RBS Mathematics
MATHEMATICS-IV, KAS-302/402
Pre University Model Questions

Short Type Questions.

1. Form the partial differential equation from $z=(x+a)(y+b)$ by eliminating arbitrary constants.
2. Form the P.D.E. by eliminating a and b from $z=(x^2+a)(y^2+b)$.
3. Form the partial differential equation from $z=f(x^2+y^2)$ by eliminating arbitrary function.
4. Form the partial differential equation from $z=f(x+ct)+g(x-ct)$ by eliminating arbitrary functions.
5. Solve $z=px+qy+\sqrt{1+p^2+q^2}$.
6. Solve: $(D^2-2DD'+D'^2)z=0$.
7. Solve $(D^2+D'^2-p^2)z=0$.
8. Show that the equation $z_{xx}+2xz_{xy}+(1-y^2)z_{yy}=0$ is elliptic for values of x and y in the region $x^2+y^2<1$, parabolic on the boundary and hyperbolic outside the region.
9. Classify the equation $(1-x^2)\frac{\partial^2 z}{\partial x^2}-2xy\frac{\partial^2 z}{\partial x\partial y}+(1-y^2)\frac{\partial^2 z}{\partial y^2}-2z=0$.
10. Write down the two dimensional wave equation.
11. Write down the equation of steady state heat conduction in the rectangular plate.
12. Write the normal equations of the curve $y=ax+bx^2$.
13. Write the normal equations of the curve $2^x=ax^2+bx+c$ by least square method.
14. In $y=a+bx, \sum x=50, \sum y=80, \sum xy=1030, \sum x^2=750$ *and* $n=10$, *then* $a=..., b=..$
15. Define skewness and kurtosis in brief.
16. Find M.G.F. of Binomial distribution.
17. Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that both events A and B occurs in 0.14. Find the probability that neither A nor B occurs.
18. State the Addition law of probability.
19. State the Multiplication law of probability.
20. What do you mean by random variable?. Explain with suitable example.
21. If the sum of the mean and variance of a Binomial distribution of 5 trials is 9/5, find P(X ≥ 1).
22. Define level of significance and confidence interval.
23. What do you mean by errors in sampling?
24. What is the expected frequencies of 2×2 contingency table $\boxed{\begin{matrix} a & b \\ c & d \end{matrix}}$.
25. Write a short note on χ^2-test as a test of goodness of fit.
26. Characterize the following partial differential equation into elliptic, parabolic and hyperbolic equations $A\frac{\partial^2 u}{\partial x^2}+2B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}=F(x,y,u,u_x,u_y)$. Here A, B, C may be functions of x and y.

Type- Long type problems(Section B and Section-C)

1. Form the partial differential equations from (i) $x^2+y^2+(z-c)^2=a^2$ where a and c are arbitrary constants (ii) $f\left(x+y+z, x^2+y^2+z^2\right)=0$.

   Ans. (i) $yp-xq=0$. (ii) $(y-z)p+(z-x)q=x-y$.

2. Solve the partial differential equation $x(y^2+z)p-y(x^2+z)q=z(x^2-y^2)$ where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$. **Ans:** $\phi(x^2+y^2-2z, xyz)=0$.

3. Solve the following by Charpit's method $\left(p^2+q^2\right)y=qz$. Ans. $z^2-a^2y^2=(ax+b)^2$.

4. Solve the following by Charpit's method $px+qy=pq$. Ans. $az=\frac{(y+ax)^2}{2}+b$

5. Solve the following PDE **(i)** $(D^2-2DD')z=\sin x\cos 2y$

   (ii) $(D^2-DD')z=\cos 2y(\sin x+\cos x)$ **(iii)** $r+2s+t=2(y-x)+\sin(x-y)$

   (iv) $(D^2-6DD'+9D'^2)z=12x^2+36xy$ **(v)** $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}-6\frac{\partial^2 z}{\partial y^2}=y\cos x$

   Ans.(i) $f_1(y)+f_2(y+x)+\frac{1}{6}\sin(x+2y)-\frac{1}{10}\sin(x-2y)$

   (ii) $f_1(y)+f_2(y+x)+\frac{1}{2}[\sin(x+2y)+\cos(x+2y)]-\frac{1}{6}[\sin(x-2y)+\cos(x-2y)]$

   (iii) $f_1(y-x)+xf_2(y-x)+x^2(y-x)+\frac{x^2}{2}\sin(x-y)$

   (iv) $f_1(y+3x)+xf_2(y+3x)+6x^3y+x^4$ **(v)** $f_1(y+2x)+f_2(y-3x)-y\cos x+\sin x$.

6. Solve the following linear partial differential equations:

   (i) $(D-3D'-2)^3z=6e^{2x}\sin(3x+y)$ **(ii)** $(D^2+2DD'+D'^2-2D-2D')z=\sin(x+2y)$

   (iii) $(D^2-D'^2-3D+3D')z=xy+e^{x+2y}$

   Ans: (i) $e^{2x}f_1(y+3x)+xe^{2x}f_2(y+3x)+x^2e^{2x}\tan(y+3x)$

   (ii) $f_1(y-x)+e^{2x}f_2(y-x)+\frac{1}{39}[2\cos(x+2y)-3\sin(x+2y)]$

   (iii) $f_1(y+x)+e^{3x}f_2(y-x)-\frac{1}{3}\left[\frac{x^2y}{2}+\frac{xy}{3}+\frac{x^2}{3}+\frac{x^3}{6}+\frac{2x}{9}\right]-xe^{x+2y}$

7. Solve the linear partial differential equation

$$x^2\frac{\partial^2 z}{\partial x^2}-4xy\frac{\partial^2 z}{\partial x\partial y}+4y^2\frac{\partial^2 z}{\partial y^2}+6y\frac{\partial z}{\partial y}=x^3y^4$$

   Ans. $z=g_1\left(yx^2\right)+xg_2\left(yx^2\right)+\frac{1}{30}x^3y^4$.

8. Solve the following equation by method of separation of variables: $\frac{\partial^2 u}{\partial x\partial t}=e^{-t}\cos x$, given that $u=0$ when $t=0$ and $\frac{\partial u}{\partial t}=0$ when $x=0$. Ans: $\sin x(1-e^{-t})$.

9. Solve the P.D.E. by separation of variables method, $u_{xx} = u_y + 2u$, given that $u(0, y) = 0, \frac{\partial}{\partial x} u(0, y) = 1 + e^{-3y}$. Ans. $u(x, y) = \frac{1}{\sqrt{2}} \sinh\left(x\sqrt{2}\right) + e^{-3y} \sin x.$

10. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = A\sin\left(\frac{\pi x}{l}\right)$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = A\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right)$.

11. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3 \frac{\pi x}{l}$. If it is released from this position, find the displacement $y(x, t)$.

Ans: $\frac{3y_0}{4}\sin\frac{\pi x}{l}\cos\frac{\pi ct}{l} - \frac{y_0}{4}\sin\frac{3\pi x}{l}\cos\frac{3\pi ct}{l}$

12. A tightly stretched string with fixed end points x=0 and x=l is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points an initial velocity $\lambda x(l - x)$, find the displacement of the string at any distance x from one end at any time t.

13. Find the temperature in a bar of length 2 whose ends are kept at zero and lateral surfaces insulated if the initial temperature is $\sin\frac{\pi x}{2} + 3\sin\frac{n\pi x}{2}$. Ans. $u(x,t) = \sin\frac{\pi x}{l}.e^{\frac{-\pi^2 c^2 t}{l^2}} + 3\sin\frac{5\pi x}{l}.e^{\frac{-25\pi^2 c^2 t}{l^2}}$

14. An insulated rod of length l has its ends A and B maintained at $0^0 C$ and $100^0 C$ respectively until steady state condition prevail. If B is suddenly reduced to $0^0 C$ and maintained at $0^0 C$. Find the temperature at a distance x from A at time t. Find also the temperature if the change consists of raising the temperature of A to $20^0 C$ and reducing that of B to $80^0 C$. Ans

$$u(x,t) = 20 + \frac{60}{l}x - \frac{80}{\pi}\sum_{n=2,4}^{\infty}\frac{1}{n}\sin\frac{n\pi x}{l}e^{-\left(\frac{n\pi c}{l}\right)^2 t} = 20 + \frac{60}{l}x - \frac{40}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}\sin\frac{2m\pi x}{l}e^{\frac{-4c^2 m^2 \pi^2 t}{l^2}}$$

15. Use the method of separation of variables method to solve the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to boundary conditions $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin\left(\frac{n\pi x}{l}\right)$.

Ans. $u(x, y) = \frac{\sinh\left(\frac{n\pi y}{l}\right)}{\sinh\left(\frac{n\pi a}{l}\right)}\sin\left(\frac{n\pi x}{l}\right)$.

16. Solve Laplace Eqn. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in the xy plane with $u(x,0) = 0, u(x,b) = 0, u(0, y) = 0, \; u(a, y) = f(y)$ parallel to y axis.

Ans. $u(x, y) = \Sigma b_n \frac{n\pi y}{b}.\sin\frac{n\pi x}{b}$ where $b_n = \frac{2}{b\sin h\left(\frac{n\pi a}{b}\right)}\int_0^b f(y).\sin\frac{n\pi y}{b}dy$.

17. Use the method of least squares to fit the curve: $y = \frac{c_0}{x} + c_1\sqrt{x}$ to the following table of values:

x	0.1	0.2	0.4	0.5	1	2
y	21	11	7	6	5	6

Ans. $y = \frac{1.97327}{x} + 3.28182\sqrt{x}.$

18. Fit a second degree parabola to the following data by least squares method:

x	1	2	3	4	5
y	1090	1220	1390	1625	1915

Ans. $y = 27.5x^2 + 40.5x + 1024.$

19. The following table gives the results of the measurements of train resistance, V is the velocity in miles per hour, R is the resistance in pounds per ton;

V:	20	40	60	80	100	120
R:	5.5	9.1	14.9	22.8	33.3	46.0

If R is related to V by the relation $R = a + bV + cV^2$, find $a, b,$ and $c.$

Ans. a=4.35, b=0.00241, and c=0.0028705.

20. Obtain the moment generating function of the random variable x having probability distribution $F(x) = \begin{cases} x, & for\, 0 < x < 1 \\ 2 - x, & for\, 1 < x < 2 \\ 0, & elsewhere \end{cases}$. Also determine mean ν_1, ν_2 and variance μ_2.

21. Find the moment generating function of the discrete Poisson distribution given by $P(x) = e^{-\lambda}\frac{\lambda^x}{\lfloor x}$. Also find first and second moments about mean.

22. Find the moment generating function of the continuous distribution given by $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, $-\infty < x < \infty.$

23. In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$, $20x - 9y = 107$ respectively. Calculate $\overline{x}, \overline{y}$ and the coefficient of co-relation between x and y. If Variance of x=9 then find the standard deviation of y

24. If θ is the angle between the two regression lines, show that $\tan\theta = \frac{1-r^2}{r}\frac{\sigma_x\sigma_y}{\sigma_x^2 + \sigma_y^2}$. Also If the coefficient of correlation between two variables x and y is 0.5 and the acute angle between their lines of regression is $\tan^{-1}(3/5)$, show that $\sigma_x = \frac{1}{2}\sigma_y$.

25. The regression lines of y on x and x on y are respectively $y = ax + b$ and $x = cy + d$. Show that $\frac{\sigma_y}{\sigma_x} = \sqrt{\frac{a}{c}}, \overline{x} = \frac{bc+d}{1-ac}\, and, \overline{y} = \frac{ad+b}{1-ac}$.

26. In a certain distribution, the first four moments about a point are -1.5, 17, -30 and 108. Calculate the moments about mean, $\beta_1 and\, \beta_2$; and state whether the distribution is leptokurtic or platykurtic ?

27. The following table represents the height of a batch of 100 students. Calculate skewness and kurtosis.

Height(in cm)	59	61	63	65	67	69	71	73	75
No. of Students	0	2	6	20	40	20	8	2	2

28. Psychological tests of intelligence and of engineering ability were applied to 10 students. Here a record of ungrouped data showing intelligence ratio (I.R.) and engineering ratio (E.R.). Calculate the co-efficient of correlation.

Student	A	B	C	D	E	F	G	H	I	J
I.R.	105	104	102	101	100	99	98	96	93	92
E.R.	101	103	100	98	95	96	104	92	97	94

Ans. 0.59.

29. A can hit a target 3 times in 5 shots, B 2 times in 5 shots and C 3 times in 4 shots. They fire a volley. What is the probability that (i) two shots hit, (ii) atleast two shots hit? Ans. (i) 0.45 (ii) 0.63.
30. In a bolt factory, machines A, B and C manufacture 25%, 35% and 40% of the total. Of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B or C? Ans.25/69; 28/69 ; 16/69.
31. There are three bags : first containing 1 white, 2 red, 3green balls; second 2 white, 3 red, 1 green balls and third 3 white, 1 red, 2 green balls. Two balls are drawn from a bag chosen at random. These are found to be one white and one red. Find the probability that the balls so drawn came from second bag. Ans.6/11.
32. A random variable X has the following probability function:

x	0	1	2	3	4	5	6	7
p(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i)Find the value of k (ii) Evaluate P(X< 6), P($X \geq 6$) (iii) P(0<X<5).

Ans.(i) 1/10 (ii) 81/100; 19/100 (iii)4/5.

33. The probability density p(x) of a continuous random variable is given by

$$p(x) = y_0 e^{-|x|}, \quad -\infty < x < \infty.$$

Prove that $y_0 = 1/2$. Find mean and variance of the distribution. Ans.0.2.

34. The frequency function of a continuous random variable is given by

$$f(x) = y_0 x(2-x), \quad 0 \leq x \leq 2.$$

Find the value of y_0, mean and variance of x. Ans.3/4; 1; 1/5.

35. The probability that a pen manufactured by a company will be defective is 1/10. If 12 such pens are manufactured, find the probability that (a) Exactly two will be defective. (b) at least two will be defective. (c) none will be defective. Ans.(a) 0.2301 (b) 0.3412 (c) 0.2833.
36. Fit the binomial distribution to the following frequency distribution:

x: 0 1 2 3 4 5 6

f:	13	25	52	58	32	16	4

Ans. $200(0.554+0.446)^6$.

37. Fit a Poisson distribution to the set of observations:

x	0	1	2	3	4
f	122	60	15	2	1

Ans. $P(\mathrm{x}=\mathrm{r})=200\dfrac{(0.5)^r}{r!}e^{-0.5}$ where $r=0,1,2,3,4.$

Theoretical frequencies are 121, 61, 15, 2, and 0.

38. Show that in a Poisson distribution with unit mean, mean deviation about mean is $\left(\dfrac{2}{e}\right)$ times the standard deviation.
39. A car-hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days (i) on which there is no demand, (ii) on which demand is refused. ($e^{-1.5}=0.2231$). Ans.0.2231; 0.1913.
40. If X is a normal variate with mean 30 and S.D. 5, find the probabilities that (i) $26 \le X \le 40,$ (ii) $X \ge 45,$ and (iii) $|X-30|>5.$ Ans. (i) 0.7653; (ii) 0.0014; (iii) 0.3174
41. In Normal distribution, 31% of the items are under 45 and 8% are above 64. Find the mean and standard deviation of the distribution. It is given that if $f(t)=\dfrac{1}{\sqrt{2\pi}}\int_0^t e^{-\frac{1}{2}x^2}dx$ then f(0.5)=0.19 and f(1.4)=0.42. Ans. $\sigma=10$ and μ=50.
42. In a test of 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to be burn for (a) more than 2150 hours, (b) less than 1950 hours and (c) more than 1920 hours and but less than 2160 hours. Ans. (a) 67 (b) 184 (c) 1909 nearly.
43. Prove that for a normal distribution, mean deviation from the mean equals to $\dfrac{4}{5}$ of the standard deviation approximately.
44. Fit a Poisson distribution to the following data and test for its goodness of fit at level of significance 0.05.

x	0	1	2	3	4
f	419	352	154	56	19

45. Using χ^2-test , find out whether there is any association between income level and type of the schooling:

Income	Public School	Govt. School
low	200	400
high	1000	400

Given that $\chi^2_{.05}=3.84$ for one degree of freedom.

46. A machine is producing bolts of which a certain fraction is defective. A random sample of 400 is taken from a large batch and is found to contain 30 defective bolts. Does this indicate that the proportion of defectives is larger than that claimed by the manufacturer where the manufacturer claims that only 5% of his product are defective. Find 95% confidence limits of the proportion of defective bolts in batch.

47. Two independent samples of sizes 7 and 9 have the following values:

Sample A	10	12	10	13	14	11	10		
Sample B	10	13	15	12	10	14	11	12	11

Test whether the difference between the mean is significant. [$t_{.01}(14)=2.98$]

48. Two independent sample of sizes 7 and 6 had the following values:

Sample A	28	30	32	33	31	29	34
Sample B	29	30	30	24	27	28	

Examine whether the samples have been drawn from normal population having the same variance.