# E0 230 CMO Assignment 1

## Instructions

1. Read all instructions before starting the assignment.
   Read them again before submitting the assignment.
2. Submissions that do not adhere to the instructions will be given **zero marks**.
3. This is an **individual assignment**, all work including code should be your own.
4. **Do not use AI generated code.**
5. Any form of academic dishonesty will be treated as per the IISc and CSA academic integrity policy.
6. You will submit a **Microsoft Office Form**, a `PDF` file, a `csv` file and a **single** `.py` file for this assignment. Upload the files directly; **do not zip them**.
   (a) Microsoft Office Form link: `https://forms.cloud.microsoft/r/WdwhgNGPcP`. Only one response per person. **Ensure you click 'Save my response to edit' to be able to edit your answer until deadline.**
   (b) Name the `pdf` file **CMO2025A1_vwxyz.pdf** where vwxyz is your five-digit SR number. The pdf file should be generated using the **LaTeX template provided in the Class Materials folder** in the Files Section of the Team. Your pdf should include all values, justifications, and graphs (if any) that you are asked to report.
   (c) Name the `.py` file **CMO2025A1_vwxyz.py** where vwxyz is your five-digit SR number. Your python file should contain all code that you used to solve this assignment. **Only `.py` files are accepted**; no notebooks.
   (d) Name the `.csv` file **CMO2025A1_vwxyz.csv** where vwxyz is your five-digit SR number. Your csv file should contain only numbers and new line characters.
7. We will not open or evaluate any other file submitted.
8. Incomplete submissions (without the `pdf` or `.py` file) will not evaluated.
9. This assignment is due by 23:59, 10 September.
10. Late submissions will incur a 20% penalty per day.

## Oracle Instructions

1. Use the Oracle in a Unix-like terminal.
2. Windows users must run it via Windows Subsystem for Linux (WSL).
3. Run all code for this assignment in a virtual environment with Python version 3.10.
4. Unzip the Oracle into the directory you are working in.
5. Do not modify the `oracle_2025A1` folder.
6. Place your `.py` file and the `oracle_2025A1` folder in the same directory.
7. Import the Oracle using `from oracle_2025A1 import oq1,oq2f,oq2g,oq3`.
8. Call the Oracle:
   - `Q_a,Q_b,Q_c,Q_d,Q_e = oq1(12345)`
   - `f = oq2f(12345,x)` and `g = oq2g(12345,x)` where `x` is a $5 \times 1$ NumPy array.
   - `A,b = oq3(12345)`
9. Set up the Oracle by 23:59, 6 September.
10. No support will be provided for Oracle issues after this deadline.

## Question 1

Consider the quadratic function

$$f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T Q\mathbf{x} + \mathbf{b}^T\mathbf{x}$$

where $Q$ is a symmetric positive definite matrix and $\mathbf{b}$ is a vector in $\mathbb{R}^2$. Fix $b = (1, 1)^T$. Query the oracle with your five digit SR number to obtain five different Q matrices.

Report:

1. Minimize $f(x)$ using exact line search. Report $\mathbf{x}^*$ and $f(\mathbf{x}^*)$ for each $Q$. (3 marks)
2. Analytically solve for $x^*$ and plot $||x^{(k)} - x^*||$. (3 marks)
3. Explain the convergence behavior in part 2 with sufficient justification. (4 marks)

## Question 2

You are given access to an oracle $\mathcal{O}$ that returns $f(x)$ and an oracle $\mathcal{O}'$ that returns $\nabla f(x))$ when queried with your five digit SR number and $x \in \mathbb{R}^5$. Perform gradient descent using line search using the following techniques:

1. Armijo condition

2. Armijo-Goldstein condition
3. Wolfe condition
4. Backtracking

REPORT:

1. What condition can be used to decide when to stop gradient descent? (1 mark)
2. The $x^*$ and $f(x^*)$ from each method after 1000 iterations or convergence, whichever is earlier. (3 marks)
3. A single plot of the step sizes taken by each method at each iteration. (2 marks)
4. How many oracle calls did each method take? (2 marks)
5. Compare the different line search methods. (1 mark)

## Question 3

In this problem, you are tasked with solving a linear system $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{A}$ is in $\mathbb{R}^{m \times n}$. Pose this as a convex optimization problem. Use any line search algorithm you coded up for Question 2 to solve this linear system.

REPORT:

1. State the optimization problem with appropriate justification. (3 marks)
2. Query the oracle with your five digit SR number to obtain $\mathbf{A}$ and $\mathbf{b}$. Solve the linear system using the optimization problem in part 1 and submit the column vector $\mathbf{x}^*$ as a csv file. (2 marks)
3. Comment on the solution to the optimization problem if (1.5 marks)
   (a) $m < n$
   (b) $m = n$
   (c) $m > n$
4. Let $\mathbf{A}$ be full rank and square. Compare the complexity of: (2 marks)
   (a) Using matrix inversion to solve $\mathbf{Ax} = \mathbf{b}$
   (b) Using the optimization problem stated in part 1 to solve $\mathbf{Ax} = \mathbf{b}$
5. Generate random $\mathbf{A} \in \mathbb{R}^{m \times m}$ and $\mathbf{b} \in \mathbb{R}^m$ for several different $m$ in range $[2^1, 2^{16}]$. Tabulate the time it takes to solve the linear system using matrix inversion and optimization. (1.5 marks)