Resource Management Techniques

CAE - I

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CSE - 'C' sec.

Part B

6A. Given the LPP

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

subject to $x_1 + 4x_2 \leq 420$

$3x_1 + 2x_3 \leq 460$

$x_1 + 2x_2 + x_3 \leq 430$

$x_1, x_2, x_3 \geq 0$

By introducing nonnegative slack variables s_1, s_2, s_3, the standard form of the LPP becomes

Maximize $Z = 3x_1 + 2x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$

subject to $x_1 + 4x_2 + 0x_3 + s_1 + 0s_2 + 0s_3 = 420$

$3x_1 + 0x_2 + 2x_3 + 0s_1 + s_2 + 0s_3 = 460$

$x_1 + 2x_2 + x_3 + 0s_1 + 0s_2 + s_3 = 430$

and $x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Since there are 3 equations with 6 variables, the initial basic feasible.

$s_1 = 420$, $s_2 = 460$, $s_3 = 430$ where $x_1 = x_2 = x_3 = 0$

Initial Iteration

		c_j	(3	2	5	0	0	0)	
C_B	Y_B	x_B	x_1	x_2	x_3	s_1	s_2	s_3	θ
0	s_1	420	1	4	0	1	0	0	–
0	s_2	460	3	0	(2)	0	1	0	$\frac{460}{2} = 230$
0	s_3	430	1	2	1	0	0	1	$\frac{430}{1} = 430$
$z_j - c_j$		0	-3	-2	-5	0	0	0	

$\therefore (z_j - c_j) = -5$ is most negative.

to find $(z_3 - c_3) = -5$ is most negative.

non-basic variable x_3 enters into basics.

To find leaving variable

find the ratio $\theta = \min \left\{ \frac{x_{Bi}}{a_{ij}}, a_{ij} > 0 \right\}$

$= \min \left\{ \frac{x_{Bi}}{a_{i3}}, a_{i3} > 0 \right\}$

$\theta = \min \left\{ \frac{460}{2}, \frac{430}{1} \right\} = \min \{230, 430\} = 230$

$\therefore$ The leaving variable s_2 which corresponds to the minimum ratio $\theta = 230$.

$\therefore$ old pivot equation $\div$ pivot element

$= (460 \quad 3 \quad 0 \quad 2 \quad 0 \quad 1 \quad 0) \div 2$

$= 230 \quad \frac{3}{2} \quad 0 \quad 1 \quad 0 \quad \frac{1}{2} \quad 0.$

New S_1 equation = old S_1 equation $-$ $\begin{pmatrix} \text{its entering} \\ \text{column} \\ \text{coefficient} \end{pmatrix} \times \begin{pmatrix} \text{New} \\ \text{pivot} \\ \text{equation} \end{pmatrix}$

$$\begin{array}{rrrrrrrr} = & 420 & 1 & 4 & 0 & 1 & 0 & 0 \\ (-) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline & 420 & 1 & 4 & 0 & 1 & 0 & 0 \\ \hline \end{array}$$

New S_3 equation = old S_3 equation $-$ $\begin{pmatrix} \text{its entering} \\ \text{column} \\ \text{coefficient} \end{pmatrix} \times \begin{pmatrix} \text{New} \\ \text{pivot} \\ \text{equation} \end{pmatrix}$

$$\begin{array}{rrrrrrrr} = & 430 & 1 & 2 & 1 & 0 & 0 & 1 \\ (-) & 230 & \frac{3}{2} & 0 & 1 & 0 & \frac{1}{2} & 0 \\ \hline & 200 & \frac{-1}{2} & 2 & 0 & 0 & \frac{-1}{2} & 1 \\ \hline \end{array}$$

New $(Z_j - C_j)$ eqn = old $(Z_j - C_j)$ equation $-$ $\begin{pmatrix} \text{its entering} \\ \text{column} \\ \text{coefficient} \end{pmatrix} \times \begin{pmatrix} \text{New} \\ \text{pivot} \\ \text{equation} \end{pmatrix}$

$$\begin{array}{rrrrrrrr} = & 0 & -3 & -2 & -5 & 0 & 0 & \\ (-) & -1150 & \frac{-15}{2} & 0 & -5 & 0 & \frac{-5}{2} & 0 \\ \hline & 1150 & \frac{9}{2} & -2 & 0 & 0 & \frac{5}{2} & 0 \\ \hline \end{array}$$

The improved basic feasible solution is given in the following simplex table.

Then the first iteration will be

		C_j	3	2	5	0	0	0	
C_B	X_B	x_B	x_1	x_2	x_3	S_1	S_2	S_3	θ
0	S_1	420	1	4	0	1	0	0	$\frac{420}{4} = 105$
5	x_3	230	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	—
0	S_3	200	$-\frac{1}{2}$	(2)	0	0	$(-\frac{1}{2})$	1	$\frac{200}{2} = 100$
$Z_j - C_j$		1150	$9/2$	-2	0	0	$\frac{5}{2}$	0	

Since there is an $(Z_2 - C_2) = -2$

the current basic feasible solution is in not optimal.

Second iteration:-

		C_j	3	2	5	0	0	0
C_B	X_B	x_B	x_1	x_2	x_3	S_1	S_2	S_3
0	S_1	20	2	0	0	1	1	-2
5	x_3	230	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0
2	x_2	100	$-\frac{1}{4}$	1	0	0	$-\frac{1}{4}$	$\frac{1}{2}$
$Z_j - C_j$		1350	4	0	0	0	2	1

$\therefore$ Since all $(Z_j - C_j) \geq 0$ the current basic feasible solution is optimal.

$\therefore$ The optimal solution is Max

$Z = 1350$

$x_1 = 0$

$x_2 = 100$

$x_3 = 230$

8A. Given

Task	A	B	C	D	E	F	G.
M_1	3	8	7	4	9	8	7
M_2	4	3	2	5	1	4	3
M_3.	6	7	5	11	5	6	2.

Then

Timeon.

H [Machine 1 + Machine 2	3	8	7	4	9	8	7
	4	3	2	5	1	4	3
K [Machine 3	6	7	5	11	5	6	2.

$\Rightarrow$ Min. time on Machine 3 is 5. Max time on 2 is 5.

$\Rightarrow$ Min time on Machine 3 $\geq$ Max time on machine 2

$\therefore$ The required condition to convert this into a two machine Problem is satisfied.

Let H and K be the two imaginary machine then

Machine/Task	M_1+M_2 H	M_2+M_3 K	Order of condition
× A			(2)
B	7	10	(6)
× C	11	10	
D.	9	7	(3)
	9.	6	(4)

Machine / Task	M_1+M_2 H	M_2+M_3 K	order of cancellation
E	10	6	(1)
F	12	10	(7)
G	10	15	(5)

$M_1 \to M_2$

Optimal sequence. | A | D | G | F | B | C | E |

To find the total elapsed time.

Job	Machine-I Time in	Machine-I Time out	Machine-II Time in	Machine-II Time out	Machine III Time in	Machine III Time out
A	0	3	3	7	7	13
D	3	7	7	12 (2)	13	3+2+5+4 24
G	7	14	14	17 (5)	24	36
F	14	22	22	26 (4)	36	42
B	22	30	30	33 (4)	42	49
C	30	37	37	39	49	54
E	37	46	46	47	54	$\boxed{59}$

The minimum time elapsed = 59 hrs.

Idle time on Machine 1 = 59 − 46 = 13 hrs.

Idle time on Machine 2 = 3 + 2 + 5 + 4 + 4 + 7 + (59 − 47)

= 3 + 2 + 5 + 4 + 4 + 7 + 12 = 37 hrs.

Idle time on Machine 3 = 7 hrs.

Part-A

1A. Slack variables:- slack variables represents an unused quantity of resources: it is added to less than (or) equal type of constraints in order to get an equality constraint.

Surplus variables :- A surplus variable represents the amount by which solution values exceed a resource. These are also called as 'Negative slack variables'. Surplus variables like slack variables carry a zero cofficient in the objective function. It is added to greater than (or) equal to (≥) type constraints in order to get an equality constraint.

Artificial variables:-

Artificial variables are added to those constraints with the equality (=) and greater than (or) equal to (≥) sign. An artificial variable is added to constraints to get an initial solution to an LP problem.

2A). Optimal solution - a feasible solution is said to be the optimal solution if it minimize total transportation cost. Balanced Transportation Problem- a transportation problem in which the total supply from all sources is equal to the total demand in all the destinations.

3A) Mathematical formulation of an assignment problem can be stated as follows.

$$z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

where $x_{ij} = \begin{cases} 1, & \text{if } i\text{th person is assigned to the } j\text{th job.} \\ 0, & \text{if } i\text{th person is not assigned the } j\text{th job.} \end{cases}$

subject to conditions

$$\sum_{i=1}^{n} x_{ij} = 1, \; j = 1, 2 \dots n$$

Any feasible solution of an Assignment problem consists $(2n-1)$ variables of which the $(n-1)$ variables are zero.

4A) Idle time is a period time associated with employees waiting. Idle time is a period of time in which an asset is ready and available but not doing anything productive. This why Idle time is sometimes referred to as waiting time. Idle time is when a machine is waiting for input material.

5A) Economic order quantity (EOQ) is the ideal order quantity a company should purchase to minimize inventory costs such as holding costs, shortage costs, order costs etc...