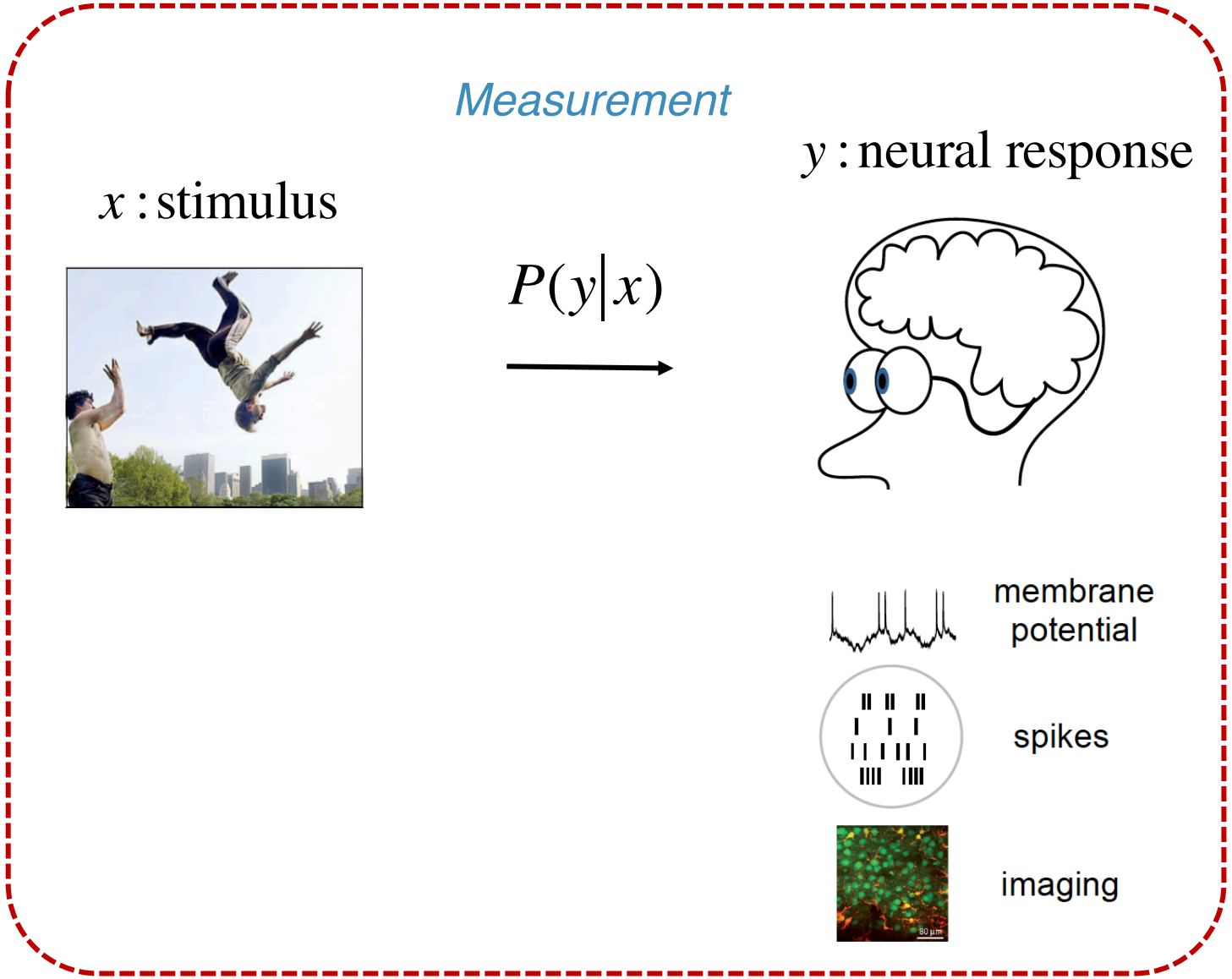
Bayesian Decoding

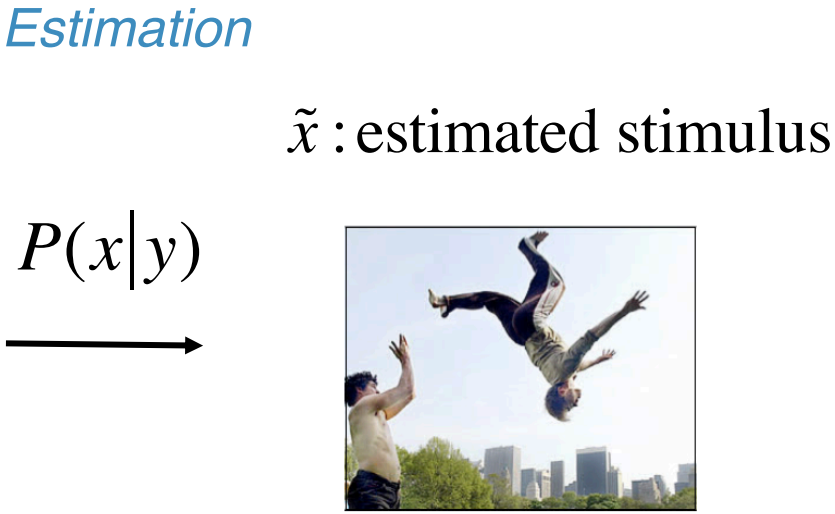
is simple!

Wenbo Tang

March 22, 2021

Neural coding problem



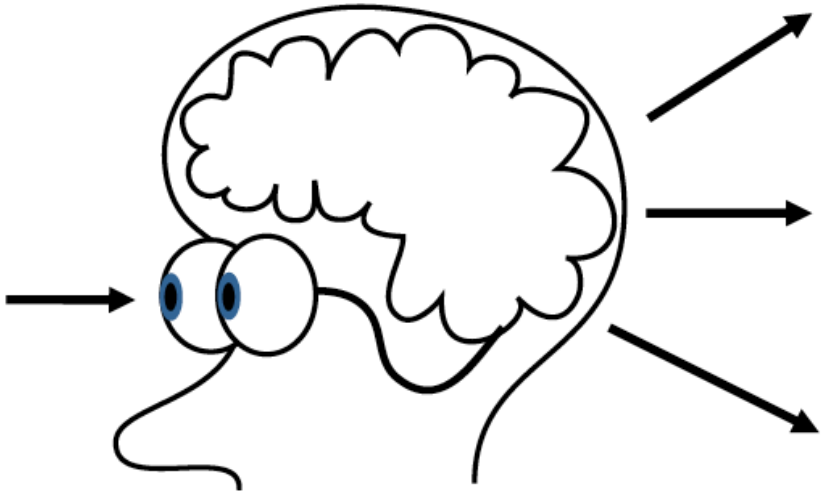


[Jonathan Pillow]

Neural coding problem: encoding

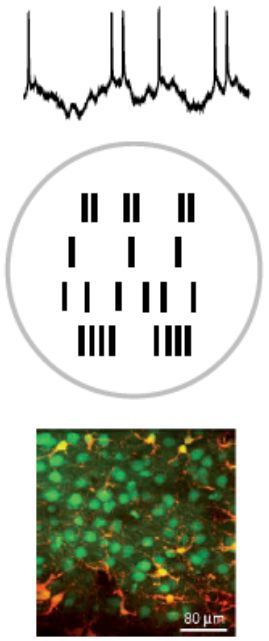
stimulus

x



Encoding model:

$P(y|x)$



membrane potential

spikes

imaging

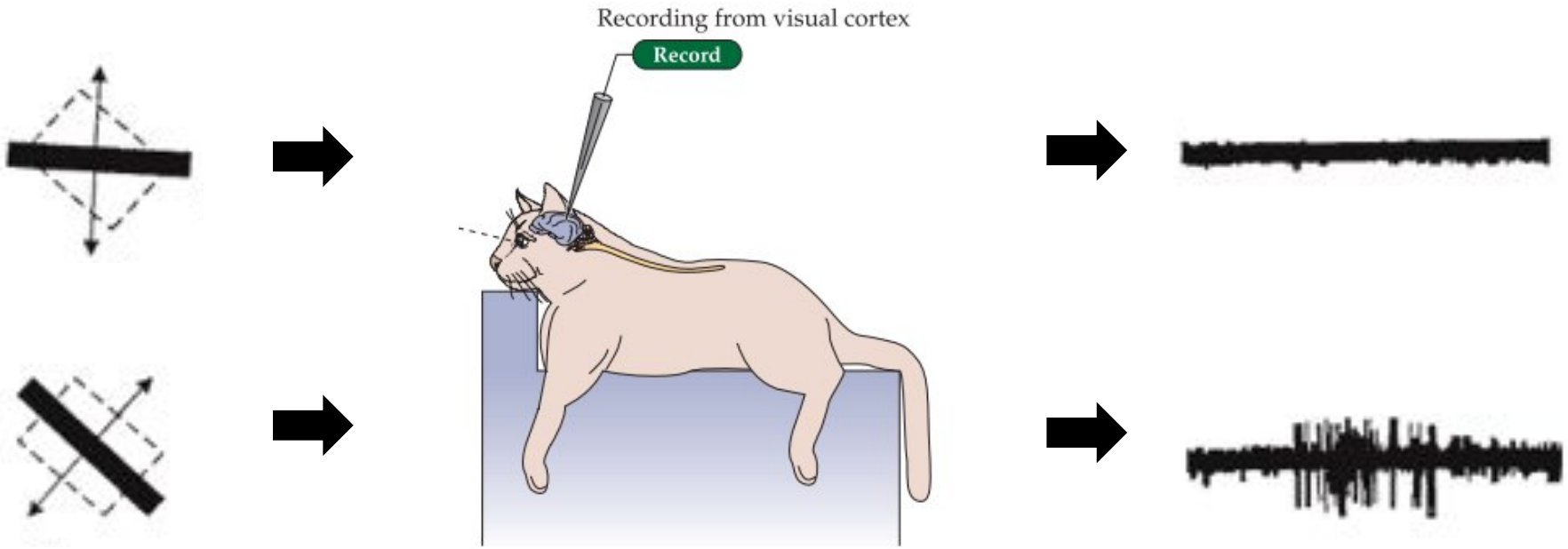
neural activity

y

Given a stimulus, predict the neural response

[Jonathan Pillow]

Encoding problem: orientation response in V1



What the neuronal response will be?

[Hubel & Wiesel, 1968]

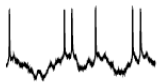
Neural coding problem

x : stimulus

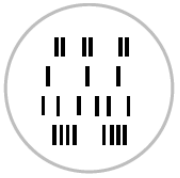
Measurement

$P(y|x)$

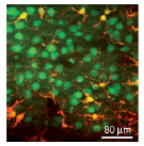
y : neural response



membrane potential



spikes



imaging

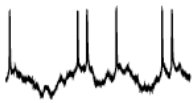
Estimation

$P(x|y)$

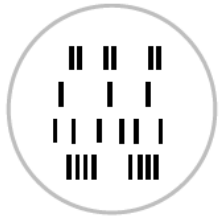
$\tilde{x}$: estimated stimulus

[Jonathan Pillow]

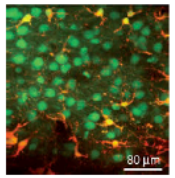
Neural coding problem: decoding



membrane potential



spikes



imaging

neural activity

y

x : stimulus

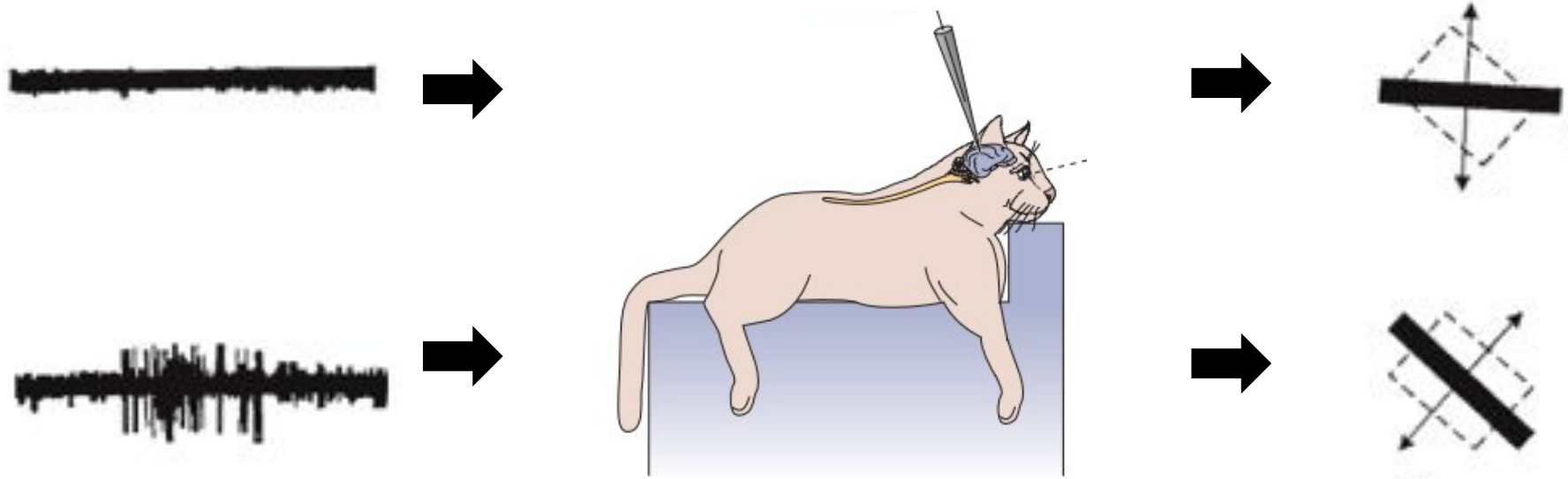
Decoding model:

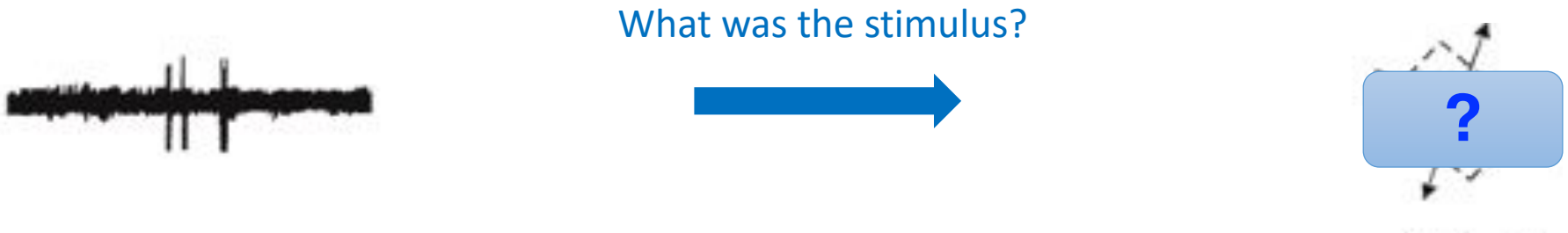
$$P(x|y)$$

Given a neural response, what was the stimulus?

[Jonathan Pillow]

Decoding problem: estimate orientation of the stimulus



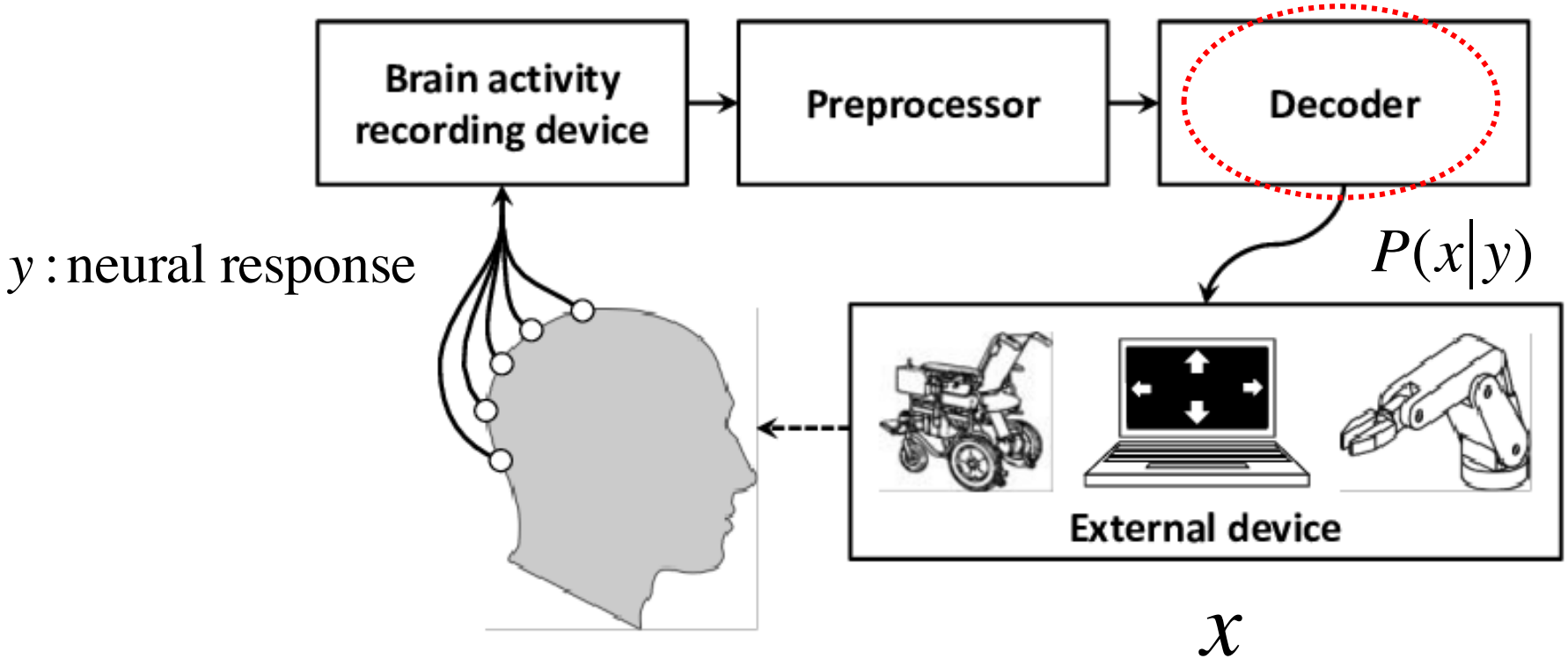


[Hubel & Wiesel, 1968]

Why decoding?

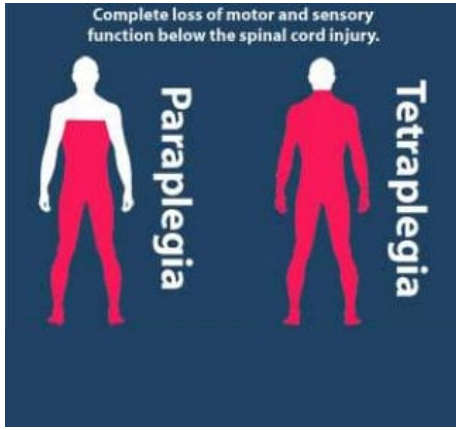
1. “mind reading”

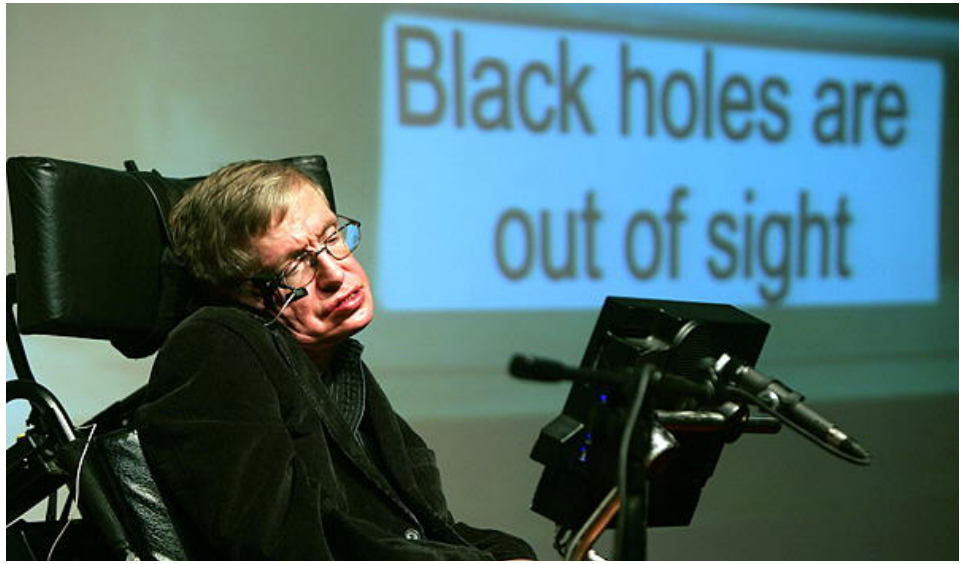
Brain-computer interface (BCI)



Why decoding?

1. “mind reading”





ALS

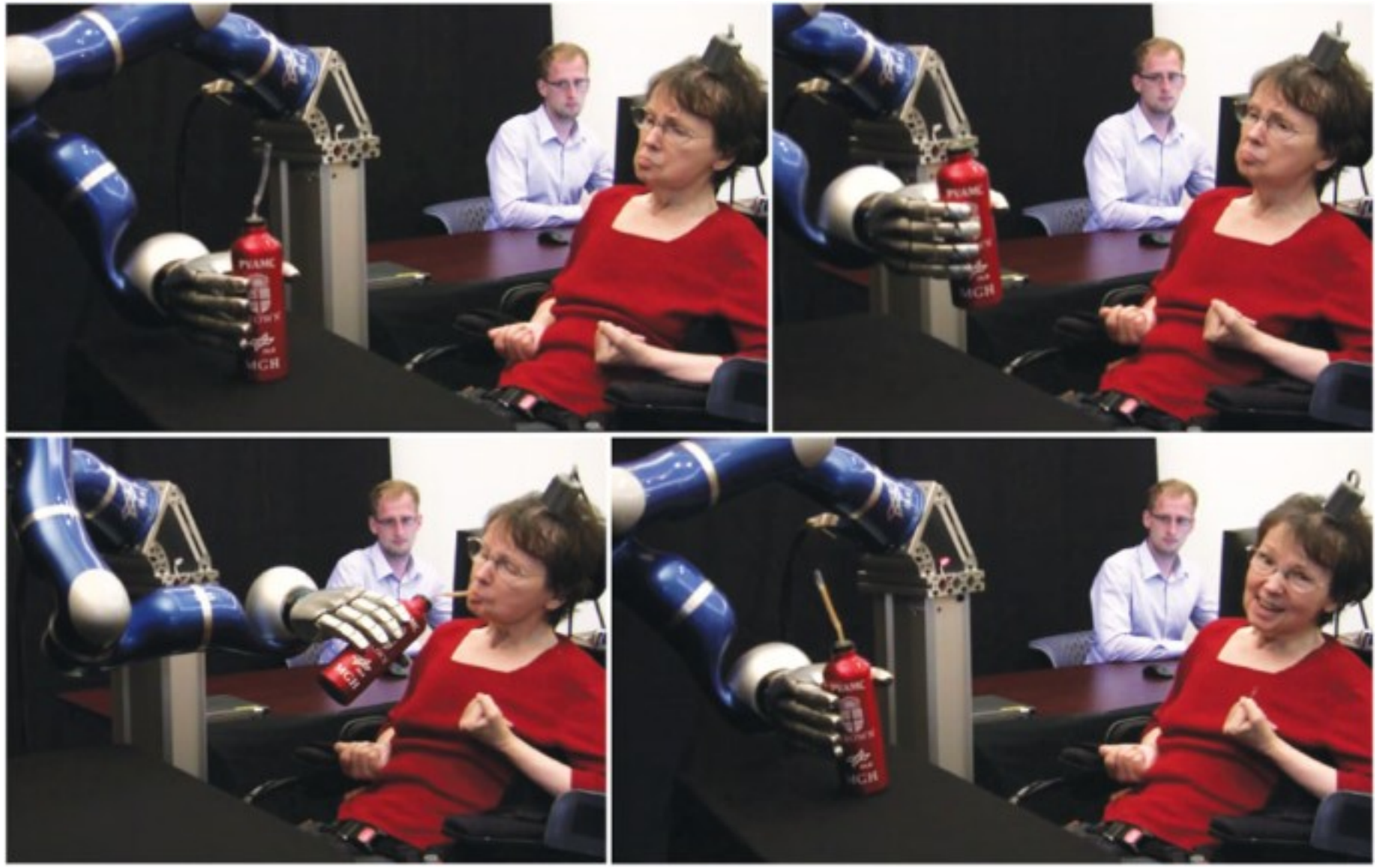
(Clinatec/Juliette Treillet/PA; exoskeleton)

Tetraplegia

Why decoding?

Why decoding?

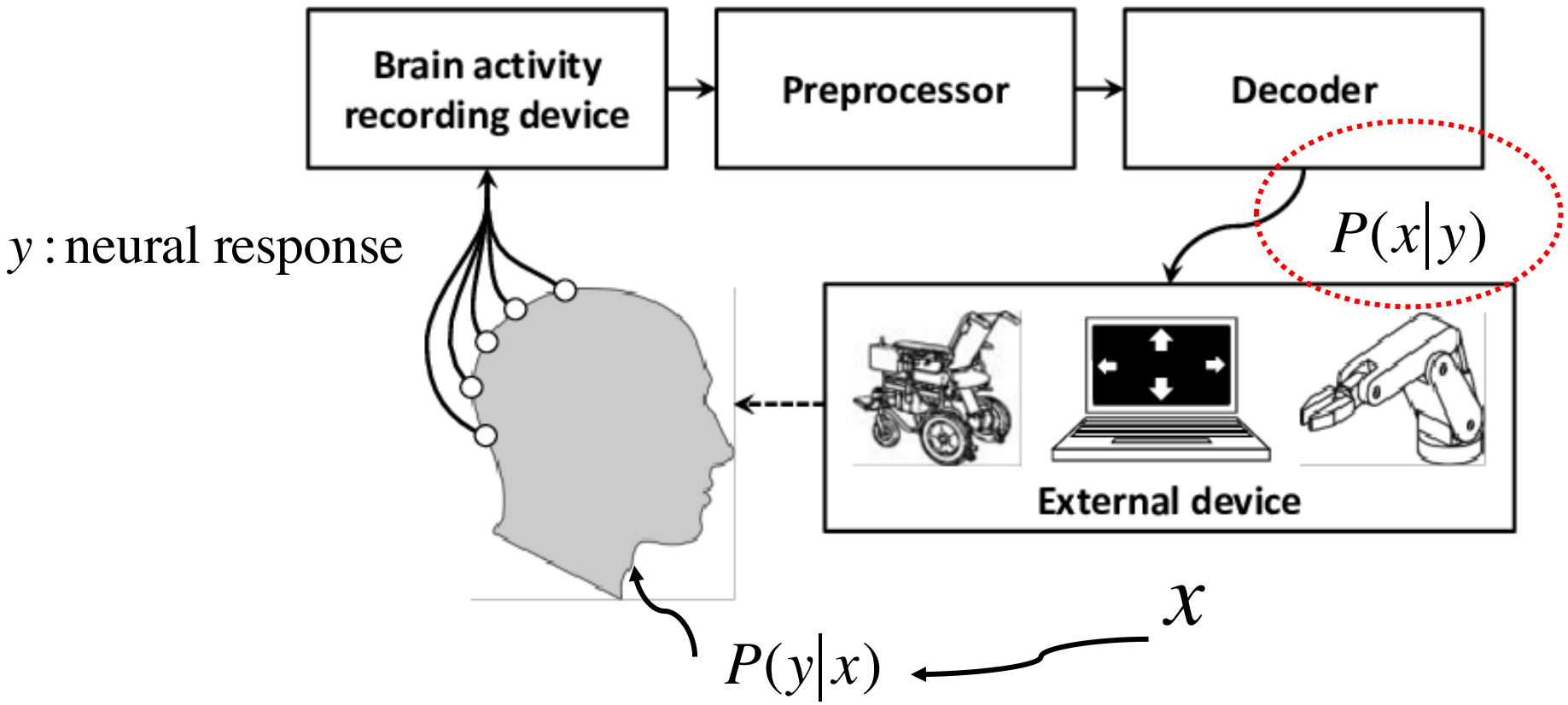
1. “min



Why decoding?

1. “mind reading”

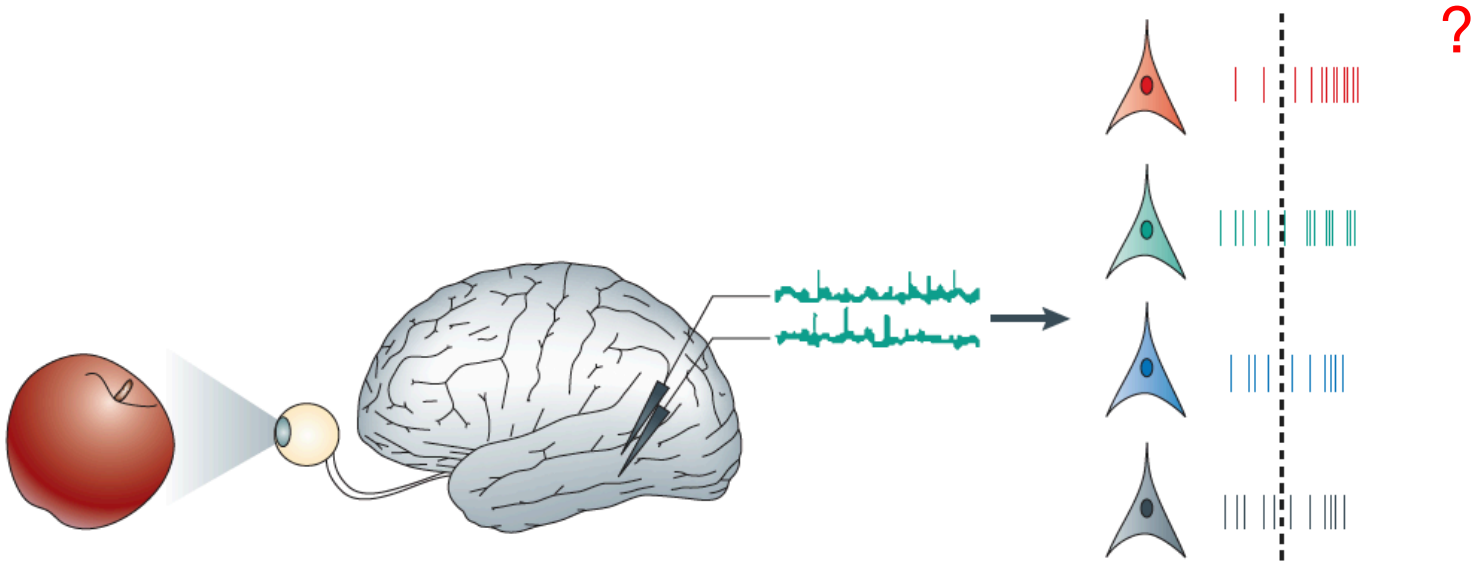
Brain-computer interface (BCI)



Why decoding?

1. “mind reading”

2. How well stimulus-related information is preserved in the neural response?



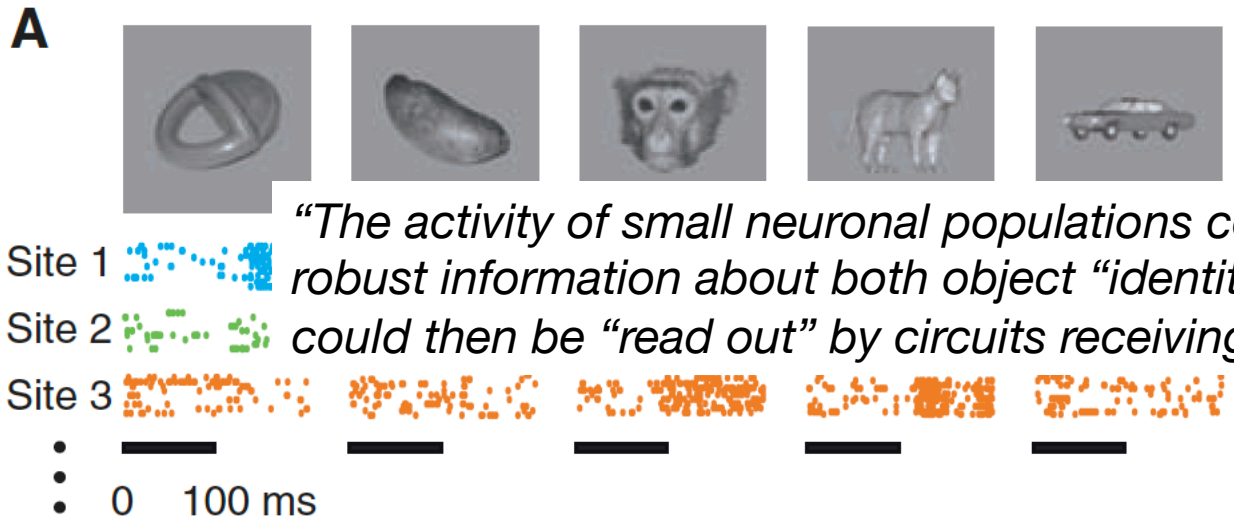
Why decoding?

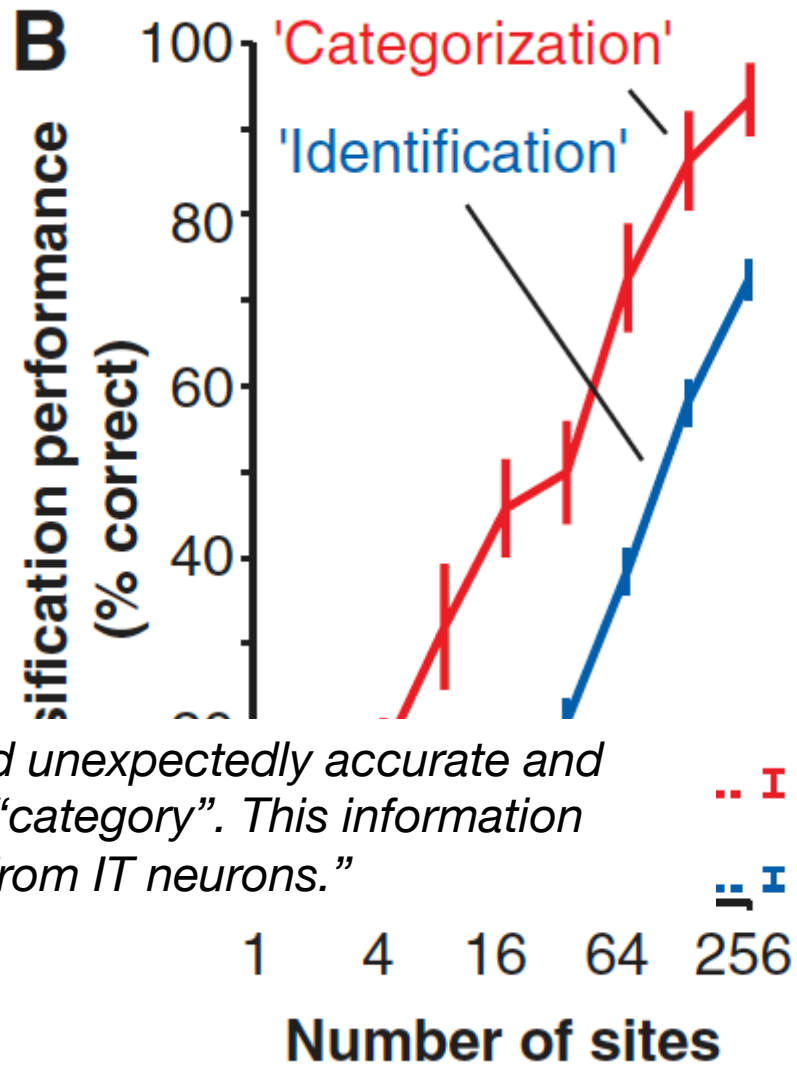
Science

Fast Readout of Object Identity from Macaque Inferior Temporal Cortex

Chou P. Hung, Gabriel Kreiman, Tomaso Poggio and James J. DiCarlo

Science **310** (5749), 863-866.
DOI: 10.1126/science.1117593





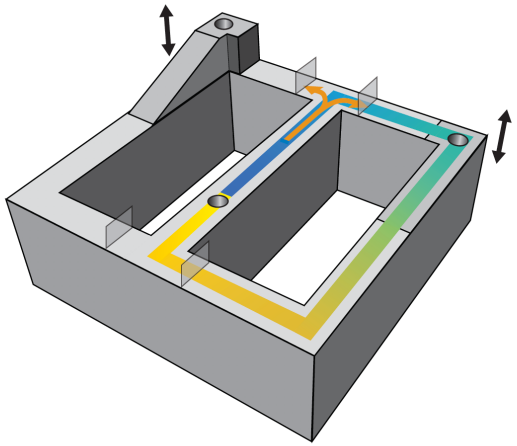
"The activity of small neuronal populations contained unexpectedly accurate and robust information about both object "identity" and "category". This information could then be "read out" by circuits receiving input from IT neurons."

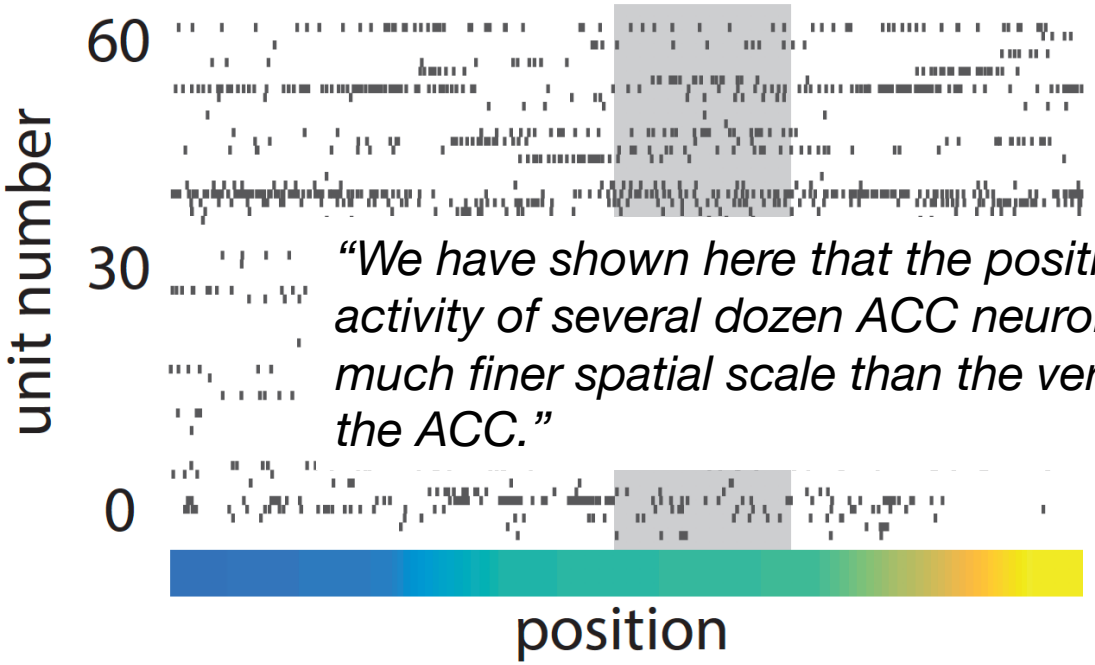
Why decoding?

eLIFE
elife.elifesciences.org

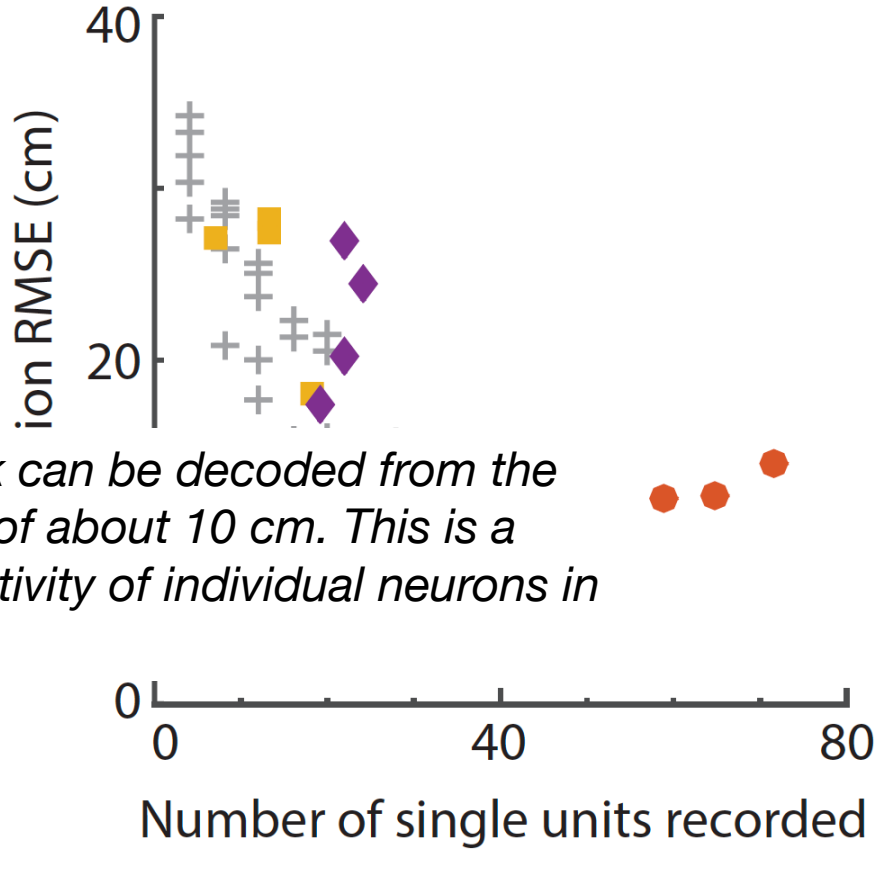
Rat anterior cingulate cortex recalls features of remote reward locations after disfavoured reinforcements

Ali Mashhoori†, Saeedeh Hashemnia†, Bruce L McNaughton, David R Euston, Aaron J Gruber*





"We have shown here that the position of a rat on a track can be decoded from the activity of several dozen ACC neurons with an accuracy of about 10 cm. This is a much finer spatial scale than the very broad spatial sensitivity of individual neurons in the ACC."



Bayesian theorem: link decoding and encoding

x : stimulus

y : neural activity

Encoding model
(likelihood)

Stimulus distribution
(prior)

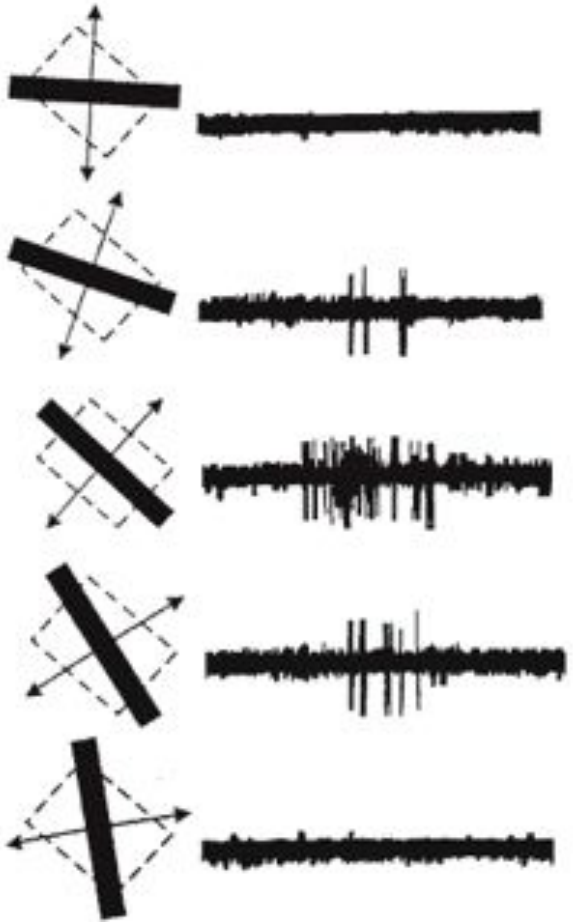
$$P(x|y) = \frac{P(y|x)P(x)}{P(y)}$$

Decoding model
(posterior)

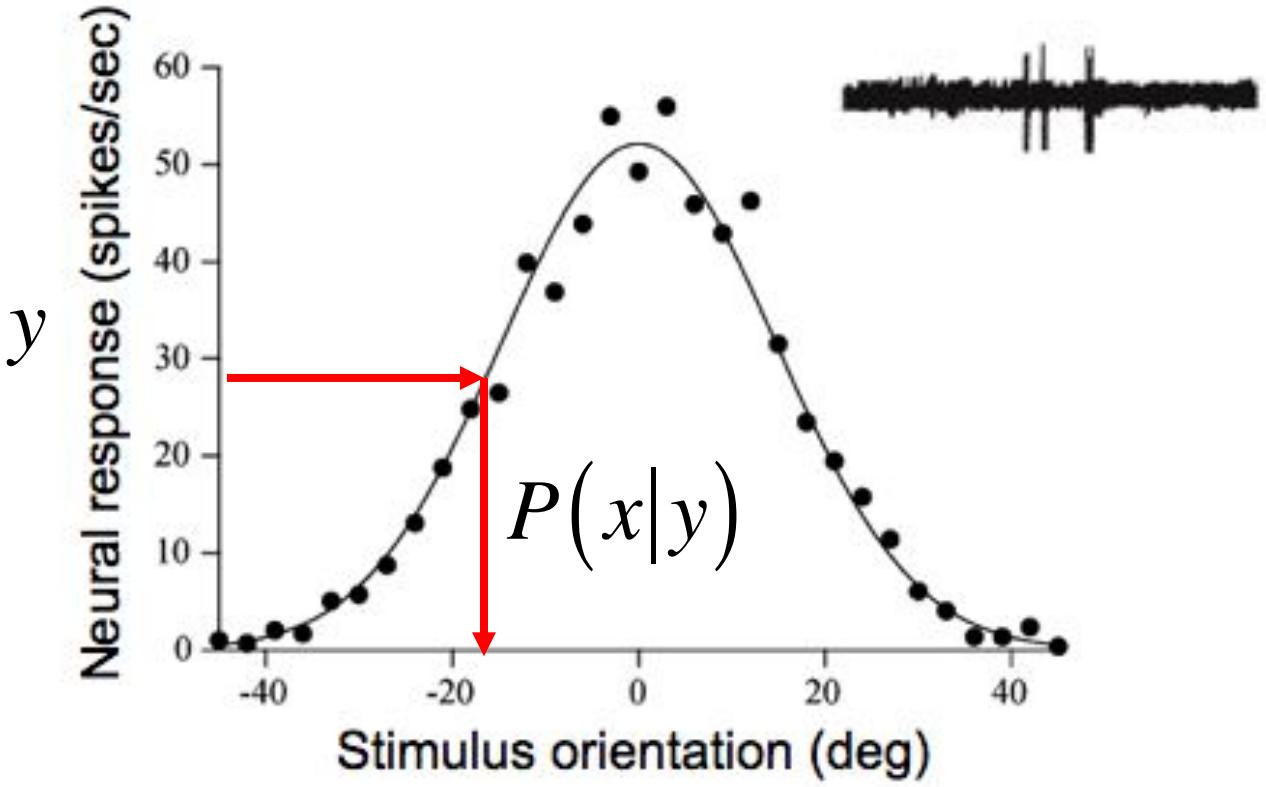
Neural response distribution
(partition function; make sure the sum is 1)

“Simple way” to do decoding

x y



$P(y|x)$

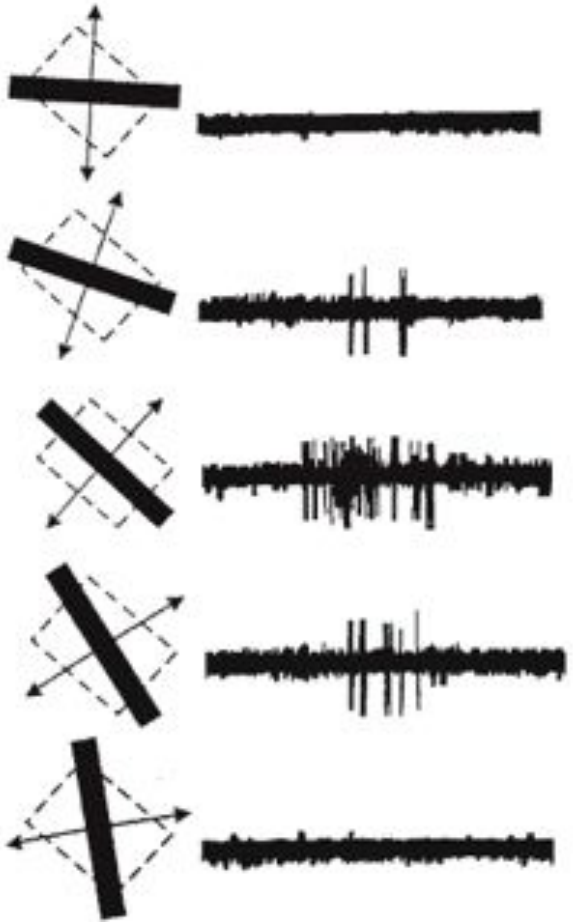


x

[Hubel & Wiesel, 1968]

“Simple way” to do decoding

x y



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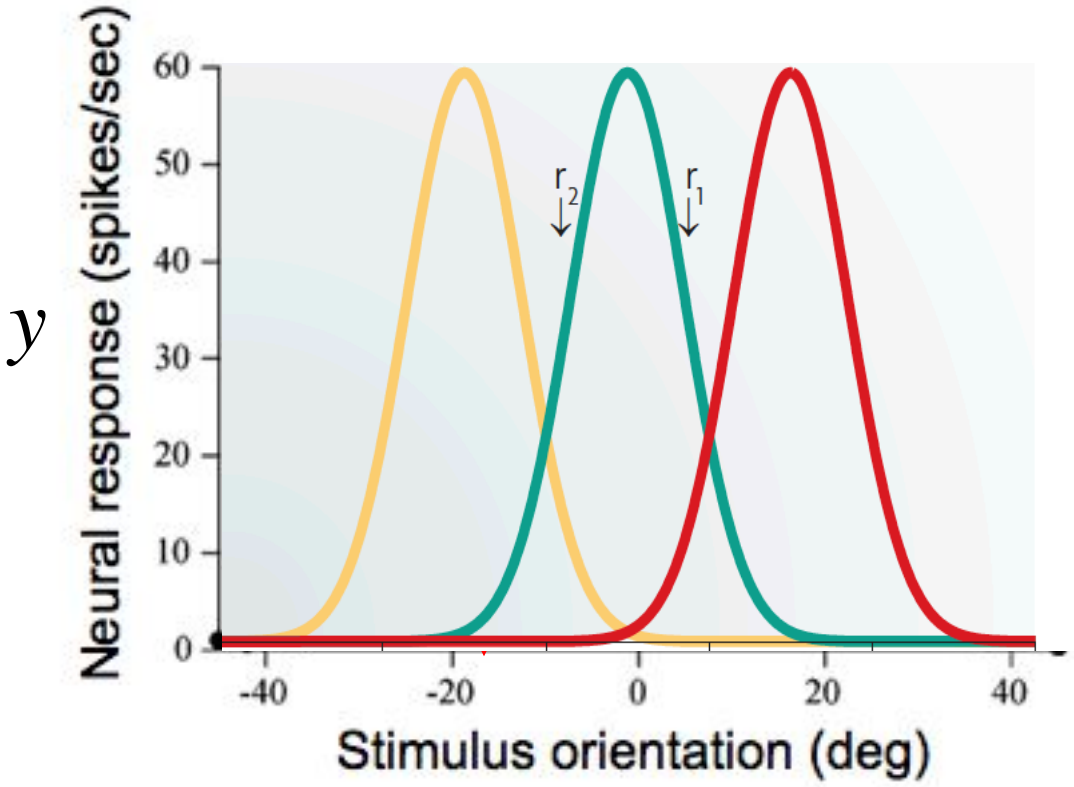


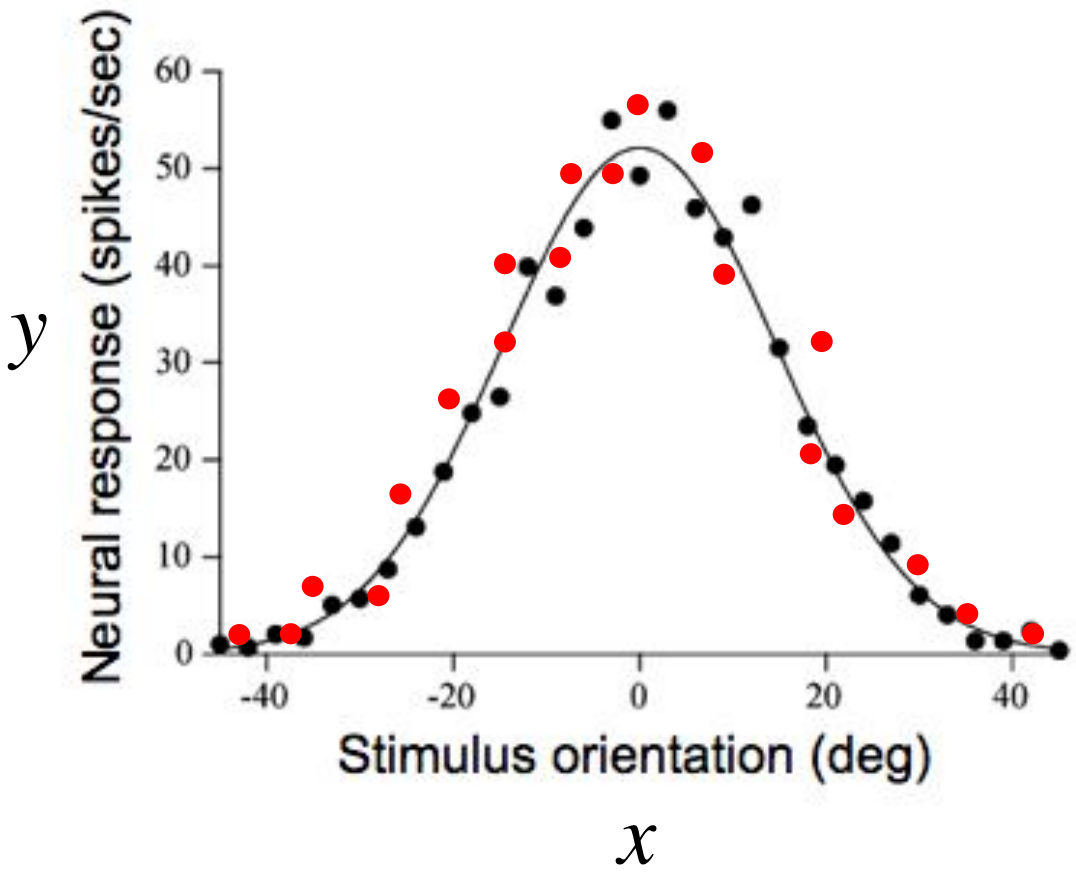
y

x

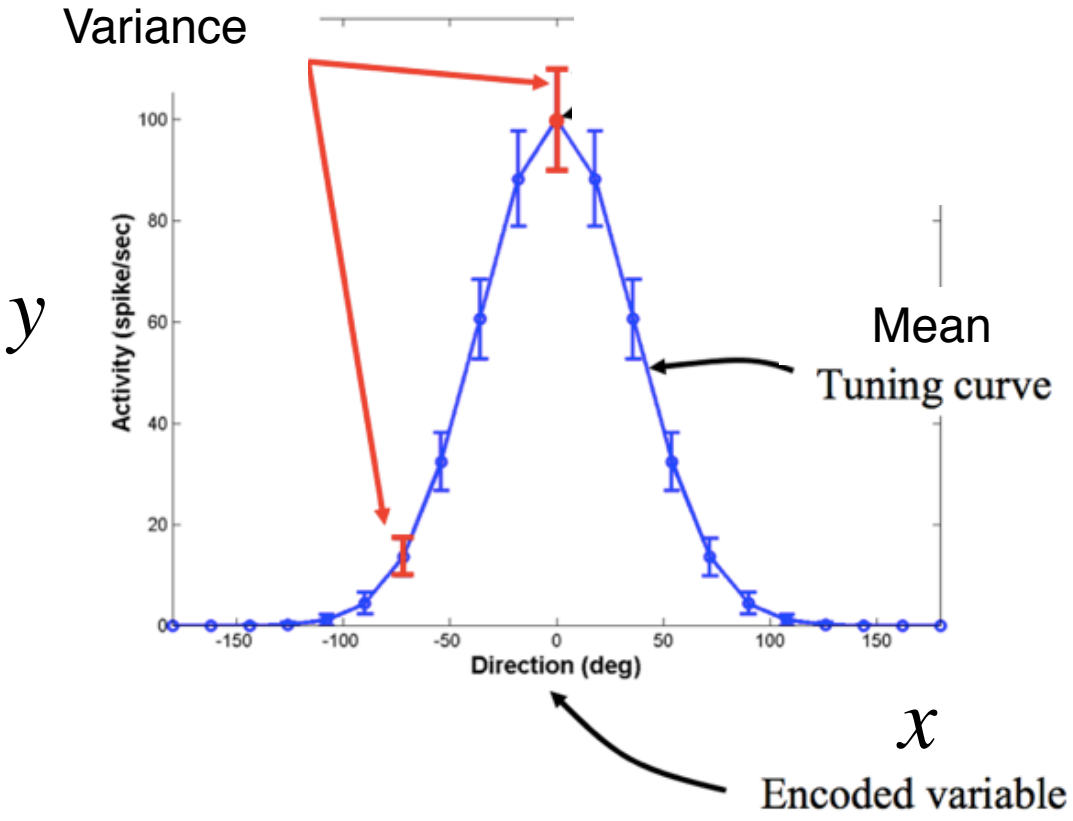
[Hubel & Wiesel, 1968]

Variability

$$P(y|x)$$



$$P(y|x)$$

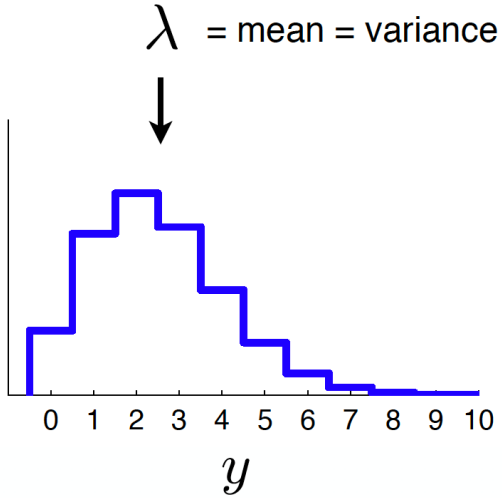


Poisson Variability

Stimulus

Spike rate $\lambda = \tau f(x)$

Spike count $y \sim Poiss(\lambda)$ $\quad \frac{1}{y!}\lambda^y e^{-\lambda}$



Example: real spike data from a prefrontal cortical neuron during 2-min sleep

`spikeTimes:` A (3266 X 1) vector, spike time (in ms) of 3266 spikes from a single PFC neuron

1. Plot the distribution of inter-spike intervals.
2. Fit it with a Poisson distribution (exponential function), and compare with the fits with a linear function and an inverse function. Which one fits the best?

Poisson Variability

Stimulus

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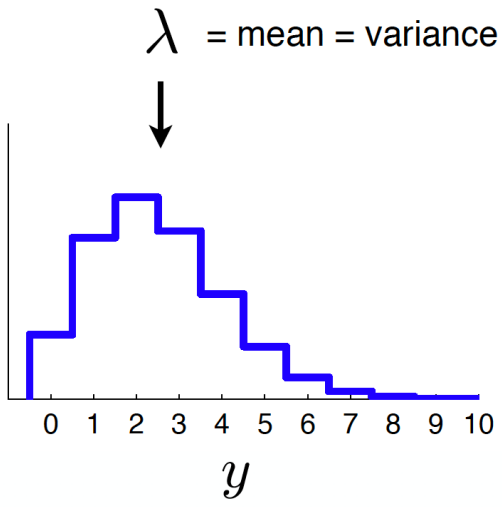

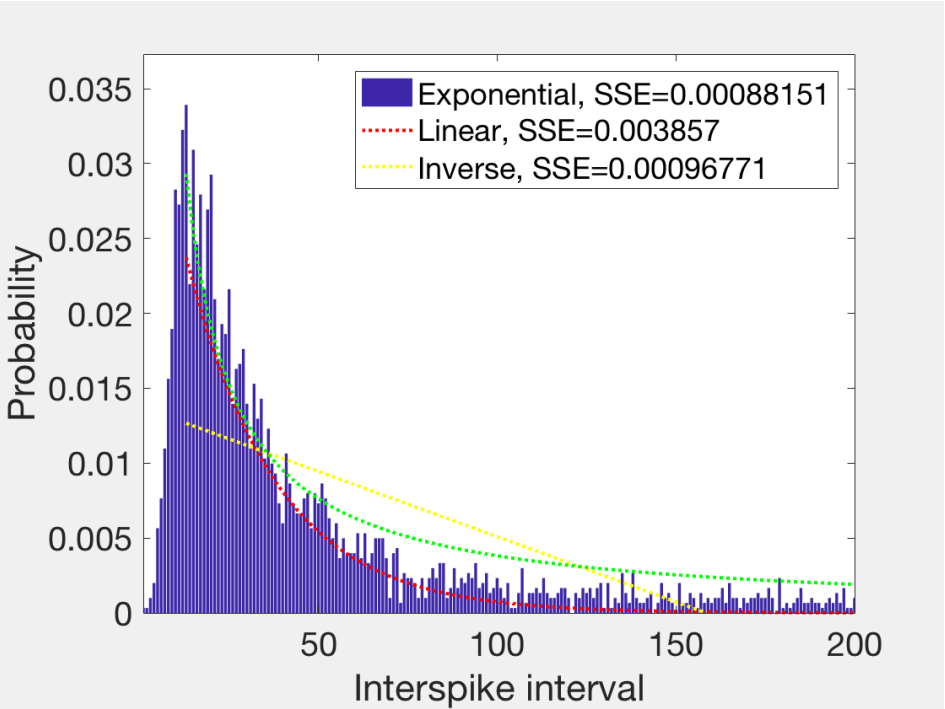

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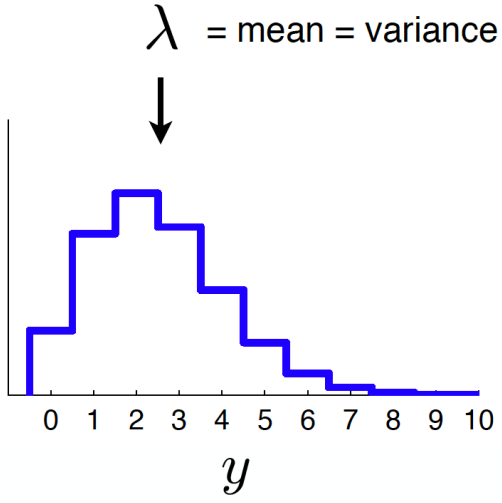


Encoding model: $P(y|x) = \frac{1}{y!}\lambda^y e^{-\lambda} = \frac{1}{y!}(\tau f(x))^y e^{-\tau f(x)}$

$$\text{mean}(y) = \lambda = \tau f(x)$$

$$\text{var}(y) = \lambda = \tau f(x)$$

Poisson Neuron

Encoding model: $$P(y|x) = \frac{1}{y!}\lambda^y e^{-\lambda} = \frac{1}{y!}(\tau f(x))^y e^{-\tau f(x)}$$

Decoding model: $$P(x|y) = \frac{P(y|x)P(x)}{P(y)}$$

Assuming stable neural response across time: $$P(x|y) \propto P(y|x)P(x)$$

Assuming the distribution of stimuli (i.e. *P(x)*, **the "prior"**) is uniform:

Decoding model $$P(x|y) \propto P(y|x)$$ Encoding model

Population of Poisson Neurons

Decoding model

$$P(x|y) \propto P(y|x)$$

Encoding model

$$P(x|y) = C(x) \bullet P(y|x)$$

normalization factor, so that $\sum_x P(x|y) = 1$

Population of neurons (*N*): $\mathbf{y} = \{y_1, y_2, \ldots, y_N\}$

$$P(x|\mathbf{y})$$

Independent Poisson variability in a population

Population of neurons (*N*): $\mathbf{y} = \{y_1, y_2, \ldots, y_N\}$

Assume each neuron fires independently:

$$P(\mathbf{y}|x) = P(y_1|x)P(y_1|x)\ldots P(y_1|x) = \prod_{i=1}^{N} P(y_i|x)$$

$$P(\mathbf{y}|x) = \prod_{i=1}^{N} \frac{1}{y_i!}\left(\tau f_i(x)\right)^{y_i} e^{-\tau f_i(x)}$$

Independent Poisson variability in a population

$$P(\mathbf{y}|x) = \prod_{i=1}^{N} \frac{1}{y_i!} f_i(x)^{y_i} e^{-f_i(x)}$$

$$P(x|\mathbf{y}) = C(x) \bullet P(\mathbf{y}|x)$$

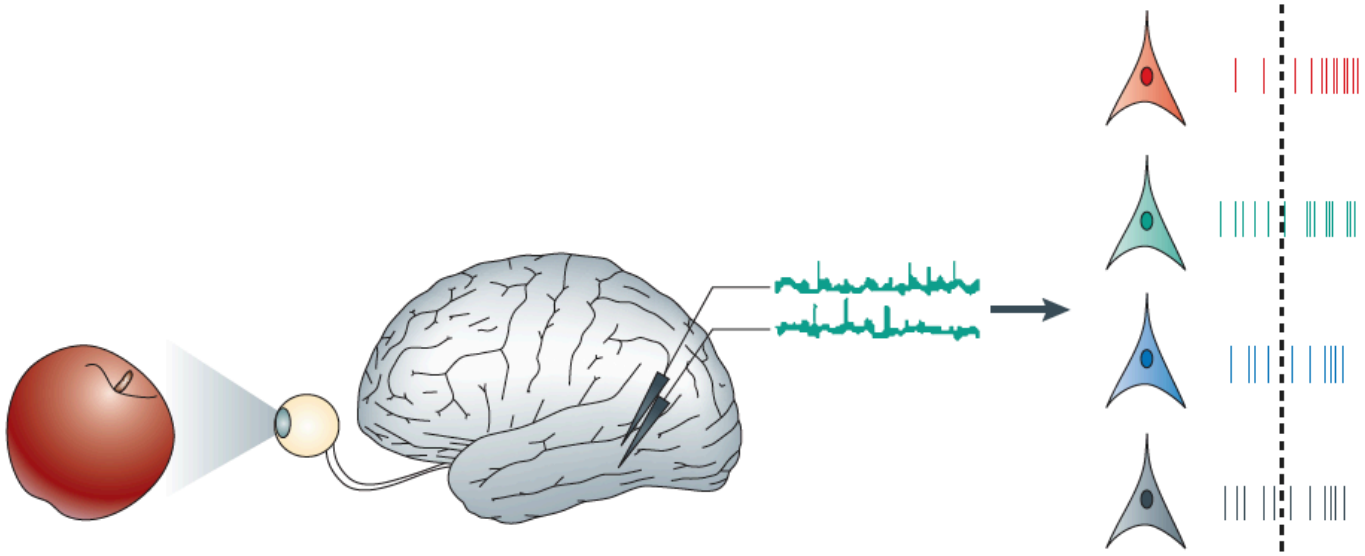
Decoding model

$$P(x|\mathbf{y}) = C(x) \prod_{i=1}^{N} \frac{1}{y_i!} (\tau f_i(x))^{y_i} e^{-\tau f_i(x)} = \boxed{C'(x) \prod_{i=1}^{N} (\tau f_i(x))^{y_i} e^{-\sum_i \tau f_i(x)}}$$

normalization factor, so that $\sum_x P(x|\mathbf{y}) = 1$

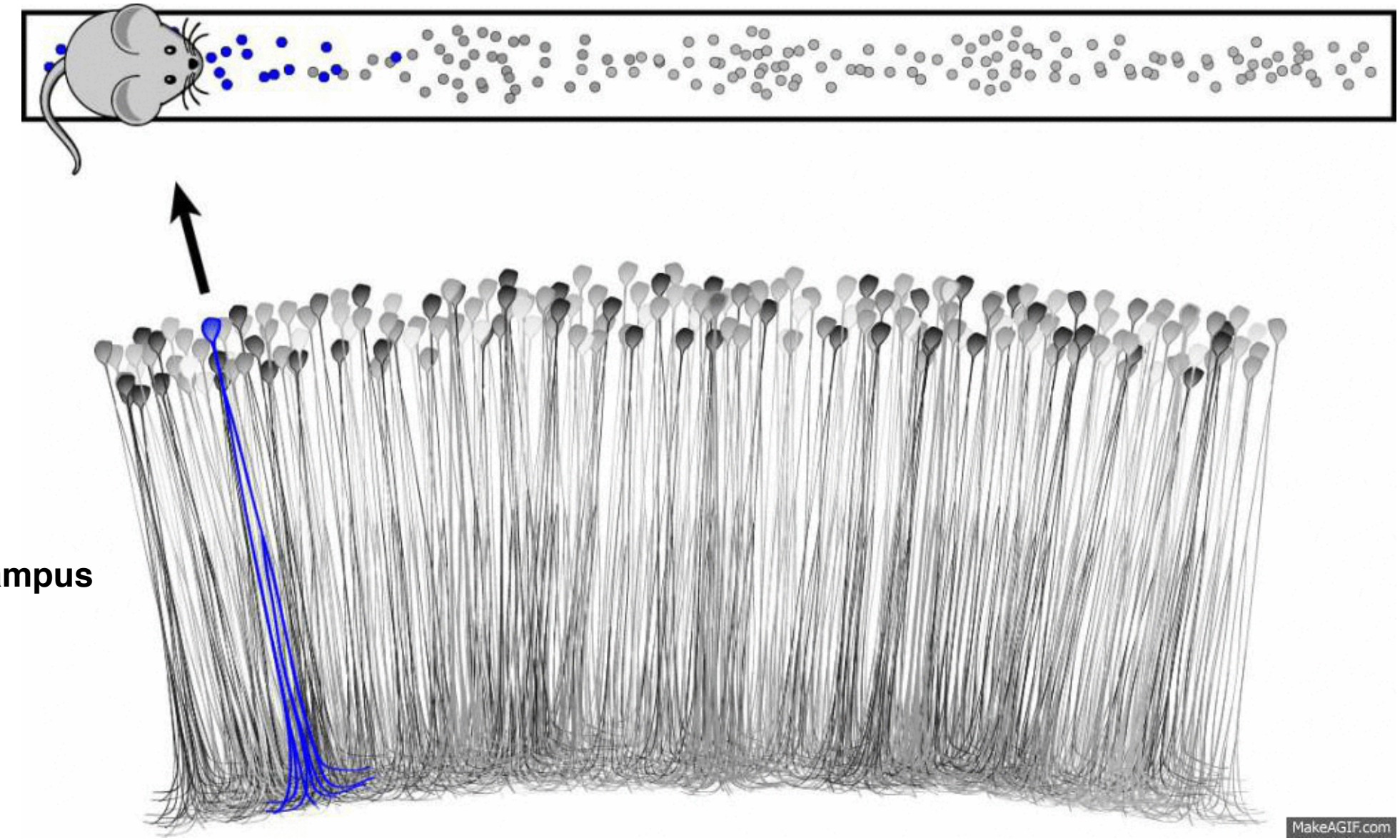
Independent Poisson variability in a population

$$P(x|\mathbf{y}) = C(x)\prod_{i=1}^{N}\frac{1}{y_i!}\left(\tau f_i(x)\right)^{y_i} e^{-\tau f_i(x)} = C'(x)\prod_{i=1}^{N}\left(\tau f_i(x)\right)^{y_i} e^{-\sum_i \tau f_i(x)}$$



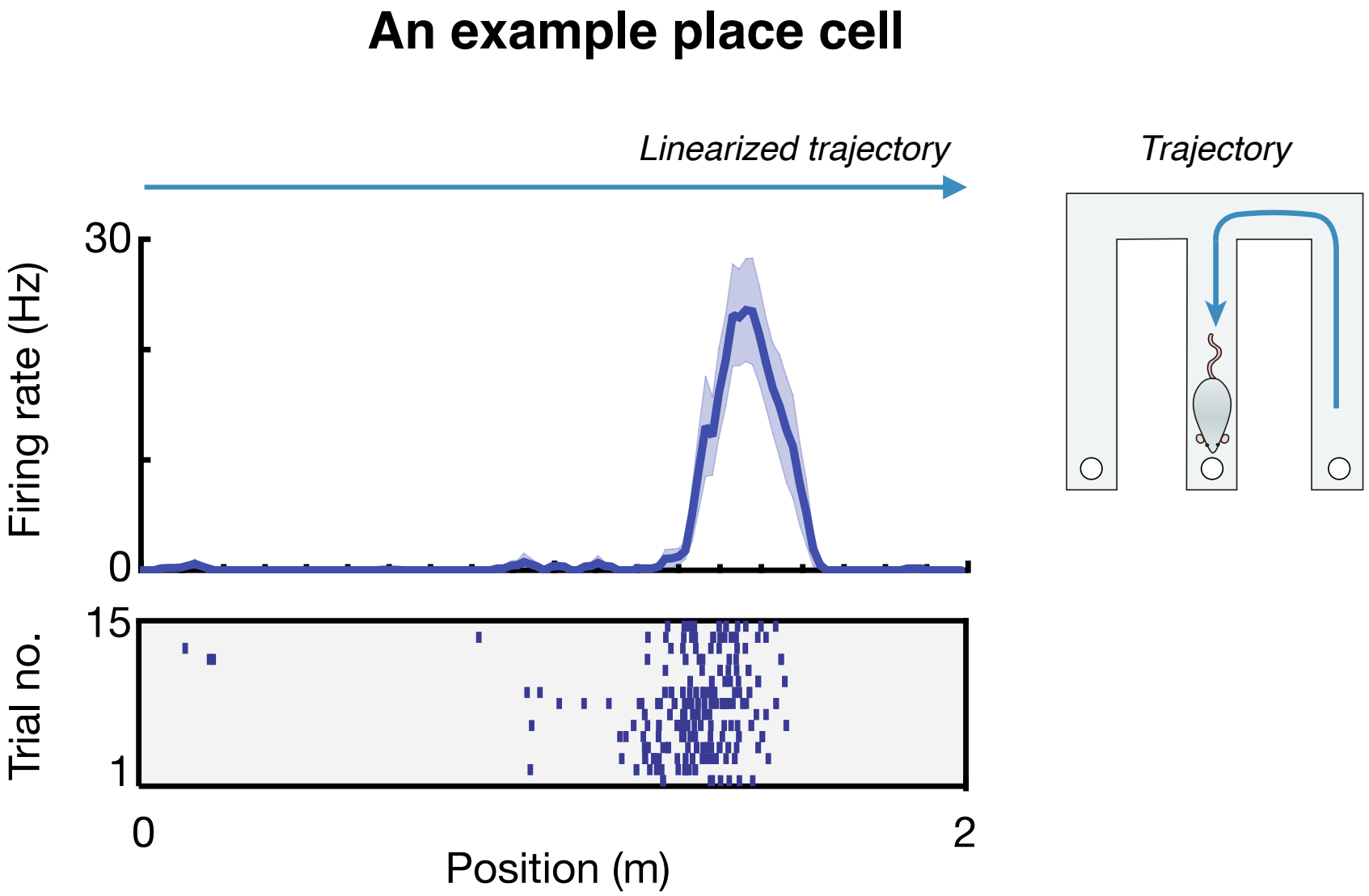
It is an apple!

Example: decoding positions from place cells



Hippocampus

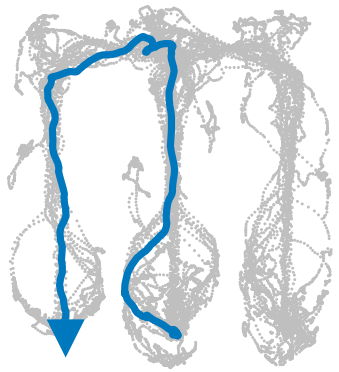
Example: decoding positions from place cells

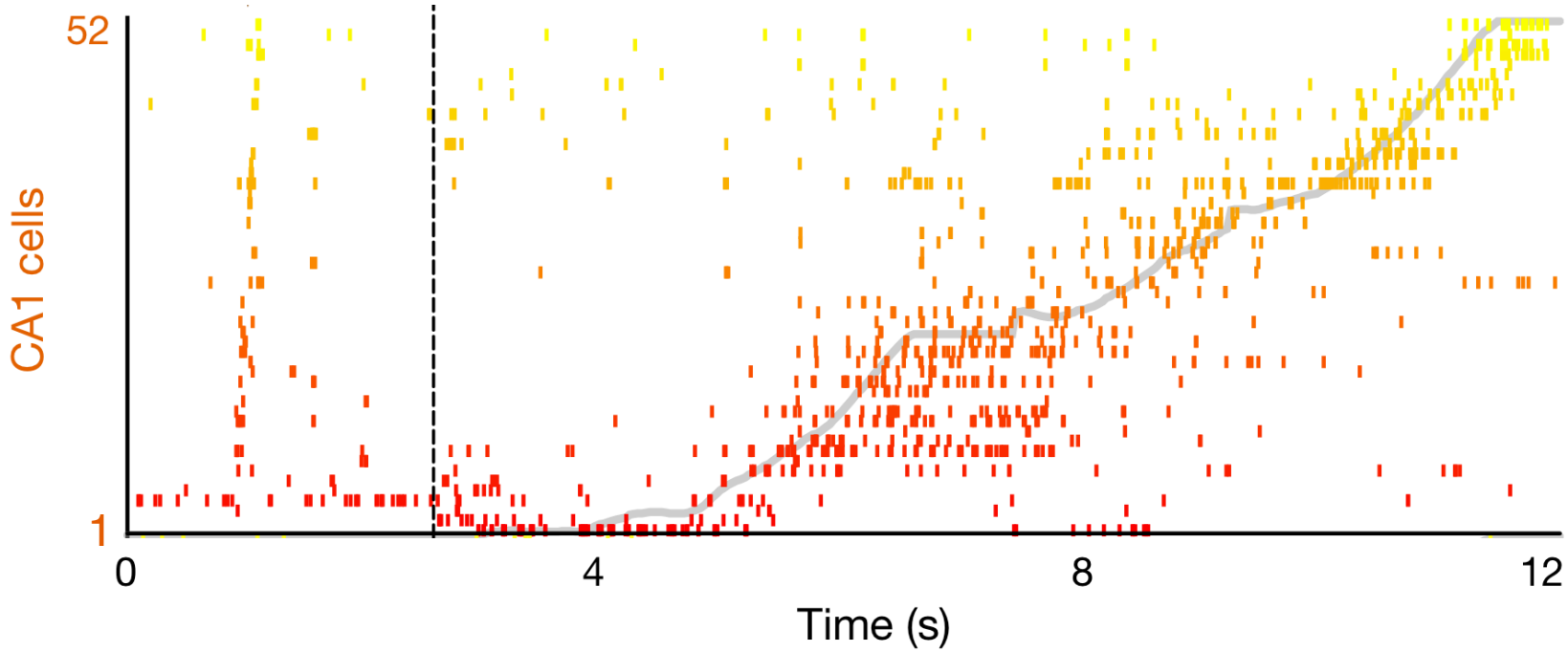


[Tang, Shin & Jadhav, 2021]

Example: decoding positions from place cells

The data





[Tang, Shin & Jadhav, 2021]

Example: decoding positions from place cells

The data structure:

`fx_placefield:` $f(x)$ — Tuning curve, or place field of each cell (*x*: position, in cm)

A (100 X 52) matrix, 100 position bins, 52 place cells

`x_position:` x — A (100 X 1) vector, 100 position bins of the *f(x)*

`truex_position:` x_t — Actual position of the current trial.

A (416 X 2) matrix as [time, position]
time = 0, running start; time < 0, standing at the reward well

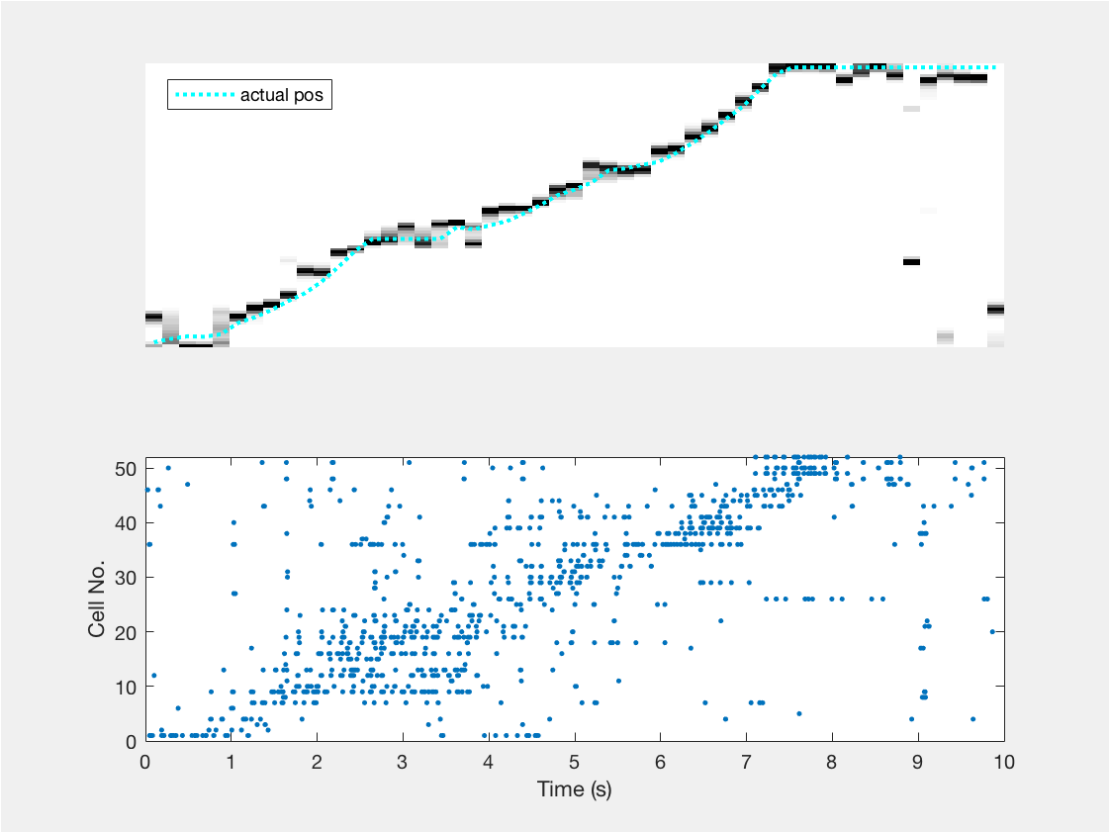
`y_spiketrain:` $\mathbf{y}_t$ — Population spike trains.

A (1450 X 2) matrix as [cell ID, spike time], Cell ID ∈ {1,2,…,52}

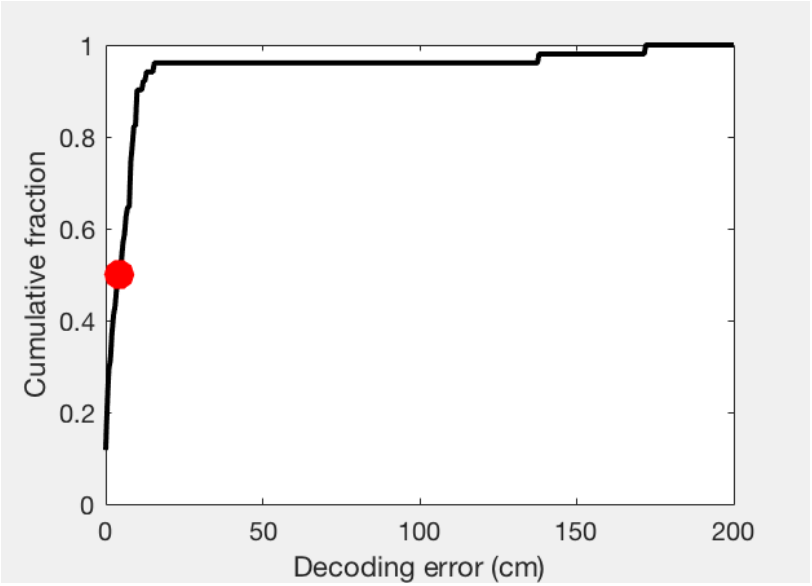
$$P(x|\mathbf{y}) = C'(x)\prod_{i=1}^{N}\left(\tau f_i(x)\right)^{y_i} e^{-\sum_i \tau f_i(x)}, \tau = 200ms$$

Example: decoding positions from place cells

Decoding result

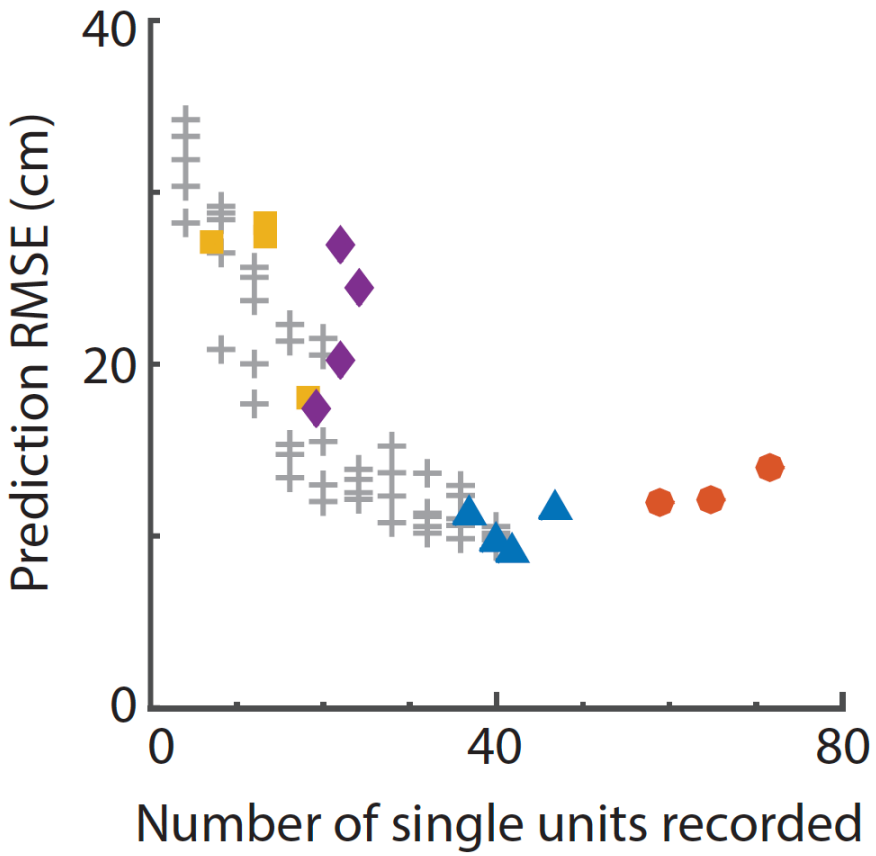


Decoding error
(median = 4.45 cm)



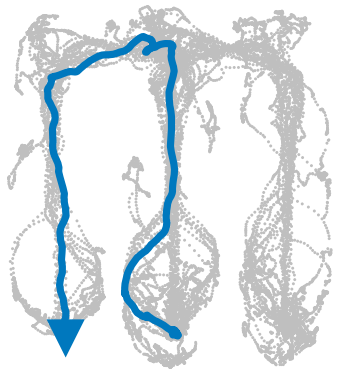
Example: decoding positions from place cells

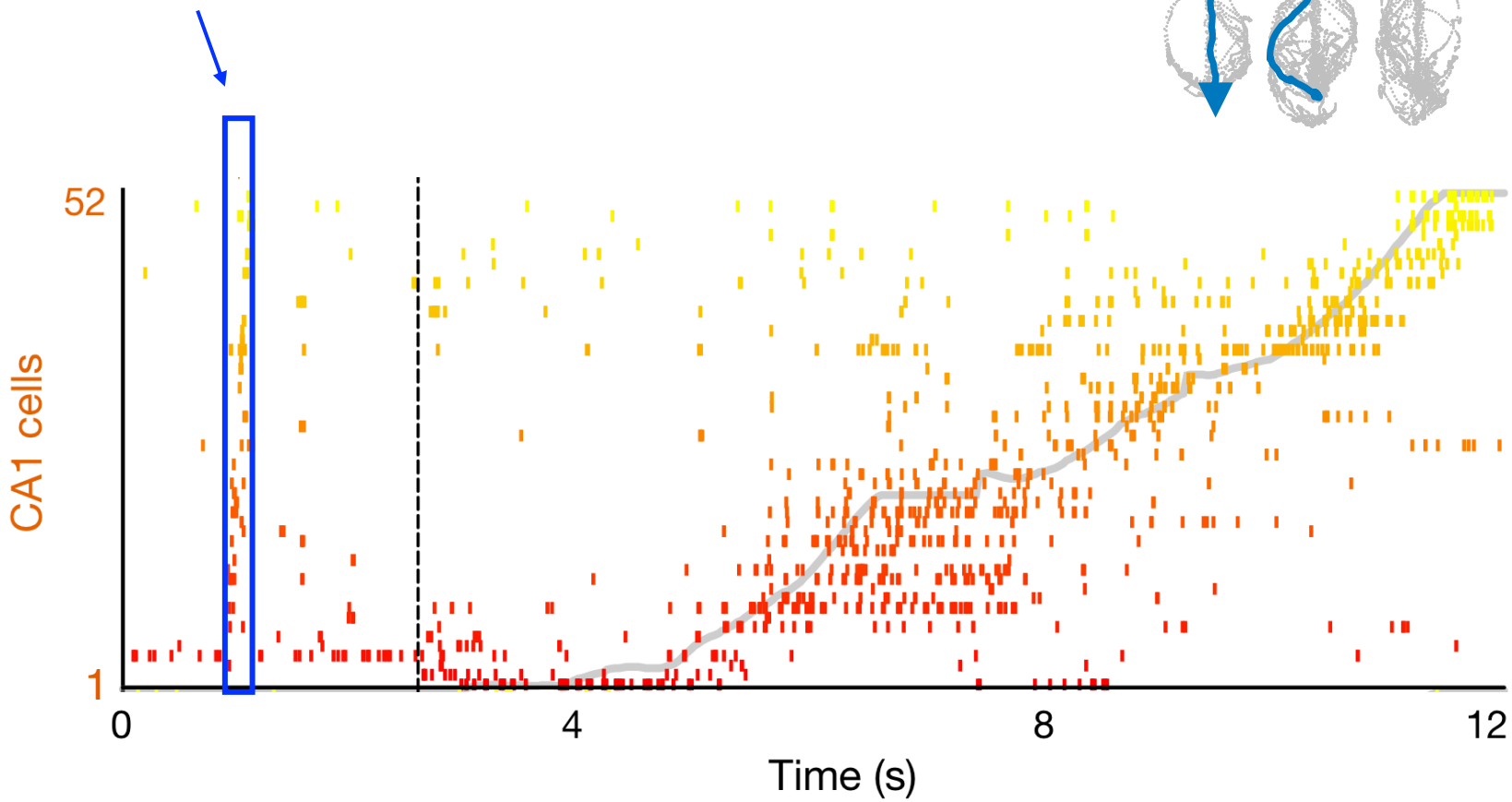
Challenge:



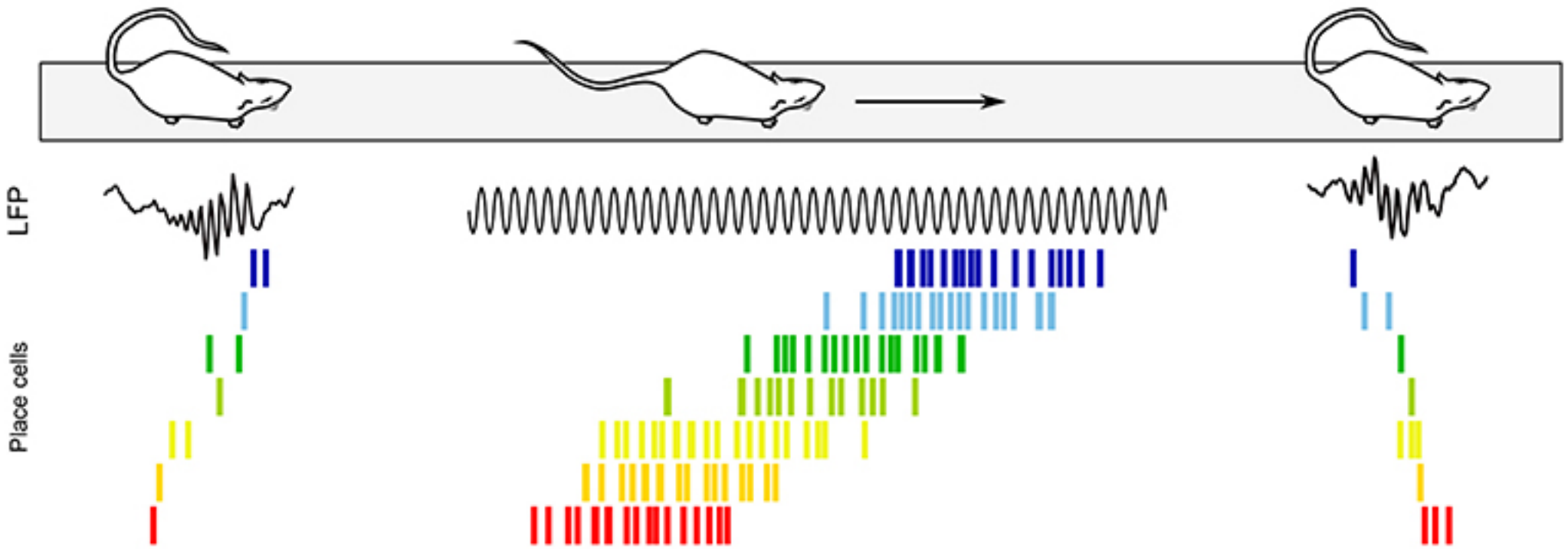
What more can you do?

Burst event: a faster version of the slow sequence?





Example: replay decoding



[Drieu and Zugaro, 2019]

Example: replay decoding

Position decoding

$$P(x|\mathbf{y}) = C'(x)\prod_{i=1}^{N}\left(\tau f_i(x)\right)^{y_i} e^{-\sum_i \tau f_i(x)}, \tau = 200ms$$

Slow time window

Replay decoding

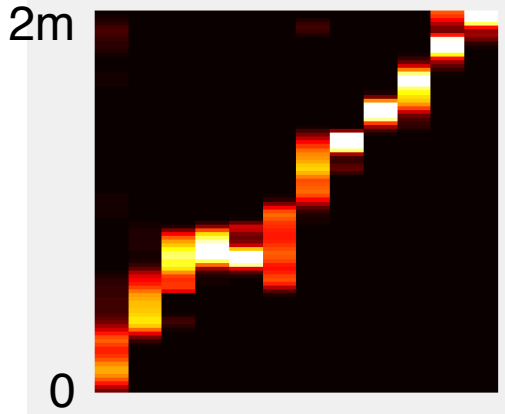
$$P(x|\mathbf{y}) = C'(x)\prod_{i=1}^{N}\left(\tau f_i(x)\right)^{y_i} e^{-\sum_i \tau f_i(x)}, \tau = 20ms$$

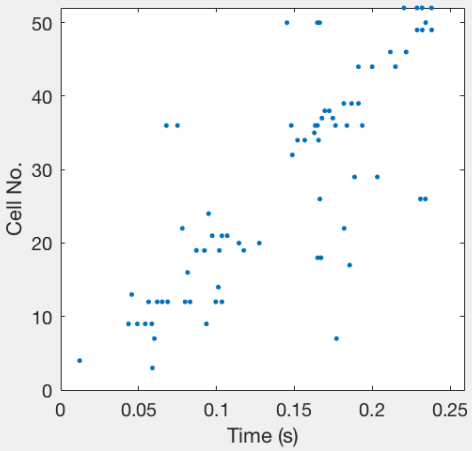
fast time window

`replay_time:` [start, end] time of the burst event

Example: replay decoding

Decoding result





A sweep went from 0 to 2m within ~250ms!

Example: decoding positions from place cells

Challenges:

Can you find some other “bursting” events?

What do their decoding plots look like?

Do they look sequential?

How to quantitatively measure it (is it significantly “sequential”?)?

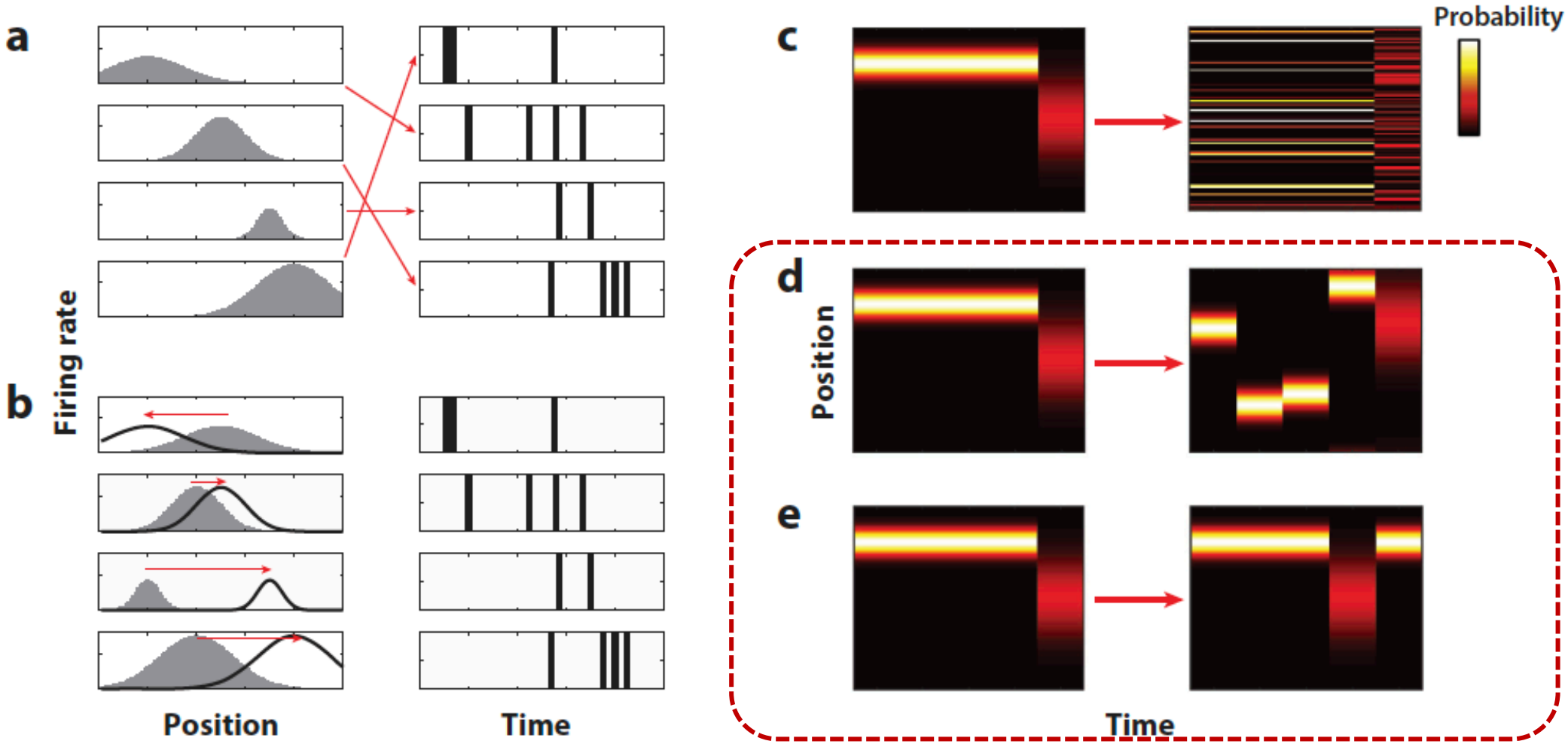
Weighted correlation

$$\bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} \quad \bar{y} = \frac{\sum_{i=1}^{n} w_i y_i}{\sum_{i=1}^{n} w_i} \tag{1}$$

$$s_{x,y} = \frac{\sum_{i=1}^{n} w_i (x_i - \bar{x}_i)(y_i - \bar{y}_i)}{\sum_{i=1}^{n} w_i} \quad s_x = \frac{\sum_{i=1}^{n} w_i (x_i - \bar{x})}{w_i} \quad s_y = \frac{\sum_{i=1}^{n} w_i (y_i - \bar{y})}{w_i} \tag{2}$$

$$r(x, y, w) = \frac{s_{x,y}}{\sqrt{s_x s_y}} \tag{3}$$

Shuffling: permutation tests



[Foster, 2017]