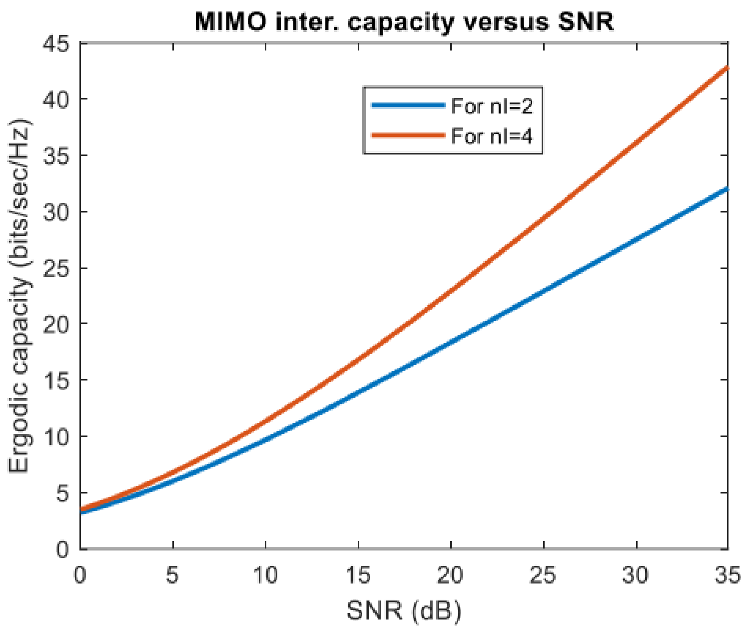

Figure 6.3: Increase in MIMO interference Capacity Vs SNR curve with interfering antennas

8. Discussion and Conclusion

a) Discussion: All of the figures depict the fluctuation in capacity in relation to SNR for various transmitting, receiving, and interfering antenna values. The channel capacity of a MIMO interference channel is determined by all factors, and we can improve the capacity by adding or decreasing the number of antennas. Interference is an adversary of every signal and should be avoided for optimal signal delivery.

b) Conclusion: From our investigation, we can draw the following conclusions:

- **The effect of interfering antenna on the capacity is more than another antenna.** An interfering antenna has a bigger impact on capacity than other antennas. When numerous antennas are functioning in a shared communication environment, the presence of interfering antennas can have a major impact on the channel's total capacity. Interfering antennas introduce new signal components into the mix, resulting in increased noise and probable signal deterioration.

- **Number of interfering antennas should be reduced to get high capacity of any channel.** Interference degrades signal quality and increases the risk of mistakes, lowering the possible data rate. As a result, the signal-to-interference-plus-noise ratio (SINR) improves, allowing for higher data rates and increased overall channel capacity.

- **Increased receiving antenna number gives us better SNR as well as the capacity.** Increasing the number of receiving antennas (using multiple-input, multiple-output, or MIMO techniques) can significantly improve both SNR and capacity. As the number of receiving antennas grows, the system can take advantage of spatial diversity and multipath propagation, resulting in higher SNR.

Finally, comprehending the statistical features of the MIMO channel as revealed by the ergodic capacity analyses is critical for building robust and efficient digital communication systems.