

Probability (for background see 3.pdf, which is Ch. 3 of the stats book)

- a) Definition of probability
 - i) Sample space S containing finite number of points s_i ($i = 1, \dots, n$)
 - ii) $p_i > 0$ for $i = 1, \dots, n$ and $\sum_{i=1}^n p_i = 1$
- b) Negation
 - i) $\sim p_i = 1 - p_i$
- c) Independence
 - i) $\Pr(AB) = \Pr(A)\Pr(B)$
- d) Conditional Probability
 - i) $\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)}$
 - (1) Conditional probability that A has occurred given that B has occurred (proportion of events within A and B)
 - (2) Or ... outcomes in event B that also belong to event A
- e) Bayes rule
 - Let the events A_1, \dots, A_k form a partition of the space S such that
 - (1) $\Pr(A_j) > 0$ for $j = 1, \dots, k$ and let B be any event such that $\Pr(B) > 0$, then
$$\Pr(A_i|B) = \frac{\Pr(A_i)\Pr(B|A_i)}{\sum_{j=1}^k \Pr(A_j)\Pr(B|A_j)} = \frac{\Pr(A_i)\Pr(B|A_i)}{\Pr(B)}$$
 - (2) $\Pr(A_i)$ is the prior,
 $\Pr(A_i|B)$ is the posterior.

Random variables

- i) “a random variable is a quantity whose values are random and to which a probability distribution is assigned” or
- ii) . “... a measurable function from a sample space to the measurable space of possible values of the variable.” (Wikipedia)
- iii) Set of possible outcomes resulting from a sampling of an event.

Distribution Functions

- a) Probability Distribution Function (pdf)
 - (1) Function $f(x)$ describing probability of getting various values of x :

Requires that $\int_{-\infty}^{\infty} f(u)du = 1$

b) Cumulative Distribution Function (cdf)

Function $F(x)$ describing cumulative probability of getting various values of x :

$$F(x) = P(X < x) = \int_{-\infty}^x f(u)du$$

Simulating arbitrary CDF:

1. Draw random number R between 0 and 1.
2. Find x coordinate of CDF that corresponds to y coordinate of R .
3. Repeat for each desired sample.

Example:

$g(x) = 1, 2, 3 \text{ or } 4;$

$f(x) = .1 * x$

Draw $f(x)$, $F(x)$, take samples