Experiment No. 01

1. Problem Statement: The probability density function of the signal-to-noise ratio of Rayleigh Fading SISO is given by

$$f_\gamma(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(\frac{\gamma}{\bar{\gamma}}\right), \gamma > 0$$

(a) Derive the expressions of

(i) The cumulative distribution function of γ.
(ii) The moment generation function of γ.
(iii) The amount of fading in Rayleigh fading SISO channel.

(b) write the programs of

(i) The probability density function of γ.
(ii) The cumulative distribution function of γ.
(iii) The moment generation function of γ.

(c) Explain the numerical results of

(i) The probability density function of γ.
(ii) The cumulative distribution function of γ.
(iii) The moment generation function of γ.

2. Derivation of Cumulative Distribution Function (CDF) of γ

The formula for finding CDF is given by-

$$\boldsymbol{F_\gamma(\gamma) = \int_0^\gamma f_\gamma(\gamma)\, d\gamma \ldots\ldots\ldots (1)}$$

By using the above formula, we can find the expression of CDF as follows-

$$\Rightarrow F_\gamma(\gamma) = \int_0^\gamma \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}}\, d\gamma$$

$$= \frac{1}{\bar{\gamma}} \int_0^\gamma e^{-\frac{\gamma}{\bar{\gamma}}}\, d\gamma$$

$$= \frac{1}{\bar{\gamma}} \left[e^{-\frac{\gamma}{\bar{\gamma}}} . \frac{1}{-\frac{1}{\bar{\gamma}}} \right]_0^\gamma$$

$$= -\left[e^{-\frac{\gamma}{\bar{\gamma}}} . \right]_0^\gamma$$

$$= -\left[e^{-\frac{\gamma}{\bar{\gamma}}} - e^0 \right]$$

$$= -\left[e^{-\frac{\gamma}{\bar{\gamma}}} - 1 \right]$$

$$\boldsymbol{\Rightarrow F_\gamma(\gamma) = 1 - e^{-\frac{\gamma}{\bar{\gamma}}} \ldots\ldots\ldots\ldots (2)}$$

Equation-2 is the Cumulative Distribution Function (CDF) of Rayleigh Fading SISO channel.

3. Derivation of Moment Generating Function (MGF) of γ

The formula for finding MGF is given by-

$$M_\gamma(s) = \int_0^\infty f_\gamma(\gamma) e^{s\gamma}\, d\gamma$$

By using the above formula, we can find the expression of MGF as follows-

$$M_\gamma(s) = \int_0^\infty \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}} e^{s\gamma}\, d\gamma$$
$$= \frac{1}{\bar{\gamma}} \int_0^\infty e^{-\frac{\gamma}{\bar{\gamma}}} e^{s\gamma}\, d\gamma$$
$$= \frac{1}{\bar{\gamma}} \int_0^\infty e^{\left(s\gamma - \frac{\gamma}{\bar{\gamma}}\right)}\, d\gamma$$
$$= \frac{1}{\bar{\gamma}} \int_0^\infty e^{-\left(\frac{1}{\bar{\gamma}} - s\right)\gamma}\, d\gamma$$
$$= \frac{1}{\bar{\gamma}} \int_0^\infty e^{-\left(\frac{1-s\bar{\gamma}}{\bar{\gamma}}\right)\gamma}\, d\gamma$$
$$= \frac{1}{\bar{\gamma}} . \frac{1}{-\left(\frac{1-s\bar{\gamma}}{\bar{\gamma}}\right)} \left[e^{-\left(\frac{1-s\bar{\gamma}}{\bar{\gamma}}\right)\gamma}\right]_0^\infty$$
$$= -\frac{1}{(1-s\bar{\gamma})}\left[e^{-\infty} - e^0\right]$$
$$= -\frac{1}{(1-s\bar{\gamma})}[0-1]$$
$$= \frac{1}{(1-s\bar{\gamma})}$$
$$\Rightarrow \boldsymbol{M_\gamma(s) = (1-s\bar{\gamma})^{-1}} \dots \dots \dots \dots \dots \mathbf{(3)}$$

Equation-3 is the Moment Generating Function (MGF) of Rayleigh Fading SISO channel.

4. Derivation of Amount of Fading (AF) of γ

The formula for finding AF is given by-

$$AF = \frac{\left.\frac{d^2 M_\gamma(s)}{ds^2}\right|_{s=0} - \left(\left.\frac{dM_\gamma(s)}{ds}\right|_{s=0}\right)^2}{\left(\left.\frac{dM_\gamma(s)}{ds}\right|_{s=0}\right)^2} \dots \dots \dots (4)$$

Now, $\frac{d\left(M_\gamma(s)\right)}{ds} = \frac{d}{ds}(1-s\bar{\gamma})^{-1} = \frac{\bar{\gamma}}{(1-s\bar{\gamma})^2}$

And, $\frac{d^2\left(M_\gamma(s)\right)}{ds^2} = \frac{d}{ds}\left\{\frac{\bar{\gamma}}{(1-s\bar{\gamma})^2}\right\} = \frac{2\bar{\gamma}^2}{(1-s\bar{\gamma})^3}$

$$\left.\frac{d\left(M_\gamma(s)\right)}{ds}\right|_{s=0} = \frac{\bar{\gamma}}{(1-0.\bar{\gamma})^2} = \bar{\gamma}$$

And, $\left.\frac{d^2\left(M_\gamma(s)\right)}{ds^2}\right|_{s=0} = \frac{2\bar{\gamma}^2}{(1-0.\bar{\gamma})^3} = 2\bar{\gamma}^2$

Now, putting these values in equation 4, we get

$$AF = \frac{2\bar{\gamma}^2 - \bar{\gamma}^2}{\bar{\gamma}^2} = 1$$

$$\boldsymbol{AF = 1} \dots \dots \dots \mathbf{(5)}$$

So, the Amount of Fading of Rayleigh Fading SISO channel is 1.

5. Program for probability density function of γ

```
SNRADB = 5;
snra = 10^(SNRADB/10);
strm = OpenWrite["D:\PDF.txt"];
For[SNRdB = 0, SNRdB < 35.1, SNRdB++,
  snr = 10^(SNRdB/10) // N;
  PDF1 = 1/snra × Exp[-snr/snra];
  Print["SNR=", SNRdB, "dB\t", PDF1];
  Write[strm, PDF1];
 ];
Close[strm];
```

6. Program for cumulative distribution function of γ

```
SNRADB = -5;
snra = 10^(SNRADB/10);
strm = OpenWrite["D:\CDF.txt"];
For[SNRdB = 0, SNRdB < 23.1, SNRdB++,
  snr = 10^(SNRdB/10) // N;
  CDF1 = 1 - Exp[-snr/snra];
  Print["SNR=", SNRdB, "dB\t", CDF1];
  Write[strm, CDF1];
 ];
Close[strm];
```

7. Program for moment generation function of *γ*

```
SNRADB = -1;
snra = 10^(SNRADB/10);
strm = OpenWrite["D:\MGF.txt"];
For[s = -15, s < 3.1, s = s + 0.5,
  SN = 1 - s × snra // N;
  MGF = 1/SN;
  Print["S=", s, "\t", MGF];
  Write[strm, MGF];
 ];
Close[strm];
```

8. Numerical results of probability density function of γ

a) Numerical data

SNR	PDF value at SNRADB=5	PDF value at SNRADB=7	PDF value at SNRADB=10
0	0.230496336	0.163435674	0.090483742
1	0.212375421	0.15520659	0.088170959
2	0.191574257	0.145433356	0.085343208
3	0.168259156	0.133999832	0.081911873
4	0.142898487	0.120875185	0.077787562
5	0.116333694	0.10616435	0.072889341
6	0.089795721	0.09016285	0.067159005
7	0.064817105	0.073401599	0.060581099
8	0.043000013	0.056657269	0.053208217
9	0.02565084	0.040896829	0.045188469
10	0.013385675	0.027131174	0.036787944
11	0.005902589	0.016184586	0.0283959
12	0.002105579	0.00844579	0.020496968
13	0.000575158	0.003724282	0.013597798
14	0.000112273	0.00132853	0.008111508
15	1.436E-05	0.0003629	0.004232922
16	1.078E-06	7.084E-05	0.001866562
17	4.139E-08	9.058E-06	0.000665842

SNR	PDF value at SNRADB=5	PDF value at SNRADB=7	PDF value at SNRADB=10
18	6.834E-10	6.801E-07	0.000181881
19	3.900E-12	2.612E-08	3.5504E-05
20	5.840E-15	4.312E-10	4.540E-06
21	1.623E-18	2.460E-12	3.408E-07
22	5.416E-23	3.685E-15	1.300E-08
23	1.253E-28	1.024E-18	2.161E-10
24	1.006E-35	3.418E-23	1.233E-12
25	1.176E-44	7.904E-29	1.847E-15
26	6.692E-56	6.350E-36	5.134E-19
27	4.666E-70	7.423E-45	1.713E-23
28	7.028E-88	4.222E-56	3.962E-29
29	2.571E-110	2.944E-70	3.182E-36
30	1.459E-138	4.435E-88	3.720E-45
31	4.020E-174	1.622E-110	2.116E-56
32	6.873E-219	9.205E-139	1.476E-70
33	3.011E-275	2.537E-174	2.223E-88
34	3.37000728189298	4.337E-219	8.131E-111
35	1.605159142565173	1.900E-275	4.613E-139

b) Graphical representation

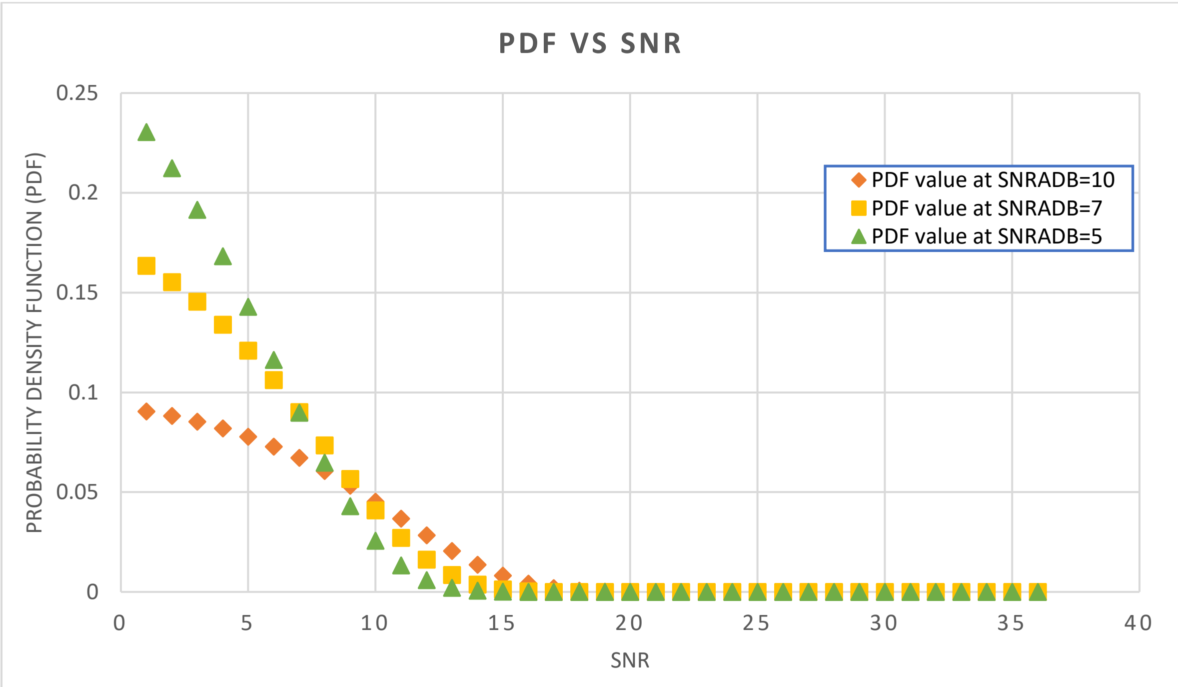

Figure 1.1. Probability Density Function vs Signal to Noise Ratio plot for Rayleigh fading SISO channel.

(b) Description of Figure 1.1: This is a plot of the probability density function (***PDF***) vs signal-to-noise ratio (***SNR*** or ***g***) for various values of the average SNR (SNRADB). This graph depicts the impact of SNR on PDF.

The graph above demonstrates that as the SNR value increases, the PDF value falls. The rate at which the PDF value falls is determined by the average SNR value. When the SNRADB value is lower (e.g., 5) the PDF value declines faster than when the SNRADB value is larger (e.g., 10 or 15). When the SNR is low, noise dominates the received signal, and the fading envelope undergoes deep fades. As the SNR rises, the fading envelope becomes less severe, causing the PDF to move toward higher envelope values.

9. Numerical results of cumulative distribution function of γ

a) Numerical data

SNR	PDF value at SNRADB=-5	PDF value at SNRADB=-3	PDF value at SNRADB=-1
0	0.95767078	0.86402202	0.716040998
1	0.981334375	0.918884923	0.795030316
2	0.993341575	0.95767078	0.86402202
3	0.998181191	0.981334375	0.918884923
4	0.999644961	0.993341575	0.95767078
5	0.9999546	0.998181191	0.981334375
6	0.999996592	0.999644961	0.993341575
7	0.999999869	0.9999546	0.998181191
8	0.999999998	0.999996592	0.999644961
9	1	0.999999869	0.9999546
10	1	0.999999998	0.999996592
11	1	1	0.999999869

SNR	PDF value at SNRADB=-5	PDF value at SNRADB=-3	PDF value at SNRADB=-1
12	1	1	0.999999998
13	1	1	1
14	1	1	1
15	1	1	1
16	1	1	1
17	1	1	1
18	1	1	1
19	1	1	1
20	1	1	1
21	1	1	1
22	1	1	1
23	1	1	1

b) Graphical representation

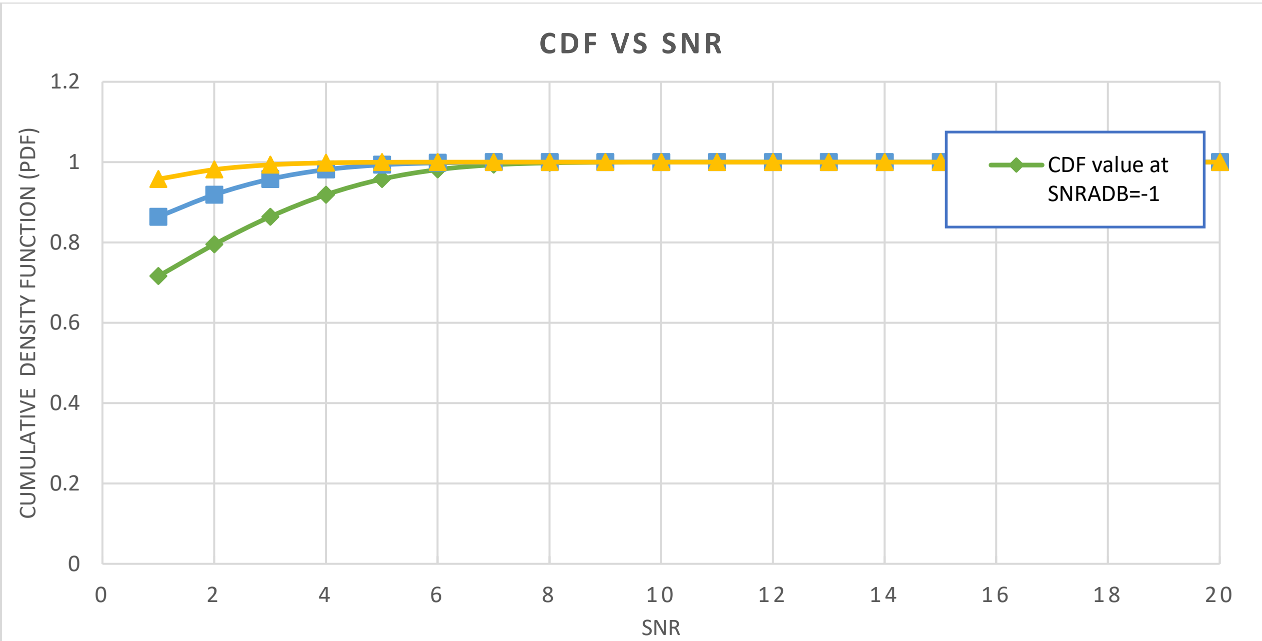

Figure 1.2. Cumulative Distribution Function vs Signal to Noise Ratio plot for Rayleigh fading SISO channel.

c) Description of Figure 1.2: This is a visualization of the cumulative distribution function (***CDF***) vs signal-to-noise ratio (***SNR*** or ***g***) for a certain average value of SNR (***SNRADB***). This graph depicts the impact of SNR on CDF.

The CDF vs. SNR graph shows the cumulative chance of encountering various fading envelope values at different Signal to Noise Ratio (***SNR***) levels. For example, if the CDF value at SNR = 10 dB is 0.8, it means that there is an 80% chance that the fading envelope R is smaller than or equal to r at SNR = 10 dB, where the fading envelope is denoted as R and its cumulative distribution function (CDF) as FR(r). The following trends were detected in the CDF vs. SNR graph:

The CDF climbs slowly and steadily at low SNR levels, indicating a larger likelihood of meeting small fading envelope values. As SNR grows, the slope of the CDF becomes steeper, indicating a lower likelihood of encountering small envelope values.

The graph also shows that as the value of SNRADB falls (e.g., -5), the rate of growing CDF increases faster than at higher levels of SNRADB (e.g., -3 or -1). Finally, all of the CDF values saturated at 1. The signal may be lost at SNR 0 db.

10. Numerical results of moment generation function of γ

a) Numerical data

SNR	MGF value at SNRADB=-5	MGF value at SNRADB=-1
-15	0.174112395	0.077429804
-15	0.174112395	0.077429804
-14.5	0.179041329	0.079886501
-14	0.184257458	0.082504199
-13.5	0.189786638	0.08529926
-13	0.195657922	0.088290343
-12.5	0.201904074	0.091498818
-12	0.208562179	0.094949279
-11.5	0.215674381	0.098670176
-11	0.223288777	0.102694598
-10.5	0.231460503	0.107061263
-10	0.240253073	0.11181577
-9.5	0.249740035	0.117012189
-9	0.260007027	0.122715135
-8.5	0.271154379	0.129002463
-8	0.283300394	0.135968847
-7.5	0.296585567	0.143730578
-7	0.311178042	0.152432108
-6.5	0.327280769	0.162255125

SNR	MGF value at SNRADB=-5	MGF value at SNRADB=-1
-5.5	0.365063068	0.186261178
-5	0.387425887	0.201140824
-4.5	0.412707265	0.218604222
-4	0.44151844	0.239388338
-3.5	0.474654138	0.264539849
-3	0.513167019	0.295596962
-2.5	0.55848156	0.334916306
-2	0.612574113	0.386300775
-1.5	0.67826884	0.456310057
-1	0.759746927	0.557311634
-0.5	0.863472941	0.71573553
0	1	1
0.5	1.187809111	1.658826272
1	1.462475296	4.862116094
1.5	1.902376321	-5.222140671
2	2.72075922	-1.698783674
2.5	4.774851773	-1.014383361

b) Graphical representation

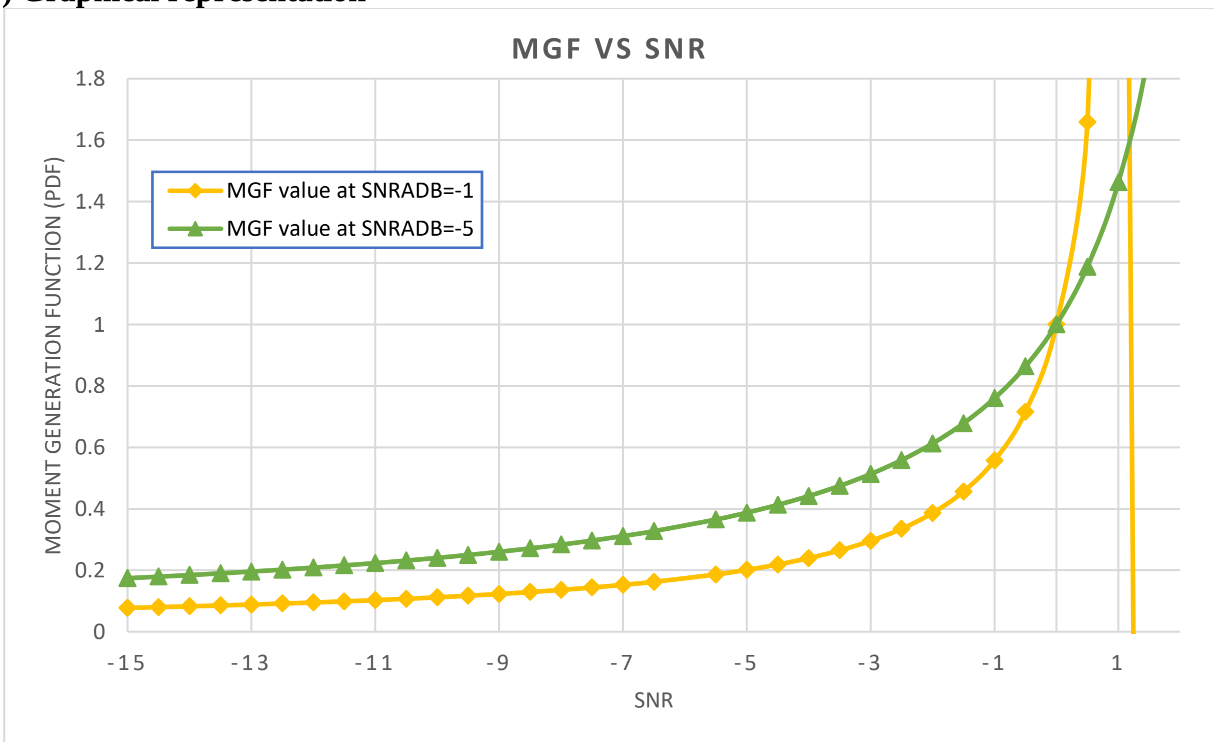

Figure 1.3. Moment Generation Function vs Variable (*s*) plot for Rayleigh fading SISO channel.

c) Description of Figure 1.3: This is a visualization of the Moment Generating Function (***MGF***) as a function of variable ***s*** for a given value of SNR average (***SNRADB***). This graph depicts the impact of SNR on MGF. The graph above demonstrates that as the "s" value increases, so does the MGF value. The rate of growth of the MGF value at the negative section of s is determined by the average SNR value. When SNRADB (e.g., -5) is small, the MGF value increases slower than when SNRADB is greater (e.g., -3 or -1), but in the positive portion of s, the situation is completely reversed. This means that the lower-valued SNRADB MGF function hits saturation sooner than other SNRADB values.

11. Discussion and Conclusion

a) Discussion: The peak MGF value at 0 dB is determined by the average SNR value. The initial value of MGF (at 0 dB) was higher at lower SNRADB values than at higher SNRADB values.

The influence of SNR on ***PDF*** is depicted in **Figure 1.1**. The PDF value falls as the SNR increases. Similar characteristics are observed for various SNR average values.

The effect of SNR on ***CDF*** is depicted in **Figure 1.2**. According to the instantaneous SNR magnitude, the CDF value may increase or decrease.

The effect of SNR on MGF is depicted in **Figure 1.3**. The ***MGF*** value rises as the SNR rises. Similar characteristics are observed for various SNR average values.

For the Rayleigh Fading SISO channel amount of fading is unity and independent of SNR.

b) Conclusion: From our investigation, we can draw the following conclusions:

- The channel parameters, as represented by the PDF, CDF, and MGF, are functions of the channel's instantaneous SNR. The instantaneous SNR is critical in determining the statistical behavior of the fading envelope and the reliability of the communication link.

- Greater SNR is preferable for a good communication channel. When the SNR is high, the signal power outnumbers the noise power, resulting in better communication performance. This increases the likelihood of encountering greater envelope values, reduces fading effects, and resulting in a more dependable wireless communication link.

- The amount of fading for the Rayleigh-fading SISO channel is independent of the SNR. The fading envelope has a unity amount of fading, which means that its magnitude varies around its mean value without being affected by SNR variations.

- The PDF, CDF, and MGF are all affected by the channel's average SNR. The shape of these curves varies depending on the average SNR's constant parameter.

Finally, comprehending the statistical features of the Rayleigh-fading SISO channel as revealed by the PDF, CDF, and MGF analyses is critical for building robust and efficient digital communication systems.