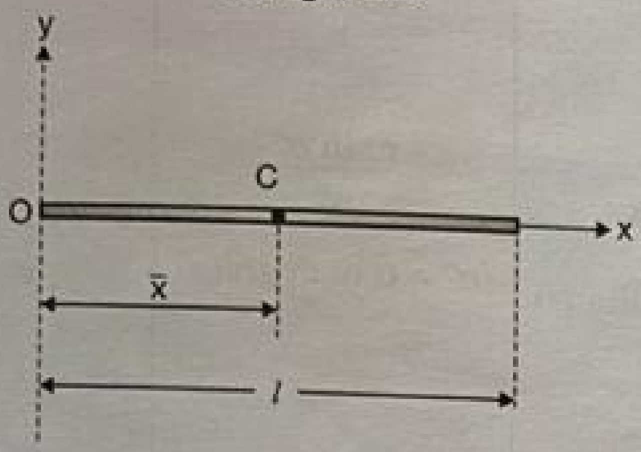
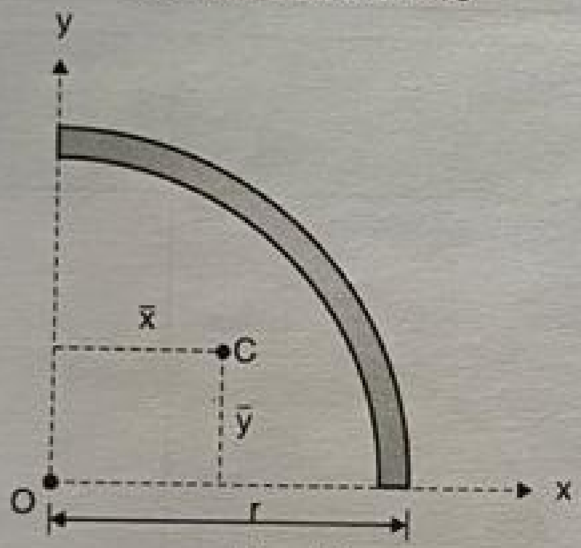
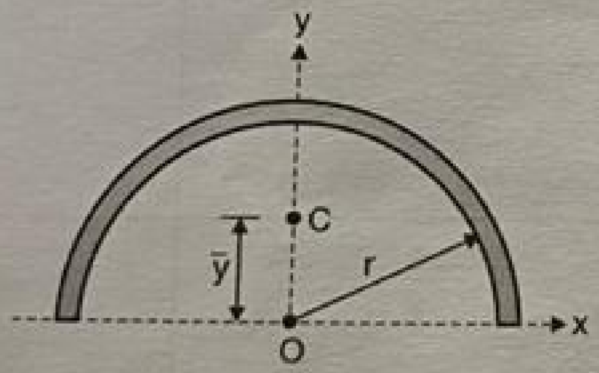
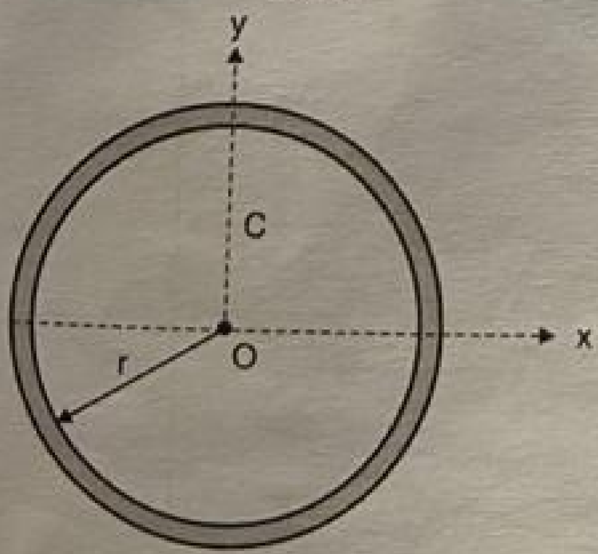
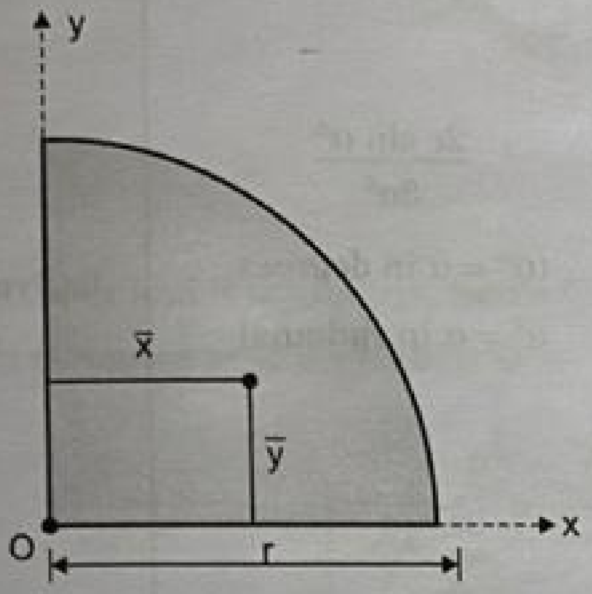
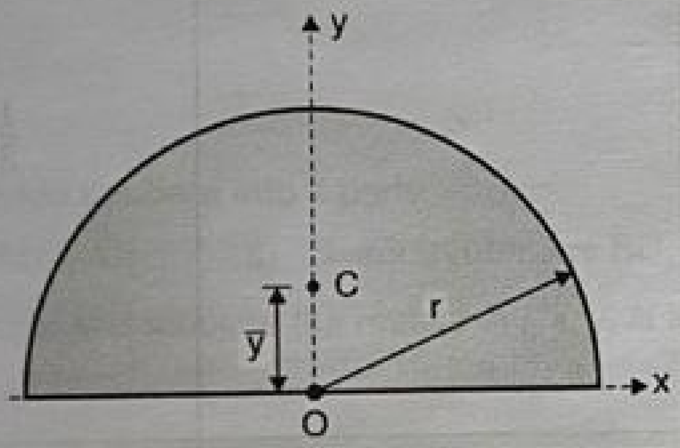
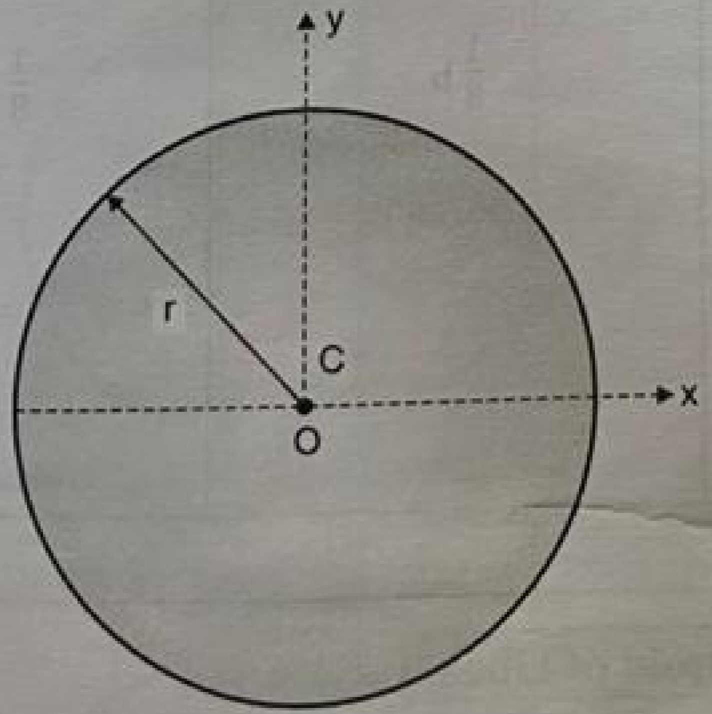
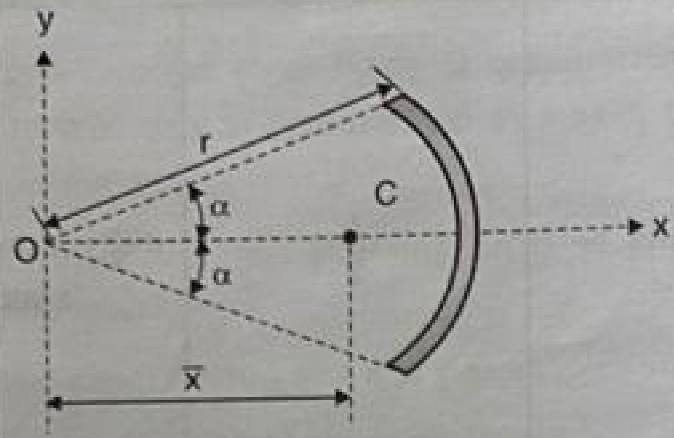
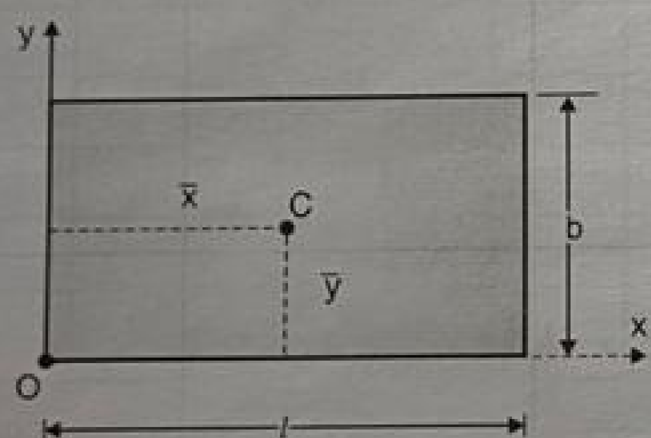
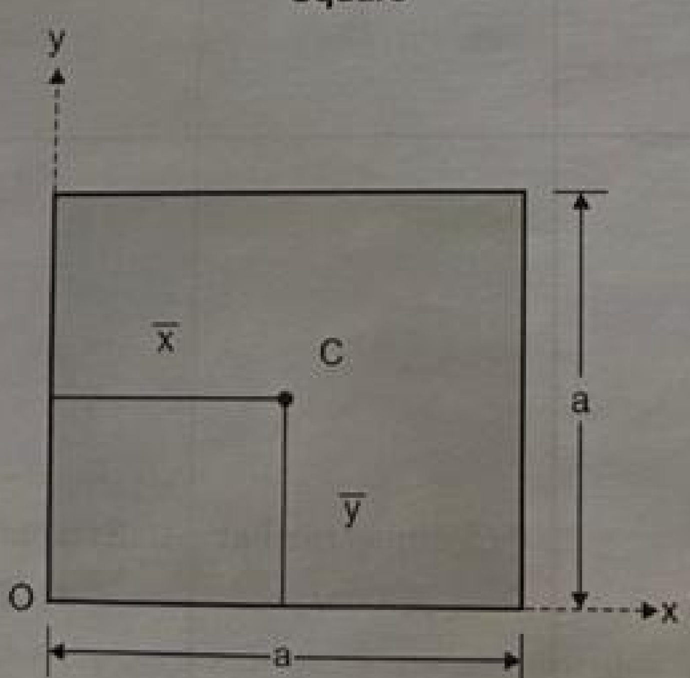
5.6 Centroid of Common Geometrical Shapes of Lines :

Sr. No.	Shape	Length (l)	$\bar{x}$	$\bar{y}$
1	**Straight line**  **Fig. (a)**	l	$\frac{l}{2}$	0 (Symmetrical at x-axis)
2	**Quarter Circular Arc**  **Fig. (b)**	$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$
3	**Semi circular arc**  **Fig. (c)**	πr	0 (Symmetrical at y-axis)	$\frac{2r}{\pi}$
4	**Circle**  **Fig. (d)**	$2\pi r$	0 (Symmetrical at y-axis)	0 (Symmetrical at x-axis)

Sr. No.	Shape	Area (A)	$\overline{x}$	$\overline{y}$
3	**Quarter circle**  **Fig. (c)**	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
4	**Semi-circle**  **Fig. (d)**	$\frac{\pi r^2}{2}$	0 (Symmetrical at y-axis)	$\frac{4r}{3\pi}$
5	**Circle**  **Fig. (e)**	πr^2	0 (Symmetrical at y-axis)	0 (Symmetrical at x-axis)

Sr. No.	Shape	Length (l)	$\bar{x}$	$\bar{y}$
5	Arc of a circle  Fig. (e)	$2r\alpha^c$ ($\alpha^c = \alpha$ in radians)	$\dfrac{r \sin \alpha^\circ}{\alpha^c}$ ($\alpha^\circ = \alpha$ in degrees)	0 (Symmetr x-axi

5.7 Centroids of Common Geometrical Shapes of Areas :

Sr. No.	Shape	Area (A)	$\bar{x}$	$\bar{y}$
1	Rectangle  Fig. (a)	lb	$\dfrac{l}{2}$	$\dfrac{b}{2}$
2	Square  Fig. (b)	a^2	$\dfrac{a}{2}$	$\dfrac{a}{2}$

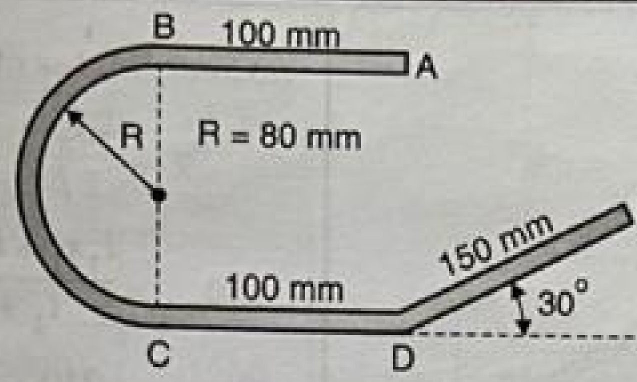


Fig. P. 5.8.1(a)

Soln. :

Wire is 1-D body and it is uniform, hence C.G. coincides with centroid of line.

Step 1 : Select reference axes w.r.t. point 'C' .

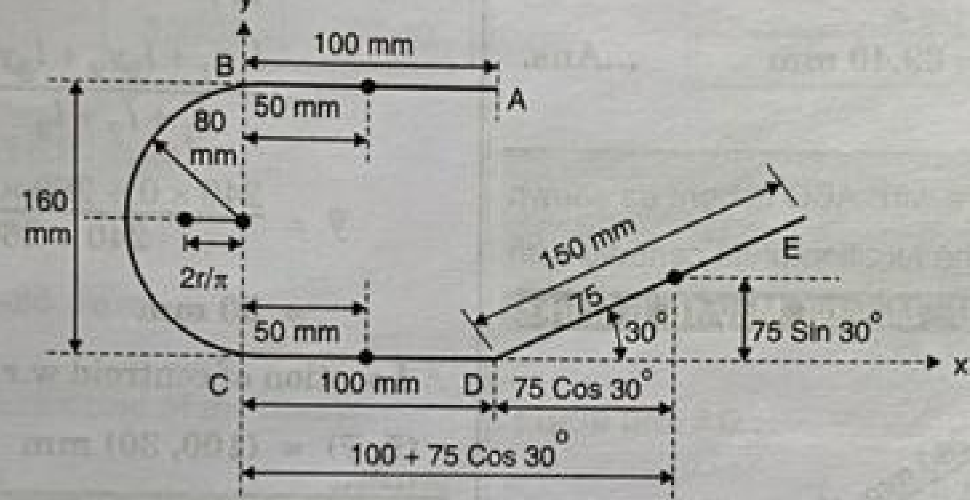


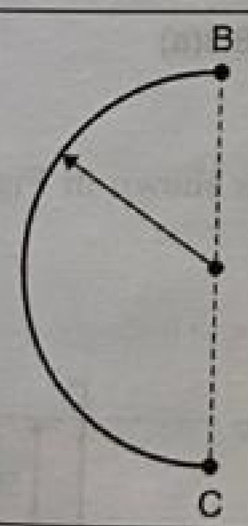
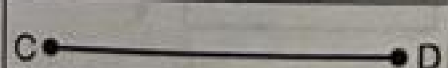
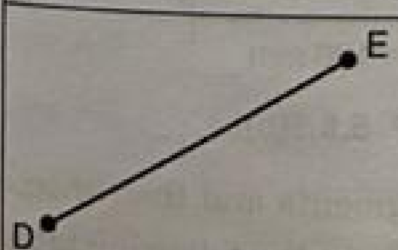
Fig. P. 5.8.1(b)

Step 2 : Divide the bent into 4 parts :

(1) Line AB (2) Semicircular arc BC (3) Line CD (4) Line DE

Step 3 : Obtain and tabulate the results of *l*, x, y, *l*x and *l*y.

Table P. 5.8.1

Line	l (mm)	x (mm)	y (mm)	l . x (mm^2)	l . y (mm^2)
	100	50	160	5000	16000
	πr $= \pi \times 80$ $= 251.33$	$-\left(\frac{2r}{\pi}\right) = -\left(\frac{2 \times 80}{\pi}\right) = -50.93$	80	–12800.13	20106.40
	100	50	0	5000	0
	150	$100 + 75 \cos 30° = 164.95$	$75 \sin 30° = 37.50$	24742.5	5625
	601.33			**21942.37**	**41731.4**

Step 4 : Take Summation of l, lx and ly.

$$\Sigma l = 601.33 \text{ mm}$$
$$\Sigma lx = 21942.37 \text{ mm}^2$$
$$\Sigma ly = 41731.4 \text{ mm}^2$$

Step 5 : Co-ordinates of centre of gravity w.r.t. point 'C' are;

$$\bar{x} = \frac{\Sigma l \cdot x}{\Sigma l} = \frac{21942.37}{601.33} = \mathbf{36.49 \text{ mm}} \qquad \text{...Ans.}$$

$$\bar{y} = \frac{\Sigma l \cdot y}{\Sigma l} = \frac{41731.4}{601.33} = \mathbf{69.40 \text{ mm}} \qquad \text{...Ans.}$$

Ex. 5.8.2 : A thin homogeneous wire ABC is bent as shown in Fig. P. 5.8.2(a). Determine the location of its centroid with respect to A. **SPPU : May 08, May 16, 6 Marks**

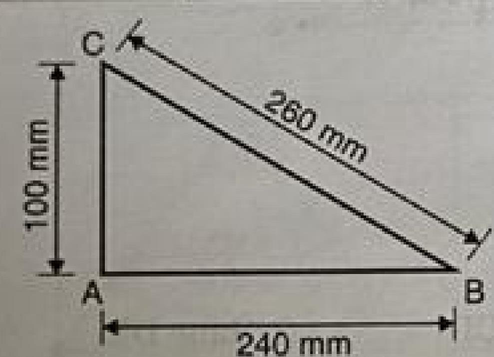


Fig. P. 5.8.2(a)

Soln. :

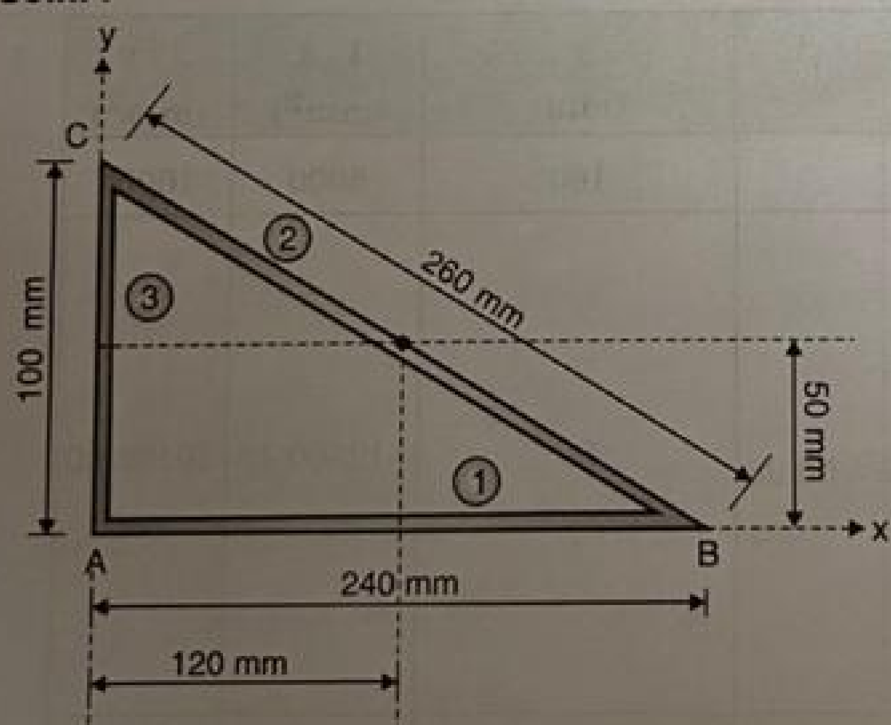


Fig. P. 5.8.2(b)

Dividing the bent into three parts, AB, BC and CA as (1), (2) and (3) respectively.

$l_1 = 240$ mm,	$x_1 = 120$ mm,	$y_1 = 0$
$l_2 = 260$ mm,	$x_2 = 120$ mm,	$y_2 = 50$ mm
$l_3 = 100$ mm,	$x_3 = 0$,	$y_3 = 50$ mm

$\therefore$ Co-ordinates of centroid are given by,

$$\bar{x} = \frac{\Sigma l_x}{\Sigma l} = \frac{l_1 x_1 + l_2 x_2 + l_3 x_3}{l_1 + l_2 + l_3}$$

$$= \frac{240 \times 120 + 260 \times 120 + 100 \times 0}{240 + 260 + 100}$$

$$= \mathbf{100 \text{ mm}}$$

$$\bar{y} = \frac{\Sigma l_y}{\Sigma l} = \frac{l_1 y_1 + l_2 y_2 + l_3 y_3}{l_1 + l_2 + l_3}$$

$$\bar{y} = \frac{240 \times 0 + 260 \times 50 + 100 \times 50}{240 + 260 + 100}$$

$$= \mathbf{30 \text{ mm}}$$

$\therefore$ Location of centroid w.r.t. point A is

$$(\bar{x}, \bar{y}) = \mathbf{(100, 30) \text{ mm}}$$

Ex. 5.8.3 : A thin rod is bend into a shape OABC in Fig. P. 5.8.3(a). Determine the centroid of with respect to origin O. **SPPU : May**

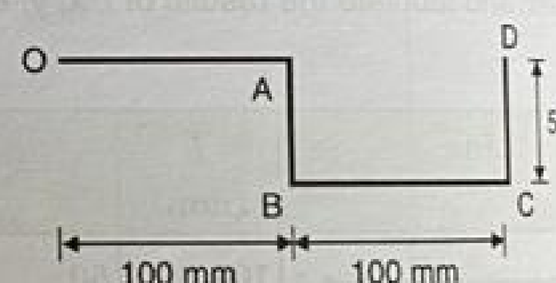


Fig. P. 5.8.3(a)

Soln. :

Selecting x and y axes as shown in F w.r.t. 'O'.

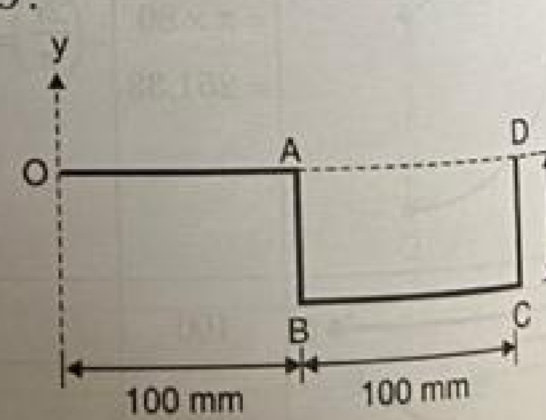


Fig. P. 5.8.3(b)

The length of line segments and the x-axis and y-axis are shown in the follow