Unit No - II

Centroid / centre of Gravity

* Gravity axis:-

Line of action of the gravitational force acting on the body.

* Centre of Gravity:-

The point through which the whole ~~weight~~ mass or weight of the body acts, irrespective of its position is known as centre of Gravity.

* Centre of mass:-

The point where the entire mass of a body is supposed to be concentrated.

* Centroid :- (geometrical centre of plane Areas)

The plane figures like triangle, quadrilateral, circle have only areas but no mass. The centre of area of such figures is known as centroid.

* Procedure to find centroid of composite Areas & Lines

- Select suitable co-ordinate axes if not given.
- Divide given area into different parts having known length & area & centroid distances.
- If the section is symmetrical about x axis, then we can directly find $\bar{y}$.
- If the section is symmetrical about y axis, then we can directly find $\bar{x}$.

Let $\bar{x}$ = x co-ordinate of centre of gravity OR distance of C.G. of the area from y axis.

$\bar{y}$ = y-co-ordinate of centre of gravity OR Distance of C.G. of the area from x-axis.

let $a_1, a_2, a_3 \cdots$ are the areas into which whole figure is divided,

Then

$$\bar{x} = \frac{A_1 x_1 \pm A_2 x_2 \pm \cdots \pm A_n x_n}{A_1 \pm A_2 \pm \cdots \pm A_n}$$

$$\bar{y} = \frac{A_1 y_1 \pm A_2 y_2 \pm \cdots \pm A_n y_n}{A_1 \pm A_2 \pm \cdots \pm A_n}$$

where,

$x_1, x_2, x_3 \cdots$ = distances of centroid of parts $1, 2, 3, \cdots$ from Y axis.

&

$y_1, y_2, y_3 \cdots$ = distance of centroid of parts $1, 2, 3 \cdots$ from x axis.

For line segments & wire bents.

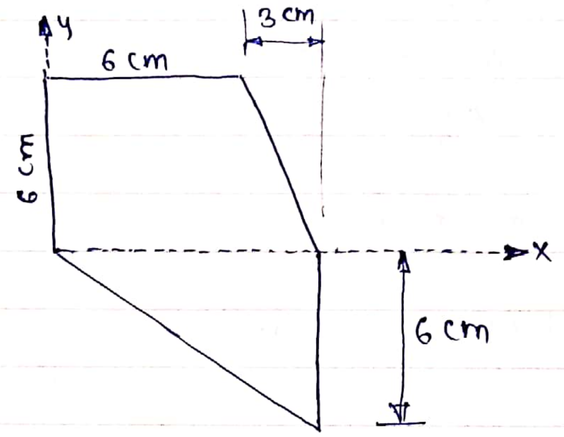
Replace the area (A) by length (L)

$$\therefore \bar{X} = \frac{L_1 x_1 \pm L_2 x_2 \pm \cdots \pm L_n x_n}{L_1 \pm L_2 \pm \cdots \pm L_n}$$

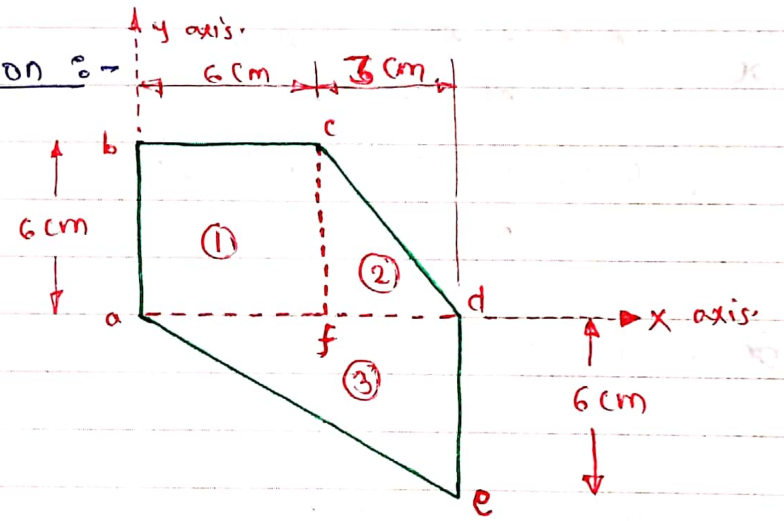
$$\therefore \bar{Y} = \frac{L_1 y_1 \pm L_2 y_2 \pm \cdots \pm L_n y_n}{L_1 \pm L_2 \pm \cdots \pm L_n}$$

* Problems on the centroid of Areas *.

① Locate the centroid of area of the following figure w.r.t. given axis.



⇒ Solution :-



Let us divide the given area as shown above into 3 simple parts.

Consider Area ① i.e. square abcd,

$\therefore$ Area of □ abcd $= A_1 = 6 \times 6 = 36 \text{ cm}^2$

$x_1 = \frac{6}{2} = 3 \text{ cm}$

$y_1 = \frac{6}{2} = 3 \text{ cm.}$

consider Area ② i.e. Δ^{le} cfd,

$\therefore A_2 =$ Area of Δ cfd $= \frac{1}{2} \times df \times cf$

$= \frac{1}{2} \times 3 \times 6$

$A_2 = 9 \text{ cm}^2$

$x_2 = 6 + \frac{b}{3} = 6 + \frac{3}{3} = 7 \text{ cm.}$

$y_2 = \frac{h}{3} = \frac{6}{3} = 2 \text{ cm.}$

Consider Area ③ i.e. Δ ade,

$\therefore$ Area of Δ ade $= \frac{1}{2} \times b \times h$

$A_3 = \frac{1}{2} \times 9 \times 6$

$A_3 = 27 \text{ cm}^2$

$x_3 = \frac{2}{3} \times 9 = 6 \text{ cm.}$

$y_3 = -\frac{1}{3} \times 6 = -2 \text{ cm.}$

Now,

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$= \frac{(36 \times 3) + (9 \times 7) + (27 \times 6)}{36 + 9 + 27}$$

$$\bar{x} = 4.625 \text{ cm.}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

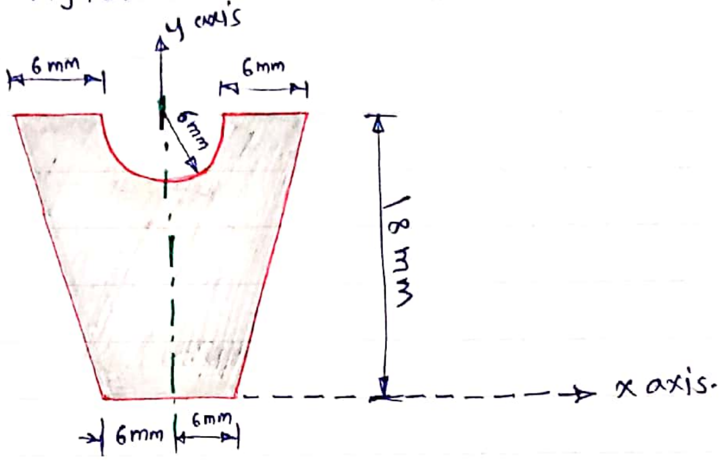
$$= \frac{(36 \times 3) + (9 \times 2) + (27 \times (-2))}{36 + 9 + 27}$$

$$\bar{y} = 1 \text{ cm.}$$

$\therefore$ co-ordinate of centroid of the given area are $\bar{x} = 4.625$ cm (from y axis)

$\bar{y} = 1$ cm (from x axis)

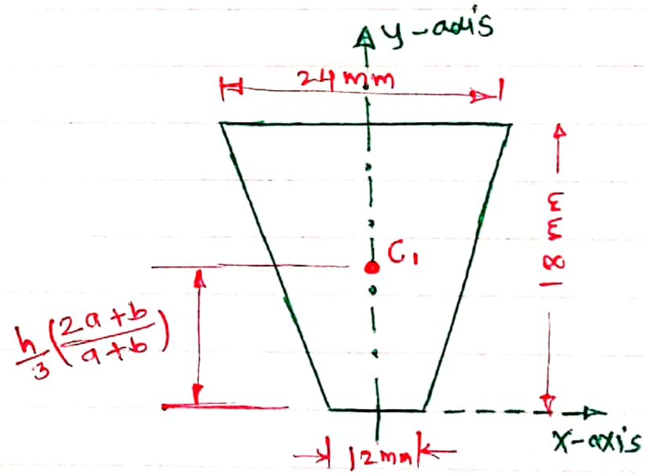
② Determine the Y co-ordinate of the centroid of the shaded area as shown in figure.



⇒ Consider Trapezoid As area ① & semicircle as Area ②

∴ For trapezoid & semicircle, shaded area is symmetrical about y axis, & thus, as y axis is passing through symmetry line, $\boxed{\bar{x} = 0}$

1] For Trapezoid :-



$A_1 = \left(\frac{a+b}{2}\right) h$

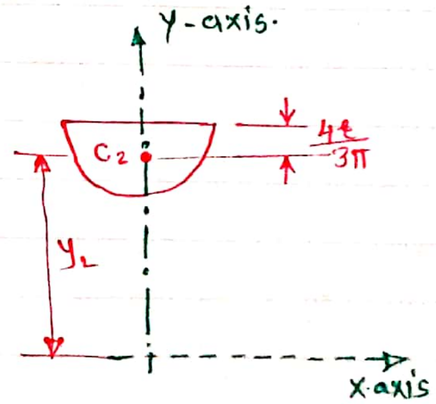
$= \left(\frac{24+12}{2}\right) \times 18$

$A_1 = 324 \text{ mm}^2$

$y_1 = \left(\frac{2a+b}{a+b}\right) \frac{h}{3}$

$y_1 = \frac{(2 \times 24) + 12}{24+12} \times \frac{18}{3} = 10 \text{mm}.$

② for semicircle.



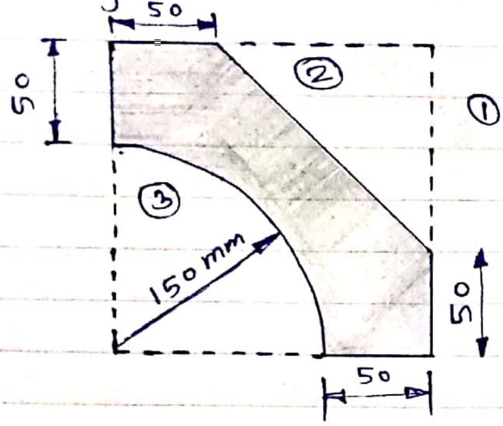
$A_2 = \frac{\pi r^2}{2} = \frac{\pi \times 6^2}{2} = 56.55 \text{ mm}^2$

$y_2 = 18 - \frac{4r}{3\pi} = 18 - \frac{4 \times 6}{3\pi} = 15.45 \text{mm}.$

$\therefore \bar{Y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{(324 \times 10) - (56.55 \times 15.45)}{324 - 56.55} =$

$\therefore \boxed{\bar{Y} = 8.84 \text{ mm}}$

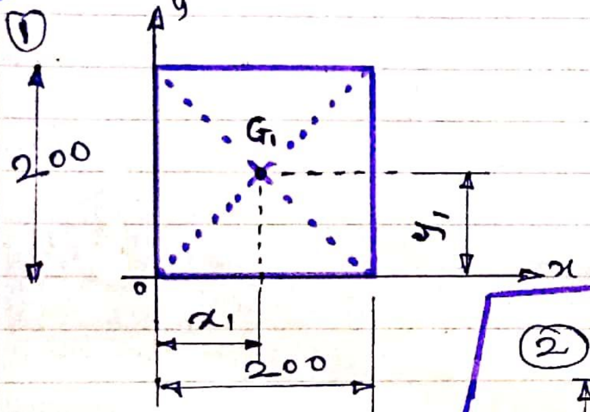
Locate the centroid of shaded Lamina as shown in figure.



Let us split given composite area into simple parts.

Area ① - Square
Area ② - Triangle
Area ③ - quarter circle.

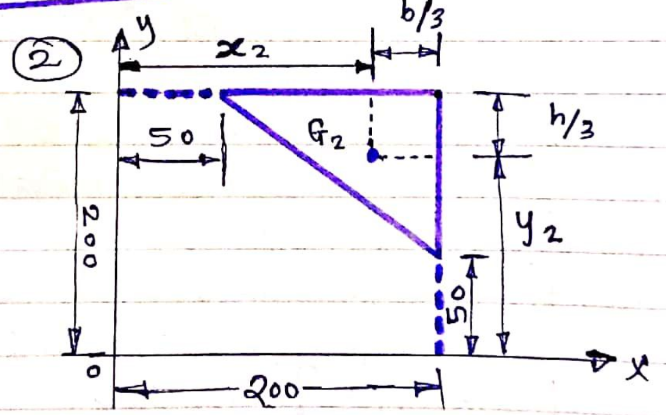
①



$A_1 = 200^2 = 40000 \text{ mm}^2$

$x_1 = y_1 = \frac{200}{2} = 100 \text{ mm}.$

②



$\therefore A_2 = \frac{1}{2} \times b h$

$= \frac{1}{2} \times 150 \times 150$

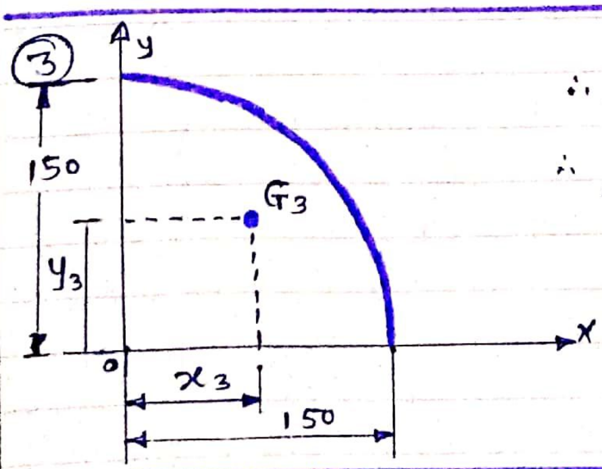
$= 11250 \text{ mm}^2$

$\therefore x_2 = 200 - \frac{b}{3}$

$x_2 = 200 - \frac{150}{3} = 150 \text{ mm}$

$\therefore y_2 = 200 - \frac{h}{3} = 200 - \frac{150}{3} = 150 \text{ mm}.$

③



$\therefore A_3 = \frac{\pi r^2}{4} = \frac{\pi}{4} \times 150^2$

$\therefore A_3 = 17671.46 \text{ mm}^2$

$\therefore x_3 = y_3 = \frac{4r}{3\pi} = \frac{4 \times 150}{3\pi}$

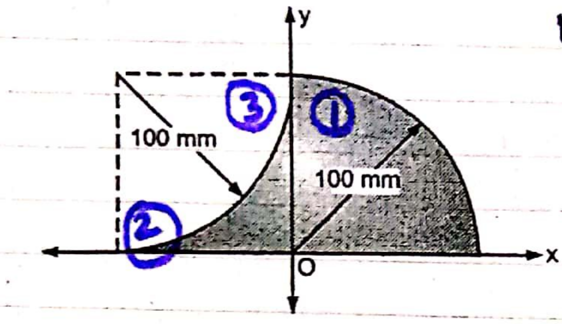
$\therefore x_3 = y_3 = 63.66 \text{ mm}$

$\bar{X} = \frac{A_1 x_1 - A_2 x_2 - A_3 x_3}{A_1 - A_2 - A_3} = 107.91 \text{ mm}$

$\bar{Y} = \frac{A_1 y_1 - A_2 y_2 - A_3 y_3}{A_1 - A_2 - A_3} = 107.91 \text{ mm}$

Centroid of Shaded lamina.

Locate the centroid of shaded Lamina as shown in fig.



Consider the composite Area shown in figure.

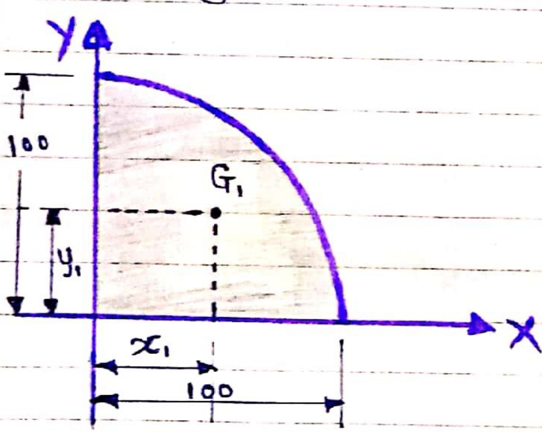
let us split this composite Area into simple parts.

Thus Area ① - quarter circle (shaded)

Area ② - square

Area ③ - quarter circle (Non-shaded).

* Area ①



$A_1 = \frac{\pi r^2}{4} = \frac{\pi}{4} \times 100^2 = 7854 \text{ mm}^2$

$x_1 = \frac{4r}{3\pi} = \frac{4 \times 100}{3\pi} = 42.44 \text{ mm}$

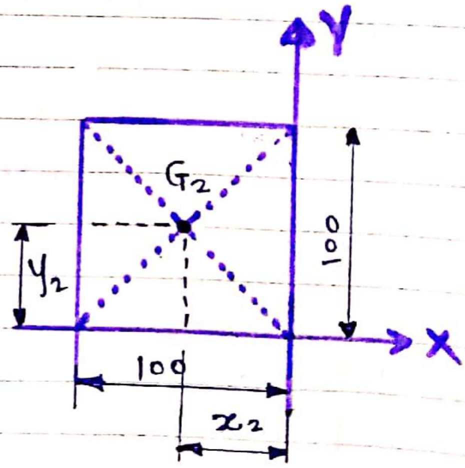
$y_1 = \frac{4r}{3\pi} = 42.44 \text{ mm.}$

* Area ②

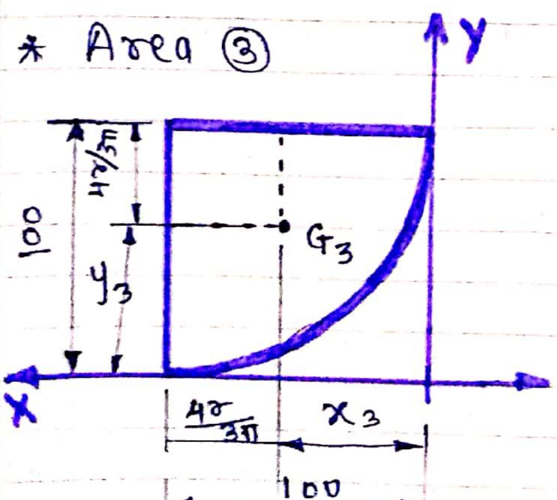
$A_2 = 100 \times 100 = 10000 \text{ mm}^2$

$x_2 = \frac{100}{2} = -50 \text{ mm.}$

$y_2 = \frac{100}{2} = 50 \text{ mm}$



* Area ③



$\therefore A_3 = \frac{\pi r^2}{4} = \frac{\pi}{4} \times 100^2$

$= 7854 \text{ mm}^2$

$x_3 = -\left(100 - \frac{4r}{3\pi}\right)$

$= -57.56 \text{ mm}$

$y_3 = \left(100 - \frac{4r}{3\pi}\right)$

$= 57.56 \text{ mm.}$

∴ Centroid of shaded portion will be,

$$\therefore \bar{X} = \frac{A_1 X_1 + A_2 X_2 - A_3 X_3}{A_1 + A_2 - A_3}$$

$$\therefore = \frac{(7854 \times 42.44) + (10000 \times (-50)) - (7854 \times (-57.56))}{7854 + 10000 - 7854}$$

$$= \frac{285400}{10000}$$

$$\boxed{\bar{X} = 28.54 \text{ mm}}$$

$$\therefore \bar{Y} = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - A_3}$$

$$\therefore = \frac{(7854 \times 42.44) + (10000 \times 50) - (7854 \times 57.56)}{7854 + 10000 - 7854}$$

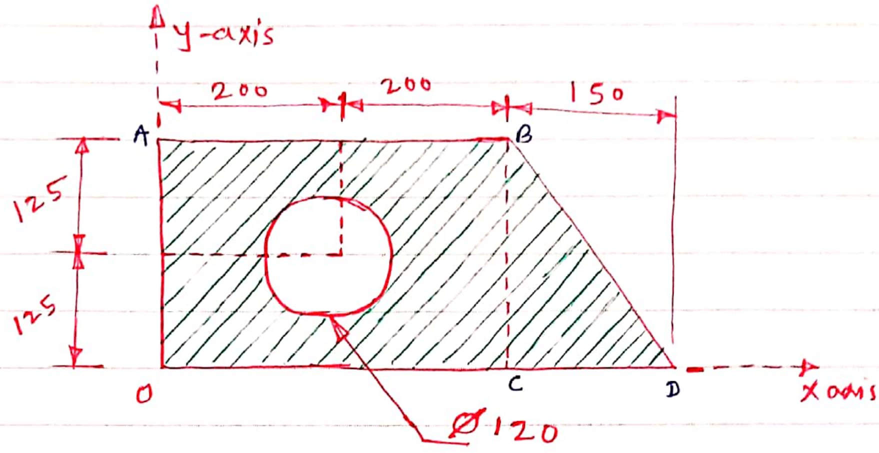
$$= \frac{381247.52}{10000}$$

$$\boxed{\bar{Y} = 38.12 \text{ mm}}$$

Ans

Centroid of shaded lamina will be at (28.54 mm, 38.12 mm).

(3) Determine the x co-ordinate of the centroid of shaded area with respect to origin O as shown in figure.



All diamensions are in mm.

① Consider Rectangle OABC

$\therefore$ Area $= A_1 = 400 \times 250 = 100000\ mm^2$

$x_1 = 200\ mm.$

② Consider Right angle Δ BCD.

$\therefore$ Area $= A_2 = \frac{1}{2} \times 150 \times 250$

$= 18750\ mm^2$

$x_2 = 400 + \frac{b}{3}$

$= 400 + \frac{150}{3}$

$x_2 = 450 mm.$

③ Consider circle of ϕ 120 mm.

Area $= A_3 = \pi r^2$

$A_3 = \pi \times 60^2 = 11309.73\ mm^2$

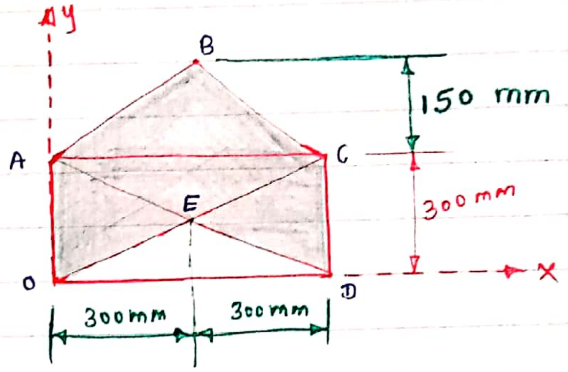
$x_3 = 200\ mm.$

Thus $\therefore$ $\bar{x} = \frac{A_1 \cdot x_1 + A_2 \cdot x_2 - (A_3 \times x_3)}{A_1 + A_2 - A_3}$

$\therefore = \frac{(100000 \times 200) + (18750 \times 450) - (11903.73 \times 200)}{100000 + 18750 - 11309.73}$

$\therefore \boxed{\bar{x} = 242.52\ mm}$

(4) Determine the co-ordinate of centroid of the shaded area as shown in figure.



⇒

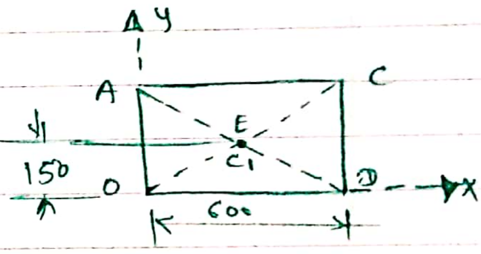
The given figure of shaded area is symmetrical about y-axis. Thus for shaded area,

$$\boxed{\bar{x} = 300\text{ mm}}$$

Let us divide the given diagram into 3 parts,

① - Rectangle OACD ② Δ ABC ③ Δ OED.

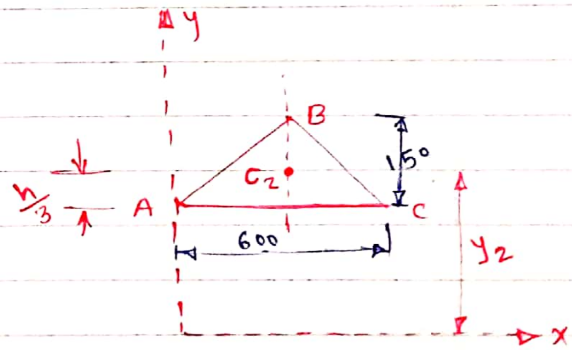
① For Rectangle OACD



$A_1 = 600 \times 300 = 180000\text{ mm}^2$

$y_1 = \frac{300}{2} = 150\text{ mm}.$

② For Δ ABC



$A_2 = \frac{1}{2} \times b \times h$

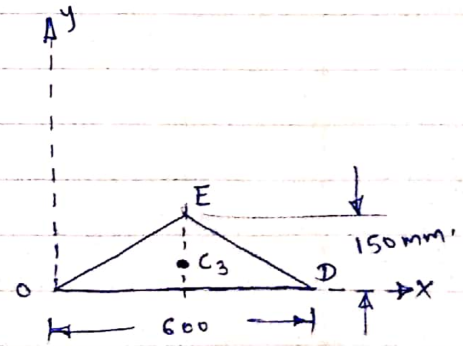
$= \frac{1}{2} \times 600 \times 150$

$= 45000\text{ mm}^2$

$y_2 = 300 + \frac{h}{3}$

$y_2 = 300 + (150/3) = 350\text{ mm}.$

③ For OED



$A_3 = \frac{1}{2} \times 600 \times 150 = 45000\text{ mm}^2$

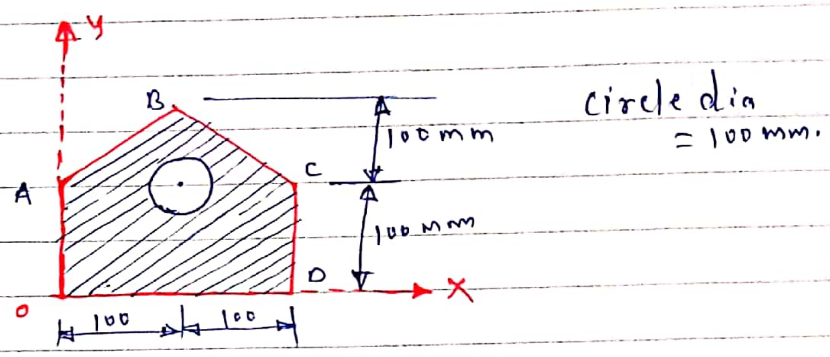
$y_3 = \frac{h}{3} = 150/3 = 50\text{ mm}.$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - A_3}$$

$$= \frac{(180000 \times 150) + (45000 \times 350) - (45000 \times 50)}{180000 + 45000 - 45000}$$

$$\boxed{\bar{y} = 225\text{ mm}}$$

(5) Determine the Y co-ordinate of centroid of the shaded area as shown in figure.



$\Longrightarrow$

(1) Consider Rectangle OACD.

$\therefore A_1 = 200 \times 100 = 20000 \text{ mm}^2$

$x_1 = 100 \text{ mm}$

$y_1 = 50 \text{ mm}.$

(2) consider ΔABC

$A_2 = \frac{1}{2} \times AC \times \text{height}$

$= \frac{1}{2} \times 200 \times 100$

$A_2 = 10000 \text{ mm}^2$

$y_2 = 100 + \frac{h}{3} = 100 + \frac{100}{3} = 133.33 \text{ mm}.$

(3) For circle.

$A_3 = \pi r^2 = \pi \times 50^2$

$\therefore A_3 = 7853.98 \text{ mm}^2$

$y_3 = 100 \text{ mm}.$

$\therefore$

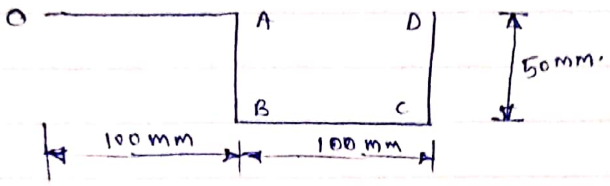
$$\bar{Y} = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - A_3}$$

$$= \frac{20000 \times 50 + 10000 \times 133.33 - 7853.98 \times 100}{20000 + 10000 - 7853.98}$$

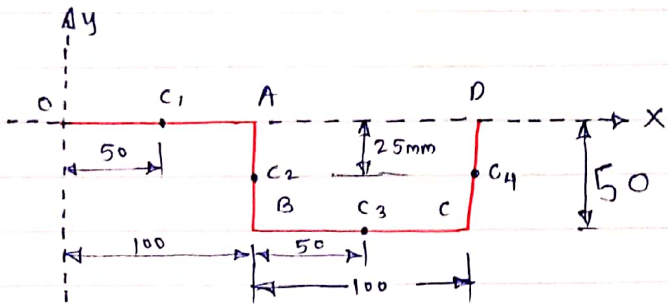
$$\boxed{\bar{Y} = 69.89 \text{ mm}}$$

Examples on wire bents - centroid

① A thin rod is bent into a shape OABCD as shown in figure. Determine the centroid of bent rod w.r.t. origin 'O'.



⟹



① Line segment OA

Length $= L_1 = 100$ mm.

$x_1 = 50$ mm, $y_1 = 0$ mm $\}$ co-ordinate of point C_1

② Line segment AB

Length $= L_2 = 50$ mm

$x_2 = 100$ mm, $y_2 = -25$ mm $\}$ co-ordinate of C_2.

③ Line segment BC

$L_3 = 100$ mm

$x_3 = 150$ mm, $y_3 = -50$ mm $\}$ co-ordinate of C_3

④ Line segment CD

$L_4 = 50$ mm

$x_4 = 200$ mm, $y_4 = -25$ mm $\}$ co-ordinate of C_4.

Thus

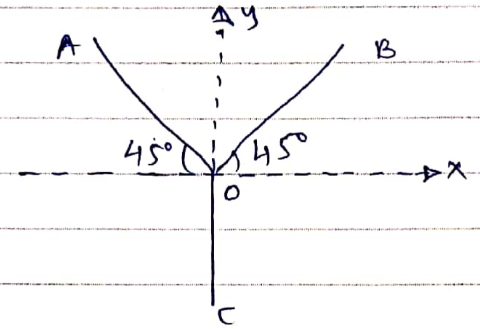
$$\bar{X} = \frac{L_1x_1 + L_2x_2 + L_3x_3 + L_4x_4}{L_1 + L_2 + L_3 + L_4} = \frac{(100 \times 50) + (50 \times 100) + (100 \times 150) + (50 \times 200)}{100 + 50 + 100 + 50}$$

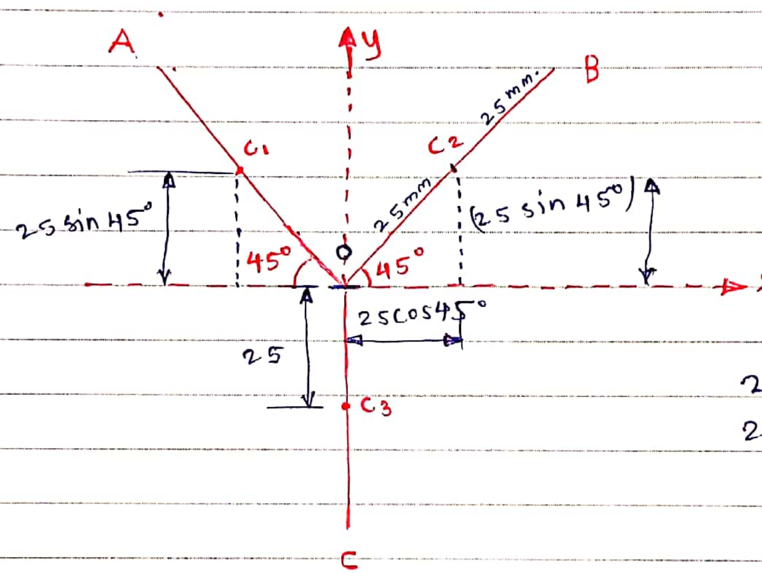
$$\therefore \boxed{\bar{X} = 116.67 \text{ mm}}$$

$$\bar{Y} = \frac{L_1y_1 + L_2y_2 + L_3y_3 + L_4y_4}{L_1 + L_2 + L_3 + L_4} = \frac{(100 \times 0) + [50 \times (-25)] + [100 \times (-50)] + [50 \times (-25)]}{100 + 50 + 100 + 50}$$

$$\boxed{\bar{Y} = -25 \text{ mm}}$$

② A slender rod is welded into the shape as shown in figure. locate the position of centroid of the rod w.r.t. origin 'O' if AO = BO = CO = 50 mm.





25 cos 45 = 17.68 mm

25 sin 45 = 17.68 mm

Line segment	Length	Centroid	x - cordinate	y - coordinate.
Line AO	$L_1 = 50$ mm	C_1	$x_1 = -17.68$ mm	$y_1 = 17.68$ mm
Line BO	$L_2 = 50$ mm	C_2	$x_2 = 17.68$ mm	$y_2 = 17.68$ mm
Line CO	$L_3 = 50$ mm	C_3	$x_3 = 00$ mm	$y_3 = -25$ mm.

Thus,

$$\bar{X} = \frac{L_1 x_1 + L_2 x_2 + L_3 x_3}{L_1 + L_2 + L_3}$$

$$\bar{x} = \frac{50 \times (-17.68) + 50 \times 17.68 + 50 \times 00}{50 + 50 + 50}$$

$\therefore \boxed{\bar{X} = 00 \text{ mm}}$ from origin 'O'.

$$\bar{Y} = \frac{L_1 y_1 + L_2 y_2 + L_3 y_3}{L_1 + L_2 + L_3}$$

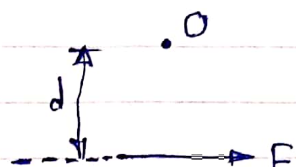
$$= \frac{50 \times 17.68 + 50 \times 17.68 + [50 \times (-25)]}{50 + 50 + 50}$$

$\therefore \boxed{\bar{y} = 3.45 \text{ mm}}$ from origin 'O'.

MOMENT OF INERTIA :-

* Moment of force About any point is the product of - magnitude of Force and perpendicular distance of line of action of Force from the point.

$$M_o = F \times d.$$



This is known as first moment of force.

* When This product Mo i.e. $(F \times d)$ is again multiplied with distance 'd', Then

$$F \times d \times d = F \times d^2$$

$F \times d^2$ is known as second moment of force or Moment of inertia of Force.

* When Force is replaced by Area of figure or mass of the body, Then second moment is known as <u>area moment of inertia</u> or mass <u>moment of Inertia</u>.

A] Area moment of Inertia (I).

The second moment of area about a particular axis is known as " area moment of inertia" about that Axis.

Unit - mm^4, cm^4, m^4 ...

First moment of Area = Area x distance
= $mm^2 \times mm$
= mm^3

Second moment of area = moment (1st) of Area x distance
I
= Area x distance x distance
= $mm^3 \times mm$
= mm^4.

B] Mass Moment of Inertia.

The second moment of mass of the body about a particular axis is known as "mass moment of inertia" about that axis.

unit - $kg \cdot mm^2$, $kg \cdot m^2$

$\therefore$ M.I. = mass × distance × distance

$= kg \times m \times m$

$= kg \times m^2$

Radius of Gyration :-

If entire mass of body be assumed to be concentrated at a certain point which is ~~at~~ located at a distance k from given axis such that

$$I = M.I. = Mass \times k^2$$

$$\therefore k = \sqrt{\frac{M.I.}{mass}} = \sqrt{\frac{I}{M}}$$

Then,

Distance k is known as Radius of Gyration.

Thus Radius of Gyration is defined as the "Distance from the axis of Reference where whole mass of the body is assumed to be concentrated.

* For plane figures having negligible mass, we can consider area for finding Radius of gyration.

Thus

M.I. of an area (Plane figure)

$$I = Ak^2$$

$$\therefore K = \sqrt{\frac{I}{A}}$$

Where, K_{xx}, K_{yy}, K_{zz} is Radius of gyration of any area or body about x, y, z axis Respectively.

$$K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{I_{xx}}{m}}$$

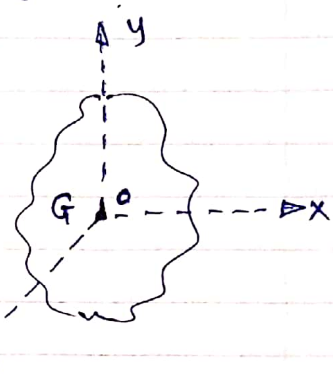
$$K_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{I_{yy}}{m}}$$

$$K_{zz} = \sqrt{\frac{I_{zz}}{A}} = \sqrt{\frac{I_{zz}}{m}}$$

Perpendicular Axis Theorem

It states that, If I_{xx} and I_{yy} be the moments of Inertia of Plane Figure or plane Section about two axis x & y which are perpendicular to each other at point O, Then Moment of Inertia of that plane section about z axis which is perpendicular to the plane [& (x & y)] is given by:

$$I_{zz} = I_{xx} + I_{yy}.$$



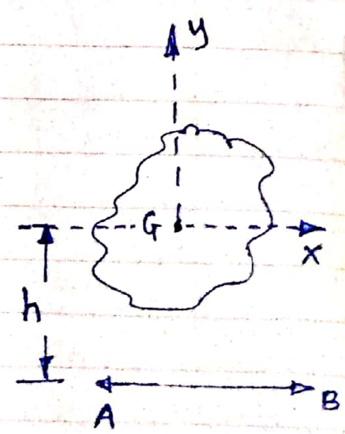
I_{zz} = M.I. of plane area about z axis Passing through C.G.
I_{xx} = M.I. of plane area about x axis passing through C.G.
I_{yy} = M.I. of plane area about y axis passing through C.G.

I_{zz} = is also called as Polar moment of Inertia (J).

Parallel axis theorem

It states that moment of Inertia of plane area about an axis passing through its centre of gravity is denoted by I_G, Then moment of Inertia of that area about any other axis AB which is parallel to the first is given by:

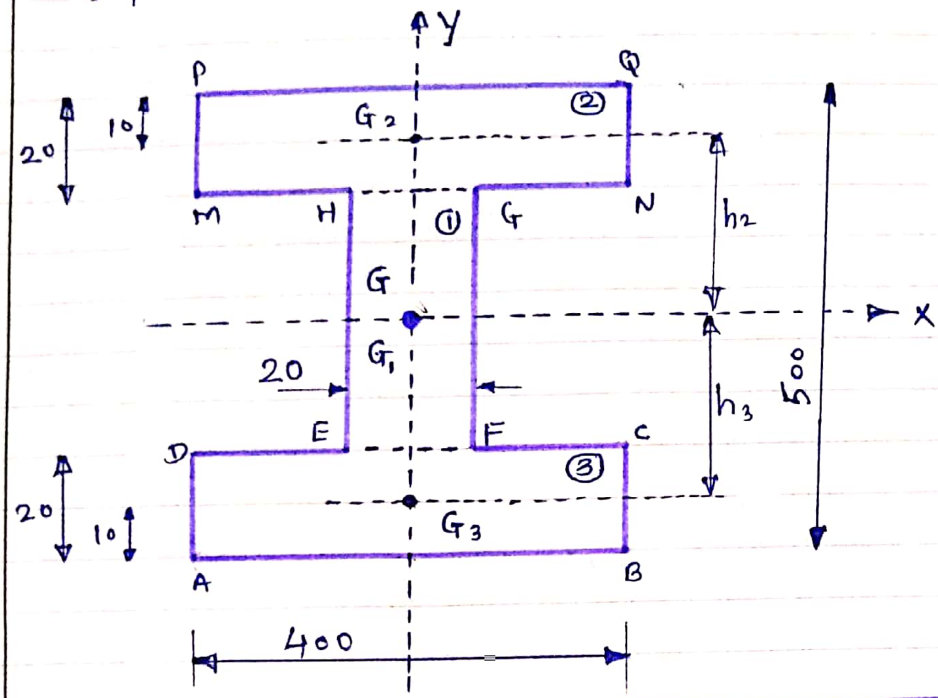
$$I_{AB} = I_G + Ah^2$$



Where

I_{AB} = M.I. of plane area about Axis AB.
I_G = I_{xx} or I_{yy} = M.I. of plane area about x or y axis passing through its C.G.
A = Area of plane figure
h = Distance betn centre of gravity of area & axis AB.

Determine Moment of Inertia for I-section about x & Y axis as shown in figure.



* Consider Area 1

□ EFGH.

M.I. of this Rectangle about xx and yy axis will be,

$$I_{xx_1} = \frac{bd^3}{12} = \frac{20 \times 460^3}{12}$$

$$I_{xx_1} = 162226666.7 \ mm^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{460 \times 20^3}{12}$$

$$I_{yy_1} = 306666.67 \ mm^4$$

* Consider Rectangle PQMN Area 2

Using parallel Axis theorem

$$I_{xx_2} = I_{G_2} + A_2 h_2^2$$

$$= \frac{bd^3}{12} + (b \times d) \times h_2^2$$

$$= \frac{400 \times 20^3}{12} + (400 \times 20 \times 240^2)$$

$$I_{xx_2} = 461066666.7 \ mm^4$$

$$I_{yy_2} = I_{G_2}$$

$$= \frac{db^3}{12} = \frac{20 \times 400^3}{12}$$

$$= 106666666.7 \ mm^4$$

Consider Rectangle ABCD Area 3

$$I_{xx_3} = 461066666.7 \ mm^4$$

$$I_{yy_3} = 106666666.7 \ mm^4.$$

∴ M.I. of I-Section will be

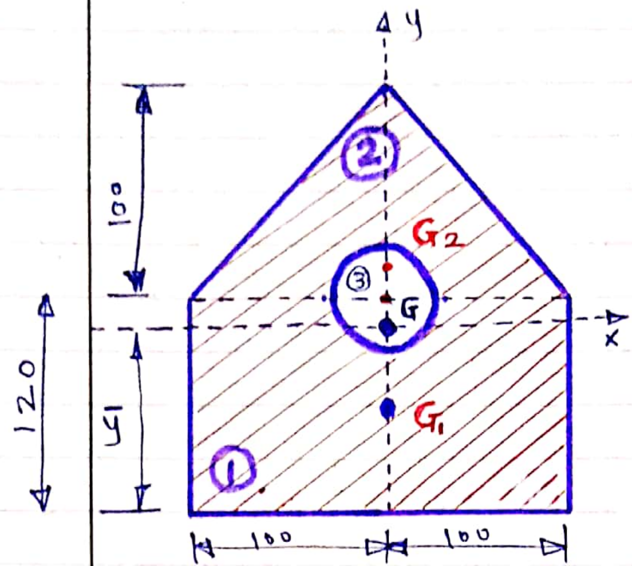
$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

$$I_{xx} = 1084.36 \times 10^6 \ mm^4$$

$$I_{yy} = I_{yy_1} + I_{yy_2} + I_{yy_3}$$

$$= 213.64 \times 10^6 \ mm^4.$$

Determine the Moment of inertia of shaded area as shown in figure about its centroidal axes.



As this figure is symmetrical about y axis,

$$\boxed{\bar{x} = 100 \text{ mm}}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - A_3}$$

Here, Area (1) = Rectangle
Area (2) = Triangle
Area (3) = Circle.

$$\therefore \bar{y} = \frac{(120 \times 200 \times 60) + (\frac{1}{2} \times 200 \times 100 \times 153.33) - (\frac{\pi}{4} \times 90^2 \times 120)}{(120 \times 200) + (\frac{1}{2} \times 200 \times 100) - (\frac{\pi}{4} \times 90^2)}$$

$$\boxed{\bar{y} = 79.95 \text{ mm}}$$

$\therefore$ To find M.I. of shaded portion, let G is the centroid of shaded area which is at $\bar{y} = 79.95$ mm from Base.

$\therefore$ M.I. of shaded portion @ x-x axis passing through its centroid G will be,

I_{xx} = (M.I. of Rectangle about x-x) + (M.I. of Triangle about x-x) − (M.I. of circle about x-x axis)

$$I_{xx} = I_{xx_1} + I_{xx_2} - I_{xx_3}$$

$$= (I_{G_1} + A_1 h_1^2) + (I_{G_2} + A_2 h_2^2) - (I_{G_3} + A_3 h_3^2)$$

$$= \left[\frac{200 \times 120^3}{12} + 200 \times 120 \times (79.95 - 60)^2\right] +$$

$$\left[\frac{200 \times 100^3}{36} + \frac{1}{2} \times 200 \times 100 \times (153.33 - 79.95)^2\right] -$$

$$\left[\frac{\pi}{64} \times 90^4 + \frac{\pi}{4} \times 90^2 \times (120 - 79.95)^2\right]$$

$\therefore I_{xx} = 38352060 + 59401799.56 - 13424846.35$

$I_{xx} = 84329013.21 \ mm^4.$

M.I. of shaded portion about y-y axis passing through its centroid G will be,

I_{yy} = (M.I of Rectangle about y-y axis) + (M.I. of Triangle about y-y axis) − (M.I. of circle about y-y axis).

$$I_{yy} = I_{yy_1} + I_{yy_2} - I_{yy_3}$$

$$= \frac{db^3}{12} + \frac{hb^3}{48} - \frac{\pi}{64} d^4$$

$$= \frac{120 \times 200^3}{12} + \frac{100 \times 200^3}{48} - \frac{\pi}{64} \times 90^4$$

$$= 80000000 + 16666666.67 - 3220623.34$$

$I_{yy} = 93446043.33 \ mm^4.$

FRICTION :-

- It is a tangential force developed between the bodies (or two surfaces) which are in contact when one body moves or tends to move over another body (or surface).

- Frictional force always acts in the opposite direction of motion or in the opposite direction of impending motion.

Impending motion :-
It is the state of body in which the body is on the verge of motion i.e. About to move.

Frictional force is denoted by F_s or F_K.

Characteristics of frictional force:

① It is Reactive and self adjusting force.
② Frictional force increases when external force is increased upto limiting condition.
At the limiting condition partical try to impend the motion.
when partical is in motion, Frictional force decreases with increase in external force.

③ Frictional force always acts in the opposite direction of motion or in the opposite direction of impending motion.
④ upto limiting condition, Frictional force is equal in magnitude of external force.

* **Friction:**

Friction is defined as the resistive force which acts in the opposite direction of motion of body.

* **Classification of friction :-**

1] Static friction

2] Kinetic friction - a) Sliding friction
  b) Rolling friction.

Static Friction:-

Friction experienced by a body when it is at rest condition.

Kinetic friction:-

- Friction experienced by a body when it moves.
- When one body slides over another, then friction experienced by a body is sliding friction.
- When one body Rolls over another, then friction experienced by a body is called as rolling friction.

* **Laws of Friction [Coulomb's Law]**

A] For static friction :-

1] Frictional force always acts in the opposite dirn in which body tends to move.

2] Frictional force is directly proportional to Normal Reaction.

3] Frictional force is equal to net force acting in the direction in which object tends to move.

4] Frictional force does not depends on the surface area of contact.

5] Frictional force depends upon Roughness of surface.

B] For kinetic Friction :-

1] Frictional Force always acts in opposite direction in which body moves.
2] Frictional force is directly proportional to Normal reaction.
3] For low speeds, Frictional Force is constant.
4] for Higher speeds, Frictional force decreases with increasing speed and vice-versa.

* Coefficient of Friction :-

A] Coefficient of static Friction. (μ_s) :-

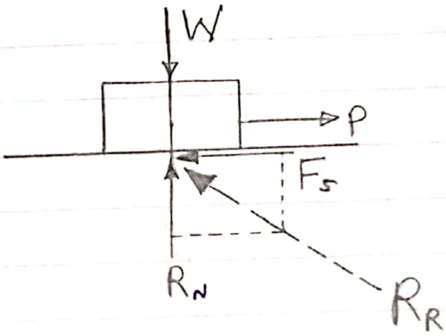
From culomb's law,

Magnitude of Frictional force is directly proportional to Normal reaction.

$\therefore F_s \propto R_N$

$\therefore \boxed{F_s = \mu_s R_N}$

$\therefore \boxed{\mu_s = \frac{F_s}{R_N}}$



R_R = Resultant Reaction.

B] Coefficient of kinetic friction (μ_k) :-

$F_k \propto R_N$

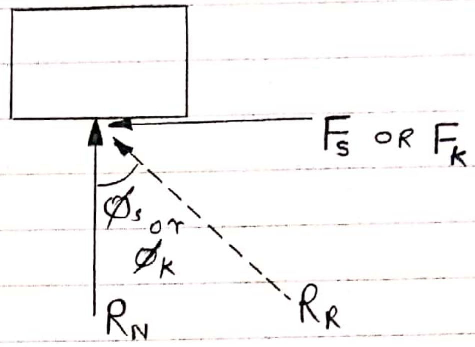
$\therefore \boxed{F_k = \mu_k \cdot R_N}$

$\therefore \boxed{\mu_k = \frac{F_k}{R_N}}$

* Angle of friction :- (ϕ)

Angle of static friction (ϕ_s) :-

For the body at rest, the angle betn Normal reaction and Resultant reaction is called as angle of static friction. (ϕ_s).

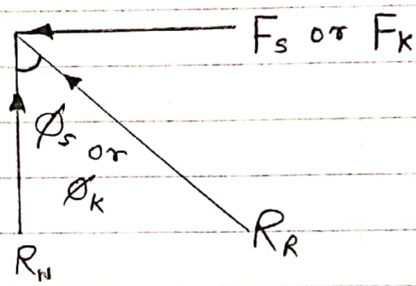


$$\therefore \tan(\phi_s) = \frac{F_s}{R_N}$$

$$\therefore \tan \phi_s = \mu_s$$

$$\therefore \boxed{\mu_s = \tan \phi_s} \quad \&$$

$$\boxed{\phi_s = \tan^{-1} \mu_s}$$



Angle of kinetic Friction (ϕ_K) :-

For the body in motion, angle made by normal reaction with resultant reaction is called as angle of kinetic friction. (ϕ_K).

$$\therefore \tan \phi_K = \frac{F_K}{R_N}$$

$$\therefore \tan \phi_K = \mu_K$$

$$\therefore \boxed{\mu_K = \tan \phi_K} \quad \&$$

$$\boxed{\phi_K = \tan^{-1} \mu_K}$$

* Limiting Friction / Limiting equilibrium :-

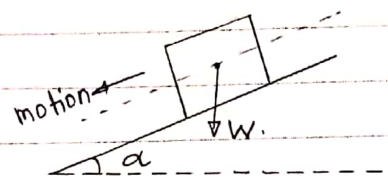
It is the max. value of Frictional force when body just begins to slide over a surface.

OR

It is the max. value of frictional force when body is just on the verge of sliding.

Angle of Repose:- (α)

It is the angle made by inclined surface with the horizontal, when the block kept on inclined plane just begins to slide down the plane due to its self weight.

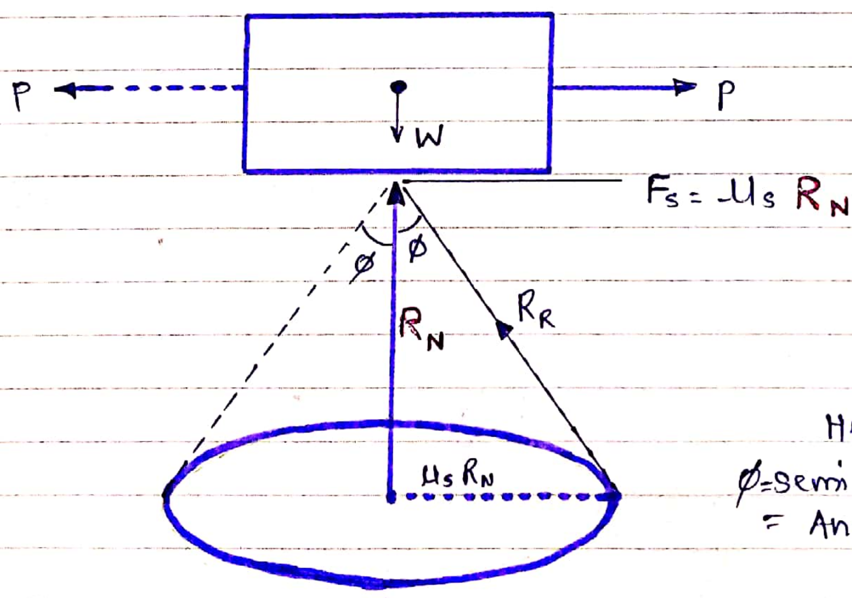


Angle of friction (static) and angle of repose are equal.

$\therefore$ $\boxed{\phi_s = \alpha}$

$\therefore$ $\boxed{\tan\phi_s = \tan\alpha = \mu_s}$

Cone of Friction.



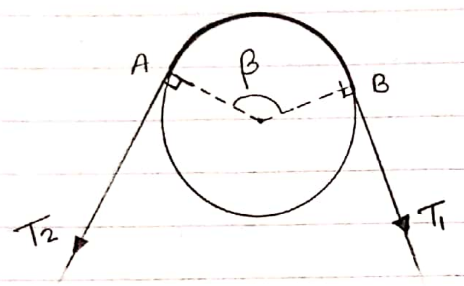
Here
ϕ = semi vertex angle
= Angle of friction

In the limiting state of equilibrium, of block, if line of action of applied Force P is rotated in a horizontal plane, then line of action of Resultant Reaction R_R will Also Rotate. Due to rotation, line of action of R_R will generate a cone which is known as Cone of Friction.

Altitude or Height of cone = Normal Reaction = R_N
Base Radius = Limiting frictional Force = $F_s = \mu_s R_N$
Generator of cone = Slant Height = R_R = Resultant Reaction.

★ Belt friction :-

① Flat Belt :-



Let, T_1 = Tension in Tight side of Belt
T_2 = Tension in slack side of belt.
β = Lap angle or angle of contact in radians.
μ_s = coeff. of static friction.
μ_k = coeff. of kinetic friction.

When belt is about to slip :-

$$\frac{T_1}{T_2} = e^{\mu_s \beta} \quad \text{-----}(T_1 > T_2)$$

when Belt is actually slipping (in motion):-

$$\frac{T_1}{T_2} = e^{\mu_k \beta} \quad \text{----} \; T_1 > T_2)$$

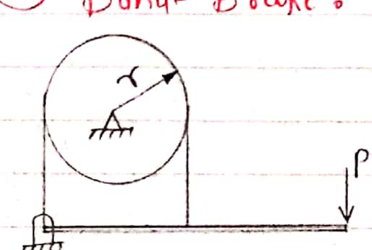
② V-Belt

$$\frac{T_1}{T_2} = e^{\mu_s \beta \cdot \operatorname{cosec}(\alpha/2)}$$

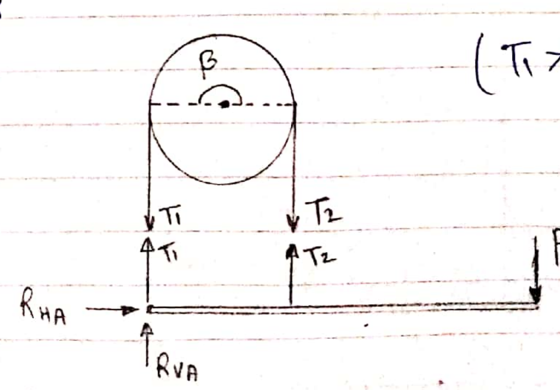
where, α = Angle of v groove or belt.

③ Band-Brake :-

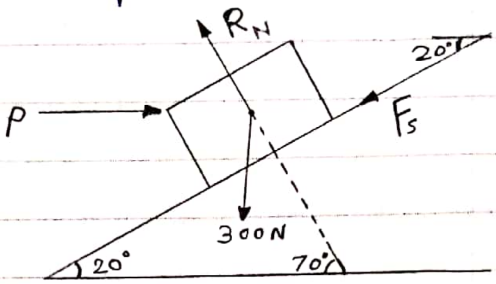
$$\frac{T_1}{T_2} = e^{\mu\beta} \qquad (T_1 > T_2)$$



Braking Torque
$T_B = (T_1 - T_2)\, r.$



Determine the Horizontal force P needed to just start moving the 300 N crate up the plane as shown in figure. Take $\mu_s = 0.1$.



⇒ for the equilibrium,

$\therefore \Sigma f_x = 0.$

$\therefore P - R_N \cos 70 - F_s \cos 20 = 0$

$\therefore P - R_N \cos 70 - \mu_s . R_N \cos 20 = 0$

$\therefore P - 0.342 R_N - 0.094 R_N = 0$

$\therefore \boxed{P = 0.436 R_N}$ --- (1)

$\Sigma f_y = 0$

$\therefore -300 + R_N \sin 70 - F_s \sin 20 = 0$

$\therefore -300 + R_N \sin 70 - \mu_s . R_N \sin 20 = 0$

$\therefore -300 + R_N \sin 70 - 0.1 R_N \sin 20 = 0$

$\therefore -300 + 0.94 R_N - 0.0342 R_N = 0$

$\therefore 0.9058 R_N = 300$

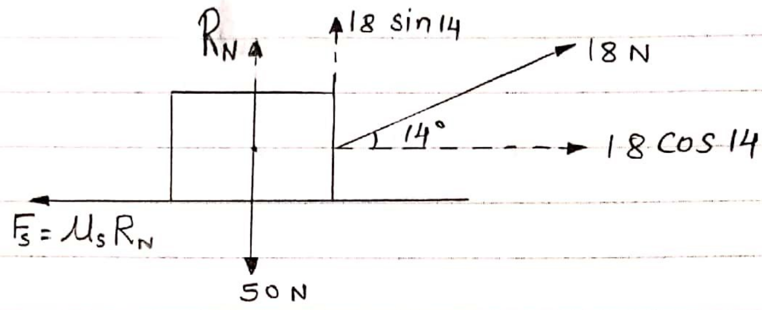
$\therefore \boxed{R_N = 331.2 \text{ N}}$

from eqn (1)

$P = 0.436 \times 331.2$

$\boxed{P = 144.4 \text{ N}}$ - force Required.

A body of weight 50 N is ~~travelled~~ placed along a rough horizontal plane. A pull of 18 N acting at an angle of 14° with the Horizontal, find the coefficient of static friction.



Considering the equilibrium of block,

$\therefore \Sigma F_x = 0$

$\therefore \quad 18 \cos 14 - F_s = 0$

$\therefore \quad \boxed{F_s = 17.46 \text{ N}}$

$\therefore \Sigma F_y = 0$

$R_N + 18 \sin 14 - 50 = 0$

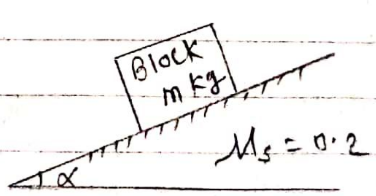
$\boxed{R_N = 45.64 \text{ N}}$

As $F_s = \mu_s . R_N$

$17.46 = \mu_s \times 45.64$

$\therefore \boxed{\mu_s = 0.386}$

A block of mass m rests on a frictional plane which makes an angle α with Horizontal as shown. If the coeff. of friction between the block & frictional plane is 0.2, determine angle α for limiting friction.



⇒ For limiting friction;

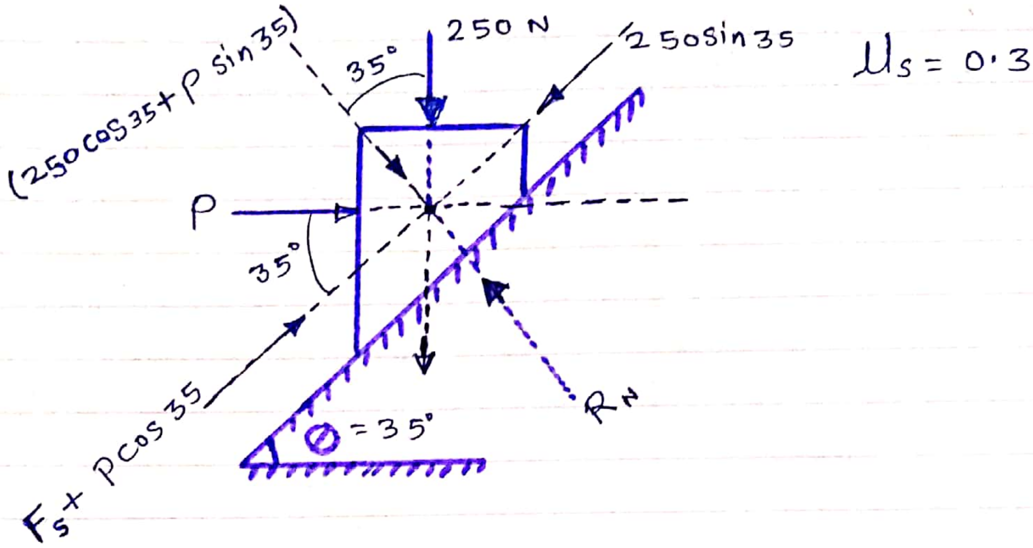
$\tan\alpha = \tan\phi_s = \mu_s$

$\therefore \alpha = \tan^{-1} \mu_s$

$\alpha = \tan^{-1} 0.2$

$\therefore \boxed{\alpha = 11.31°}$

Determine whether the block shown is in equilibrium and find the magnitude and direction of friction force when $\theta = 35°$ and $P = 100\,N$.



let us select inclined plane as x-x axis & perpendicular to plane as y-y axis.

let F_s = static frictional force which is required to maintain equilibrium.

$\therefore \quad \Sigma F_x = 0$ — Resolving forces along x-axis

$\therefore \; F_s + 100\cos 35 - 250\sin 35 = 0$

$\therefore \quad F_s - 61.48 = 0$

$\therefore \quad \boxed{F_s = 61.48\ N}$ — frictional force

$\Sigma F_y = 0$ - Resolving forces along y axis.

$\therefore -(250\cos 35 + P\sin 35) + R_N = 0$

$\therefore \quad -250\cos 35 - 100\sin 35 + R_N = 0$

$-262.15 + R_N = 0$

$\therefore \quad \boxed{R_N = 262.15\ N}$ -- Normal Reaction

Maximum frictional force that can be developed is given by,

$F_{s\,max} = \mu_s \times R_N = 0.3 \times 262.15$

$\boxed{F_{s\,max} = 78.65\ N}$

As Frictional force required to maintain equilibrium (61.48 N) is less than max. Frictional Force, $\therefore$ Equilibrium will be maintained.

Uniform Ladder AB has a length of 8 m and mass 24 kg. End A is on Horizontal floor and end B rests against verticall wall. A man of mass of 60 kg has to climb this ladder. At what position from the base, will he induce the slipping of Ladder. Take $\mu_s = 0.34$ at all Contact surface.

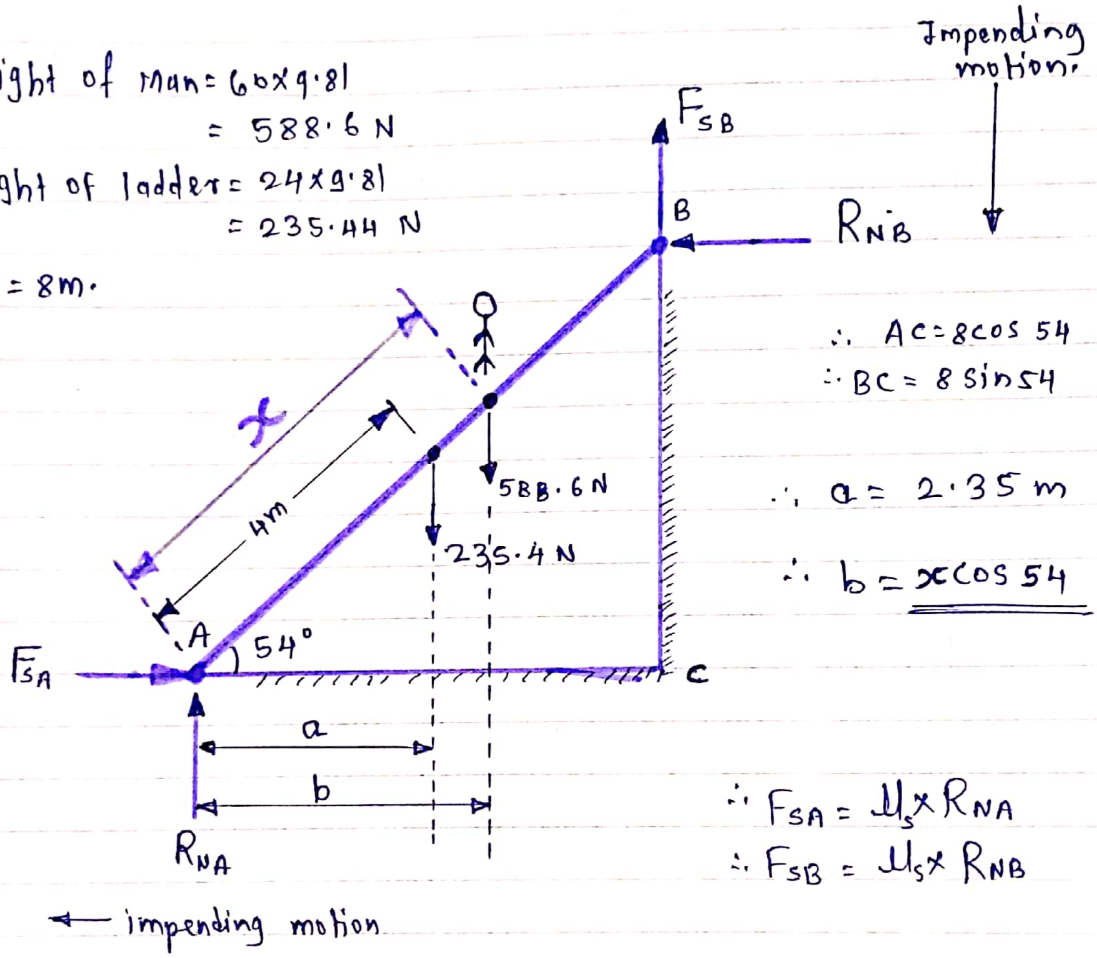
Weight of Man $= 60 \times 9.81$

$= 588.6$ N

Weight of ladder $= 24 \times 9.81$

$= 235.44$ N

AB = 8m.



$\therefore AC = 8\cos 54$

$\therefore BC = 8\sin 54$

$\therefore a = 2.35$ m

$\therefore b = x\cos 54$

$\therefore F_{SA} = \mu_s \times R_{NA}$

$\therefore F_{SB} = \mu_s \times R_{NB}$

Considering the equilibrium of ladder,

$\therefore \Sigma F_x = 0$

$\therefore F_{SA} - R_{NB} = 0$

$\therefore (\mu_s R_{NA}) - R_{NB} = 0$

$\therefore 0.34\, R_{NA} = R_{NB} \quad -①$

$\therefore \Sigma F_y = 0$

$\therefore R_{NA} - 235.44 - 588.6 + F_{SB} = 0$

$\therefore R_{NA} - 824.04 + (\mu_s \times R_{NB}) = 0$

$\therefore R_{NA} - 824.04 + 0.34\, R_{NB} = 0$

$\therefore 0.34\, R_{NB} + R_{NA} = 824.04$

$\therefore 0.34\,(0.34\, R_{NA}) + R_{NA} = 824.04$

$\therefore 0.1156\, R_{NA} + R_{NA} = 824.04$

$\therefore R_{NA} = \dfrac{824.04}{1.1156}$

$\therefore \boxed{R_{NA} = 738.65 \text{ N}}$

From eqn ①

$R_{NB} = 0.34 \times 738.65$

$\therefore \boxed{R_{NB} = 251.14 \text{ N}}$

Taking moment about point A,

$\therefore \Sigma M_A = 0$

$\therefore (235.44 \times 2.35) + (588.6 \times x \cos 54) - (F_{SB} \times 8 \cos 54) - (R_{NB} \times 8 \sin 54) = 0$

$\therefore 553.284 + 345.97 x - (\mu_s \times R_{NB} \times 4.7) - (6.47 R_{NB}) = 0$

$\therefore 553.284 + 345.97 x - 1.598 R_{NB} - 6.47 R_{NB} = 0$

$\therefore 345.97 x - 8.068 R_{NB} = -553.284$

$\therefore 345.97 x - 8.068 \times 251.14 = -553.284$

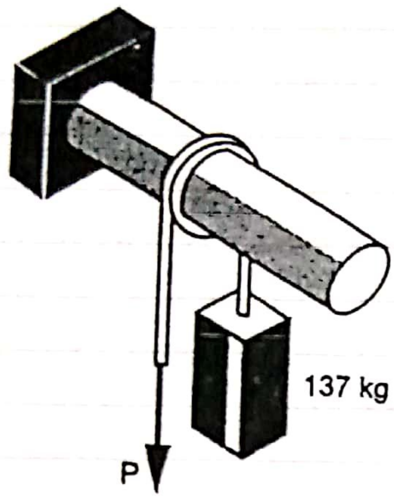
$\therefore 345.97 x = 1472.913$

$$\boxed{x = 4.257 \text{ m}}$$

Ans :

At $x = 4.257$ m from point A, along the ladder, man can climb without slipping.

Beyond this distance, ladder will be slipping.



A 137 kg block is supported by a rope which is wrapped 1.5 times around a horizontal rod. Knowing that coeffi of static Friction betn rope and rod is 0.15, Determine the range of values of P for which equilibrium is maintained.

Weight of block = 137 × 9.81 = 1343.97 N.

Angle of contact = LAP Angle

$\therefore \quad \beta = 1.5 \times 360 = 540^\circ = 540 \times \frac{\pi}{180}$

$\therefore \quad \beta = 3\pi$ radians

A) Case I :- When P > 1343.97 N,

Force P tends to move the rope downward and block moves upward. Thus, the part of rope on which force P is acting is tight side and rope to which block is attached is slack side.

$\therefore \quad \frac{T_1}{T_2} = e^{\mu_s \beta} \qquad \therefore \quad \frac{P}{1343.97} = e^{0.15 \times 3\pi}$

$\therefore \quad P = 5525.34$ N.

B] Case - II : When P < 1343.97 N

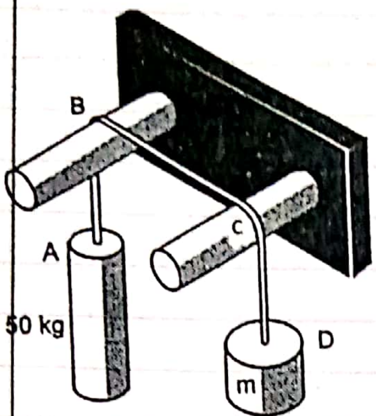
$\therefore$ Force P is less than 1343.97 N, so block will move downward & the side of Rope on which P acts will move upward.

$\therefore$ Rope side to which block is attached is tight side and another side is slack side.

$\therefore \quad \frac{T_1}{T_2} = e^{\mu_s \beta} \qquad \therefore \quad \frac{1343.97}{P} = e^{0.15 \times 3\pi}$

$\therefore \quad P = 326.92$ N.

$\therefore$ Range of P for equilibrium is : $326.92\text{ N} \leq P \leq 5525\text{ N}$

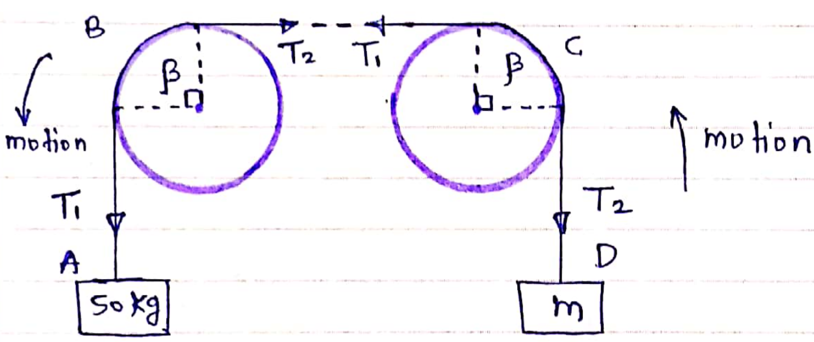


Two cylinders are connected by a rope that passes over two fixed rods as shown. Knowing that the coefficient of static friction between the rope and the rods is 0.40, determine the range of the mass m of cylinder D for which equilibrium is maintained.

Weight of cylinder $= 50 \times 9.81 = 490.5$ N
Lap Angle $= \beta = 90° = \frac{\pi}{2}$ radian.
$\mu_s = 0.40$

1] Case - I :
weight of cylinder A > weight of cylinder B.

In this case cylinder A will tends to move downward while B will tends to move upward.



Consider part AB of the Rope. As AB Rope is moving down, Tension in tight side $= T_1 = 50 \times 9.81 = 490.5$ N
Tension in slack side $= T_2 =$ unknown.

$\therefore \frac{T_1}{T_2} = e^{\mu_s \beta}$ $\quad \therefore \frac{490.5}{T_2} = e^{(0.4 \times \frac{\pi}{2})} = 1.874$

$\therefore \frac{490.5}{1.874} = T_2 \quad \therefore \boxed{T_2 = 261.74 \text{ N}}$

Consider Part CD, of the rope. As rope CD is moving upward, Tension in tight side, $T_1 = T_2$ of rope AB
Tension in tight side, $T_1 = 261.74$ N
Tension in slack side $= T_2 = (m \times 9.81)$

$\therefore \frac{T_1}{T_2} = e^{\mu_s \beta}$

$\therefore \frac{261.74}{(m \times 9.81)} = e^{(0.4 \times \frac{\pi}{2})}$

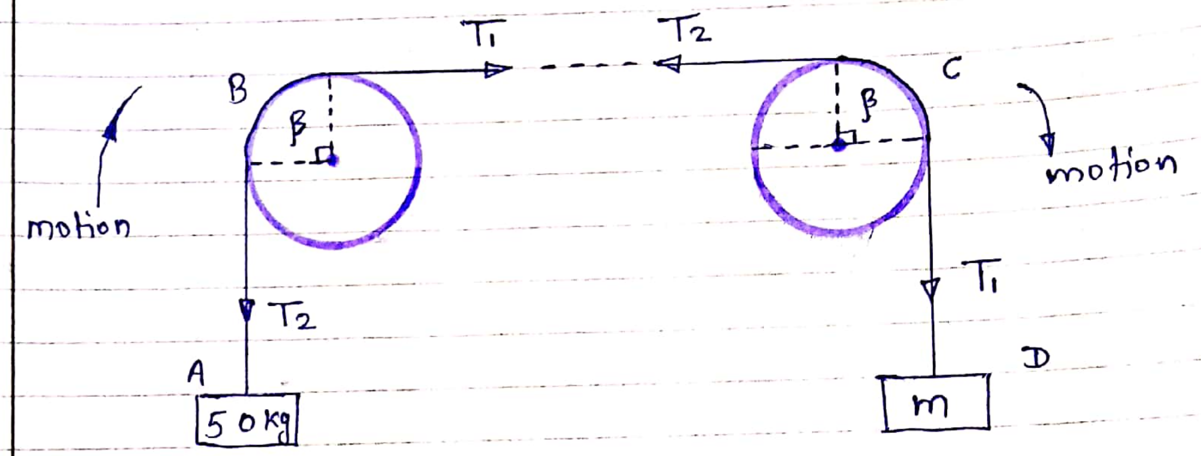
$\therefore \frac{261.74}{(m \times 9.81)} = 1.874$

$\therefore \boxed{m = 14.24 \text{ kg}}$

B] Case - II] :-

weight of cylinder A < weight of cylinder B

In this case, cylinder A will ~~~~ tends to move upward while cylinder B will tend to move downward.



Consider part AB of the rope, moving upwards

Tension in tight side = T_1

Tension in slack side = side A = 50×9.81 N = 490.5 N

$\therefore \frac{T_1}{T_2} = e^{\mu_s \beta}$

$\therefore \frac{T_1}{490.5} = e^{(0.4 \times \frac{\pi}{2})}$

$\therefore \frac{T_1}{490.5} = 1.874$

$\therefore \boxed{T_1 = 919.197 \text{ N}}$

Consider Part CD of rope, Rope CD is moving downwards.

Tension in tight side = T_1 = $(m \times 9.81)$ N

Tension in slack side = T_2 = T_1 of Rope AB = 919.197 N

$\therefore \frac{T_1}{T_2} = e^{\mu_s \beta}$

$\therefore \frac{m \times 9.81}{919.197} = e^{(0.4 \times \frac{\pi}{2})}$

$\therefore \frac{m \times 9.81}{919.197} = 1.874$

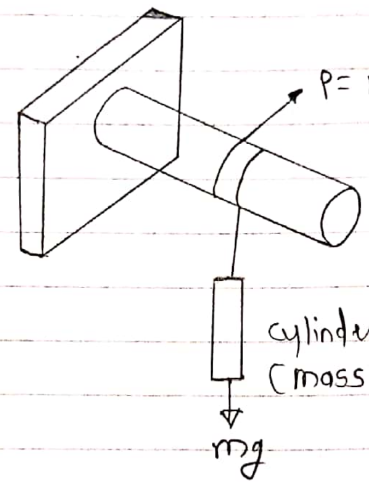
$\therefore \boxed{m = 175.59 \text{ kg}}$

Range of mass m for equilibrium to be maintained is

$14.23 \text{ kg} \leq m \leq 175.59 \text{ kg}$

A force $P = mg/6$ is required to lower the cylinder with ~~load~~ cord making 1.25 turns around the fixed shaft. Determine the coeff. of friction "μ_s" betn the cord and the shaft. Refer the fig.

→ Lap angle,



$$\beta = 1.25 \times 2\pi$$

$$= 2.5\pi \text{ radians}$$

for Flat Belt,

$$\frac{T_1}{T_2} = e^{\mu_s \beta}$$

$$\therefore \frac{mg}{P} = e^{\mu_s \beta}$$

$$\therefore \frac{mg}{(mg/6)} = e^{\mu_s \times 2.5\pi}$$

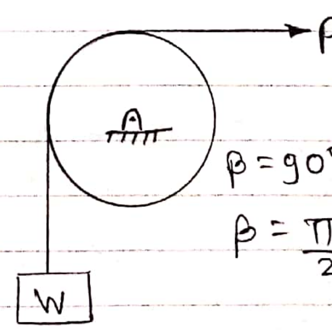
$$\therefore 6 = e^{\mu_s \times 2.5\pi}$$

$$\therefore 1.79 = \mu_s \times 2.5\pi$$

$$\therefore \boxed{\mu_s = 0.23}$$

Determine the range of P for the equilibrium of block of weight W as shown in fig. The coeff. of friction betn rope and pulley is 0.2.

→



$\beta = 90^\circ$

$\beta = \frac{\pi}{2}$ radians

case I :- when P is Max.

$$\frac{T_1}{T_2} = e^{\mu_s \beta}$$

$$\frac{P_{max}}{W} = e^{0.2 \times \pi/2}$$

$$\therefore \boxed{P_{max} = 1.369\ W}$$

Case-II :- when P is min.

$$\frac{T_1}{T_2} = e^{\mu_s \beta} \qquad \therefore \frac{W}{P_{min}} = e^{0.2 \times \pi/2}$$

$$\therefore P_{min} = 0.73\ W.$$

for equilibrium, P must be betn $0.73W \le P \le 1.369W$.