Diffraction

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Unit I Wave Optics

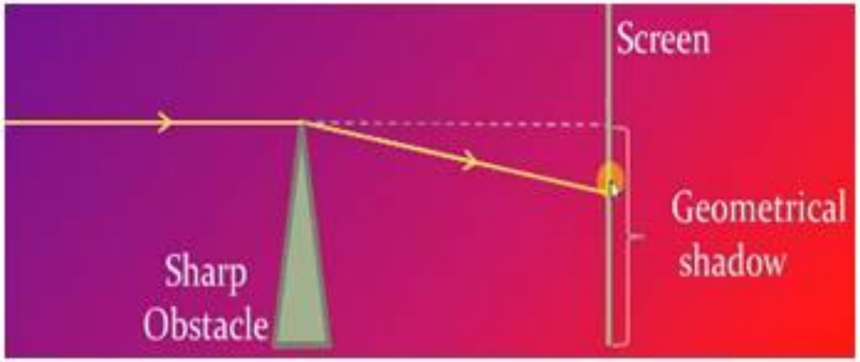
Interference - Introduction to electromagnetic waves and electromagnetic spectrum - Interference in thin film of uniform thickness (with derivation) - Interference in thin film wedge shape (qualitative) - Applications of interference: testing optical flatness, anti-reflection coating

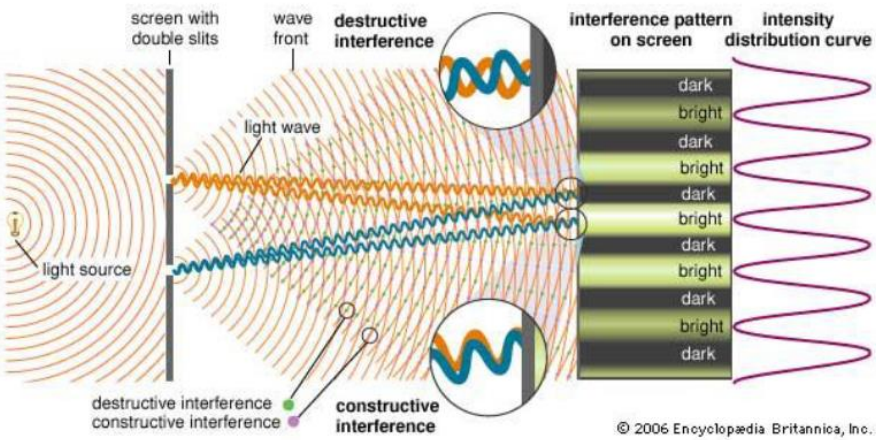
Diffraction - Diffraction of light - Diffraction at a single slit, conditions for principal maxima and minima, diffraction pattern - Diffraction grating, conditions for principal maxima and minima starting from resultant amplitude equations, diffraction pattern - Rayleigh's criterion for resolution, resolving power of telescope and grating

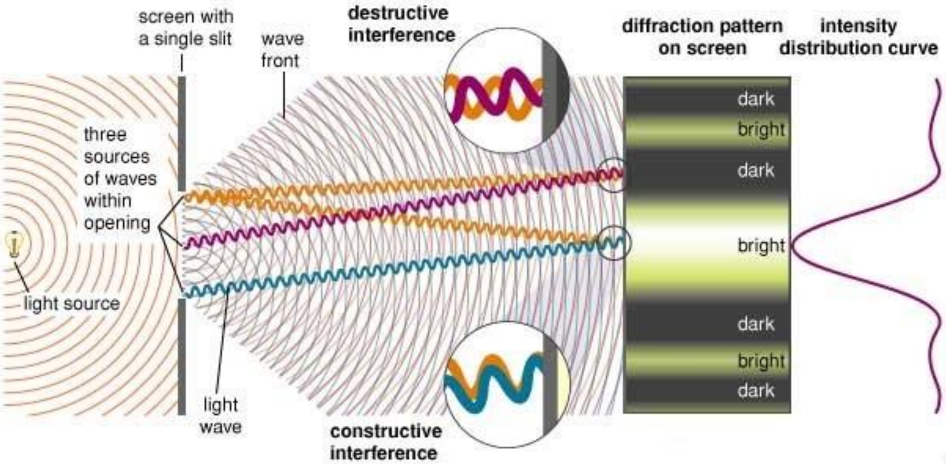
Polarization - Polarization of light, Malus law - Double refraction, Huygen's theory of double refraction Applications of polarization: LCD

Diffraction of light

The bending of light at the edge of an obstacle and hence its encroachment into the region of geometrical shadow is known as "diffraction of light".



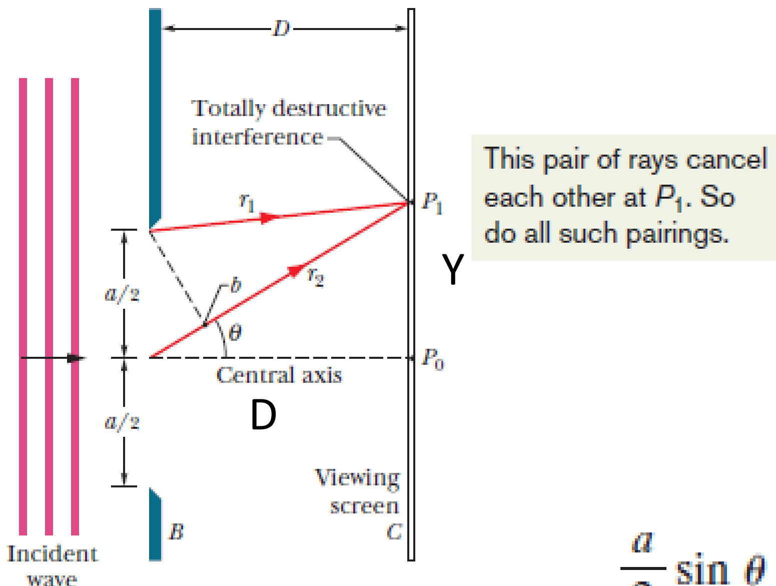




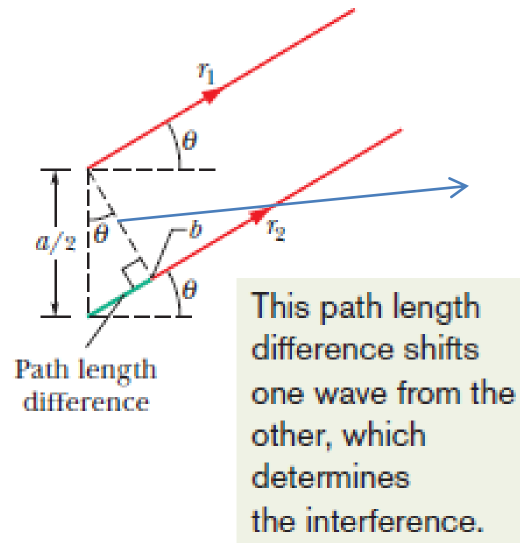
Two Set-up For Diffraction

Fresnel Diffraction	Fraunhofer diffraction
1:If the source of light and screen is at a finite distance from the obstacle, then the diffraction called Fresnel diffraction.	1:If the source of light and screen is at infinite distance from the obstacle then the diffraction is called Fraunhofer diffraction.
2:The corresponding rays are not parallel.	2:The corresponding rays are not parallel.
3:The wavefronts falling on the obstacle are not plane.	3:The wavefronts falling on the obstacle are planes.

Single slit Diffraction



For D >>> a we can assume



$$\sin\theta = \frac{\frac{\lambda}{2}}{\frac{a}{2}}$$

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2},$$

$$a\sin\theta = \lambda \qquad \text{(first minimum).}$$

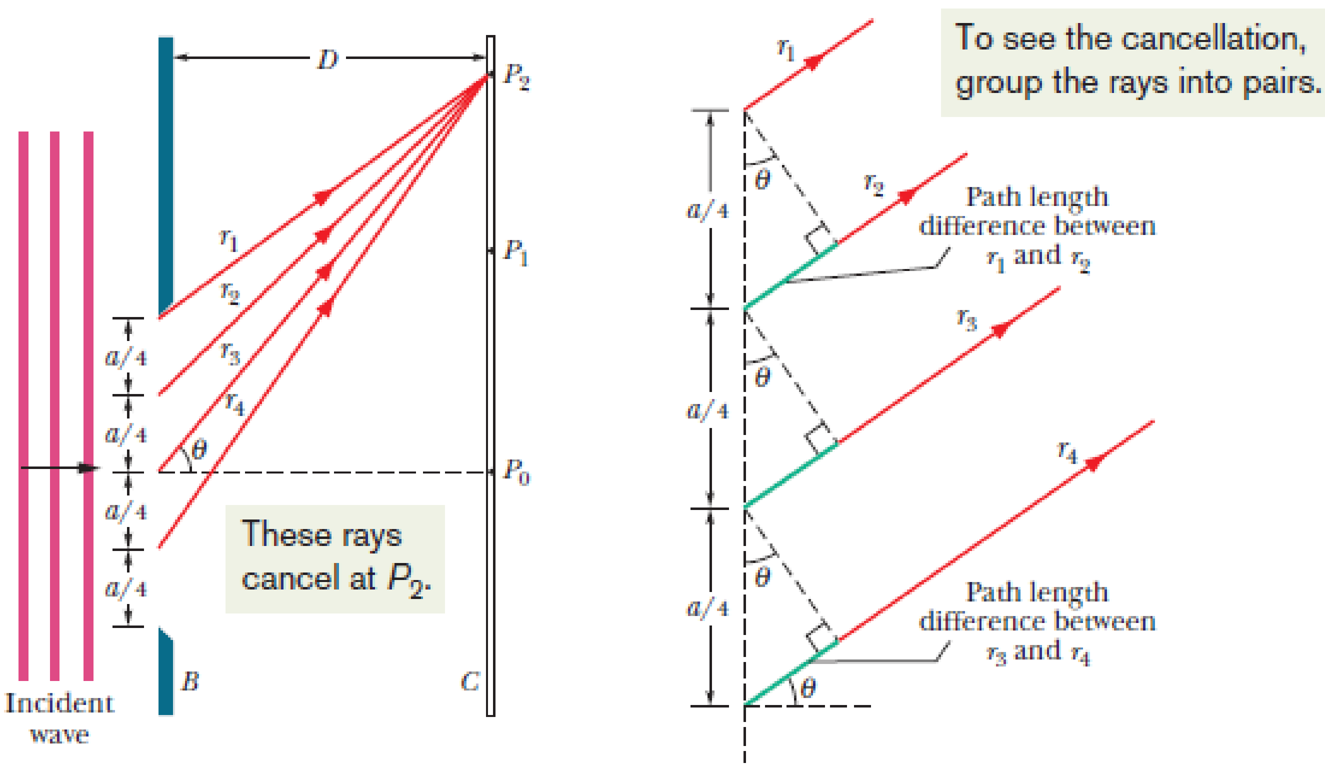
For very small θ, we can write

$$\sin\theta = tan\theta = \frac{\boldsymbol{Y}}{\boldsymbol{D}}$$

$$\frac{\boldsymbol{Y}}{\boldsymbol{D}} = \frac{\lambda}{\boldsymbol{a}}$$

$$\boldsymbol{Y} = \frac{\mathbf{D}\lambda}{\boldsymbol{a}}$$

Single slit Diffraction

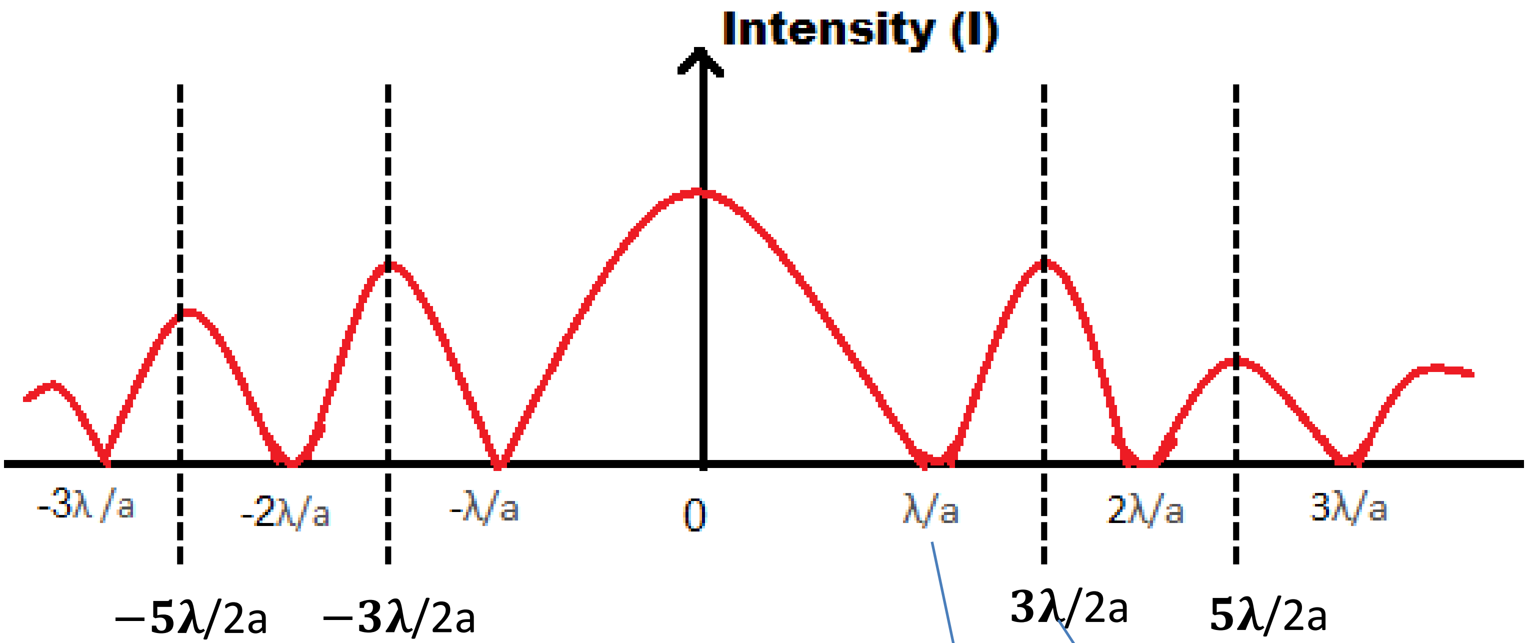


$$\sin\theta = \frac{\frac{\lambda}{2}}{\frac{a}{4}}$$

$$\frac{a}{4}\sin\theta = \frac{\lambda}{2}, \qquad a\sin\theta = 2\lambda \quad \text{(second minimum).}$$

$$a\sin\theta = m\lambda, \qquad \text{for } m = 1, 2, 3, \ldots \quad \text{(minima—dark fringes).}$$

Single slit Diffraction



First Maxima

First Minima

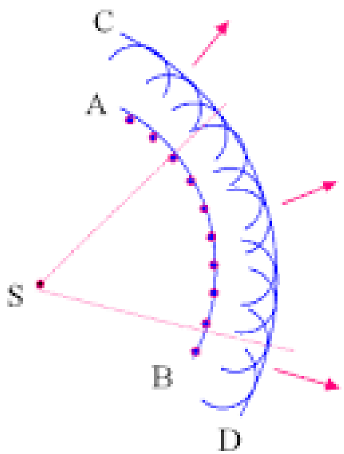
$a \sin\theta = m\lambda,$ for $m = 1, 2, 3, \ldots$ (minima—dark fringes).

$a\sin\theta = \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}$

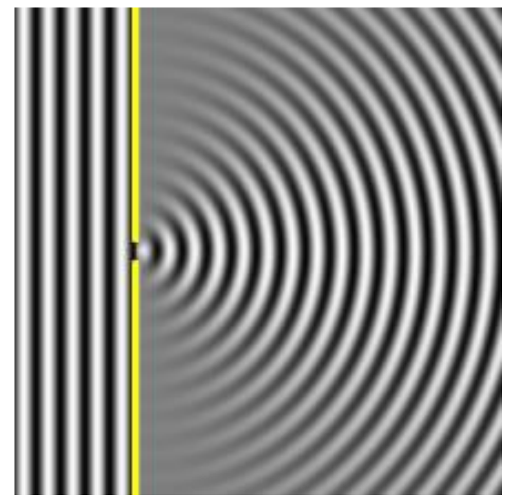
$a\sin\theta = (2m+1)\frac{\lambda}{2}$ m = 1, 2, 3 … Maxima bright fringes

A wave front is defined as a surface of constant phase of waves.

A wavelet is a wave-like oscillation with amplitude which starts at zero, increases, and then decreases back to zero.

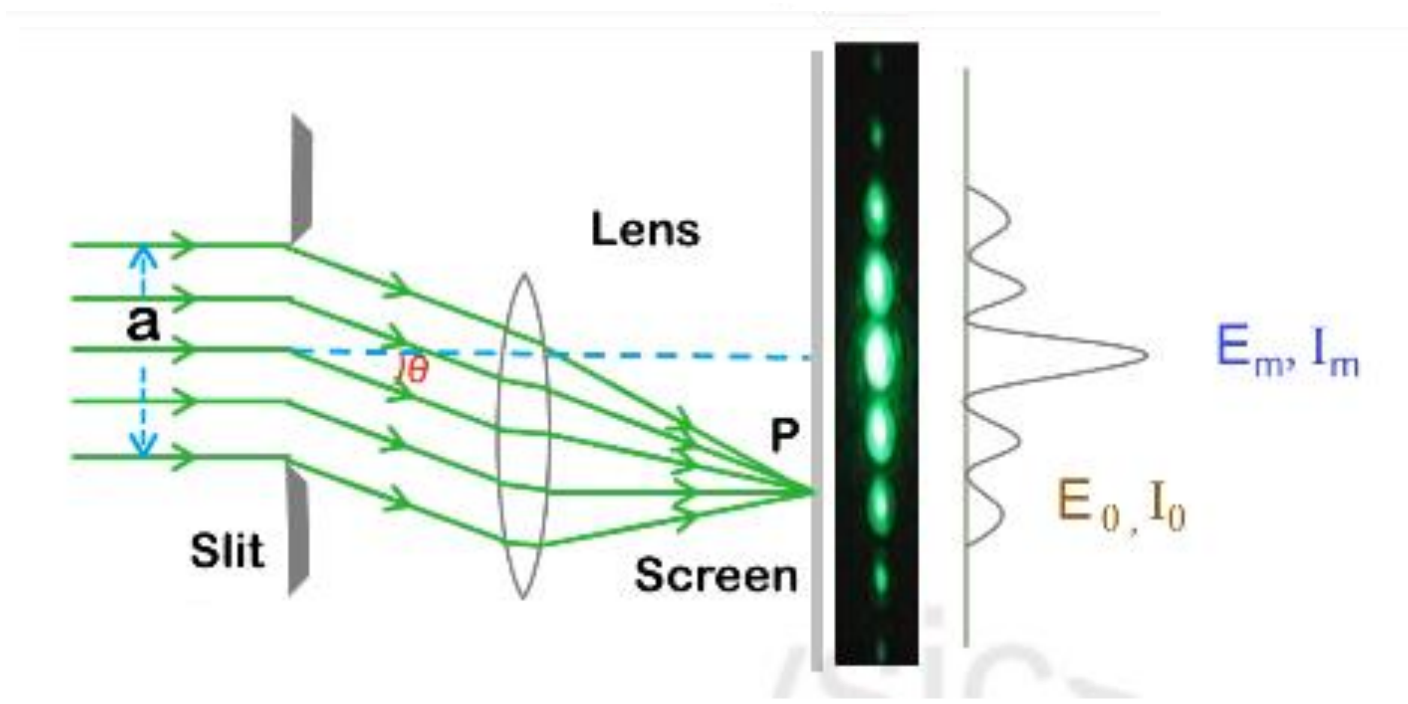


Huygens' Principle:
Each wavefront is the envelope of the wavelets. Each point on a wavefront acts as an independent source to generate wavelets for the next wavefront. AB and CD are two wavefronts.

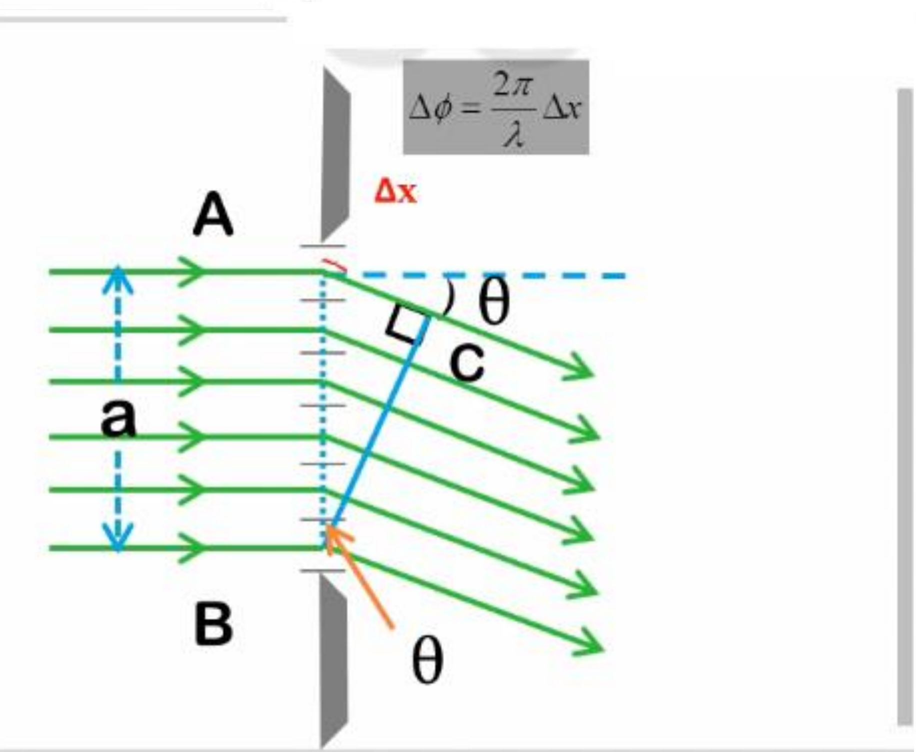


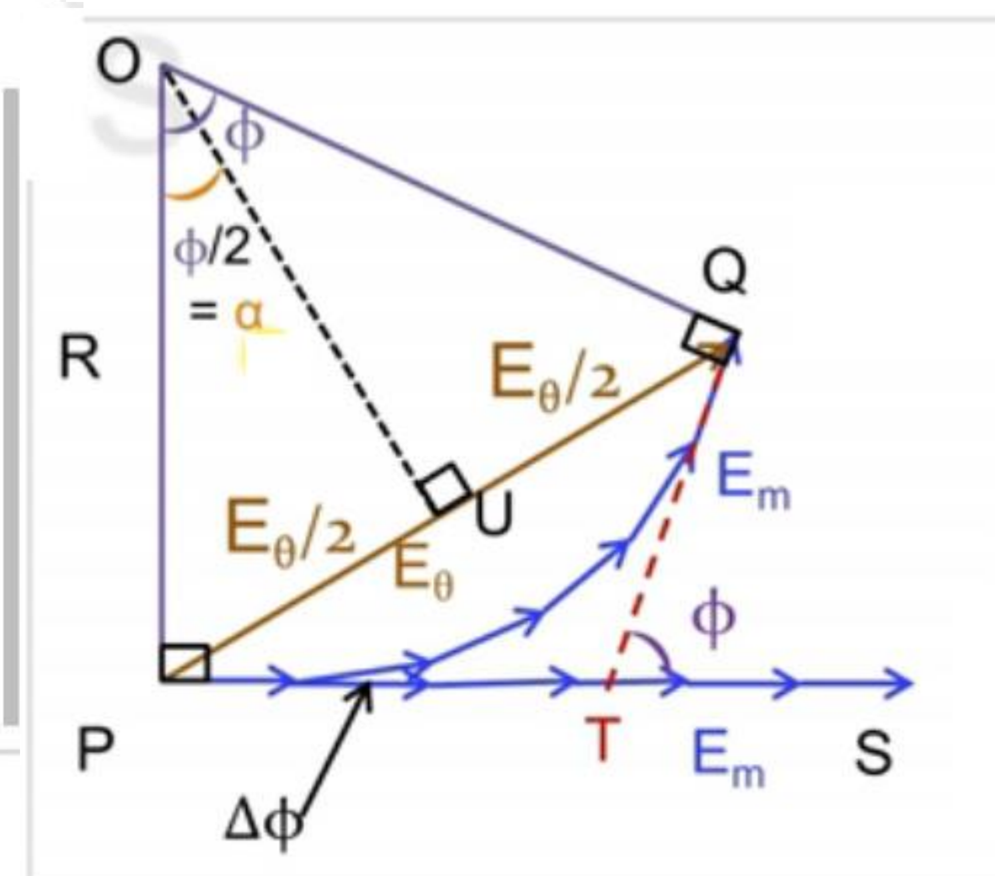
Fraunhofer Single slit Diffraction

As we can see all the plane wave front is getting incident on small slit of length a and According Huygens theory, each point on the wave front is to be considered as source of the secondary wavelets

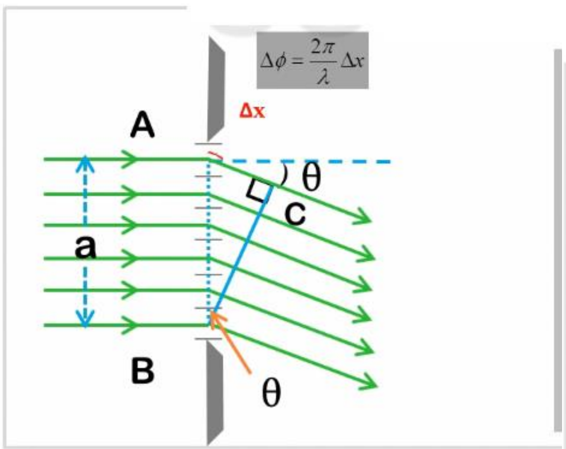


Fraunhofer Single slit Diffraction





Fraunhofer Single slit Diffraction

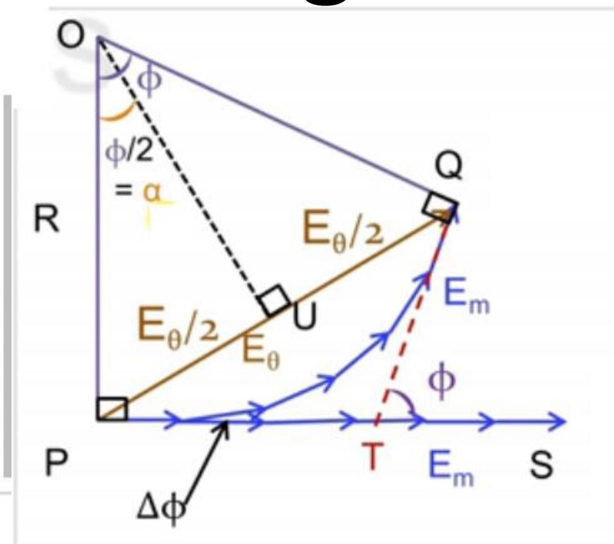


$$\Phi = \frac{2\pi}{\lambda} \text{X (path difference)}$$

$$Sin\theta = \frac{X}{a}$$

$$X = aSin\theta$$

$$\Phi = \frac{2\pi}{\lambda} aSin\theta$$



$$\alpha = \frac{\Phi}{2}$$

$$\alpha = \frac{\pi}{\lambda} aSin\theta$$

From Δ PUO

$$Sin\alpha = \frac{\frac{E_\theta}{2}}{R}$$

$$\frac{E_\theta}{2} = RSin\alpha \text{ ----(1)}$$

For sector OPQ

$$\Phi = \frac{lenght\ of\ arc}{radius}$$

$$\Phi = \frac{E_m}{R} \qquad \Phi = 2\alpha$$

$$R = \frac{E_m}{2\alpha}$$

Put in equation 1

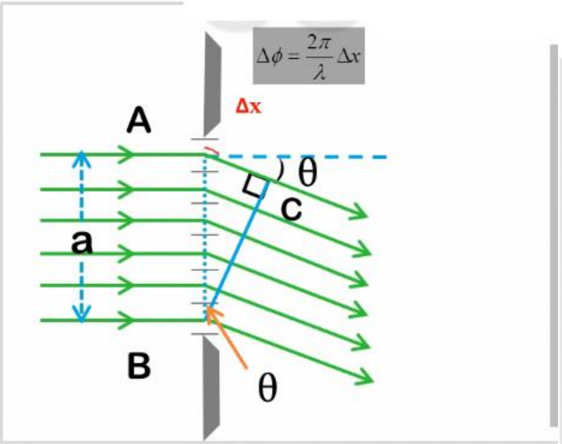
$$\frac{E_\theta}{2} = \frac{E_m}{2\alpha} Sin \text{ ----(1)}$$

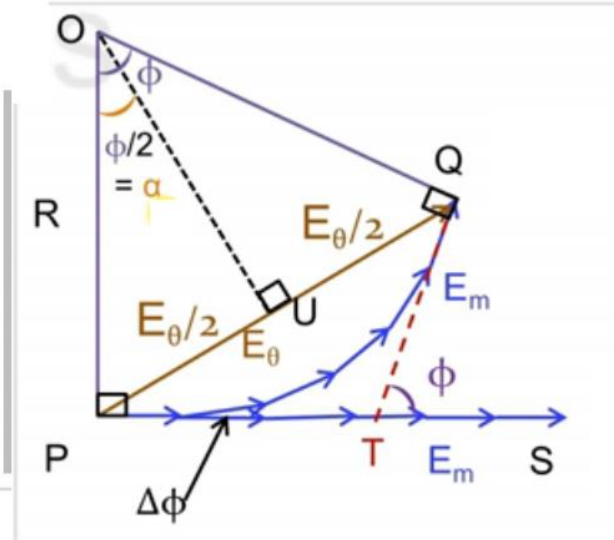
$$\boxed{E_\theta = E_m \frac{Sin\alpha}{\alpha}}$$

This is the resultant amplitude at an angle θ

Where $\alpha = \frac{\pi}{\lambda} aSin\theta$

Fraunhofer Single slit Diffraction





Now squaring amplitude equation

$$E_\theta{}^2 = E_m{}^2 \; \frac{Sin\alpha^{\,2}}{\alpha^2}$$

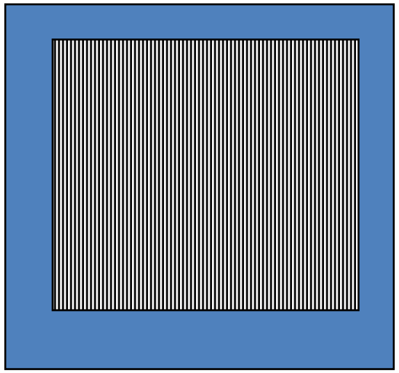
$$I = E^2$$

$$I_\theta{}^2 = I_m{}^2 \; \frac{Sin\alpha^{\,2}}{\alpha^2}$$

This is the resultant intensity
at an angle θ

Diffraction Grating

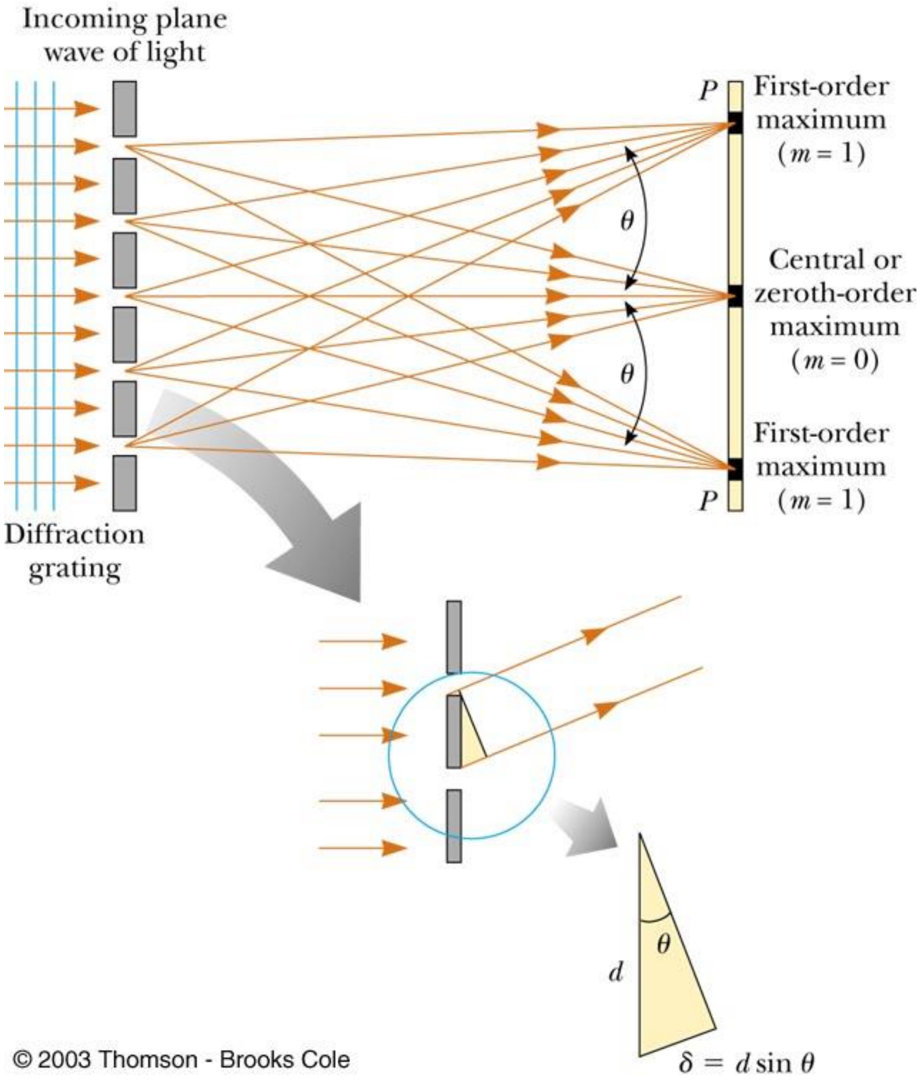
- The diffracting grating consists of many equally spaced parallel slits
  - A typical grating contains several thousand lines per centimeter
- The intensity of the pattern on the screen is the result of the combined effects of interference and diffraction

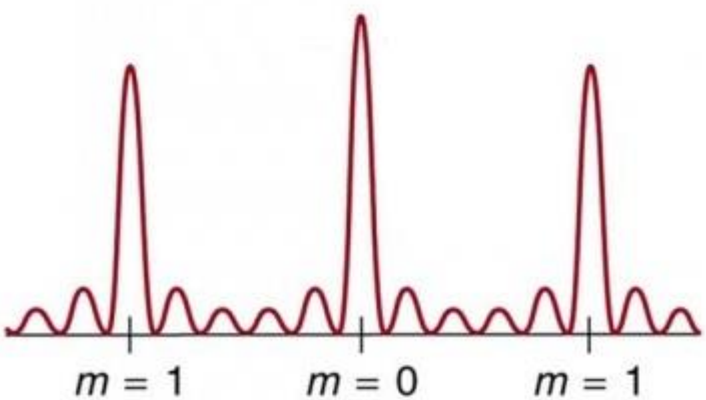


$600 mm^{-1}$

Diffraction Grating

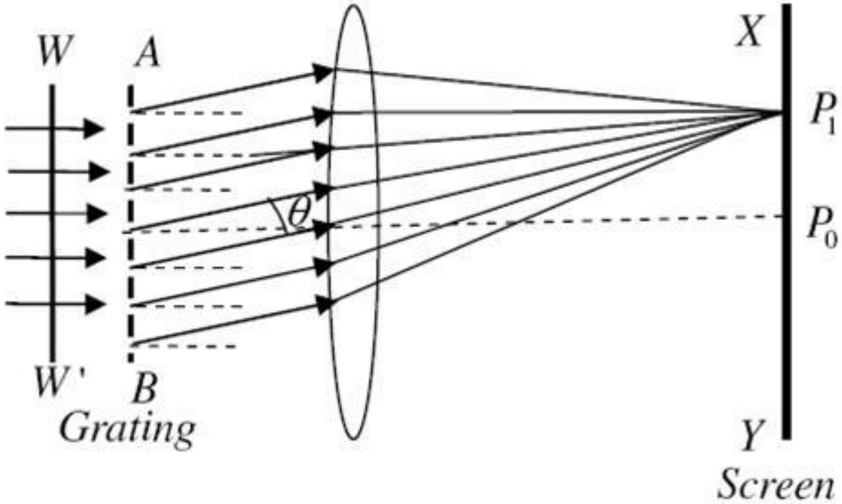
Expand our previous discussion of diffraction from a single slit; diffraction grating is an array of N slits with each slit having width “b” (b separation between slits) and center to center distance is “a”. We consider N identical slits, illuminated by an incident plane wave and determine the superposition field and irradiance at an observation point P located far from the slits





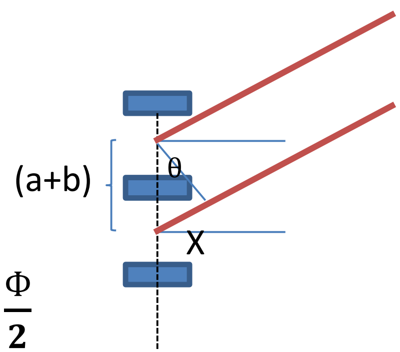
Diffraction grating, conditions for principal m
starting from resultant amplitude e

Let "a" is the width ie. separation between slits and "b" is the centre to centre distance between the slits and We consider N identical slits, illuminated by an incident plane wave and incident on diffraction grating and observe the phenomenon

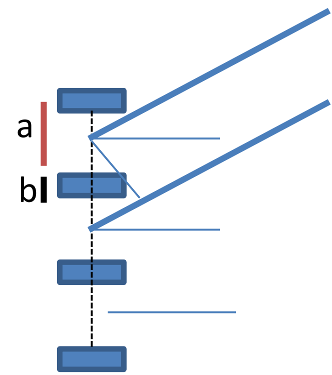


In this case we will directly start from resultant amplitude equation

$$\Phi = \frac{2\pi}{\lambda}\text{ X (path difference)}$$



$$(a+b) = \frac{1}{N}$$



$$Sin\theta = \frac{X}{(a+b)}$$

$$\beta = \frac{\Phi}{2}$$

$$X = (a+b)Sin\theta$$

$$\beta = \frac{\pi}{\lambda}(a+b)Sin\theta$$

$$\Phi = \frac{2\pi}{\lambda}(a+b)Sin\theta$$

The Resultant Amplitude for diffraction grating can be written as

$$E_\theta = E_m \frac{Sin\alpha}{\alpha} \frac{SinN\beta}{Sin\beta}$$

The corresponding intensity for diffraction grating can be written as

$$I_\theta = I_m \frac{Sin^2\alpha}{\alpha^2} \frac{Sin^2N\beta}{Sin^2\beta}$$

Condition for Principal Maxima

Remains constant

E_θ is dependant on this term

$$E_\theta = \frac{Sin\alpha}{\alpha} \frac{SinN\beta}{Sin\beta}$$

For E_θ to be maxima $Sin\beta$ must be zero

If $Sin\beta \longrightarrow 0$

$\beta = \pm\, \mathrm{m}\pi$ Where $\mathrm{m} = 0, 1, 2, ..$

We have

$$\beta = \frac{\pi}{\lambda}(a+b)Sin\theta$$

$$\frac{\pi}{\lambda}(a+b)Sin\theta = \pm\, \mathrm{m}\pi$$

$$\boxed{(a+b)Sin\theta = \pm\, \mathrm{m}\, \boldsymbol{\lambda}}$$

$$E_\theta = \frac{Sin\alpha}{\alpha} \frac{SinN\beta \longrightarrow 0}{Sin\beta \longrightarrow 0}$$

Indeterminate form

L Hospital's Rule

$$E_\theta = \frac{Sin\alpha}{\alpha} \frac{SinN\beta}{Sin\beta}$$

$$\sum_{\beta\to=\pm\, \mathrm{m}\pi} \frac{SinN\beta}{Sin\beta} = \sum_{\beta\to=\pm\, \mathrm{m}\pi} \frac{d(SinN\beta)}{d(Sin\beta)}$$

$$= \sum_{\beta\to=\pm\, \mathrm{m}\pi} \frac{NcosN(m\pi)}{cos(m\pi)}$$

$$\sum_{\beta \to = \pm m\pi} \frac{Sin N\beta}{Sin\beta} = \frac{N(-1)}{(-1)}$$

$cos(m\pi)$ = -1

Condition for Minima

$$\frac{Sin N\beta}{Sin\beta} = N$$

$$E_\theta = E_m \frac{Sin\alpha}{\alpha} \frac{Sin N\beta}{Sin\beta}$$

$$\frac{Sin^2 N\beta}{Sin^2\beta} = N^2$$

$$Sin N\beta \longrightarrow 0$$

$$N\beta = \pm m\pi$$

$$E_\theta = E_m \frac{Sin\alpha}{\alpha} N$$

$$N\left(\frac{\pi}{\lambda}(a+b)Sin\theta\right) = \pm m\pi$$

$$I_\theta = I_m \frac{Sin^2\alpha}{\alpha^2} N^2$$

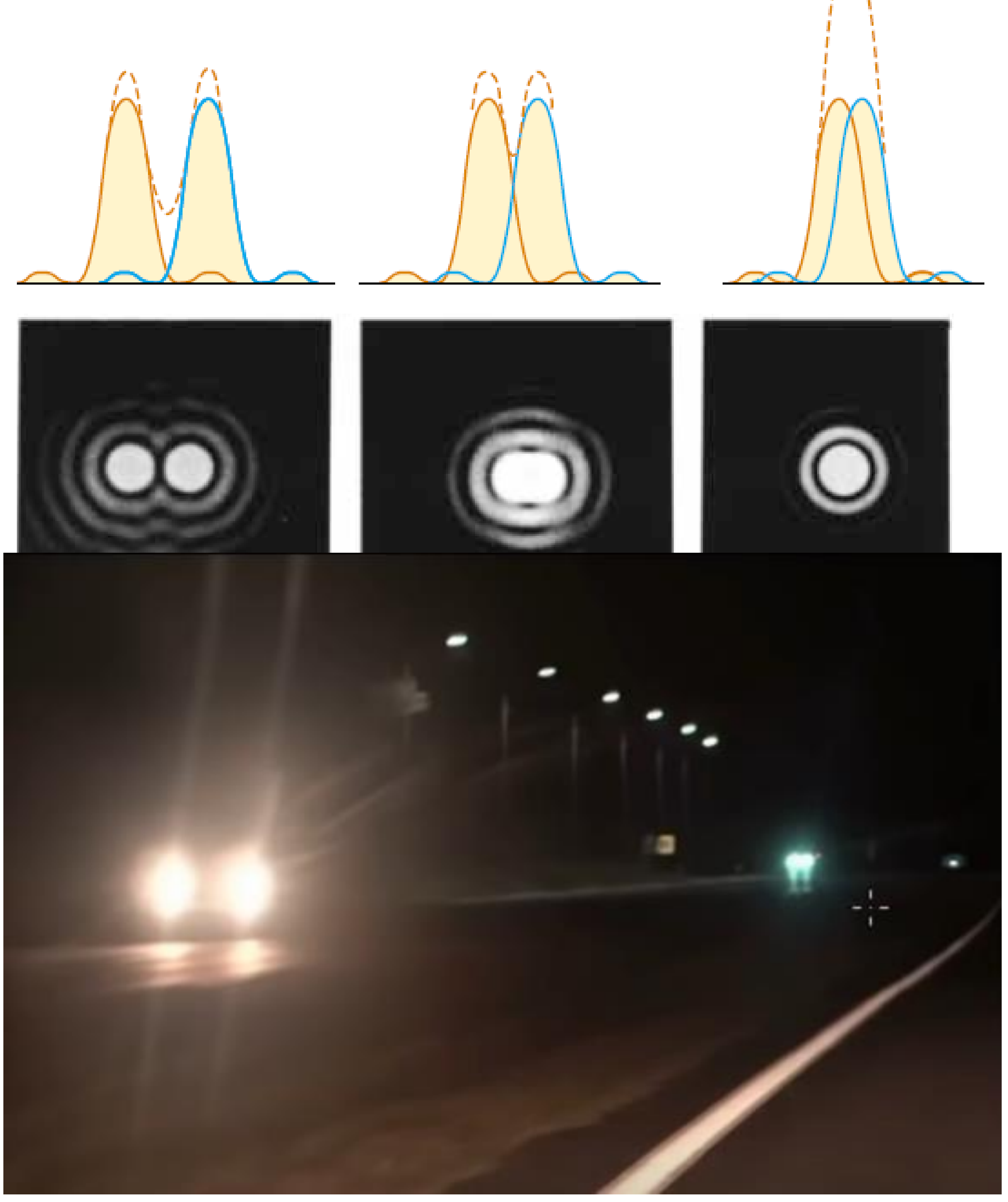
$$\boxed{N(a+b)Sin\theta = \pm m \boldsymbol{\lambda}}$$

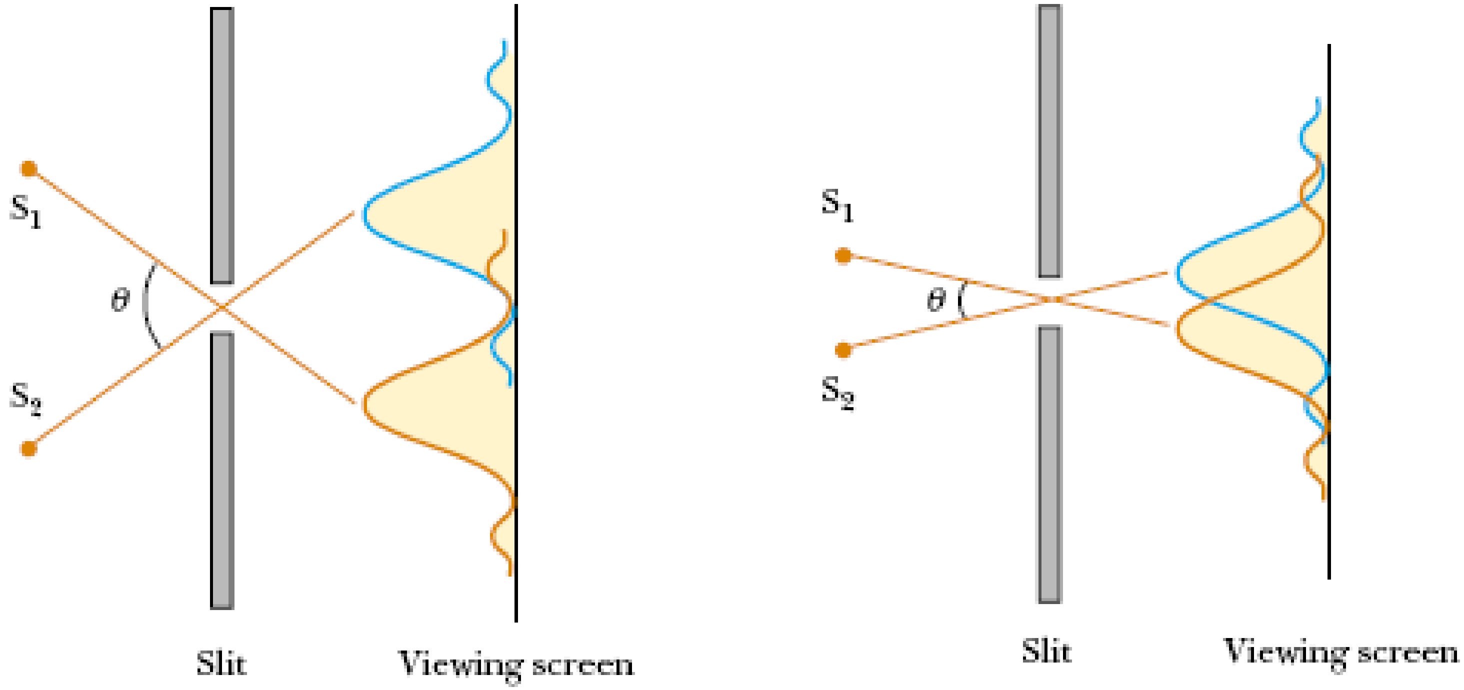
m can have all integer value excpet
$m = 0, N, 2N, 3N ..$

Otherwise it would become condition for Principal maxima

Rayleigh's criterion

- To decide when two images are resolved, the following criterion is used:
- When the central maximum of one image falls on the first minimum another image, the images are said to be just resolved.
- This limiting condition of resolution is known as Rayleigh's criterion.

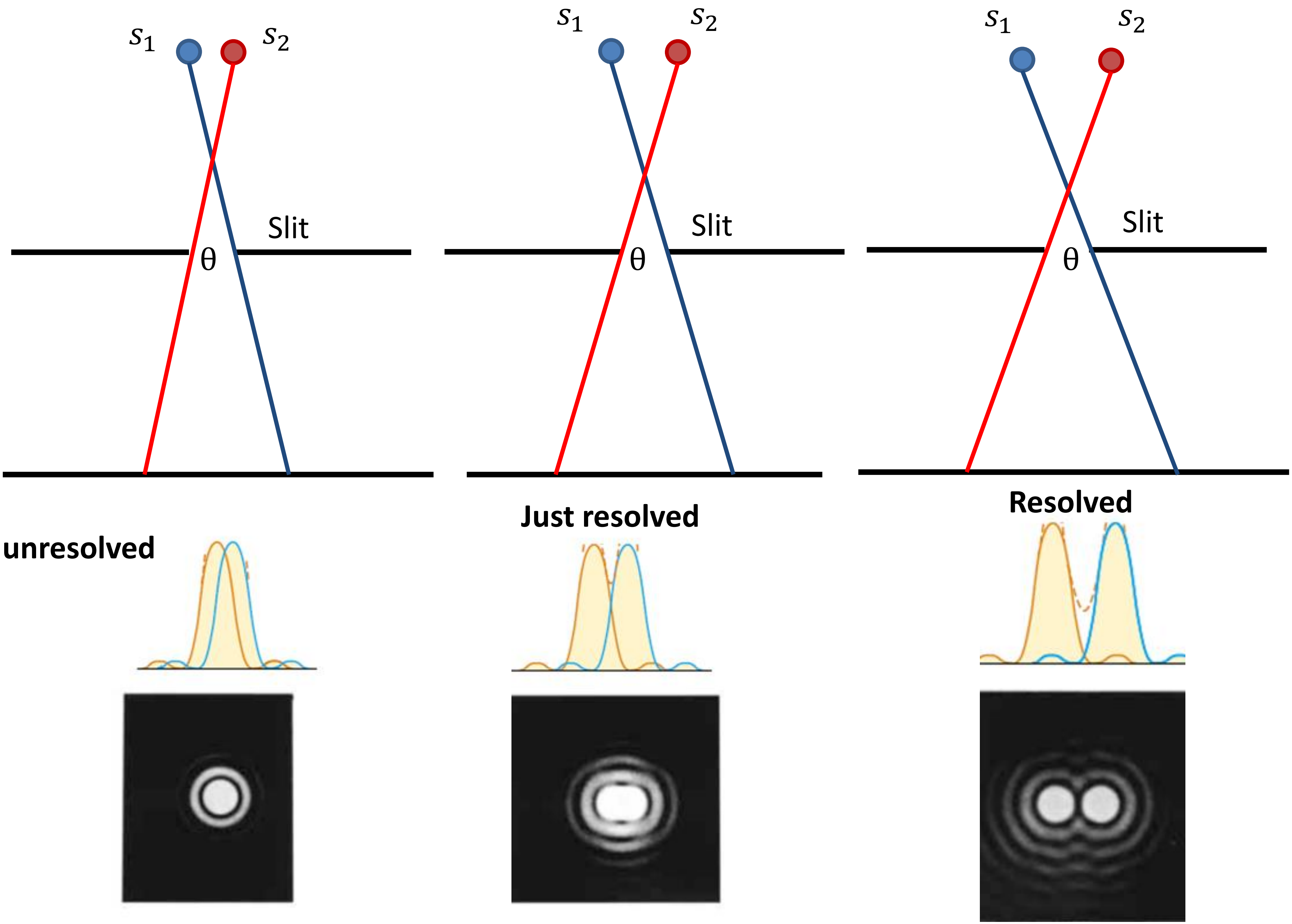




- If the two sources are <u>separated</u> enough to keep their central maxima from overlapping, their images can be distinguished and are said to be resolved.
- If the sources are <u>close</u> together, however, the two central maxima overlap and the images are not resolved.

Limit of resolution : The smallest separation (Linear of angular) between two point objects at which they appear just separated is called the limit of resolution of an optical instrument.

Resolving Power: The reciprocal of the limit of resolution of an optical instrument is known as resolving power of that instrument



s_1
s_2
Slit
θ
unresolved
s_1
s_2
Slit
θ
Just resolved
s_1
s_2
Slit
θ
Resolved

- From Rayleigh's criterion, we can determine the minimum angular separation, θ_{min} , subtended by the sources at the slit so that their images are just resolved.
- the first minimum in a single-slit diffraction pattern occurs at the angle for which

$$\sin \theta = \lambda / a$$

where a is the width of the slit. According to Rayleigh's criterion, this expression gives the smallest angular separation for which the two images are resolved.

- Because $\lambda \ll a$ in most situations,
- $\sin\theta$ is small and we can use the approximation
- $\sin\theta \approx \theta$. Therefore, the limiting angle of resolution for a slit of width a is
- $$\boldsymbol{\theta_{min} = \lambda / a}$$
- where θ_{min} is expressed in radians. Hence, the angle subtended by the two sources at the slit must be greater than λ / a if the images are to be resolved.

- The diffraction pattern of a circular aperture consists of a central circular bright disk surrounded by progressively fainter rings. The limiting angle of resolution of the circular aperture is:

$$\boldsymbol{\theta = 1.22\frac{\lambda}{D}}$$

- Where D is the diameter of the aperture.

Resolving Power of Telescope

- ***TELESCOPE***
  Telescope is used to see distant objects and therefore the amount of details given by it depends on the angle subtended at its objective by the two point objects.

  RESOLVING POWER OF A TELESCOPE
  The resolving power of a telescope is defined as the reciprocal of smallest angle subtended at the objective by the two distant objects which can be seen just as separate in the telescope.

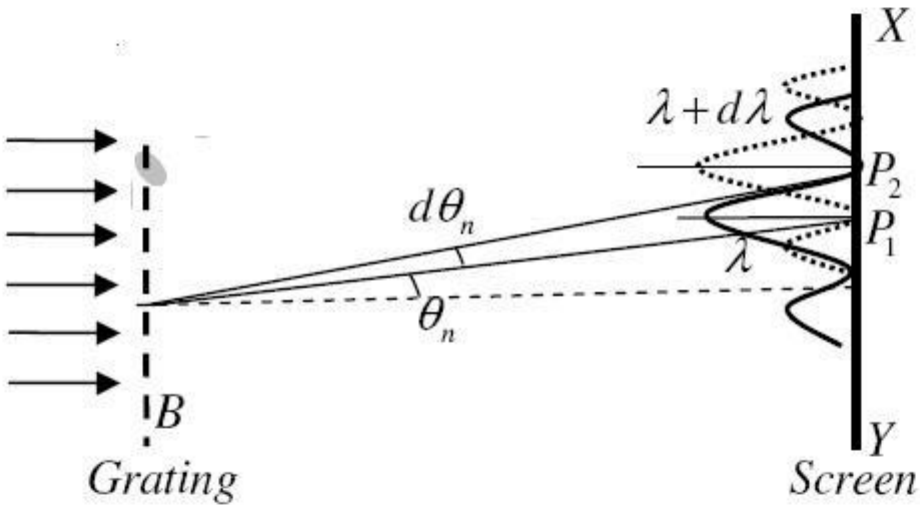
R.P=1/ θ

$$\theta = 1.22\frac{\lambda}{D}$$

$$\text{R.P} = \frac{D}{1.22 * \lambda}$$

Resolving Power of Gratings

It is defined as ***the capacity of a grating to form separate diffraction maxima of two wavelengths which are very close to each other***



Let *AB* represent the surface of a plane transmission grating having grating element *(a+b)* and *N* total number of slits. Let a beam of light having two wavelengths $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda}$+d $\boldsymbol{\lambda}$ is normally incident on the grating. Let P_1 is *nth* primary maximum of a spectral line of wavelength $\boldsymbol{\lambda}$ at an angle of diffraction $\boldsymbol{\theta}$ and $\boldsymbol{\theta}+\mathbf{d}\boldsymbol{\theta}$.

According to Rayleigh criterion, the two wavelengths will be resolved if the principal maximum $\boldsymbol{\lambda}$+d$\boldsymbol{\lambda}$ of *nth* order in a direction $\boldsymbol{\theta} + \mathbf{d}\boldsymbol{\theta}$ falls over the first minimum of *nth* order in the same direction $\boldsymbol{\theta} + \mathbf{d}\boldsymbol{\theta}$. Let us consider the first minimum of λ of *nth* order in the direction $\boldsymbol{\theta} + \mathbf{d}\boldsymbol{\theta}$ as below.

The principal maximum of $\boldsymbol{\lambda}$ in the $\boldsymbol{\theta}$ direction is given by

$$(a+b)Sin\theta = \text{m}\,\boldsymbol{\lambda} \quad \text{.........(1)}$$

The equation of minima is $(a+b)Sin\theta = \pm\ \text{m}\ \lambda$ where *m* takes all integers except *0, N, 2N, …, nN,* because for these values of *m*, the condition for maxima is satisfied. Thus first minimum adjacent to *nth* principal maximum in the direction $\theta + \text{d}\theta$ can be obtained by substituting the value of m as n(N+1) Therefore, the first minimum in the direction $\theta + \text{d}\theta$ of is given by is given by

$$N(a+b)Sin(\boldsymbol{\theta} + \mathbf{d}\boldsymbol{\theta}) = \text{m(N+1)}\boldsymbol{\lambda} \quad \text{.........(1)}$$

$$(a+b)Sin(\boldsymbol{\theta} + \mathbf{d}\boldsymbol{\theta}) = (\text{m+}\frac{1}{N})\boldsymbol{\lambda} \quad \text{.........(2)}$$

The principle maxima of $\boldsymbol{\lambda}$+d$\boldsymbol{\lambda}$ in the direction $\boldsymbol{\theta} + \mathbf{d}\boldsymbol{\theta}$ can be written as

$$(a+b)Sin(\boldsymbol{\theta} + \mathbf{d}\boldsymbol{\theta}) = \text{m}\ (\boldsymbol{\lambda}\text{+d}\boldsymbol{\lambda})\text{.........(3)}$$

Resolving Power of Gratings

Now comparing equation 2 and 3 we get

$$\left(m+\frac{1}{N}\right)\boldsymbol{\lambda} = m\,(\boldsymbol{\lambda}+d\boldsymbol{\lambda})$$

$$m\boldsymbol{\lambda}+\frac{\lambda}{N} = m\,\boldsymbol{\lambda}+md\boldsymbol{\lambda})$$

$$\frac{\lambda}{N} = md\boldsymbol{\lambda}$$

$$\boxed{\frac{\lambda}{d\lambda} = mN \quad ....(4)}$$

Thus the *resolving power is directly proportional to*

(i) The order of the spectrum *'m'*

(ii) The total number of lines on the grating *'N'*