5.6 Centroid of Common Geometrical Shapes of Lines :

Sr. No.	Shape	Length (/)	X	- y
1	Straight line		SECTION STATES	
	C X X	1	$\frac{l}{2}$	0 (Symmetrical at x-axis)
2	Fig. (a)			
2	Quarter Circular Arc	$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$
3	Fig. (b)			
	Semi circular arc	πr	0 (Symmetrical at y-axis)	$\frac{2r}{\pi}$
4	Circle Fig. (d)	$2\pi r$	0 (Symmetrical at y-axis)	0 (Symmetrical at x-axis)

Sr. No.	Shape	Area (A)	₹	y
3	Quarter circle Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
4	Semi-circle Ty Fig. (d)	$\frac{\pi r^2}{2}$	0 (Symmetrical at y-axis)	$\frac{4r}{3\pi}$
5	Circle Circle Fig. (e)	πr ²	0 (Symmetrical at y-axis)	0 (Symmetrical at x-axis)

. . . .

circle		Marin Marina	,
c) **	$2r\alpha^{c}$ $(\alpha^{c} = \alpha \text{ in radians})$	$\frac{r \sin \alpha^{\circ}}{\alpha^{c}}$	0 (Symn
The second secon		$2r\alpha^{c}$ $(\alpha^{c} = \alpha \text{ in radians})$	$2r\alpha^{c}$ $(\alpha^{c} = \alpha \text{ in radians})$ α^{c} $(\alpha^{c} = \alpha \text{ in degrees})$

5.7 Centroids of Common Geometrical Shapes of Areas :

Sr. No.	Shape	Area (A)	X	ÿ
1	Rectangle			
	X C Y	lb	$\frac{l}{2}$	
2	Fig. (a)	a^2	a	
	Square		<u>a</u> 2	
	x c			
	0			
	Fig. (b)			

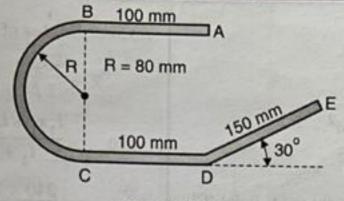


Fig. P. 5.8.1(a)

Soln.:

Wire is 1-D body and it is uniform, hence C.G. coincides with centroid of line.

Step 1: Select reference axes w.r.t. point 'C' .

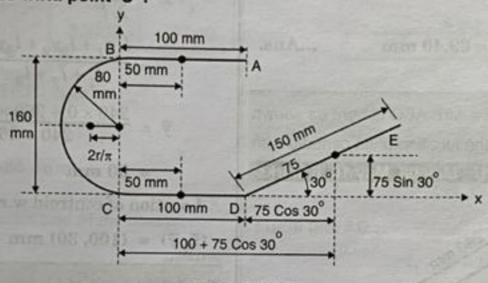


Fig. P. 5.8.1(b)

(3)

Step 2: Divide the bent into 4 parts:

(1) Line AB

(2) Semicircular arc BC

Line CD

(4) Line DE

FIR. P. E.B.ZINT

Step 3: Obtain and tabulate the results of I, x, y, Ix and Iy.

Table P. 5.8.1

Table P. 5.8.1						
Line	l (mm)	x (mm)	y (mm)	1.x (mm ²)	1.y (mm ²)	
B • A	100	50	160	5000	16000	
B	πr = π × 80 = 251.33	$-\left(\frac{2r}{\pi}\right) = -\left(\frac{2\times80}{\pi}\right) = -50.93$	80	-12800.13	20106.40	
C• D	100	50	0	5000	0	
D. E	150	100 + 75 cos 30° = 164.95	75 sin 30° = 37.50	24742.5	5625	
	601.33	mus ?		21942.37	41731.4	

Step 4: Take Summation of I, Ix and Iy.

$$\Sigma l = 601.33 \,\mathrm{mm}$$

$$\Sigma lx = 21942.37 \text{ mm}^2$$

$$\sum ly = 41731.4 \text{ mm}^2$$

Step 5: Co-ordinates of centre of gravity w.r.t. point 'C' are;

$$\bar{\mathbf{x}} = \frac{\sum l \cdot \mathbf{x}}{\sum l} = \frac{21942.37}{601.33} = 36.49 \text{ mm}$$
 ...Ans.

$$\overline{y} = \frac{\sum l \cdot y}{\sum l} = \frac{41731.4}{601.33} = 69.40 \text{ mm}$$
 ...Ans.

Ex. 5.8.2 : A thin homogeneous wire ABC is bent as shown in Fig. P. 5.8.2(a). Determine the location of its centroid with respect to A.

SPPU : May 08, May 16, 6 Marks

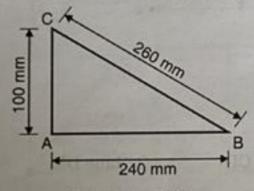


Fig. P. 5.8.2(a)



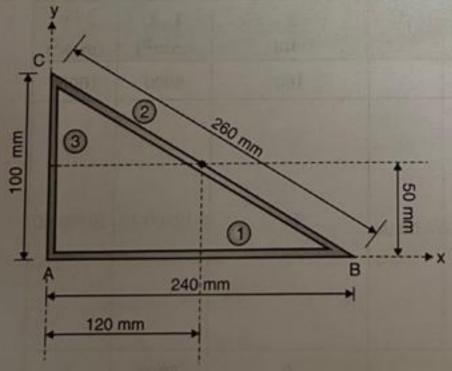


Fig. P. 5.8.2(b)

Dividing the bent into three parts, AB, BC and CA as (1), (2) and (3) respectively.

$$l_1 = 240 \text{ mm}$$
, $x_1 = 120 \text{ mm}$, $y_1 = 0$
 $l_2 = 260 \text{ mm}$, $x_2 = 120 \text{ mm}$, $y_2 = 50 \text{ mm}$
 $l_3 = 100 \text{ mm}$, $x_3 = 0$, $y_3 = 50 \text{ mm}$

.. Co-ordinates of centroid are given by,

$$\bar{\mathbf{x}} = \frac{\sum l_{\mathbf{x}}}{\sum l}$$

$$= \frac{l_{1}\mathbf{x}_{1} + l_{2}\mathbf{x}_{2} + l_{3}\mathbf{x}_{3}}{l_{1} + l_{2} + l_{3}}$$

$$= \frac{240 \times 120 + 260 \times 120 + 100 \times 0}{240 + 260 + 100}$$

$$= 100 \text{ mm}$$

$$\begin{split} \overline{y} &= \frac{\sum l_y}{\sum l} \\ &= \frac{l_1 y_1 + l_2 y_2 + l_3 y_3}{l_1 + l_2 + l_3} \\ \overline{y} &= \frac{240 \times 0 + 260 \times 50 + 100 \times 50}{240 + 260 + 100} \end{split}$$

:. Location of centroid w.r.t. point A is

$$(\bar{x}, \bar{y}) = (100, 30) \text{ mm}$$

 $= 30 \, \mathrm{mm}$

Ex. 5.8.3: A thin rod is bend into a shape OABO
in Fig. P. 5.8.3(a). Determine the centroid of the
with respect to origin O.

SPPU: May

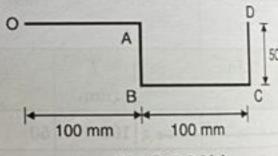


Fig. P. 5.8.3(a)

Soln.:

Selecting x and y axes as shown in F w.r.t. 'O'.

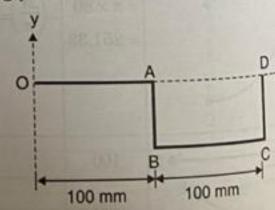


Fig. P. 5.8.3(b)

The length of line segments and the x-axis and y-axis are shown in the follow

6.8 M.I for Standard Shapes (Areas)

1. Rectangular Area:

(A) About Centroidal x-axis:

 Consider rectangle of width 'b' and depth 'd' as shown in Fig. 6.8.1.

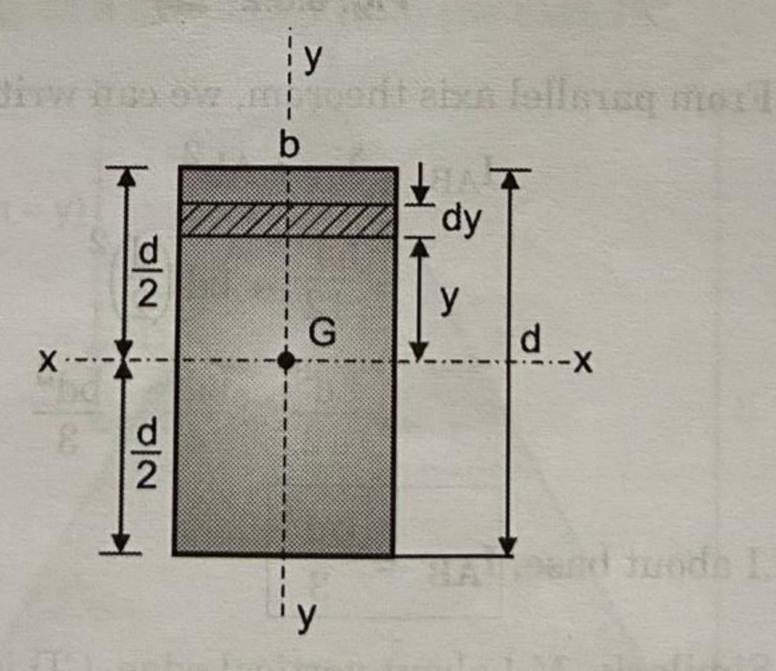


Fig. 6.8.1

Consider a small strip of width 'b' and thickness
 dy at a distance 'y' from the centroidal x-axis.

Area of elemental strip, dA = b dy Moment of inertia of elemental strip at centroidal x-axis,

$$dI_{xx} = y^2 \cdot dA$$

Total M.I about x-axis,

$$I_{xx} = \int y^{2} \cdot dA = b \int_{-d/2}^{+d/2} y^{2} \cdot dy$$

$$= b \left[\frac{y^{3}}{3} \right]_{-d/2}^{+d/2} = \frac{b}{3} \left[\frac{d^{3}}{8} + \frac{d^{3}}{8} \right]$$

$$= \frac{b}{24} (2d^{3}) = \frac{bd^{3}}{12}$$

$$I_{xx} = \frac{bd^{3}}{12}$$

Similarly,

M.I at the base i.e AB can be found by using parallel axis theorem.

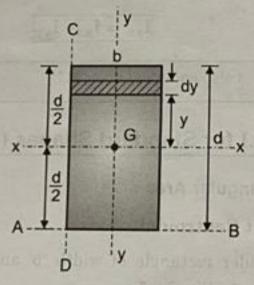


Fig. 6.8.2

From parallel axis theorem, we can write,

$$I_{AB} = I_{xx} + Ah^{2}$$

$$= \frac{bd^{3}}{12} + bd\left(\frac{d}{2}\right)^{2}$$

$$= \frac{bd^{3}}{12} + \frac{bd^{3}}{4} = \frac{bd^{3}}{3}$$

 \therefore M.I about base, $I_{AB} = \frac{bd^3}{3}$

$$I_{AB} = \frac{bd^3}{3}$$

Similarly, M.I about vertical edge, CD is

$$I_{CD} = \frac{b^3 d}{3}$$

Hollow Rectangular area:

$$I_{xx} = \left(\frac{BD^3}{12} - \frac{bd^3}{12}\right)$$

 $I_{yy} = \left(\frac{B^3D}{12} - \frac{b^3d}{12}\right)$ and

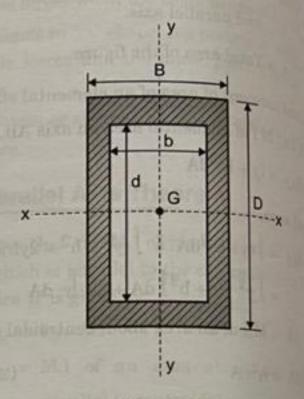


Fig. 6.8.3

3. Square:

Here,b=d=a

$$\therefore I_{xx} = I_{yy} = \frac{a^4}{12}$$

For rectangle, Ixx =

M.I o

Usi

But

D

Circular area :

Consider circular area of radius, R.

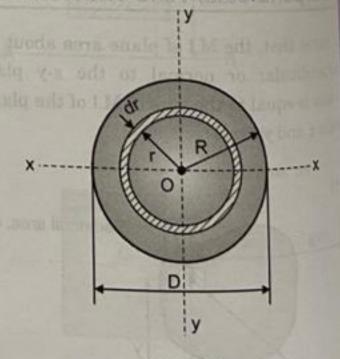


Fig. 6.8.4

Consider an elemental ring of thickness distance 'r' from the centre O.i.e. z or polar axis.

Area of elemental ring, $dA = 2\pi r dr$

M.I of an elemental ring about z-axis is

$$d I_{zz} = r^2 \cdot dA$$
$$= r^2 \cdot 2\pi r dr$$

M.I of the whole area,

$$\begin{split} I_{ZZ} &= \int\limits_{0}^{R} r^2 2\pi r dr &= 2\pi \int\limits_{0}^{R} r^3 \cdot dr = 2\pi \bigg[\frac{r^4}{4} \bigg]_{0}^{R} \\ &= 2\pi \frac{R^4}{4} = \frac{\pi R^4}{2} \end{split}$$

$$I_{zz} = \frac{\pi R^4}{2}$$

Using perpendicular axis theorem;

$$I_{zz} = I_{xx} + I_{yy}$$

 $I_{xx} = I_{yy}$ due to symmetry But

$$\therefore I_{zz} = 2I_{xx} \text{ or } 2I_{yy}$$

$$\therefore I_{xx} = I_{yy} = \frac{I_{zz}}{2} = \frac{1}{2} \left(\frac{\pi R^4}{2} \right)$$

$$I_{xx} = I_{yy} = \frac{\pi R^4}{4}$$

$$R = \frac{D}{2}$$

$$\therefore \ I_{zz} \ = \ \frac{\pi}{2} \left(\frac{D}{2} \right)^4 = \frac{\pi D^4}{32}$$

$$I_{zz} = \frac{\pi D^4}{32}$$

and
$$I_{xx} = I_{yy} = \frac{\pi D^4}{64}$$

Hollow Circular Area:

Due to symmetry,

$$I_{xx} = I_{yy} = \frac{\pi}{64} \left(D^4 - d^4 \right)$$

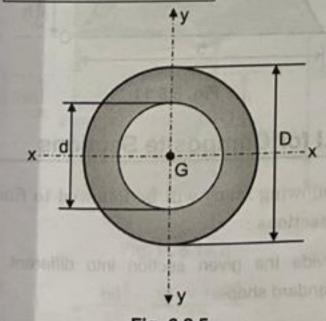


Fig. 6.8.5

6. Semi-circle

$$I_{zz} = \frac{\pi R^4}{4}$$

$$I_{xx} = I_{yy} = I_{base} = \frac{\pi R^4}{8}$$

$$= \frac{\pi D^4}{128}$$

$$I_{\rm G} = 0.11 {\rm R}^4$$

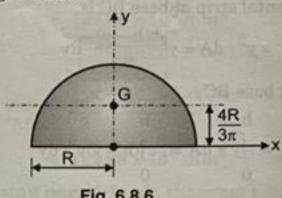


Fig. 6.8.6

7. Quarter circle:

$$I_{xx} = I_{yy} = \frac{\pi R^4}{16} = \frac{\pi D^4}{256}$$

Fig. 6.8.7

Triangle 8.

(A) About base 'BC'

Consider a triangular area with base 'b' and height 'h'.

Consider an elemental strip of width x and thickness dy at a distance y from the base BC.

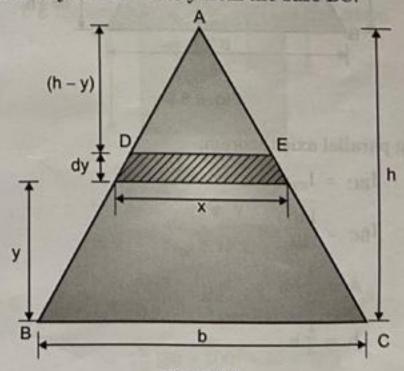


Fig. 6.8.8

$$\frac{b}{h} = \frac{x}{h - y}$$

$$x = \frac{b(h - y)}{h}$$

Area of elemental strip,
$$dA = x \cdot dy$$

$$= \frac{b(h-y)}{h} dy$$

M.I of elemental strip at base BC is

$$dI_{BC} = y^2 \cdot dA = y^2 \frac{b(h-y)}{h} \cdot dy$$

Total M.I at base BC,

$$I_{BC} = \int_{0}^{h} y^{2} \cdot dA = \frac{b}{h} \int_{0}^{h} (h - y)y^{2} \cdot dy$$

$$= \frac{b}{h} \int_{0}^{h} (hy^{2} - y^{3}) dy$$

$$= \frac{b}{h} \left[\frac{hy^{3}}{3} - \frac{y^{4}}{4} \right]_{0}^{h}$$

$$= \frac{b}{h} \left[\frac{h^{4}}{3} - \frac{h^{4}}{4} \right] = \frac{b}{h} \cdot \frac{h^{4}}{12} = \frac{bh^{3}}{12}$$

$$I_{BC} = \frac{bh^{3}}{12}$$

(B) About Centroidal x-axis:

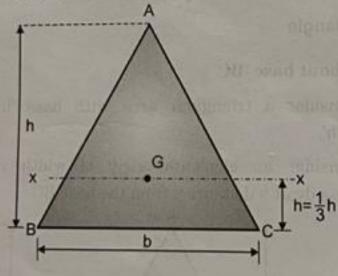


Fig. 6.8.9

Using parallel axis theorem;

$$I_{BC} = I_{xx} + Ah^{2}$$

$$I_{BC} = \frac{bh^{3}}{12}$$

$$A = \frac{1}{2}bh$$

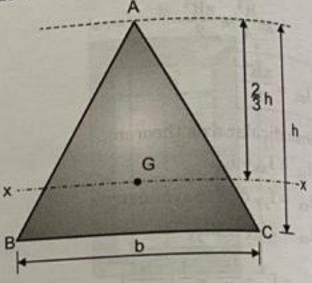
$$h = \frac{1}{3}h$$

$$\frac{bh^3}{12} = I_{xx} + \frac{bh}{2} \cdot \left(\frac{h}{3}\right)^2$$

$$I_{xx} = \frac{bh^3}{12} - \frac{bh^3}{18} = \frac{bh^3}{36}$$

$$I_{xx} = \frac{bh^3}{36}$$

(C) About Vertex 'A'



Step 5

Step 6

6.10

EX. Fig. F

So

St

Fig. 6.8.10

Again using parallel axis theorem;

$$I_{A} = I_{xx} + Ah^{2} = \frac{bh^{3}}{36} + \frac{bh}{2} \left(\frac{2h}{3}\right)^{2}$$
$$= \frac{bh^{3}}{36} + \frac{4bh^{3}}{18} = \frac{bh^{3}}{36} + \frac{2bh^{3}}{9} = \frac{bh^{3}}{4}$$

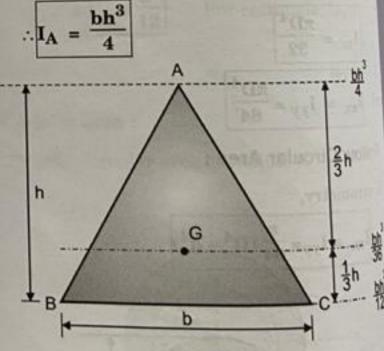


Fig. 6.8.11

6.9 M.I for Composite Sections

The following steps will be followed to find to composite sections:

Step 1: Divide the given section into different pass standard shape.

Step 2: Determine the centre of gravity of the section.

Step 3: Find M.I of each part using standard formulas

Step 4: Using parallel axis theorem, find the M.I of each part about the centroidal axes of the composite section.

Step 5: Addition of M.I of each part will give the M.I of whole section.

Step 6: If hollow section is given, M.I of inner portion is to be subtracted from the M.I of external portion.

6.10 Solved Examples

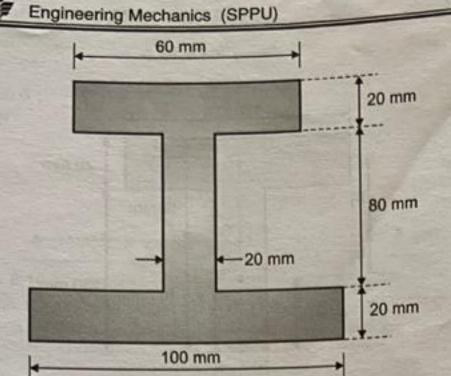


Fig. 6.10.7(a)

Soln.:

Step 1: Location of centroid of the section:

Section is symmetrical about vertical axis i.e. y-axis.

$$\vec{x} = 0$$

To find \bar{y} , divide the section into 3 rectangles.

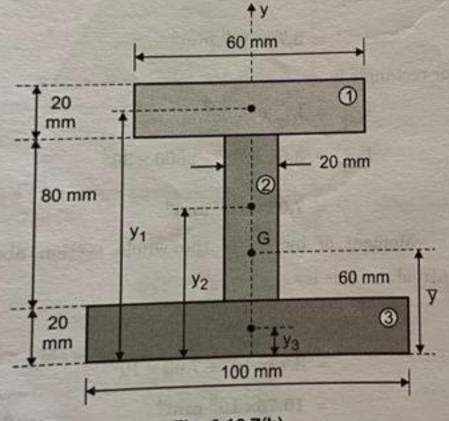


Fig. 6.10.7(b)

$$A_1 = 60 \times 20 = 1200 \text{ mm}^2$$

$$y_1 = 120 - 10 = 110 \text{ mm}$$

$$A_2 = 80 \times 20 = 1600 \text{ mm}^2$$

$$y_2 = 20 + 40 = 60 \text{ mm}$$

$$A_3 = 100 \times 20 = 2000 \text{ mm}^2$$

$$y_3 = 10 \text{ mm}$$

$$\overline{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$\therefore \overline{y} = \frac{1200 \times 110 + 1600 \times 60 + 2000 \times 10}{1200 + 1600 + 2000}$$

$$= \frac{248000}{4800} = 51.67 \text{ mm}$$

Step 2: M·I about horizontal axis passing through ca

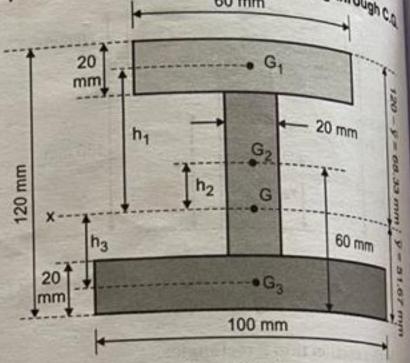


Fig. 6.10.7(c)

$$h_1 = 68.33 - 10 = 58.33 \text{ mm}$$

 $h_2 = 60 - 51.67 = 8.33 \text{ mm}$

$$h_3 = 51.67 - 10 = 41.67 \text{ mm}$$

Using parallel axis theorem;

$$I_{G_1} = \frac{60 \times 20^3}{12} = 40,000 \text{ mm}^4$$

$$I_{xx1} = I_{G_1} + A_1 h_1^2$$

$$= 40,000 + 1200 \times 58.33^2$$

$$= 40,000 + 1200 \times 58.33^{2}$$
$$= 4.123 \times 10^{6} \text{ mm}^{4}.$$

$$I_{G_2} = \frac{20 \times 80^3}{12} = 853333.33 \text{ mm}^4$$

$$I_{xx_2} = I_{G_2} + A_2 h_2^2$$

$$= 853333.33 + 1600 \times 8.33^{2}$$

$$= 0.964 \times 10^6 \text{mm}^4$$

$$I_{G_3} = \frac{100 \times 20^3}{12} = 66666.67 \text{ mm}^4$$

$$I_{xx_3} = I_{G_3} + A_3 h_3^2 = 66666.67 + 2000 \times 41.67$$

= 3.54 × 10⁶ mm⁴

M.I of the whole section,

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

$$\therefore I_{xx} = 4.123 \times 10^6 + 0.964 \times 10^6 + 3.54 \times 10^6$$

$$= 8.627 \times 10^6 \text{ mm}^4$$

Fig. 6.10.7(a)

Soln.:

Step 1: Location of centroid of the section:

100 mm

Section is symmetrical about vertical axis i.e. y-axis.

$$\vec{x} = 0$$

To find \overline{y} , divide the section into 3 rectangles.

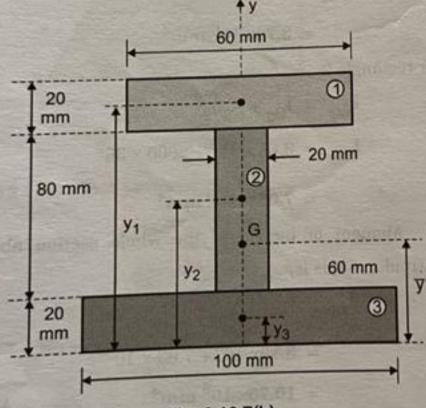


Fig. 6.10.7(b)

$$A_1 = 60 \times 20 = 1200 \text{ mm}^2$$

$$y_1 = 120 - 10 = 110 \text{ mm}$$

$$A_2 = 80 \times 20 = 1600 \text{ mm}^2$$

$$y_2 = 20 + 40 = 60 \text{ mm}$$

$$A_3 = 100 \times 20 = 2000 \text{ mm}^2$$

$$y_3 = 10 \text{ mm}$$

$$\overline{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

$$\therefore \vec{y} = \frac{1200 \times 110 + 1600 \times 60 + 2000 \times 10}{1200 + 1600 + 2000} \times 10$$

$$= \frac{248000}{4800} = 51.67 \text{ mm}$$

Step 2: M·I about horizontal axis passing through Co

6-11

20 mm

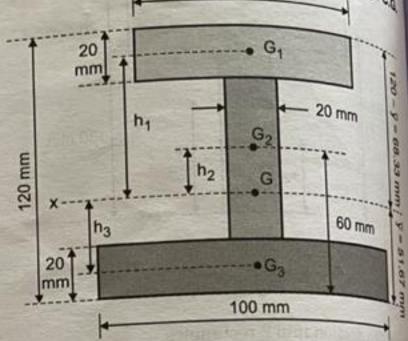


Fig. 6.10.7(c)

$$h_1 = 68.33 - 10 = 58.33 \text{ mm}$$
 $h_2 = 60 - 51.67 = 8.33 \text{ mm}$

$$h_3 = 51.67 - 10 = 41.67 \text{ mm}$$

Using parallel axis theorem;

$$I_{G_1} = \frac{60 \times 20^3}{12} = 40,000 \text{ mm}^4$$

$$I_{xx1} = I_{G_1} + A_1 h_1^2$$

$$= 40,000 + 1200 \times 58.33^{2}$$
$$= 4.123 \times 10^{6} \text{ mm}^{4}.$$

$$I_{G_2} = \frac{20 \times 80^3}{12} = 853333.33 \text{ mm}^4$$

$$I_{xx_2} = I_{G_2} + A_2 h_2^2$$

$$= 853333.33 + 1600 \times 8.33^{2}$$

$$= 0.964 \times 10^6 \text{mm}^4$$

$$I_{G_3} = \frac{100 \times 20^3}{12} = 66666.67 \text{ mm}^4$$

$$I_{xx_3} = I_{G_3} + A_3 h_3^2 = 66666.67 + 2000 \times 41.66$$

= $3.54 \times 10^6 \text{ mm}^4$

M.I of the whole section,

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

$$\therefore I_{xx} = 4.123 \times 10^6 + 0.964 \times 10^6 + 3.54 \times 10^6$$

$$= 8.627 \times 10^6 \text{ mm}^4$$

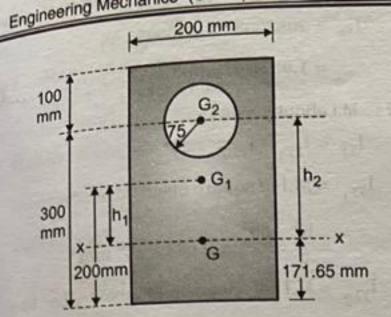


Fig. P. 6.10.9(c)

Using parallel axis theorem;

$$I_{xx_1} = I_{G_1} + A_1h_1^2$$

$$= \left(\frac{200 \times 400^3}{12}\right) + 80,000 \times 28.35^2$$

$$= 1130.96 \times 10^6 \text{ mm}^4$$

$$I_{xx_2} = I_{G_2} + A_2 h_2^2$$

$$= \frac{\pi (150)^4}{64} + 17671.46 \times 128.35^2$$

$$= 315.96 \times 10^6 \text{ mm}^4$$

: M.I of the hollow section about centroidal x-axis is,

i.e.
$$I_{xx} = 1130.96 \times 10^6 - 315.96 \times 10^6$$

= $815 \times 10^6 \text{ mm}^4$...Ans.

Ex. 6.10.10: A rectangular hole is made in a triangular section as shown in Fig. P. 6.10.10(a). Determine the moment of inertia of the section about x-x axis passing through the centre of gravity and the base BC. Section is symmetrical about y-axis.

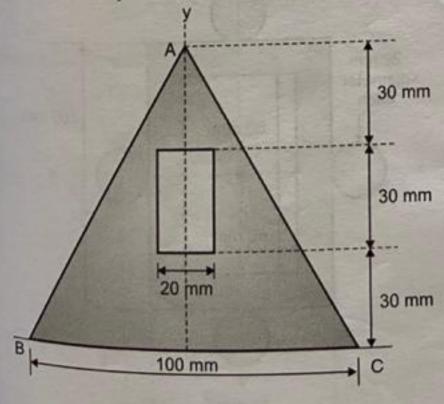


Fig. P. 6.10.10(a)

Soln. :

Step 1: Location of centre of gravity:

Section can be divided in to two parts.

- 1. AABC
- 2. Rectangle (to be removed)

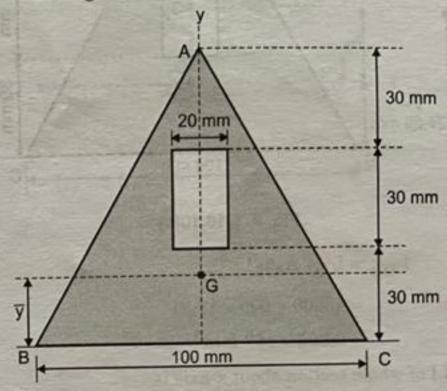


Fig. P. 6.10.10(b)

Section is symmetrical about y-axis.

$$\therefore \bar{x} = 0$$

From base of the triangle,

$$\therefore \overline{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

$$A_1 = \frac{1}{2} \times 100 \times 90 = 4500 \text{ mm}^2$$

$$y_1 = \frac{1}{3} \times 90 = 30 \text{ mm}$$

$$A_2 = 20 \times 30 = 600 \text{ mm}^2$$

$$y_2 = 30 + 15 = 45 \text{ mm}$$

$$\overline{y} = \frac{4500 \times 30 - 600 \times 45}{4500 - 600} = 27.69 \text{ mm}$$

Step 2: M.I about centroidal x-axis:

$$\begin{aligned} h_1 &= 30 - 27.69 &= 2.31 \text{ mm} \\ h_2 &= 45 - 27.69 &= 17.31 \text{ mm} \\ I_{G_1} &= \frac{bh^3}{36} = \frac{100 \times 90^3}{36} = 2.025 \times 10^6 \text{mm}^4 \\ I_{xx_1} &= I_{G_1} + A_1h_1^2 \\ &= 2.025 \times 10^6 + 4500 \times 2.31^2 \\ &= 2.049 \times 10^6 \text{ mm}^4 \end{aligned}$$

$$I_{G_1} = \frac{bd^3}{12} = \frac{20 \times 30^3}{12} = 45000 \text{ mm}^4$$

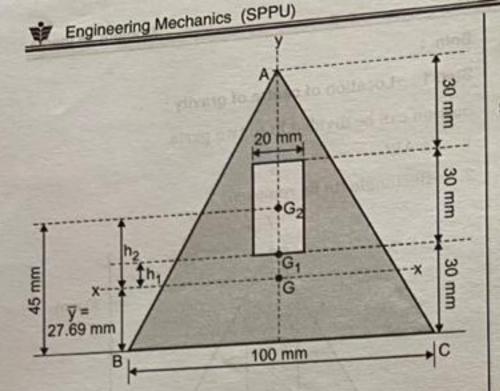


Fig. P. 6.10.10(c)

$$I_{xx_2} = I_{G_2} + A_2 h_2^2$$

= $45000 + 600 \times 17.31^2$
= $0.225 \times 10^6 \text{ mm}^4$

... M.I of whole section about x-axis is

$$\begin{split} I_{xx} &= I_{xx_1} - I_{xx_2} \\ &= 2.049 \times 10^6 - 0.225 \times 10^6 \\ &= 1.824 \times 10^6 \text{ mm}^4 \end{split}$$

Ex. 6.10.11: Determine the M.I of the shaded area shown in Fig. P. 6.10.11(a)about x and y axes.

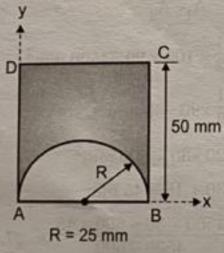


Fig. P. 6.10.11(a)

Soln.:

Step 1: M.I about x-axis:

: M.I about x-axis :
$$I_{xx} = I_{xx_1} - I_{xx_2}$$

$$I_{xx_1} = \text{M.I of square ABCD about base}$$

$$= \frac{b^4}{3} = \frac{50^4}{3} = 2083333.33 \text{ mm}^4$$

$$I_{xx_2} = \text{M.I of semi-circle about base AB}$$

$$= \frac{\pi D^4}{128} = \frac{\pi (50)^4}{128} = 153398.08 \text{ mm}^4$$

Step 2: M.I about y-axis:

$$\begin{split} I_{yy} &= I_{yy_1} - I_{yy_2} \\ I_{yy_1} &= \text{M.I of square about edge} \\ &= \frac{b^4}{3} = \frac{50^4}{3} = 2.083 \times 10^6 \text{ mm}^4 \\ I_{yy_2} &= \text{M.I of Semicircle about the line} \\ &= \text{parallel to its centroidal axis.} \end{split}$$

$$h_2 = 25 \text{ mm}$$

$$A_2 = \frac{\pi (25)^2}{2}$$

 $= 981.75 \text{ mm}^2$

GZ

Fig. P. 6.10.11()

Using parallel axis theorem;

$$I_{yy_2} = I_{G_2} + A_2 h_2^2$$

$$=\frac{\pi(50)^4}{128} + 981.75 \times 25^2$$

$$= 153.39 \times 10^3 + 613.592 \times 10^3$$

$$= 766.982 \times 10^3 \text{ mm}^4$$

$$I_{vv} = 2083 \times 10^3 - 766.982 \times 10^3$$

Ex. 6.10.12 : Find the M.I of the section show Fig. P. 6.10.12(a) about centroidal x and y axes.

