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# Power Analysis

## Overview

Power analysis is an important aspect of experimental design. It allows us to determine the sample size required to detect an effect of a given size with a given degree of confidence. Conversely, it allows us to determine the probability of detecting an effect of a given size with a given level of confidence, under sample size constraints. If the probability is unacceptably low, we would be wise to alter or abandon the experiment.

The following **four quantities** have an intimate relationship:

1. sample size
2. effect size
3. significance level =  $P(\text{Type I error})$  = probability of finding an effect that is not there
4. power =  $1 - P(\text{Type II error})$  = probability of finding an effect that is there

Given any three, we can determine the fourth.

## Power Analysis in R

The [pwr](#) package developed by Stéphane Champely, implements power analysis as outlined by [Cohen \(!1988\)](#). Some of the more important functions are listed below.

function	power calculations for
<code>pwr.2p.test</code>	two proportions (equal n)
<code>pwr.2p2n.test</code>	two proportions (unequal n)
<code>pwr.anova.test</code>	balanced one way ANOVA
<code>pwr.chisq.test</code>	chi-square test
<code>pwr.f2.test</code>	general linear model
<code>pwr.p.test</code>	proportion (one sample)
<code>pwr.r.test</code>	correlation
<code>pwr.t.test</code>	t-tests (one sample, 2 sample, paired)
<code>pwr.t2n.test</code>	t-test (two samples with unequal n)

For each of these functions, you enter three of the four quantities (effect size, sample size, significance level, power) and the fourth is calculated.

The significance level defaults to 0.05. Therefore, to calculate the significance level, given an effect size, sample size, and power, use the option "sig.level=NULL".

Specifying an [effect size](#) can be a daunting task. ES formulas and Cohen's suggestions (based on social science research) are provided below. Cohen's suggestions should only

be seen as very rough guidelines. Your own subject matter experience should be brought to bear.

(To explore confidence intervals and drawing conclusions from samples try [this interactive course on the foundations of inference](#).)

## t-tests

For t-tests, use the following functions:

`pwr.t.test(n = , d = , sig.level = , power = , type = c("two.sample", "one.sample", "paired"))`

where n is the sample size, d is the effect size, and type indicates a two-sample t-test, one-sample t-test or paired t-test. If you have unequal sample sizes, use

`pwr.t2n.test(n1 = , n2 = , d = , sig.level = , power = )`

where n1 and n2 are the sample sizes.

For t-tests, the effect size is assessed as

$$d = \frac{|\mu_1 - \mu_2|}{\sigma}$$

where  $\mu_1$  = mean of group 1  
 $\mu_2$  = mean of group 2  
 $\sigma^2$  = common error variance

Cohen suggests that d values of 0.2, 0.5, and 0.8 represent small, medium, and large effect sizes respectively.

You can specify `alternative="two.sided", "less", or "greater"` to indicate a two-tailed, or one-tailed test. A two tailed test is the default.

## ANOVA

For a one-way analysis of variance use

`pwr.anova.test(k = , n = , f = , sig.level = , power = )`

where k is the number of groups and n is the common sample size in each group.

For a one-way ANOVA effect size is measured by f where

$$f = \sqrt{\frac{\sum_{i=1}^k p_i * (\mu_i - \mu)^2}{\sigma^2}}$$

where  $p_i = n_i / N$ ,  
 $n_i$  = number of observations in group i  
 $N$  = total number of observations  
 $\mu_i$  = mean of group i  
 $\mu$  = grand mean  
 $\sigma^2$  = error variance within groups

Cohen suggests that f values of 0.1, 0.25, and 0.4 represent small, medium, and large effect sizes respectively.

## Correlations

For correlation coefficients use

`pwr.r.test(n = , r = , sig.level = , power = )`

where n is the sample size and r is the correlation. We use the population correlation coefficient as the effect size measure. Cohen suggests that r values of 0.1, 0.3, and 0.5 represent small, medium, and large effect sizes respectively.

## Linear Models

For linear models (e.g., multiple regression) use

**pwr.f2.test(u = , v = , f2 = , sig.level = , power = )**

where u and v are the numerator and denominator degrees of freedom. We use f2 as the effect size measure.

$$f^2 = \frac{R^2}{1 - R^2}$$

where  $R^2$  = population squared multiple correlation

$$f^2 = \frac{R_{AB}^2 - R_A^2}{1 - R_{AB}^2}$$

where  $R_A^2$  = variance accounted for in the population by variable set A  
 $R_{AB}^2$  = variance accounted for in the population by variable set A and B together

The first formula is appropriate when we are evaluating the impact of a set of predictors on an outcome. The second formula is appropriate when we are evaluating the impact of one set of predictors above and beyond a second set of predictors (or covariates). Cohen suggests f2 values of 0.02, 0.15, and 0.35 represent small, medium, and large effect sizes.

## Tests of Proportions

When comparing two proportions use

**pwr.2p.test(h = , n = , sig.level = , power = )**

where h is the effect size and n is the common sample size in each group.

$$h = 2 \arcsin(\sqrt{p_1}) - 2 \arcsin(\sqrt{p_2})$$

Cohen suggests that h values of 0.2, 0.5, and 0.8 represent small, medium, and large effect sizes respectively.

For unequal n's use

**pwr.2p2n.test(h = , n1 = , n2 = , sig.level = , power = )**

To test a single proportion use

**pwr.p.test(h = , n = , sig.level = power = )**

For both two sample and one sample proportion tests, you can specify `alternative="two.sided", "less", or "greater"` to indicate a two-tailed, or one-tailed test. A two tailed test is the default.

## Chi-square Tests

For chi-square tests use

**pwr.chisq.test(w = , N = , df = , sig.level = , power = )**

where w is the effect size, N is the total sample size, and df is the degrees of freedom. The effect size w is defined as

$$w = \sqrt{\frac{\sum_{i=1}^m (p_{0i} - p_{1i})^2}{p_{0i}}}$$

where  $p_{0i}$  = cell probability in  $i$ th cell under  $H_0$   
 $p_{1i}$  = cell probability in  $i$ th cell under  $H_1$

Cohen suggests that  $w$  values of 0.1, 0.3, and 0.5 represent small, medium, and large effect sizes respectively.

## Some Examples

```
library(pwr)

# For a one-way ANOVA comparing 5 groups, calculate the
# sample size needed in each group to obtain a power of
# 0.80, when the effect size is moderate (0.25) and a
# significance level of 0.05 is employed.

pwr.anova.test(k=5,f=.25,sig.level=.05,power=.8)

# What is the power of a one-tailed t-test, with a
# significance level of 0.01, 25 people in each group,
# and an effect size equal to 0.75?

pwr.t.test(n=25,d=0.75,sig.level=.01,alternative="greater")

# Using a two-tailed test proportions, and assuming a
# significance level of 0.01 and a common sample size of
# 30 for each proportion, what effect size can be detected
# with a power of .75?

pwr.2p.test(n=30,sig.level=0.01,power=0.75)
```

## Creating Power or Sample Size Plots

The functions in the [pwr](#) package can be used to generate power and sample size graphs.

```
# Plot sample size curves for detecting correlations of
# various sizes.

library(pwr)

# range of correlations
r <- seq(.1,.5,.01)
nr <- length(r)

# power values
p <- seq(.4,.9,.1)
np <- length(p)

# obtain sample sizes
samsize <- array(numeric(nr*np), dim=c(nr,np))
```

```

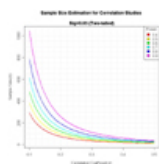
for (i in 1:np){
  for (j in 1:nr){
    result <- pwr.r.test(n = NULL, r = r[j],
      sig.level = .05, power = p[i],
      alternative = "two.sided")
    samsize[j,i] <- ceiling(result$n)
  }
}

# set up graph
xrange <- range(r)
yrange <- round(range(samsize))
colors <- rainbow(length(p))
plot(xrange, yrange, type="n",
  xlab="Correlation Coefficient (r)",
  ylab="Sample Size (n)" )

# add power curves
for (i in 1:np){
  lines(r, samsize[,i], type="l", lwd=2, col=colors[i])
}

# add annotation (grid lines, title, legend)
abline(v=0, h=seq(0,yrange[2],50), lty=2, col="grey89")
abline(h=0, v=seq(xrange[1],xrange[2],.02), lty=2,
  col="grey89")
title("Sample Size Estimation for Correlation Studies\n
  Sig=0.05 (Two-tailed)")
legend("topright", title="Power", as.character(p),
  fill=colors)

```



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