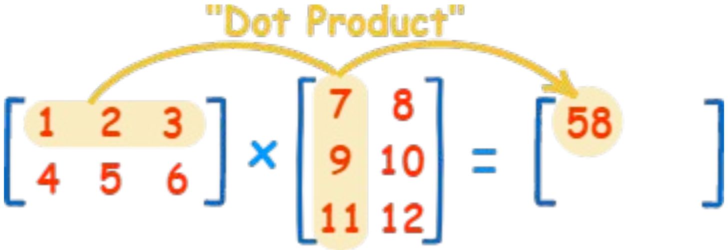
Lecture 2

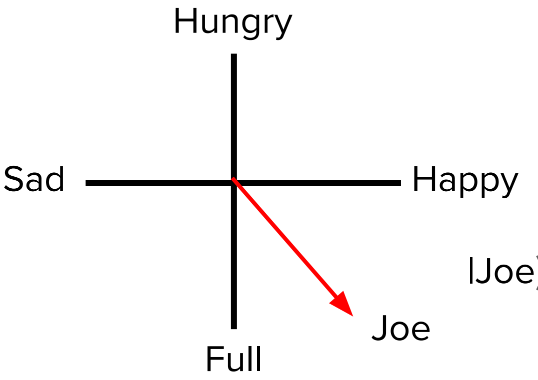
Models of Quantum Computing & Postulates 1 and 2

General Math Ideas

- Complex numbers
- Inner and Outer Products
- Vectors
- Matrix Multiplication
- Dagger



$$[A^{\dagger}]_{ij} = [A]^{*}_{ji}$$

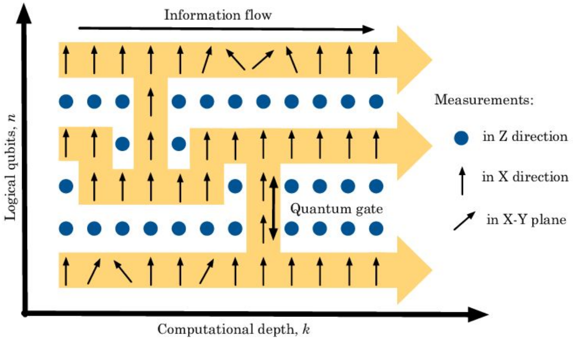


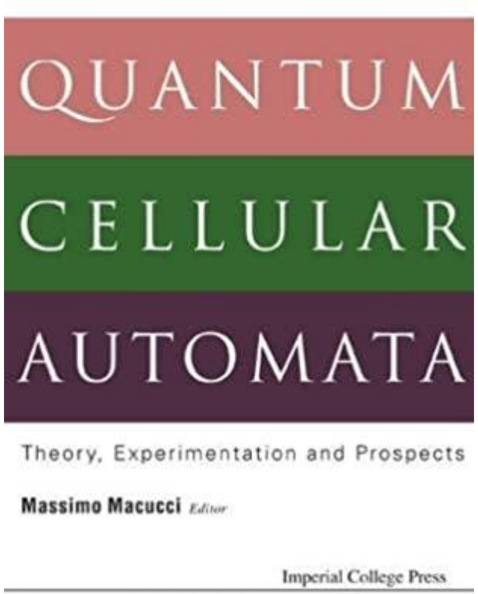
$$|Joe\rangle = \begin{pmatrix} .5 \\ 1 \end{pmatrix}$$

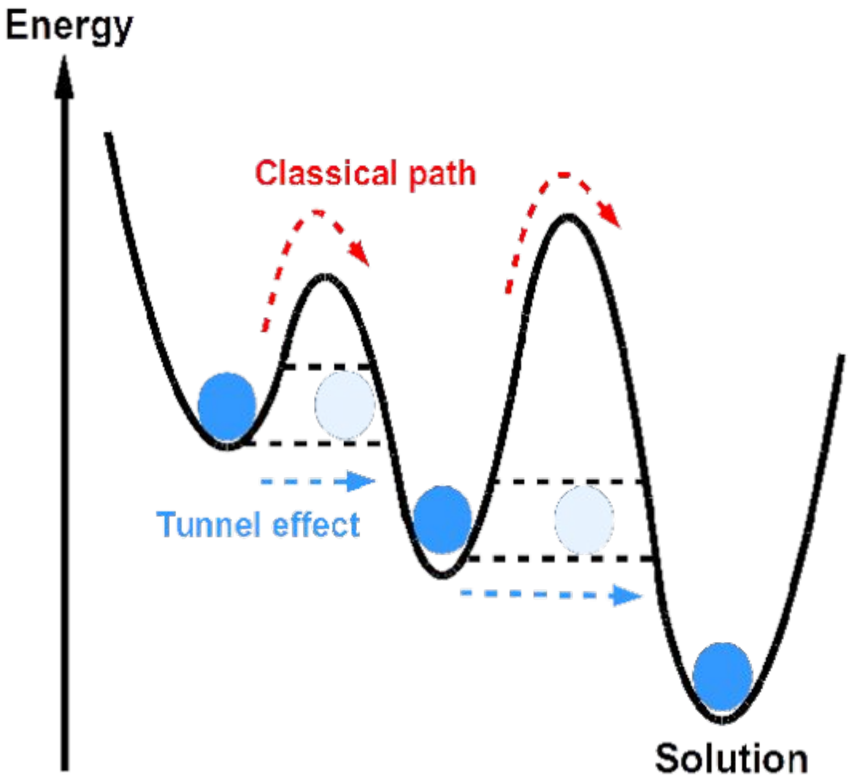
Models of Quantum Computing

Models of quantum computing

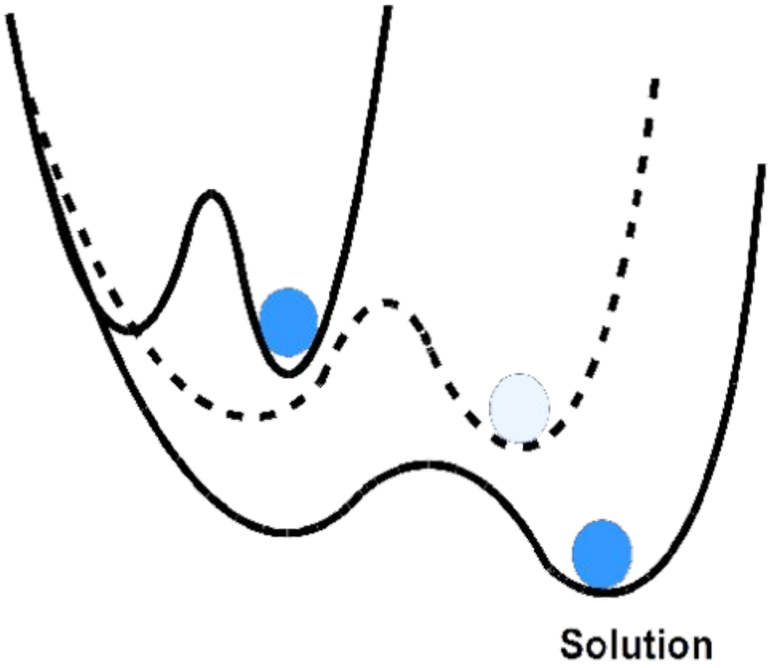
- Quantum circuit model
- Adiabatic quantum computer
- Quantum Turing machine
- One-way quantum computer
- Various quantum cellular automata
- Boson sampling







Quantum Tunnelling



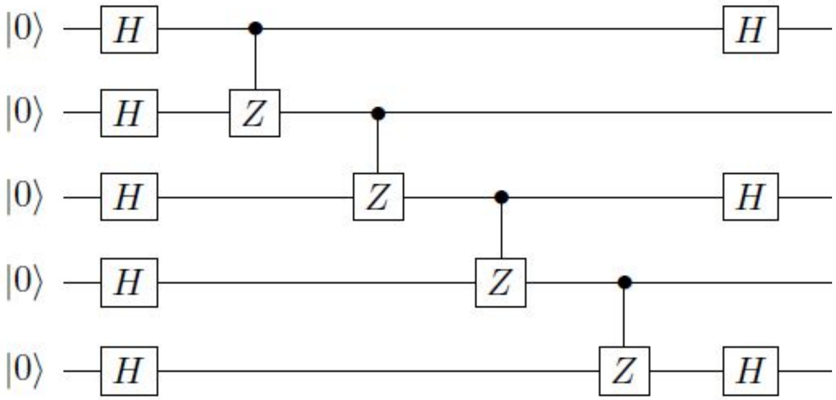
Adiabatic evolution

Quantum circuit model

A model of quantum computation that uses quantum circuits as units of computation.

In the quantum circuit model, we have:

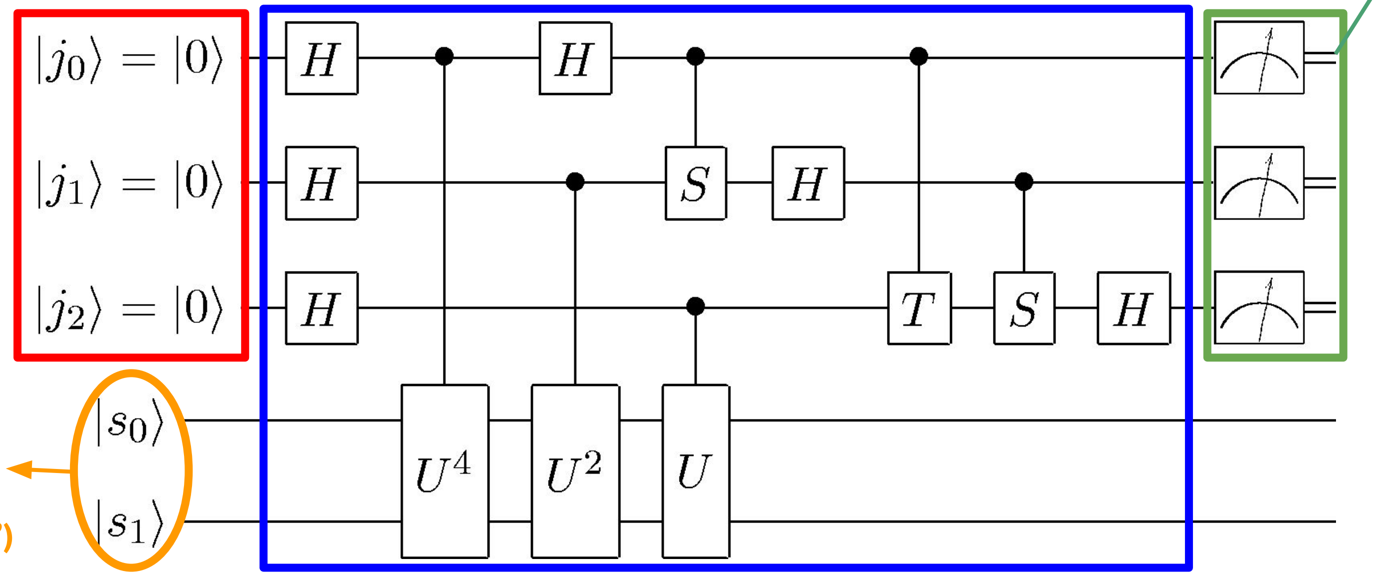
- **Qubits:** fundamental storage unit of quantum information
  - Classical analog: bits
- **Quantum Gates:** How we manipulate information in the qubits
  - Classical analog: logic gates



What is a quantum circuit?

- Stage 1: State Preparation
- Stage 2: Evolution
- Stage 3: Measurement

Double line represents <u>classical information</u>! (in bits)



Ancilla qubits (Aid computation that can be "thrown away")

Postulates of Quantum Mechanics

The Four Postulates (in brief)

1. The state of a quantum system is specified by its **state vector**
2. The evolution of a closed quantum system is described by a **unitary transformation**
3. Quantum measurements are described by a collection of **measurement operators** $\{P_m\}$
4. **Composite systems** are the tensor product of the sub-spaces

See Nielsen and Chuang for more details

Postulate 1: State Vector

Postulate 1

Associated to any isolated physical system is a complex vector space with inner product (that is, a Hilbert space) known as the state space *of the system. The system is completely described by its* state vector*, which is a unit vector in the system's state space.*

Postulate 1: Breakdown

“Associated to any isolated physical system”

- We are considering a closed system
- For example, a qubit with no noise

Postulate 1: Breakdown

“is a complex vector space with inner product (that is, a Hilbert space) known as the state space of the system”

- Hilbert space is essentially a space that can have inner products (like R3 space)

- So we know that the space a quantum system exists in must have a defined inner product

Postulate 1: Breakdown

“The system is completely described by its state vector, which is a unit vector in the system’s state space.”

- If you are given the state vector (“position” in state space), you know everything there is to know about the qubit

- However, you usually do not know which specific state vector applies to a quantum system until you make a measurement

State Vector

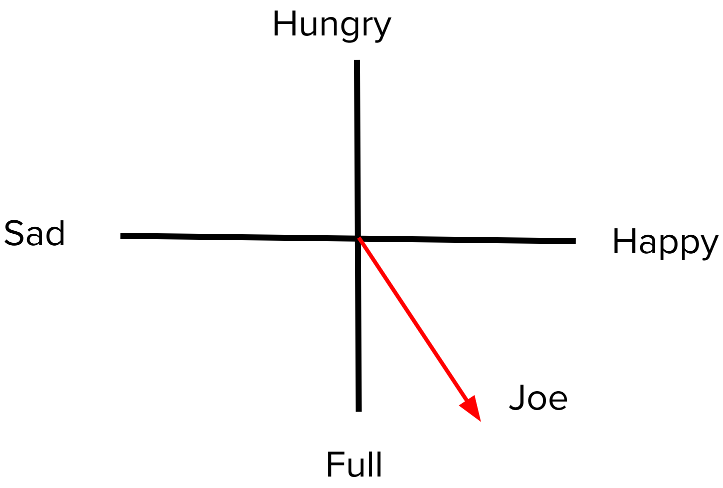
- The particular condition something is in at a certain time

$$|\psi\rangle$$

- Examples
  - A classical bit can be in a 0 state or 1 state
  - Alice can be hungry or full
  - Bob can be 5’9”, 5’10”, 5’11”, etc

State Space

- If you consider a state to be a vector, the space is all possible linear combinations of these state vectors
- You can think of a space as “the graph you can plot a state on”
- Example:



$$|\text{Joe}\rangle = \begin{pmatrix} .5 \\ 1 \end{pmatrix}$$

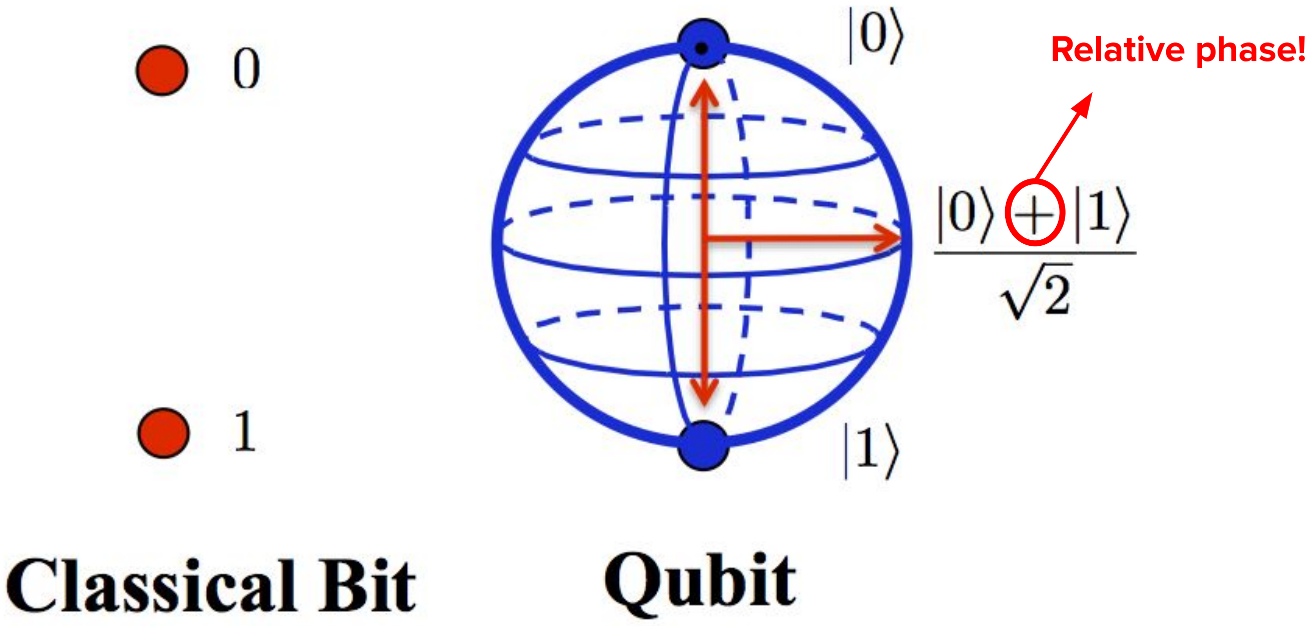
Classical Bit vs. Qubit

- A classical bit is anything that stores a 0 or 1
    - Circuit component with low or high voltage
    - Row of coins that are either heads up or tails up
    - Anything that takes on two states will work

- A qubit is anything that stores a $|0\rangle$, $|1\rangle$, or a superposition of the two

Qubit States

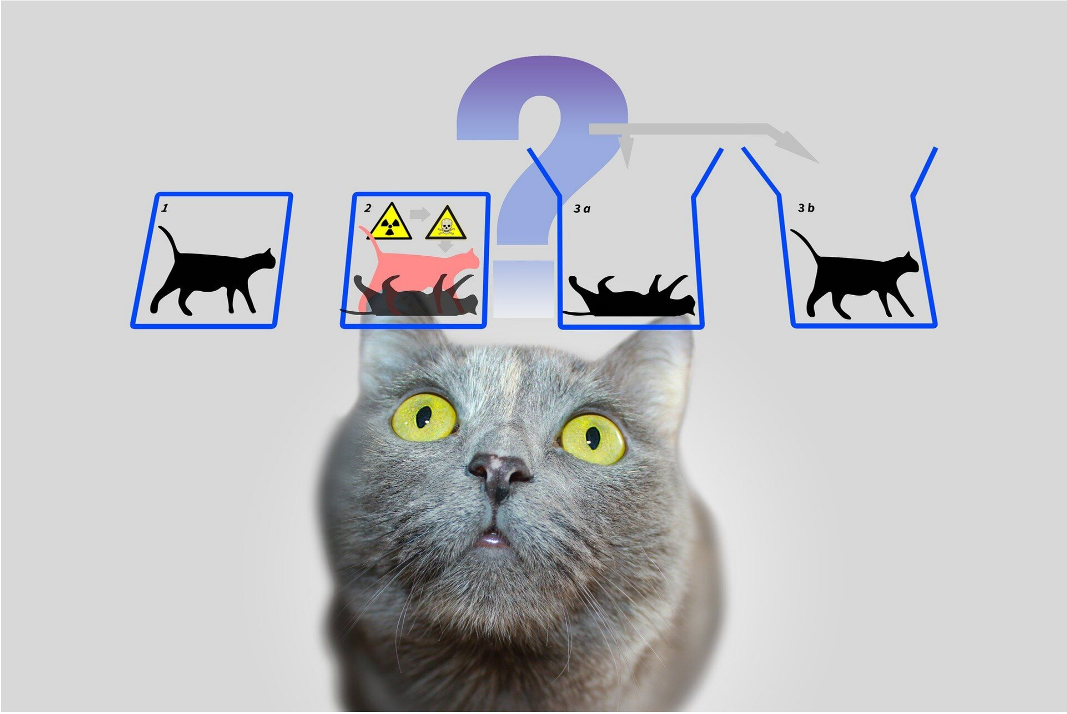
A Qubit is a unit of information



Superposition

- For simplicity, we consider this to mean a “combination”
- For example, a superposition of $|0\rangle$ and $|1\rangle$ is some “combination” of the two
- If I measure it, I have some probability of seeing $|0\rangle$, and some probability of seeing $|1\rangle$

Schrodinger’s Cat



Bloch Sphere

Visualization of a qubit's state

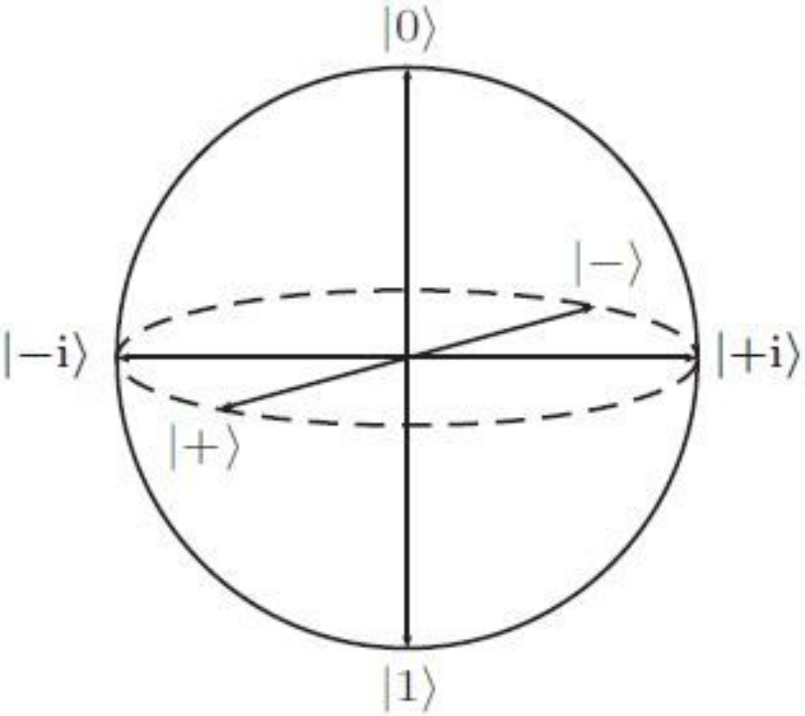
$$\left.\begin{matrix}|0\rangle \\ |1\rangle\end{matrix}\right\}$$ Computational Basis (Z Basis)

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$|+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$

$$|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$



Bloch Sphere

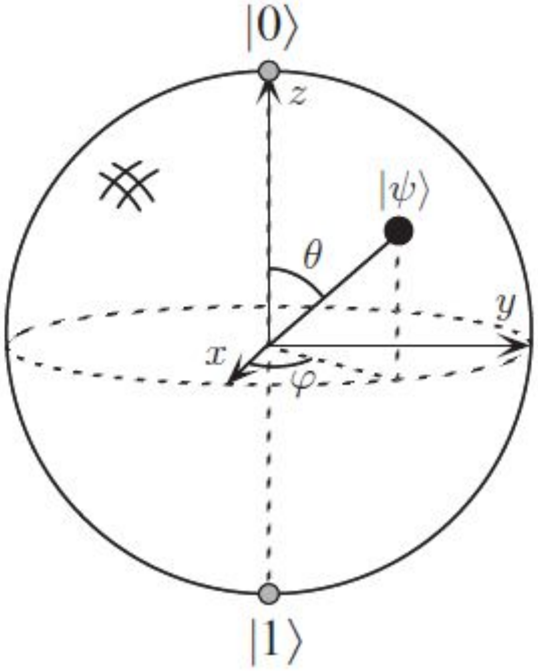
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\psi\rangle = cos\frac{\theta}{2}|0\rangle + e^{i\phi}sin\frac{\theta}{2}|1\rangle$$

$$|\psi\rangle = e^{i\gamma}(cos\frac{\theta}{2}|0\rangle + e^{i\phi}sin\frac{\theta}{2}|1\rangle)$$

Global Phase

Relative Phase



From N&C

Plotting on a Bloch Sphere

https://www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/blochsphere/blochsphere.html

Classical Bit vs. Qubit

- A classical bit is anything that stores a 0 or 1
    - Circuit component with low or high voltage
    - Row of coins that are either heads up or tails up
    - Anything that takes on two states will work

- A qubit is anything that stores a $|0\rangle$, $|1\rangle$, or a superposition of the two
    - Atomic/particle spin
    - Charge, current, or energy of a superconducting circuit
    - Light polarization
    - Can you think of other examples?

Dirac Notation for States

- States are represented using kets like $|\Psi\rangle$
- The 0 state looks like $|0\rangle$ and the 1 state is $|1\rangle$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |\alpha|^2 + |\beta|^2 = 1$$

- We can write this as a vector in the $|0\rangle$, $|1\rangle$ basis like this:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$|\Psi\rangle = |0\rangle$ $\qquad$ $|\Psi\rangle = |1\rangle$

Sanity check:
$\langle 0|1\rangle = \langle 1|0\rangle = 0$
$\langle 1|1\rangle = \langle 0|0\rangle = 1$

Born Rule

- Probability of a given outcome is square of magnitude of α or β

$$
\begin{aligned}
\langle\psi|\psi\rangle =&(\alpha^* \langle 0| + \beta^* \langle 1|)(\alpha|0\rangle + \beta|1\rangle) \\
=&\alpha^*\alpha\langle 0|0\rangle + \alpha^*\beta\langle 0|1\rangle + \beta^*\alpha\langle 1|0\rangle + \beta^*\beta\langle 1|1\rangle \\
=&|\alpha|^2 + |\beta^2| = 1
\end{aligned}
$$

- Probability of observing $|0\rangle$ is $|\alpha|^2$ and vice versa
- So if we have 1 $|0\rangle$ + 0 $|1\rangle$ then we measure $|0\rangle$ 100% of the time

Dirac Notation with Superposed States

- What if we had some state that was superposed?

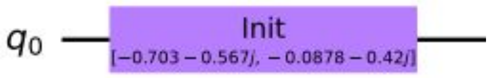
$$|\Psi\rangle = 0.8\ |0\rangle + 0.6\ |1\rangle$$

$$|0.8|^2 + |0.6|^2 = 1$$

- If we measure in the $|0\rangle$, $|1\rangle$ basis
- 64% of the time we observe $|0\rangle$
- 36% of the time we observe $|1\rangle$

Stage 1: State Preparation

Customized Initialization:



- Initialize the state of our computational (+ ancilla) qubits in the desired quantum state
    - Usually in the $|0\rangle$ state (assumed for all circuits in this course, unless otherwise specified)

$$
\boxed{\begin{aligned} |j_0\rangle &= |0\rangle \\ |j_1\rangle &= |0\rangle \\ |j_2\rangle &= |0\rangle \end{aligned}}
$$

Postulate 2: Evolution

Evolution

- Ideally, we would like a quantum computer to do something
- Postulate 1 says we are allowed to have any state $|\psi\rangle = \boldsymbol{\alpha}|\mathbf{0}\rangle + \boldsymbol{\beta}|\mathbf{1}\rangle$ as long as its normalized

$$\langle\psi|\psi\rangle = |\alpha|^2 + |\beta|^2 = 1$$

- So to **change** the state of our qubit, we need to guarantee it **remains normalized**

Changing the State of our Qubits

- Formally, we change the state of our qubits with “operators” that are represented by matrices

$$|\psi'\rangle = U|\psi\rangle$$

- A matrix U will preserve normalization (norm preservation property) if

$$\langle\psi'|\psi'\rangle = \langle\psi|U^\dagger U|\psi\rangle = \langle\psi|\psi\rangle$$

Reversibility

- We can **preserve the norm** if we require that U is **unitary**

$$U^{+}U = UU^{+} = I$$

$$U^{-1} = U^{+}$$

- This means that **every operation has to be reversible**
  - Can Command-Z

Postulate 2

The evolution of a <u>closed</u> quantum system is described by a <u>unitary transformation</u>. That is, the state $|\psi\rangle$ *of the system at time* t_1 *is related to the state* $|\psi'\rangle$ *of the system at a time* t_2 *by a unitary operator* U *which depends only on the times* t_1 *and* t_2,

$$|\psi'\rangle = U\,|\psi\rangle$$

Postulate 2: Breakdown

“The evolution of a closed quantum system”

- Again, we consider a system without outside interference or noise
- No interaction with the surrounding environment

Postulate 2: Breakdown

“[the evolution] is described by a unitary transformation”

- A unitary transformation is a transformation described by a unitary matrix
- Recall the examples of unitary matrices from earlier

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Pauli X
Bit Flip

Pauli Y

Pauli Z
Phase Flip

$a|0\rangle + b|1\rangle \rightleftarrows b|0\rangle + a|1\rangle$

$a|0\rangle + b|1\rangle \rightleftarrows a|0\rangle - b|1\rangle$

Quantum Gates vs Logic Gates

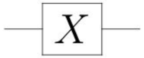
- Quantum gates can be represented as Unitary Matrices ($U^\dagger U = I$)
    - Some special quantum gates (i.e. Pauli and Hadamard) are Hermitian ($A = A^\dagger$)
- All quantum gates have to be reversible
- Classical logic gates are not necessarily reversible and information is lost as heat

Quantum Gates

- Several quantum gates to help control qubits
- Analogous to classical gates for bits

1 Qubit Gates	2 Qubit Gates	3 Qubit Gates
X Gate	CNOT Gate	Toffoli Gate
Y Gate	CZ Gate	Fredkin Gate
Z Gate	SWAP Gate	
H Gate		

Pauli X



Matrix X = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

- Pauli X Gate (NOT) flips a qubit state

$$X\,(\mathrm{a}|0\rangle + \mathrm{b}|1\rangle) = \mathrm{b}|0\rangle + \mathrm{a}|1\rangle$$

Pauli X Operation on States

Matrix X = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

- Pauli X Gate (NOT) flips a qubit state

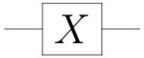
$$X\,(\mathrm{a}|0\rangle + \mathrm{b}|1\rangle) = \mathrm{b}|0\rangle + \mathrm{a}|1\rangle$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$X\,|0\rangle =$$

$$X\,|1\rangle =$$

Pauli X



Matrix $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

- Pauli X Gate (NOT) flips a qubit state

$$X\,(\mathrm{a}|0\rangle + \mathrm{b}|1\rangle) = \mathrm{b}|0\rangle + \mathrm{a}|1\rangle$$

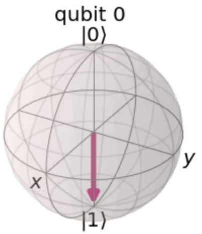
$$X\,|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

$$X\,|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

Truth Table

$\lvert\psi\rangle$	$X\,\lvert\psi\rangle$
$\lvert 0\rangle$	$\lvert 1\rangle$
$\lvert 1\rangle$	$\lvert 0\rangle$

Bloch Sphere $X\,|0\rangle$



Pauli X cont.

X can be calculated using

$$|0\rangle\langle 1| + |1\rangle\langle 0| = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \boldsymbol{X}$$

Corresponding Eigenvalues

Eigenstates of X:

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \longrightarrow +1$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \longrightarrow -1$$

Pauli X Eigen-Decomposition Check

$A\,|\Psi\rangle = \lambda\,|\Psi\rangle$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \lambda = +1$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad \lambda = -1$$

$$A = X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Pauli Y

Matrix $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

- Similar to Pauli X but adds complex $\pm \boldsymbol{i}$

$$Y\,(a|0\rangle + b|1\rangle) = -i\,(b|0\rangle) + i\,(a|1\rangle)$$

$$Y = -i\,|0\rangle\langle 1| + i\,|1\rangle\langle 0| =$$

Pauli Y

Matrix Y = $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

- Similar to Pauli X but adds complex $\pm\, \boldsymbol{i}$

$Y\,(a|0\rangle + b|1\rangle) = \text{-}i\,(b|0\rangle) + i\,(a|1\rangle)$

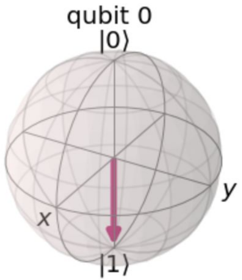
$$Y = -i\,|0\rangle\langle 1| + i\,|1\rangle\langle 0|$$

Truth Table

$\|\psi\rangle$	$Y\,\|\psi\rangle$
$\|0\rangle$	$i\,\|1\rangle$
$\|1\rangle$	$-\,i\,\|0\rangle$

Bloch Sphere $Y\,|0\rangle$

Note: Global phase i is not shown on bloch sphere



Eigenstates

$|{+i}\rangle = \sqrt{\frac{1}{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$ $\lambda = +1$

$|{-i}\rangle = \sqrt{\frac{1}{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$ $\lambda = -1$

Pauli Z

Matrix $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

- Does not change $|0\rangle$, but adds $e^{i\pi}$ = -1 phase to $|1\rangle$

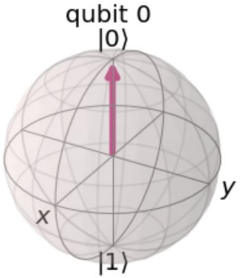
$$Z\,(a|0\rangle + b|1\rangle) = a|0\rangle - b|1\rangle$$

$$Z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

Truth Table

$\vert\psi\rangle$	$Z\,\vert\psi\rangle$
$\vert 0\rangle$	$\vert 0\rangle$
$\vert 1\rangle$	$-\vert 1\rangle$

Bloch Sphere $Z\,|0\rangle$



Eigenstates $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\lambda = +1$

$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\lambda = -1$

Important properties of Pauli matrices

- $Y = iXZ$, $Z=iYX$, $X=iZY$
- $X^2 = I$, $Y^2 = I$, $Z^2 = I$
- Anti-commute with each other: $XZ = -ZX$, $XY = -YX$, $YZ = -ZY$

1 Qubit Gates - Pauli Matrices

All Pauli Gates are Unitary and Hermitian

Operator	Truth Table	Matrix	Bra-ket Notation	Eigenstates
Pauli-X (X) $-\boxed{X}-$	$\begin{array}{c\|c} \|\psi\rangle & X\|\psi\rangle \\ \hline \|0\rangle & \|1\rangle \\ \|1\rangle & \|0\rangle \end{array}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$X = \|0\rangle\langle 1\| + \|1\rangle\langle 0\|$	$\|+\rangle$, $\|-\rangle$
Pauli-Y (Y) $-\boxed{Y}-$	$\begin{array}{c\|c} \|\psi\rangle & Y\|\psi\rangle \\ \hline \|0\rangle & i\|1\rangle \\ \|1\rangle & -i\|0\rangle \end{array}$	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	$Y = -i\ \|0\rangle\langle 1\| + i\ \|1\rangle\langle 0\|$	$\|+i\rangle$, $\|-i\rangle$
Pauli-Z (Z) $-\boxed{Z}-$	$\begin{array}{c\|c} \|\psi\rangle & Z\|\psi\rangle \\ \hline \|0\rangle & \|0\rangle \\ \|1\rangle & -\|1\rangle \end{array}$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$Z = \|0\rangle\langle 0\| - \|1\rangle\langle 1\|$	$\|0\rangle$, $\|1\rangle$

1 Qubit Gates - Hadamard

Matrix $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

- Transforms the state of the qubit between the X and Z bases
- Fundamental quantum gate
  - No classical analog
- Allows for the creation of a superposition of $|0\rangle$ and $|1\rangle$

$$H\,(\mathrm{a}|0\rangle + \mathrm{b}|1\rangle) = \mathrm{a}|+\rangle + \mathrm{b}|-\rangle$$

Truth Table

$\lvert\psi\rangle$	$H\lvert\psi\rangle$
$\lvert 0\rangle$	$\lvert +\rangle$
$\lvert 1\rangle$	$\lvert -\rangle$

H Gate Operation on States

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$H\,|0\rangle =$$

$$H\,|-\rangle =$$

H Gate Transformations

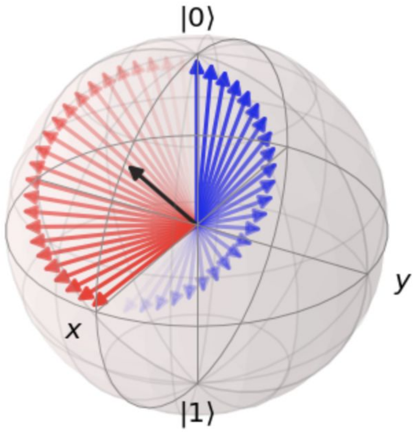
Property: $HH = I$ (Identity) Hermitian $-\boxed{H}-\boxed{H}- \;\Rightarrow\; ——$

Possible Transformations

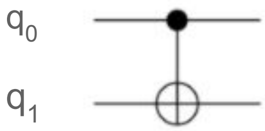
$$H|0\rangle = |+\rangle \qquad H|+\rangle = |0\rangle$$

$$H|1\rangle = |-\rangle \qquad H|-\rangle = |1\rangle$$

Bloch Sphere



2 Qubit Gate - CNOT



Matrix $CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

- q_0 is the control qubit; q_1 is the target qubit
- If the control qubit is 0, do nothing to the target qubit
- If the control qubit is 1, apply X to the target qubit

Truth Table

$\lvert\psi_0\psi_1\rangle$	$CNOT\,\lvert\psi_0\psi_1\rangle$
$\lvert 00\rangle$	$\lvert 00\rangle$
$\lvert 01\rangle$	$\lvert 01\rangle$
$\lvert 10\rangle$	$\lvert 11\rangle$
$\lvert 11\rangle$	$\lvert 10\rangle$

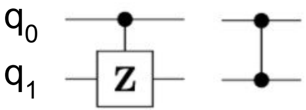
$CNOT$ = Controlled Not = CX

I

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

X

2 Qubit Gate - CZ



Matrix $CZ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

- q_0 is the control qubit; q_1 is the target qubit
- If the control qubit is 0, do nothing to the target qubit
- If control qubit is 1, apply Z to the target qubit (-1 phase change only to $|11\rangle$)

Truth Table

$\|\psi_0\psi_1\rangle$	$CZ\,\|\psi_0\psi_1\rangle$
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|10\rangle$
$\|11\rangle$	-$\|11\rangle$

$CZ =$ Controlled Z

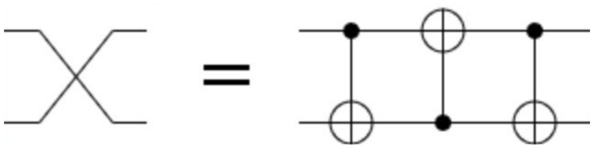
$$I \quad \begin{bmatrix} \boxed{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \boxed{\begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix}} \end{bmatrix} \quad Z$$

2 Qubit Gate - SWAP

Matrix $SWAP = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- Swaps the qubits!

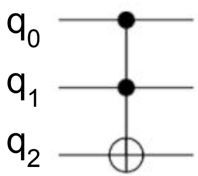
$SWAP$ can be implemented with 3 $CNOT$s:



Truth Table

$\lvert\psi_0\psi_1\rangle$	$CZ\,\lvert\psi_0\psi_1\rangle$
$\lvert 00\rangle$	$\lvert 00\rangle$
$\lvert 01\rangle$	$\lvert 10\rangle$
$\lvert 10\rangle$	$\lvert 01\rangle$
$\lvert 11\rangle$	$\lvert 11\rangle$

3 Qubit Gate - Toffoli



- CNOT with 2 controls

$$TOFF = CCNOT = CCX$$

I

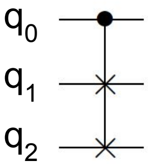
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

CX

Truth Table

$\|\psi_0\psi_1\psi_2\rangle$	$TOFF\,\|\psi_0\psi_1\psi_2\rangle$
$\|000\rangle$	$\|000\rangle$
$\|001\rangle$	$\|001\rangle$
$\|010\rangle$	$\|010\rangle$
$\|011\rangle$	$\|011\rangle$
$\|100\rangle$	$\|100\rangle$
$\|101\rangle$	$\|101\rangle$
$\|110\rangle$	$\|111\rangle$
$\|111\rangle$	$\|110\rangle$

3 Qubit Gate - Fredkin



- Controlled SWAP Gate
- q_0 needs to be 1 in order to apply SWAP on q_1 and q_2

$$
I \quad
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\quad SWAP
$$

Truth Table

$\vert\psi_0\psi_1\psi_2\rangle$	$CSWAP\,\vert\psi_0\psi_1\psi_2\rangle$
$\vert 000\rangle$	$\vert 000\rangle$
$\vert 001\rangle$	$\vert 001\rangle$
$\vert 010\rangle$	$\vert 010\rangle$
$\vert 011\rangle$	$\vert 011\rangle$
$\vert 100\rangle$	$\vert 100\rangle$
$\vert 101\rangle$	$\vert 110\rangle$
$\vert 110\rangle$	$\vert 101\rangle$
$\vert 111\rangle$	$\vert 111\rangle$

Postulate 2: Breakdown

“That is, the state $|\psi\rangle$ of the system at time t_1 is related to the state $|\psi'\rangle$ of the system at a time t_2 by a unitary operator U which depends only on the times t_1 and t_2”

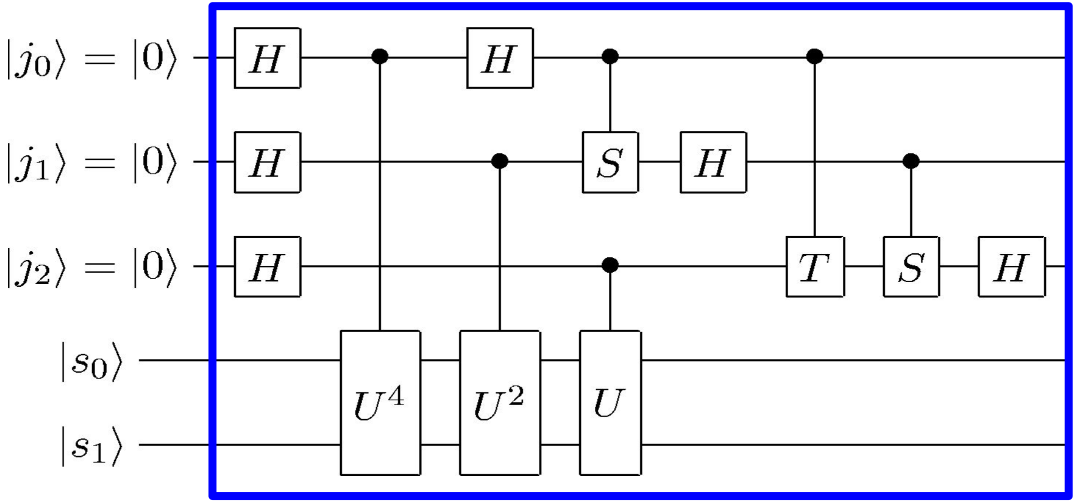
- Mostly a rephrasing of the first part
- The transformation is dependent on when the process starts and stops
- Transformations can be **continuous** (not discrete)

$$H(T|0\rangle)$$

Stage 2: State Evolution

- Propagate through the quantum circuits and carry out the computation.
    - Just like classical computers, we append logic gates to form a circuit and perform an algorithm
    - Quantum circuits have **quantum gates**

Chronological order!



Demos!

Introducing IBM Qiskit

Qiskit

Elements for building a quantum future

Qiskit is an open source SDK (software development kit) for working with quantum computers at the level of pulses, circuits and application modules.

https://qiskit.org/documentation/the_elements.html

Real devices vs Simulators

Q: Why do my results from **real devices** differ from my results from the **simulator**?

A: The simulator runs jobs as though is was in an ideal environment; one without noise or decoherence. However, when jobs are run on the real devices there is noise from the environment and decoherence, which causes the qubits to behave differently than what is intended.

Let’s Visualize Some Qubit States!

Demo Instructions

1. Download the file from Sakai ➔ Resources ➔ Demos, and save it wherever you prefer
2. Open Terminal/Command Prompt and go to the file directory by typing `cd name_of_directory`
3. Activate your Anaconda environment using `conda activate name_of_environment` in Terminal/Command Prompt
4. Open the directory in Jupyter Notebook by typing `jupyter notebook` in Terminal/Command Prompt
5. Select the file, and go to **File ➔ Rename** and **remove the .txt extension** from the end
6. Now, the file name should just have an .ipynb extension that can be opened using Jupyter Notebook

Exercises (optional)

- (Exercise 1) Practice plotting Bloch sphere in Qiskit
- (Exercise 2) Practice Eigen-decompositions
- (Exercise 3) Practice unitary transformations (especially Paulis) on paper and on the Bloch sphere in Qiskit

- Prove that the Pauli Matrices are unitary
- (In class) Prove norm preservation property of unitary matrices
- Prove properties of Pauli matrices