Experiment No. 03

1. Problem Statement: A multiple-input multiple-output (MIMO) channel is equipped with n_T transmitting antennas and n_R receiving antennas.

(a) Derive the expressions of the channel capacity.

(b) Write the programs of channel capacity for MIMO channel using MATLAB.

(**c) Explain the numerical results of** the channel capacity for MIMO channel.

2. Derivation of the channel capacity for MIMO channel

Let,

$\boldsymbol{P_T}$ **=** Transmit power
$\boldsymbol{N_0}$ **=** Noise power
$\boldsymbol{n_T}$ **=** Number of Antennas at the transmitter
$\boldsymbol{n_R}$ **=** Number of Antennas at the Receiver
$\boldsymbol{I_{nR}}$ **=** Identity Matrix

Required formula for the derivation of the capacity are -

$$(\boldsymbol{A}+\boldsymbol{B})^\dagger = \boldsymbol{A}^\dagger + \boldsymbol{B}^\dagger \quad \ldots\ldots (1)$$
$$(\boldsymbol{AB})^\dagger = \boldsymbol{B}^\dagger \boldsymbol{A}^\dagger$$
$$E[\boldsymbol{x}z^\dagger] = 0 \quad \ldots\ldots (2)$$

Since x and z are independent,

$$E(\boldsymbol{A}+\boldsymbol{B}) = E(\boldsymbol{A})+E(\boldsymbol{B}) \quad \ldots\ldots (3)$$
$$det(\alpha\boldsymbol{A}) = \alpha^n det(\boldsymbol{A}),\ n = matrix\ order$$

(i) Covariance of Transmit Signal:

$$\boldsymbol{R_x = E[xx^\dagger] = \frac{P_T}{n_T} I_{nT}}$$

(ii) Covariance of Noise Signal:

$$\boldsymbol{R_z = E[zz^\dagger] = N_0 I_{nR}}$$

(iii) Covariance of Received Signal:

$$\begin{aligned}
\boldsymbol{R}_y = E[yy^\dagger] &= E[(\boldsymbol{Hx}+\boldsymbol{z})\,(Hx+z)^\dagger] \\
&= E[(\boldsymbol{Hx}+\boldsymbol{z})\{\,(Hx)^\dagger + (z)^\dagger\}] \\
&= E[(\boldsymbol{Hx}+z)\{\,(x^\dagger H^\dagger + z^\dagger\}] \\
&= E[(Hxx^\dagger H^\dagger + Hxz^\dagger + zx^\dagger H^\dagger + zz^\dagger)] \\
&= E[Hxx^\dagger H^\dagger] + E[Hxz^\dagger] + E[zx^\dagger H^\dagger] + E[zz^\dagger] \\
&= E[Hxx^\dagger H^\dagger] + E[zz^\dagger] \\
&= HE[xx^\dagger H^\dagger] + E[zz^\dagger] \\
&= H\,\{\frac{P_T}{n_T} I_{nT}\} H^\dagger + N_0 I_{nR}
\end{aligned}$$

$\boldsymbol{R}_y = \frac{P_T}{n_T}\{HH^\dagger\} + N_0 I_{nR}$,which is the received signal covariance.

(iv) Entropy of Noise Signal:

$$\boldsymbol{H}(\boldsymbol{z}) = log_2 det(\pi e \boldsymbol{R}_z) = log_2 det(\pi e N_0 I_{nR})$$

(v) Entropy of Received Signal:

$$\boldsymbol{H}(\boldsymbol{y}) = log_2 det(\pi e \boldsymbol{R}_y) = log_2 det\{\pi e(\frac{P_T}{n_T}\{HH^{\dagger}\} + N_0 I_{nR})\}$$

(vi) Mutual Information:

$$I(\boldsymbol{x};\boldsymbol{y}) = \boldsymbol{H}(\boldsymbol{y}) - \boldsymbol{H}(\boldsymbol{z}) = log_2 det(\pi e \boldsymbol{Ry}) - log_2 det(\pi e \boldsymbol{Rz})$$

$$= log_2 \frac{det(\pi e Ry)}{det(\pi e Rz)}$$

$$= log_2 \frac{\det\{\pi e(\frac{P_T}{n_T}\{\mathrm{H}H^{\dagger}\} + N_0 I_{nR})\}}{det(\pi e N_0 I_{nR})}$$

$$= log_2 \frac{(\pi e N_0)^{n_R} \det\{(\frac{P_T}{n_T}\{\mathrm{H}H^{\dagger}\} + I_{nR})\}}{(\pi e N_0)^{n_R} det(I_{n_R})}$$

$$= log_2 \frac{\det\{(\frac{P_T}{n_T N_0}\{\mathrm{H}H^{\dagger}\} + N_0 I_{nR})\}}{1}$$

$I(\boldsymbol{x};\boldsymbol{y}) = log_2 \det\{(\frac{P_T}{n_T N_0}\{\mathrm{H}H^{\dagger}\} + N_0 I_{nR})\}$, which is the Mutual Information.

(vii) Capacity at the Receiver:

$$C = {}^{max}_{\ Q} I(x;y) = {}^{max}_{\ Q} log_2 det\{(\frac{P_T}{n_T N_0}\{HH^{\dagger}\} + N_0 I_{nR})\}$$

$C = log_2 det\{(\frac{P_T}{n_T N_0}\{HH^{\dagger}\} + N_0 I_{nR})\}$, which is the channel capacity.

3. Program for the channel capacity for MIMO channel

- **MATLAB Code for plotting the capacity**

```
% Ergodic Capacity of Nr x Nt fading MIMO Channel with
% no channel knowledge at the transmitter.
clc
clear all;
%%%%%%%% Initialization %%%%%%%%
N=30000;       % Number of Iterations for H
No=1;          % Noise Variance
%%%%%%%% Number of antennas %%%%%%%%
nt=2;          % Number of Transmit antennas
nr=2;          % Number of Receive antennas
%%%%%%%% Correlation of antennas %%%%%%%%
rhot=0;        % Correlation coefficient (Transmitter)
rhor=0;        % Correlation coefficient (Receiver)
Rt=ExponencorrMrtx(nt,rhot);    % Correlation Matrix Rt
Rr=ExponencorrMrtx(nr,rhot);    % Correlation Matrix Rr
%%%%%%%% SNR of Channel %%%%%%%%
SNRdB=[0:1:15];         % Signal-to-Noise Ratio
l=length (SNRdB);
SNR=zeros(1,l);
%%%%%%%% Creation of SNR from 1 to L %%%%%%%%
for i=1:l
    SNR(i)=10^(0.1*SNRdB(i)); %Conversion of SNR in magnitude form
    P(i)=SNR(i)*No;
end
```

```
%%%%%%%% Calculation of Ergodic capacity %%%%%%%%
Erg_Cap_unknown=zeros(1,l);
for i=1:l                        % Loop of SNR
    C_unknown=zeros(1,N);
    %%%%%%%% Calculation of capacity %%%%%%%%
    for j=1:N
        H=CGM (nr,nt,Rr,Rt);    % Generate Rayleigh Distributed fading
        C_unknown(j)=log(det(eye(nr)+(SNR(i)/nt)*(H*H')));
end
%%%%%%%% Ergodic Capacity %%%%%%%%
    Erg_Cap_unknown(i)=mean(C_unknown); %Ergodic capacity with no CSIT
%%%%%%%% Printing Data %%%%%%%%
fprintf ('% e\t% e \n',SNRdB(i),Erg_Cap_unknown(i));
end
%%%% Plotting of Ergodic capacity versus SNR  %%%%
plot (SNRdB,Erg_Cap_unknown,':r*')

xlabel ('SNR (dB)');
ylabel ('Ergodic capacity (bits/sec/Hz)');
title ('Ergodic capacity versus SNR');
grid on
hold on
```

- **To build n*n identity matrix the user defined function**

```
function out=ExponencorrMrtx(n,rho);
% generate an nxn exponential correlation matrix
A=ones(n);
for i=1:n
    for j=1:n
        if i==j
            A(i,j)=1;
        end
        if i>j
            A(i,j)=rho^(i-j);
        else A(i,j)=rho^(j-i);
        end
    end
end
out=A;
```

- **To generate relay distributed fading user defined function**

```
function out=CGM(nr,nt,Rr,Rt)
% Generate Complex Gaussian matrix
X = sqrt(0.5)*randn(nr,nt)+i*sqrt(0.5)*randn(nr,nt);
out=Rr^(1/2)*X*Rt^(1/2);
```

4. Numerical results of the channel capacity for MIMO channel

(a) Numerical data for measuring capacity at different SNR (dB)

SNR (dB)	Capacity(bits/sec/Hz)	SNR (dB)	Capacity(bits/sec/Hz)
0	1.169220	8	3.178715
1	1.353969	9	3.518419
2	1.558950	10	3.840310
3	1.781360	11	4.206453
4	2.034536	12	4.573855
5	2.297278	13	4.936794

6	2.571197	14	5.333823
7	2.861399	15	5.720873

(b) Graphical representation

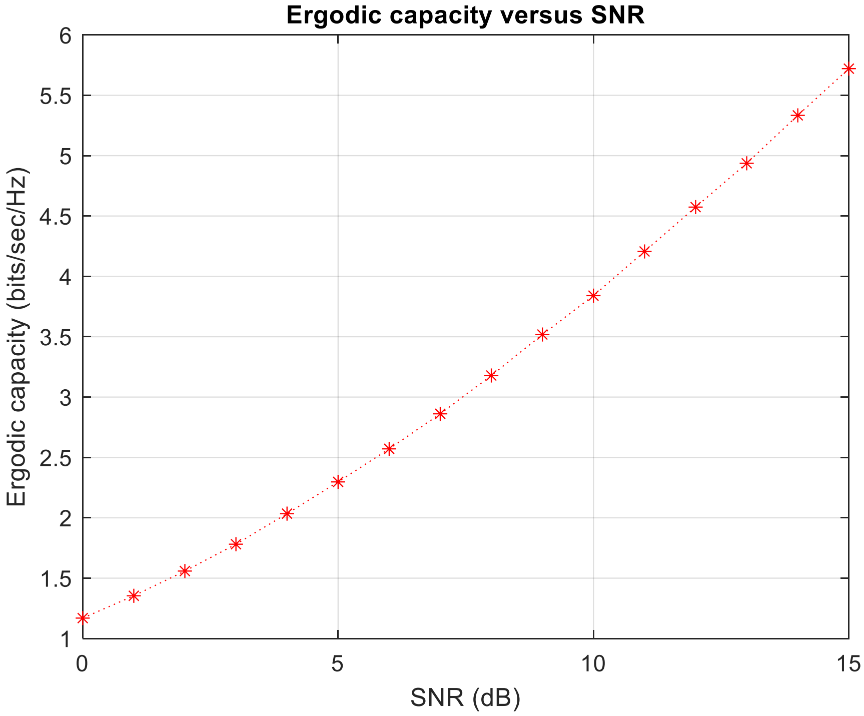

Figure 3.1. Ergodic capacity vs Signal to Noise Ratio plot for MIMO channel.

(c) Description of Figure 3.1: This is a plot of the capacity vs signal to noise ratio for various values of the average SNR (SNRADB) for MIMO) channel is equipped with n_T transmitting antennas and n_R receiving antennas. This graph depicts the impact of SNR on channel capacity. The graph above demonstrates that as the SNR value increases, the capacity rises.

6. Discussion and Conclusion

a) Discussion: The linearity property of channel ergodic capacity in response to the signal-to-noise ratio is depicted in Figure 3.1. Capacity, as we know, is the conditional data transmission rate at which the data transmission rate is maximal and the error probability is minimum. It varies depending on the case and the SNR capacity. The graph and data table illustrate that increasing the SNR increases capacity.

b) Conclusion: From our investigation, we can draw the following conclusions:

- Capacity varies depending on the channel.
- Increasing the SNR increases channel capacity.
- Noise power has an adverse influence on channel capacity.
- The number of antennas on the transmitter and receiving sides determines channel capacity.

Finally, comprehending the statistical features of the MIMO channel as revealed by the ergodic capacity analyses is critical for building robust and efficient digital communication systems.