



Mechanical & Industrial Engineering
UNIVERSITY OF TORONTO

MIE1622 : Computational Finance & Risk Management

Credit Risk Modeling and Simulation

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1.0 Purpose

This report looks at the potential losses of bonds based off credit drivers and to simulate the loss based off the credit risk on the bonds. There are a total of 100 bonds in two portfolios with 50 credit drivers and 3 scenarios. The 3 scenarios to simulate loss are 100,000 scenarios with 100,000 systemic scenarios and 100 trials of 5,000 systemic scenarios and lastly 1000 systemic scenarios with 5 idiosyncratic scenarios.

2.0 Methodology

2.1 Matlab Files

There is 1 file to generate the simulated portfolio loss due to credit risk:

1. `credit_risk_simul.m` – This is the main file that runs the different strategies, calculates portfolio value and plots the results.

2.2 Random Variables

Two types of random variables were generated: Normal Random Variables and Correlated Normal Random Variables.

- Normal Random Variables were generated with a mean of 0 and a variance of 1.
- The Correlated Random Variables were generated using the Cholesky Decomposition of the correlation matrix between credit drivers. The decomposition was then multiplied by Normal Generated Random Variables to obtain the correlated Normal Random Variables.

2.3 Credit Risk

The probabilities that a bond was going to be either upgraded or degraded was given as the area under the normal curve, as well as the gains and losses given an upgrade or degrade was given as well:

1. A Credit Z-Score was calculated using the following formula: $\text{Sensitivity}(\text{asset } i) * \text{Correlated Normal Random Variable}(\text{asset } i \text{ (credit driver } k)) + \text{stdDev}(\text{asset } i) * \text{Normal Random Variable}(0,1)(\text{asset } i)$
2. The Credit Z-Score was compared to the probabilities given as the area under the normal curve to determine if a bond would either stay the same grade, upgrade or degrade.
3. The loss was then determined from the calculated losses given based on whether the bond was determined to be upgraded, degraded or stay the same.

2.4 VaR and CVaR

The Value and Conditional Value at Risk was calculated for each portfolio losses and plotted in the figures below. They were evaluated at alphas of 99% and 99.9%.

3.0 Results

The figures for each portfolio and all 3 scenarios are shown in the Appendix.

Each scenario and each portfolio have the most frequency at around 0 which is to be expected as the probability to remain in the same grade is the highest, but each scenario has a long tail as well displaying loss worst case scenarios. The bins on the left display potential gains in best case scenarios. In all scenarios the out of sample VaR and CVaR is greater than the in sample VaR and CVaR, resulting in the in sample VaR and CVaR underestimating risk and the potential loss.

The results show sampling error is existent, and any plot shown below in Appendix it is evident that the in sample VaR significantly underestimates risk to the factor of at most ~12% between MC1 and the out of sample. This can be seen from Portfolio 1 with an Out of Sample VaR of ~40.1 million but the in-sample (MC1) scenario being ~35.5 million, giving a sampling error of 12%. This ratio exists throughout all portfolios and alphas for VaR for MC1. The sampling error for MC2 however is around 1% and isn't nearly as prevalent as the sampling error in MC1. This would mean you could potentially double your expected loss due to sampling error alone.

The model error consequently was calculated by taking the Out of Sample VaR and CVaR and the No VaR and CVaR and calculating $1 - (\text{No}/\text{VaR})$ to get the model error. The model error for the normal distribution VaR and CVaR to the out of sample true distribution proved to be ~31.5% and ~37.2% for 99% and ~42.5% and ~44.8% for 99.9% respectively for portfolio 1 (Table 1). For portfolio 2 the model error proved to be less than portfolio 1 with an error of ~24.8% and ~32.7% at 99% and ~39.3% and ~43.6% at 99.9% (Table 2). Quantile Plots can be seen in the appendix for a visual representation of the model error.

	Out of Sample	Normal	Percentage Deviation
VaR 99%	\$40,157,980.72	\$27,493,506.85	31.54%
CVaR 99%	\$48,698,330.88	\$30,572,715.30	37.22%
VaR 99.9%	\$59,899,052.42	\$34,434,766.81	42.51%
CVaR 99.9%	\$66,941,799.57	\$36,950,516.15	44.80%

Table 1- Model Error Portfolio 1

	Out of Sample	Normal	Percentage Deviation
VaR 99%	\$29,807,905.50	\$22,411,190.51	24.81%
CVaR 99%	\$36,796,053.04	\$24,769,779.74	32.68%
VaR 99.9%	\$45,722,526.14	\$27,728,005.26	39.36%
CVaR 99.9%	\$52,618,756.17	\$29,655,000.26	43.64%

Table 2- Model Error Portfolio 2

4.0 Discussion

If you report the in-sample VaR and CVaR you are underestimating your risk and the potential losses as it is not a true indication of the actual risk. By underestimating your VaR will be lower than if you reported the out of sample VaR and it will not give a true indication on how much you may lose. As a result, just from this example alone, you would be reporting a potential loss as high as 50% less than if you reported the true out of sample VaR and CVaR. This could lead to decision makers thinking that this is a worthwhile venture not realising that they have much more to lose causing the bank to lose millions. It would skew the risk vs. reward decision making as for the same reward, you are reporting a risk that is less than the real risk.

Three techniques for minimizing impacts of sampling and model errors is to take a regression of your out of sample data and try to regress to how much error there is when doing in-sample scenarios and make an adjustment for the error. The second technique you could do would be to increase your number of samples due to sampling error gets worse as your alpha approaches 1. In practice, the alpha is generally .995, there needs to be a lot of samples to get a true distribution of your sample. Generally, the number of samples used is 10,000,000 in practice (or somewhere in the millions). The last technique would be to fit a distribution to your losses and calculate VaR and CVaR according to that distributions PDF and CDF.

5.0 Appendix

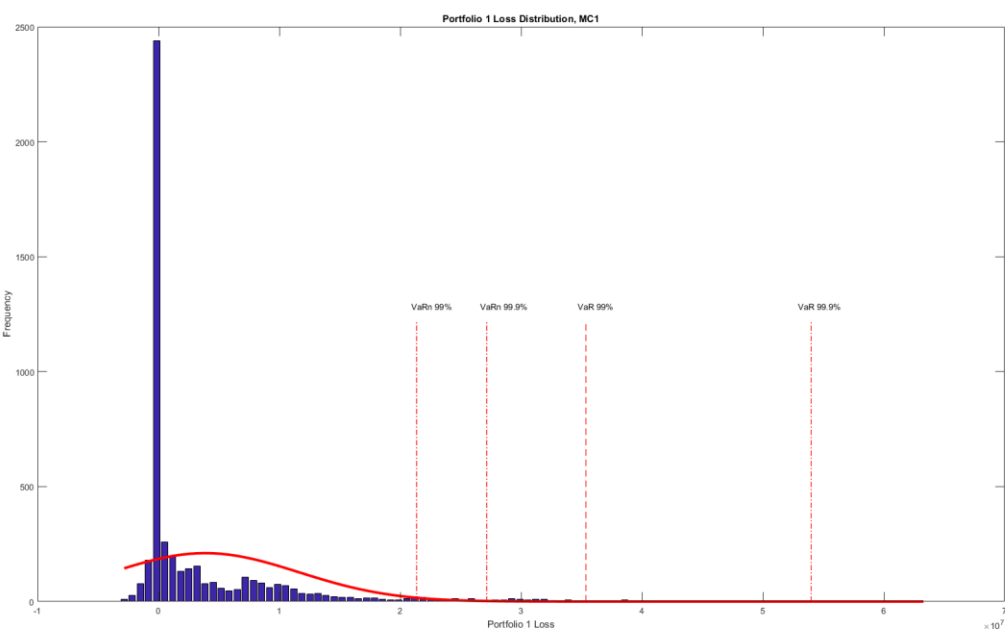


Figure 1 - MC1 Portfolio 1 Loss Distribution

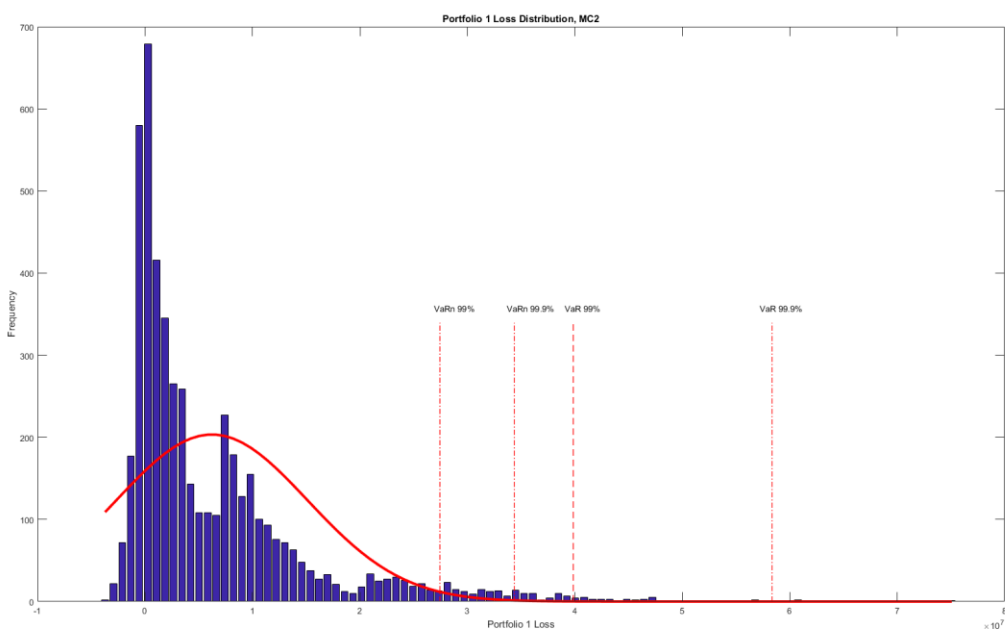


Figure 2 - MC2 Portfolio 1 Loss Distribution

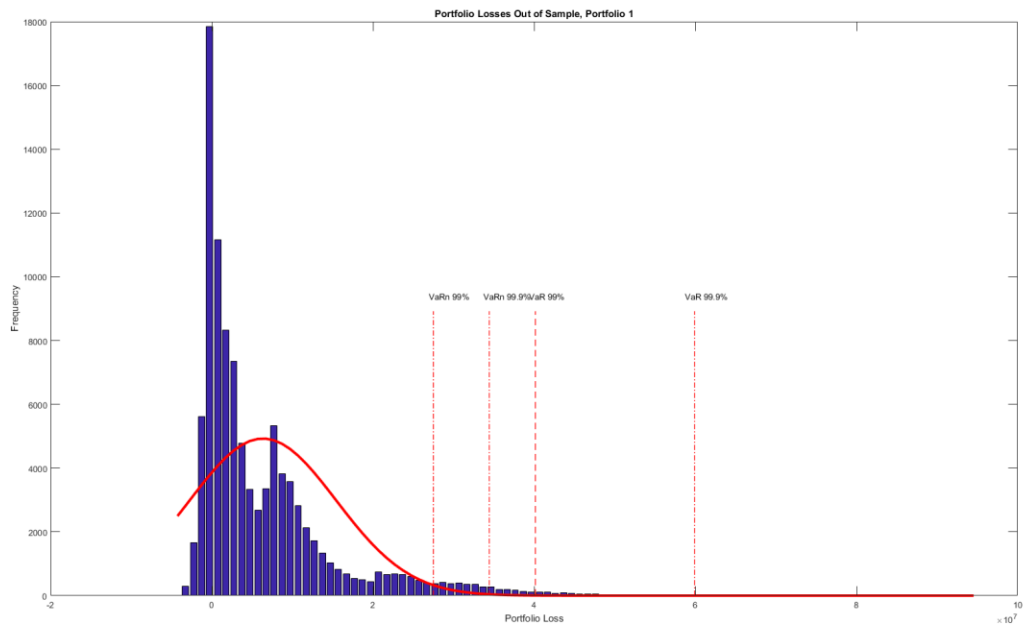


Figure 3 - Portfolio 1 Losses Out of Sample

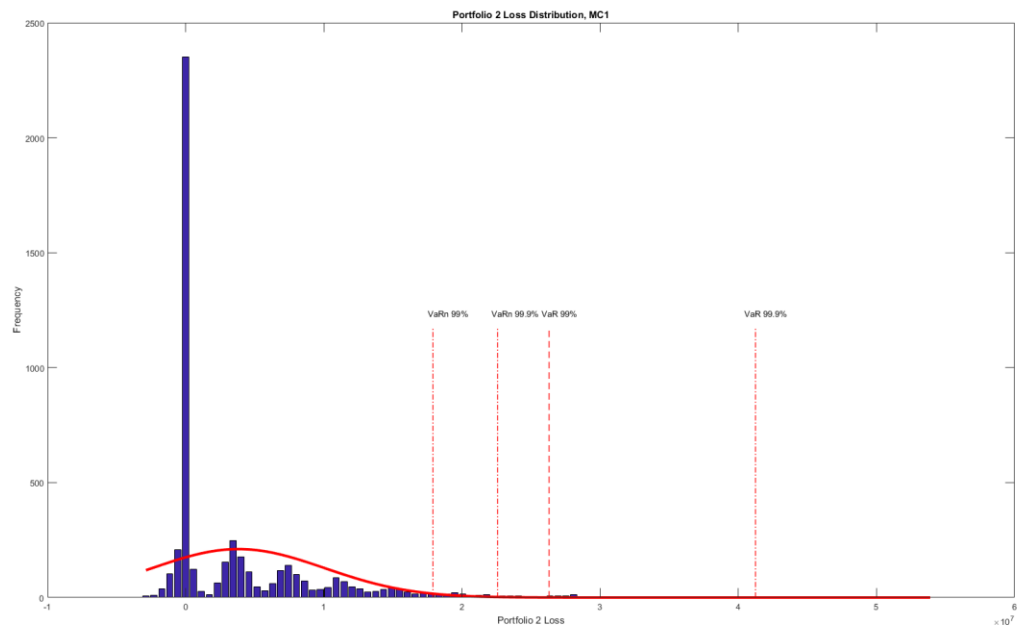


Figure 4 - MC1 Portfolio 2 Loss Distribution

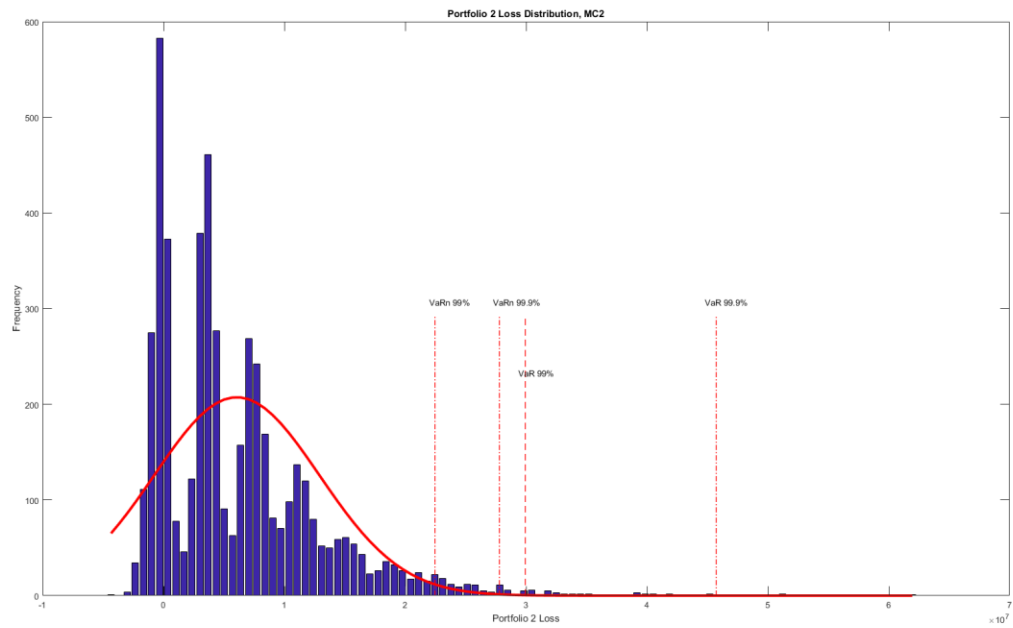


Figure 5 - MC2 Portfolio 2 Loss Distribution

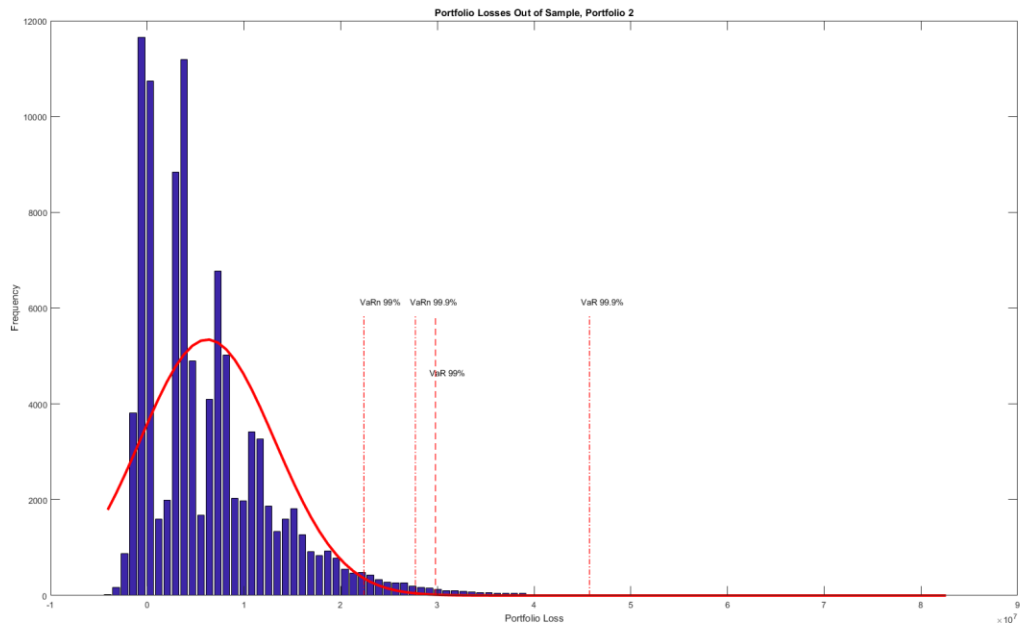


Figure 6 - Portfolio 2 Loss Distribution Out of Sample

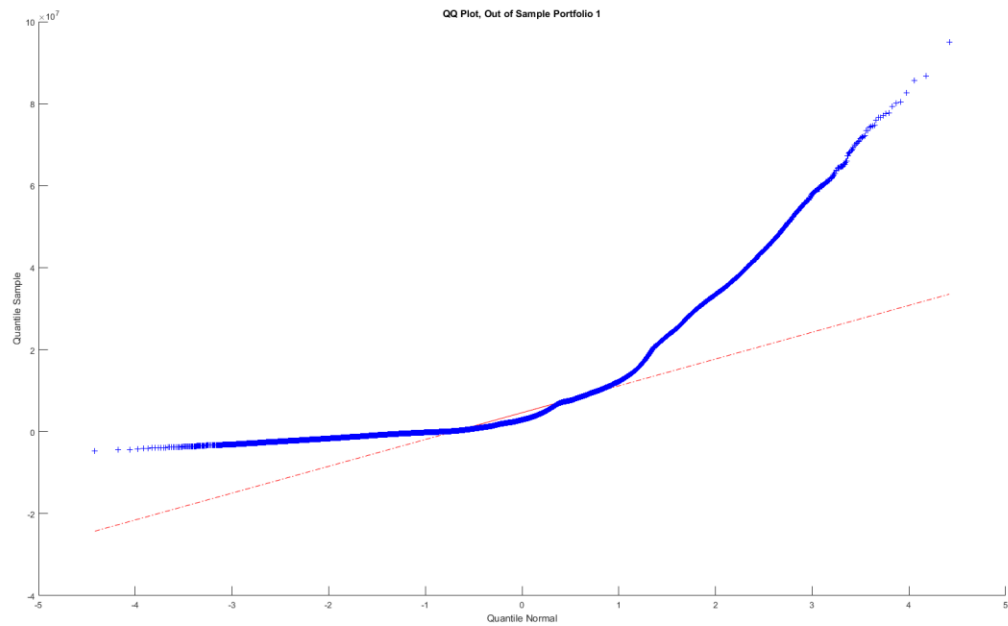


Figure 7 - Model Error Portfolio 1

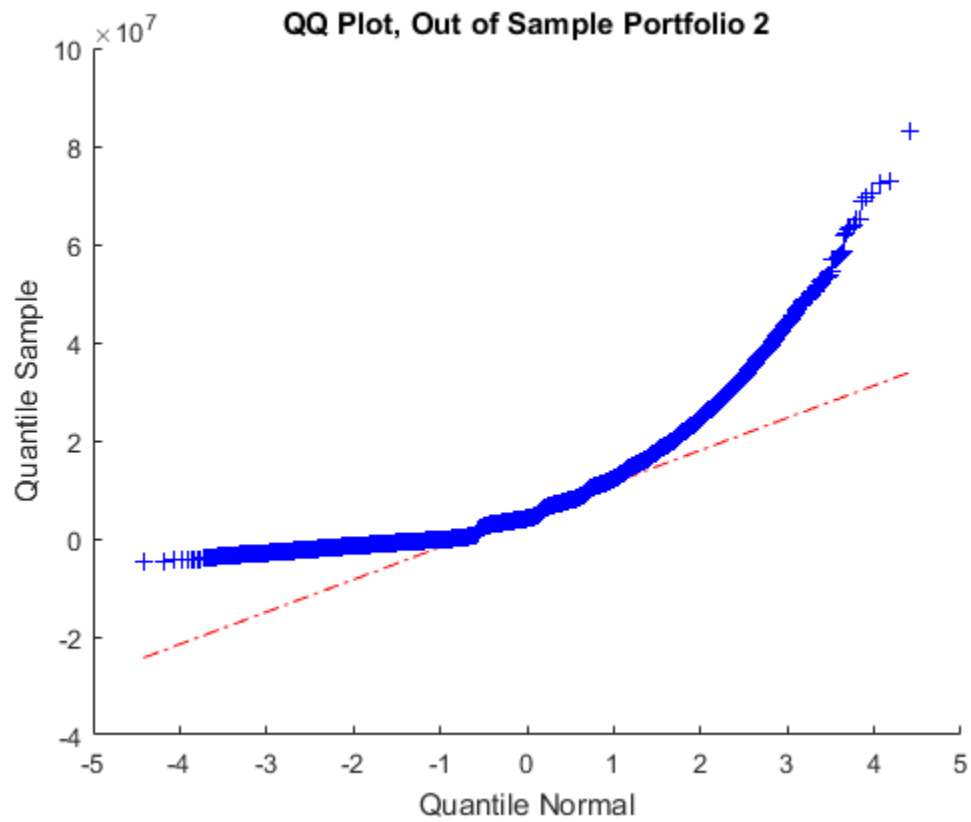


Figure 8 - Model Error Portfolio 2

Output

Portfolio 1:

Out-of-sample: VaR 99.0% = \$40157980.72, CVaR 99.0% = \$48698330.88

In-sample MC1: VaR 99.0% = \$35529756.84, CVaR 99.0% = \$43873381.54

In-sample MC2: VaR 99.0% = \$39699977.62, CVaR 99.0% = \$48369326.46

In-sample No: VaR 99.0% = \$27493506.85, CVaR 99.0% = \$30572715.30

In-sample N1: VaR 99.0% = \$21359256.74, CVaR 99.0% = \$23931128.02

In-sample N2: VaR 99.0% = \$27456528.38, CVaR 99.0% = \$30529937.62

Out-of-sample: VaR 99.9% = \$59899052.42, CVaR 99.9% = \$66941799.57

In-sample MC1: VaR 99.9% = \$54579093.37, CVaR 99.9% = \$62841148.15

In-sample MC2: VaR 99.9% = \$59165186.69, CVaR 99.9% = \$67755028.03

In-sample No: VaR 99.9% = \$34434766.81, CVaR 99.9% = \$36950516.15

In-sample N1: VaR 99.9% = \$27156859.40, CVaR 99.9% = \$29258108.33

In-sample N2: VaR 99.9% = \$34384715.54, CVaR 99.9% = \$36895726.86

Portfolio 2:

Out-of-sample: VaR 99.0% = \$29807905.50, CVaR 99.0% = \$36796053.04

In-sample MC1: VaR 99.0% = \$26406751.58, CVaR 99.0% = \$32926773.50

In-sample MC2: VaR 99.0% = \$29912490.16, CVaR 99.0% = \$36899272.24

In-sample No: VaR 99.0% = \$22411190.51, CVaR 99.0% = \$24769779.74

In-sample N1: VaR 99.0% = \$17907269.90, CVaR 99.0% = \$19986818.30

In-sample N2: VaR 99.0% = \$22467414.75, CVaR 99.0% = \$24831998.35

Out-of-sample: VaR 99.9% = \$45722526.14, CVaR 99.9% = \$52618756.17

In-sample MC1: VaR 99.9% = \$41273136.19, CVaR 99.9% = \$47586836.15

In-sample MC2: VaR 99.9% = \$45601827.40, CVaR 99.9% = \$53185143.09

In-sample No: VaR 99.9% = \$27728005.26, CVaR 99.9% = \$29655000.26

In-sample N1: VaR 99.9% = \$22595061.02, CVaR 99.9% = \$24294076.41

In-sample N2: VaR 99.9% = \$27797742.25, CVaR 99.9% = \$29729634.72