

1. Minimal Spanning Tree:

It is a special kind of tree that minimises the length or weights of edges of the tree.

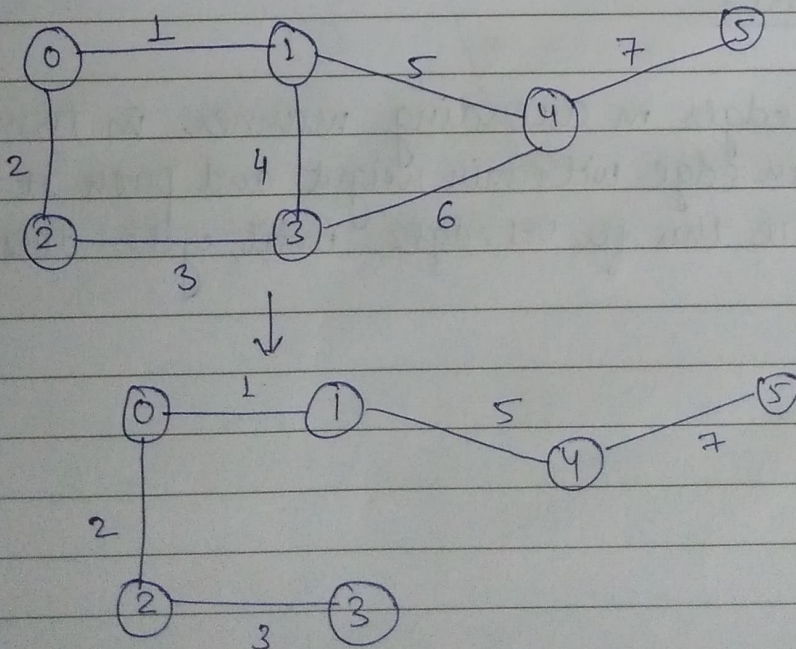
A tree is said to be spanning tree

- If it contains all the vertices.
- Spans all the vertices with $n-1$ edges.
- Is acyclic.

- A tree is minimum spanning tree if it spans the minimum weight while spanning all the vertices.

Applications:

1. Telephone
2. TV cable
3. Computer networks
4. Constructing roads while spanning several areas / cities.

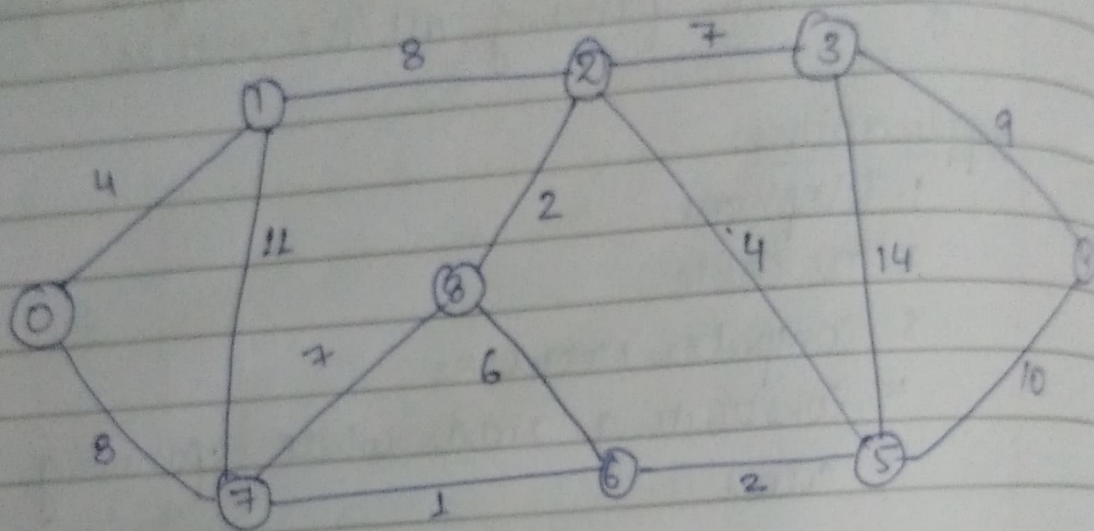


2. Time complexity

Kruskal's algorithm: $O(E \log V)$

Dijkstra's algorithm: Matrix - $O(n^2)$
Heap - $O(E \log V)$

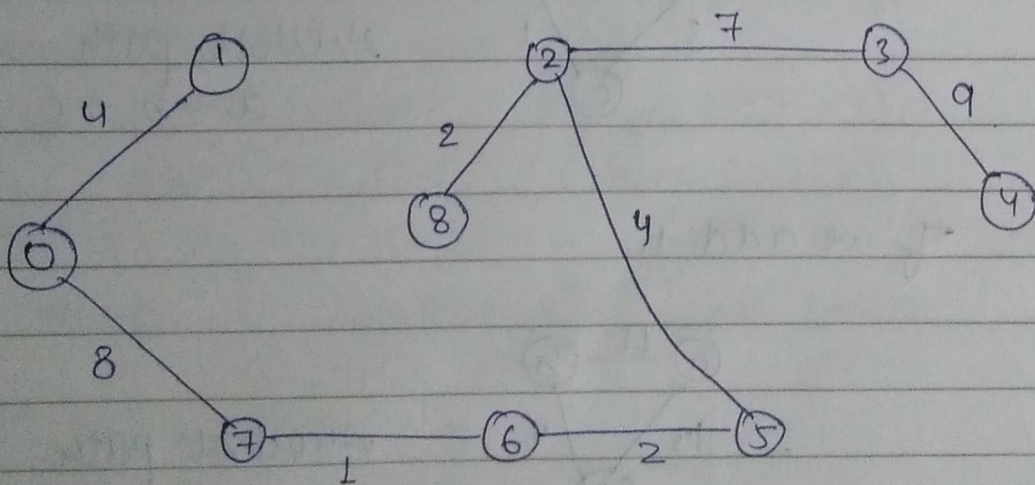
Prim's algorithm: Matrix - $O(n^2)$
Heap - $O(E \log V)$



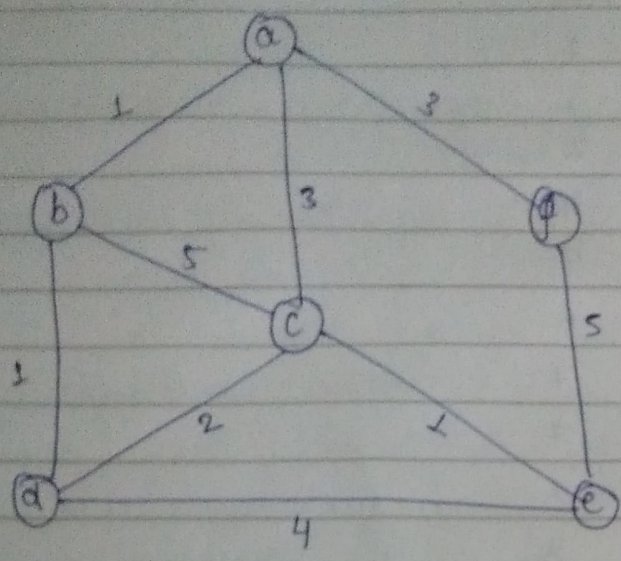
Kruskal's

- Sort edges in ascending manner in terms of weight
- Pick an edge with min weight and push it to result
- Continue this for $V-1$ edges until cycle does not come

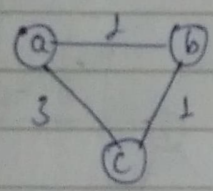
u	v	wt.
7	6	1
2	8	2
6	5	2
0	1	4
2	5	4
8	6	6
7	8	7
2	3	7
0	7	8
1	2	8
3	4	9
5	4	10
1	7	11
3	5	17.



4.

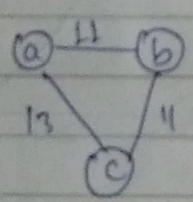


- If we add the weights of the graph by 10, Yes the shortest path can change then consider



shortest path
 $a \rightarrow b \rightarrow c$

If we add 10



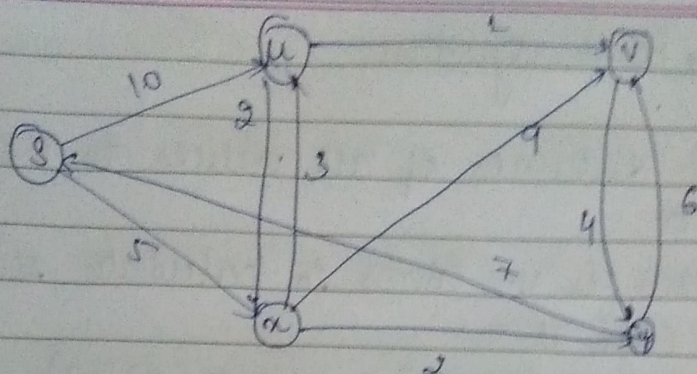
shortest path
 $a \rightarrow c$

- There is no change in the shortest path if we multiply all the weights at edges by 10.

If we multiply

let $10N$ & $10M$
 if $N < M$
 $\Rightarrow 10N < 10M$
 \therefore No change

5.



Dijkstra's algorithm:

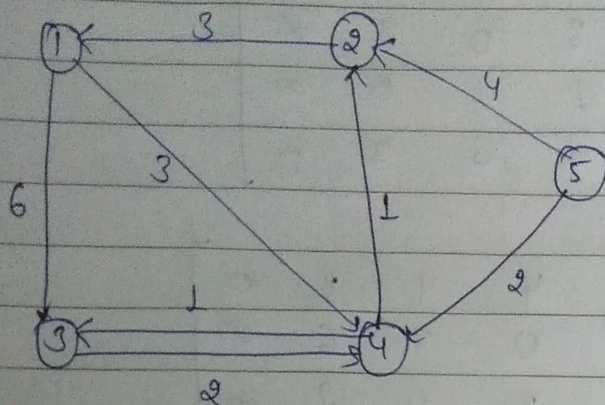
- create sptset which keeps track of vertices
- we assign all the vertices with distance infinite. Then we assign distance of source node to 0.
- While sptset does not include all the vertices.

- i. Pick a vertex which is not sptset and has min distance
- ii. Include it in sptset
- iii. update distance value of all the adjacent vertices of the above vertex using condition.

$$\text{if } (\text{dist}[v] > \text{dist}[u] + \text{graph}[u][v]) \\ \text{dist}[v] = \text{dist}[u] + \text{graph}[u][v]$$

Node	Shortest dist. from source.
s	0
u	8
a	5
v	9
y	7

6. Floyd Warshall's



$$Q = \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix}$$

$$G_4 = \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ 6 & 3 & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 6 & 3 & 3 & 2 & 0 \end{bmatrix}$$

$$G_5 = \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ 6 & 3 & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 6 & 3 & 2 & 2 & 0 \end{bmatrix}$$