19CSE453 – Natural Language Processing

Spelling Correction Edit Distance

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Spelling Correction

I am writing this email on behaf of ...

The user typed 'behaf'.

Which are some close words?

behalf

behave ....

Isolated word error correction

Pick the one that is closest to 'behaf'

How to define 'closest'? Need a distance metric

The simplest metric: edit distance

Edit Distance

The minimum edit distance between two strings is the minimum number of editing operations.

- **Insertion**
- **Deletion**
- **Substitution**

Minimum Edit Distance

Example

Edit distance from ‘intention’ to ‘execution’

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
```

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
d s s   i s
```

If each operation has a cost of 1

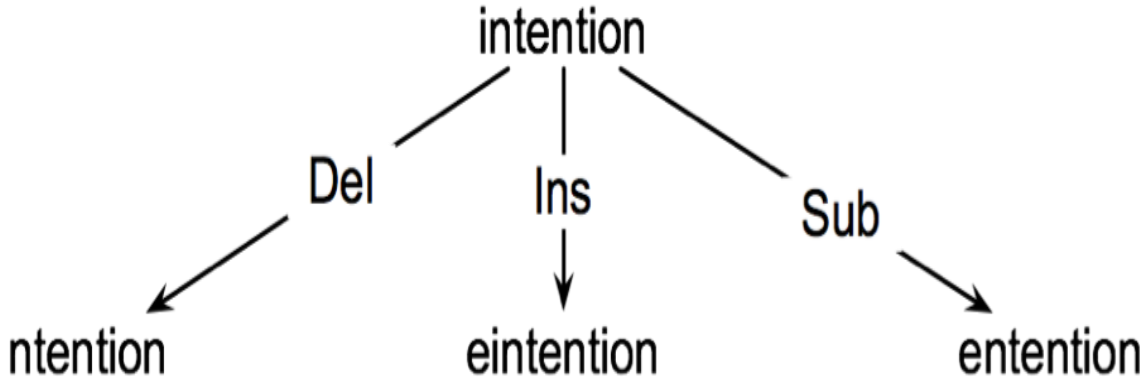
- **Distance between these is 5**

If substitution costs 2 (alternate version)

- **Distance between these is 8**

How to find the Minimum Edit Distance?

- Searching for a path (sequence of edits) from the *start string* to the *final string*:
  - **Initial state:** the word we are transforming
  - **Operators:** insert, delete, substitute
  - **Goal state:** the word we are trying to get to
  - **Path cost:** what we want to minimize: the number of edits



Minimum Edit as Search

How to navigate?

The space of all edit sequences is huge

Lot of distinct paths end up at the same state

Don’t have to keep track of all of them

Keep track of the shortest path to each state

Defining Minimum Edit Distance Matrix

For two strings

X of length n, Y of length m

We define $D(i, j)$

the edit distance between $X[1..i]$ and $Y[1..j]$

i.e., the first i characters of X and the first j characters of Y

Thus, the edit distance between X and Y is $D(n, m)$

Computing Minimum Edit Distance

Dynamic Programming

A tabular computation of $D(n, m)$

Solving problems by combining solutions to subproblems

Bottom-up

Compute $D(i, j)$ for small i, j
Compute larger $D(i, j)$ based on previously computed smaller values

Compute $D(i, j)$ for all i and j till you get to $D(n, m)$

Dynamic Programming Algorithm (Levenshtein)

Initialization

$D(i,0) = i$

$D(0,j) = j$

Recurrence Relation:

For each $i = 1 \ldots M$

For each $j = 1 \ldots N$

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } X(i) \neq Y(j) \\ 0; & \text{if } X(i) = Y(j) \end{cases} \end{cases}$$

Termination:

$D(N,M)$ is distance

The Edit Distance Table

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1									
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

delete cost
Insert cost
Substitution cost

The Edit Distance Table

N	9	8	9	10	11	12	11	10	9	**8**
O	8	7	8	9	10	11	10	9	8	9
I	7	6	7	8	9	10	9	8	9	10
T	6	5	6	7	8	9	8	9	10	11
N	5	4	5	6	7	8	9	10	11	10
E	4	3	4	5	6	7	8	9	10	9
T	3	4	5	6	7	8	7	8	9	8
N	2	3	4	5	6	7	8	7	8	7
I	1	2	3	4	5	6	7	6	7	8
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

Computing Alignments

- Computing edit distance may not be sufficient for some applications
  - ➢We often need to align characters of the two strings to each other
- We do this by keeping a “backtrace”
- Every time we enter a cell, remember where we came from When we reach the end,
  - ➢Trace back the path from the upper right corner to read off the alignment

The Edit Distance Table

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1									
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

N	9									
O	8									
I	7									
T	6									
N	5									
E	4	3	4							
T	3	4	5							
N	2	3	4							
I	1	2	3							
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

Minimum Edit with Backtrace

n	9	↓ 8	↙←↓ 9	↙←↓ 10	↙←↓ 11	↙←↓ 12	↓ 11	↓ 10	↓ 9	↙ **8**
o	8	↓ 7	↙←↓ 8	↙←↓ 9	↙←↓ 10	↙←↓ 11	↓ 10	↓ 9	↙ **8**	← 9
i	7	↓ 6	↙←↓ 7	↙←↓ 8	↙←↓ 9	↙←↓ 10	↓ 9	↙ **8**	← 9	← 10
t	6	↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ 8	↙←↓ 9	↙ **8**	← 9	← 10	←↓ 11
n	5	↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ **8**	↙←↓ 9	↙←↓ 10	↙←↓ 11	↙↓ 10
e	4	↙ 3	← 4	↙← **5**	← **6**	← 7	←↓ 8	↙←↓ 9	↙←↓ 10	↓ 9
t	3	↙←↓ 4	↙←↓ **5**	↙←↓ 6	↙←↓ 7	↙←↓ 8	↙ 7	←↓ 8	↙←↓ 9	↓ 8
n	2	↙←↓ **3**	↙←↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙←↓ 8	↓ 7	↙←↓ 8	↙ 7
i	**1**	↙←↓ 2	↙←↓ 3	↙←↓ 4	↙←↓ 5	↙←↓ 6	↙←↓ 7	↙ 6	← 7	← 8
#	**0**	1	2	3	4	5	6	7	8	9
	#	**e**	**x**	**e**	**c**	**u**	**t**	**i**	**o**	**n**

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
```

Result of Backtrace

```
I N T E * N T I O N
| | | | | | | | | |
* E X E C U T I O N
```

Performance

Time

$O(nm)$

Backtrace $O(n+m)$

Exercise

1. Find the minimum edit distance of the string “benyam” to “ephrem”

Weighted Edit Distance, Other variations

Why to add weights to the computation?

Some letters are more likely to be mistyped.

Confusion Matrix for Spelling Errors

sub[X, Y] = Substitution of X (incorrect) for Y (correct)

X \ Y (correct)	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
a	0	0	7	1	342	0	0	2	118	0	1	0	0	3	76	0	0	1	35	9	9	0	1	0	5	0
b	0	0	9	9	2	2	3	1	0	0	0	5	11	5	0	10	0	0	2	1	0	0	8	0	0	0
c	6	5	0	16	0	9	5	0	0	0	1	0	7	9	1	10	2	5	39	40	1	3	7	1	1	0
d	1	10	13	0	12	0	5	5	0	0	2	3	7	3	0	1	0	43	30	22	0	0	4	0	2	0
e	388	0	3	11	0	2	2	0	89	0	0	3	0	5	93	0	0	14	12	6	15	0	1	0	18	0
f	0	15	0	3	1	0	5	2	0	0	0	3	4	1	0	0	0	6	4	12	0	0	2	0	0	0
g	4	1	11	11	9	2	0	0	0	1	1	3	0	0	2	1	3	5	13	21	0	0	1	0	3	0
h	1	8	0	3	0	0	0	0	0	0	2	0	12	14	2	3	0	3	1	11	0	0	2	0	0	0
i	103	0	0	0	146	0	1	0	0	0	0	6	0	0	49	0	0	0	2	1	47	0	2	1	15	0
j	0	1	1	9	0	0	1	0	0	0	0	2	1	0	0	0	0	0	5	0	0	0	0	0	0	0
k	1	2	8	4	1	1	2	5	0	0	0	0	5	0	2	0	0	0	6	0	0	0	4	0	0	3
l	2	10	1	4	0	4	5	6	13	0	1	0	0	14	2	5	0	11	10	2	0	0	0	0	0	0
m	1	3	7	8	0	2	0	6	0	0	4	4	0	180	0	6	0	0	9	15	13	3	2	2	3	0
n	2	7	6	5	3	0	1	19	1	0	4	35	78	0	0	7	0	28	5	7	0	0	1	2	0	2
o	91	1	1	3	116	0	0	0	25	0	2	0	0	0	0	14	0	2	4	14	39	0	0	0	18	0
p	0	11	1	2	0	6	5	0	2	9	0	2	7	6	15	0	0	1	3	6	0	4	1	0	0	0
q	0	0	1	0	0	0	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
r	0	14	0	30	12	2	2	8	2	0	5	8	4	20	1	14	0	0	12	22	4	0	0	1	0	0
s	11	8	27	33	35	4	0	1	0	1	0	27	0	6	1	7	0	14	0	15	0	0	5	3	20	1
t	3	4	9	42	7	5	19	5	0	1	0	14	9	5	5	6	0	11	37	0	0	2	19	0	7	6
u	20	0	0	0	44	0	0	0	64	0	0	0	0	2	43	0	0	4	0	0	0	0	2	0	8	0
v	0	0	7	0	0	3	0	0	0	0	0	1	0	0	1	0	0	0	8	3	0	0	0	0	0	0
w	2	2	1	0	1	0	0	2	0	0	1	0	0	0	0	7	0	6	3	3	1	0	0	0	0	0
x	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0	0
y	0	0	2	0	15	0	1	7	15	0	0	0	2	0	6	1	0	7	36	8	5	0	0	1	0	0
z	0	0	0	7	0	0	0	0	0	0	0	7	5	0	0	0	0	2	21	3	0	0	0	0	3	0

Weighted Minimum Edit Distance

Initialization:

$$D(0,0) = 0$$

$$D(i,0) = D(i-1,0) + del[x(i)]; \quad 1 < i \leq N$$

$$D(0,j) = D(0,j-1) + ins[y(j)]; \quad 1 < j \leq M$$

Recurrence Relation:

$$D(i,j) = \min \begin{cases} D(i-1,j) + del[x(i)] \\ D(i,j-1) + ins[y(j)] \\ D(i-1,j-1) + sub[x(i),y(j)] \end{cases}$$

Termination:

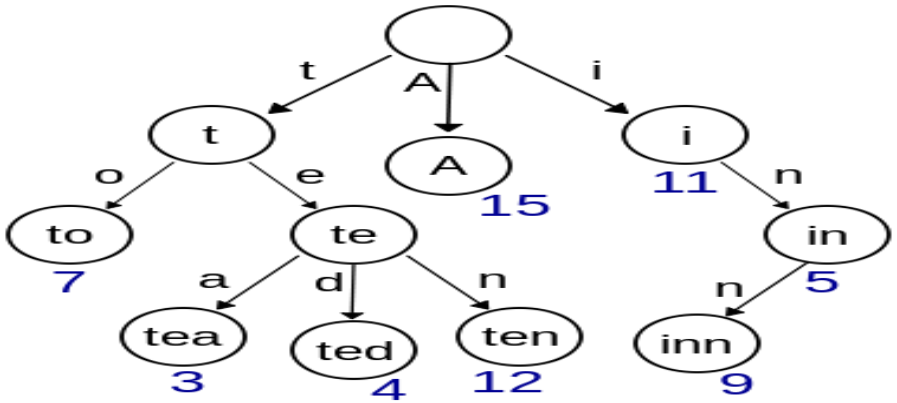
$$D(N,M) \text{ is distance}$$

How to find dictionary entries with smallest edit distance?

Naïve Method

Compute edit ditance from the query term to each dictionary term – an exhaustive search

Can be made efficient if we do it over a tree structure



How to find dictionary entries with smallest edit distance?

- Generate all possible terms with an edit distance <=2 (deletion + transpose + substitution + insertion) from the query term and search them in the dictionary.
- For a word of length 9, alphabet of size 36, this will lead to 114,324 terms to search for

How to find dictionary entries with smallest edit distance?

Symmetric Delete Spelling Correction

Generate terms with an edit distance ≤ 2 (deletes) from each dictionary term (offline)

Generate terms with an edit distance ≤ 2 (deletes) from the input terms and search in dictionary

Number of deletes within edit distance ≤ 2 for a word of length 9 will be 45

A further check is required to remove the false positives

Thank You