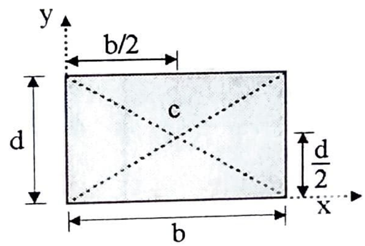
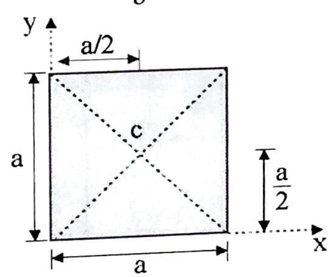
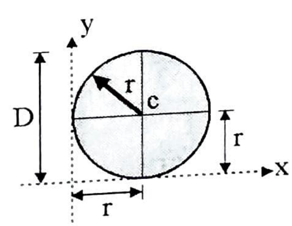
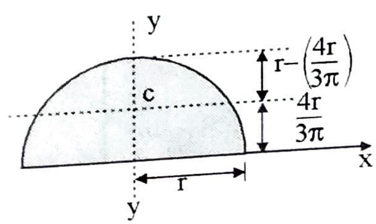
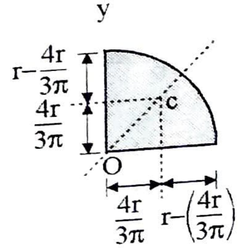
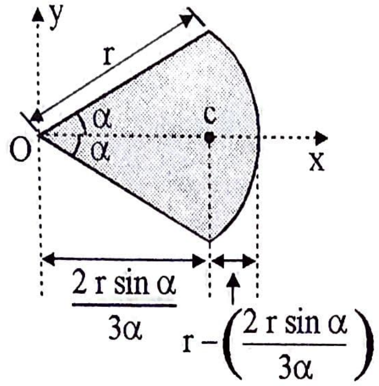
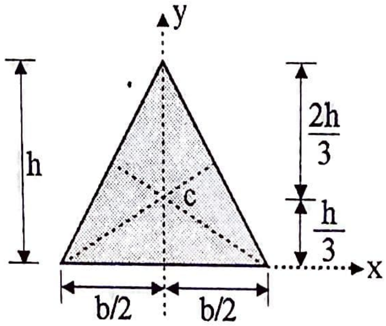
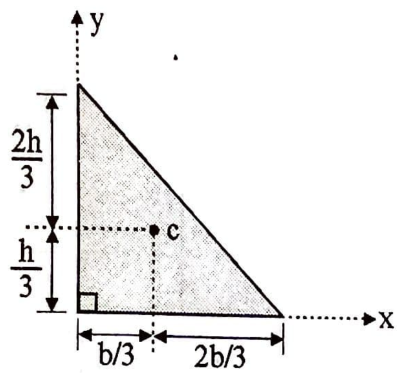
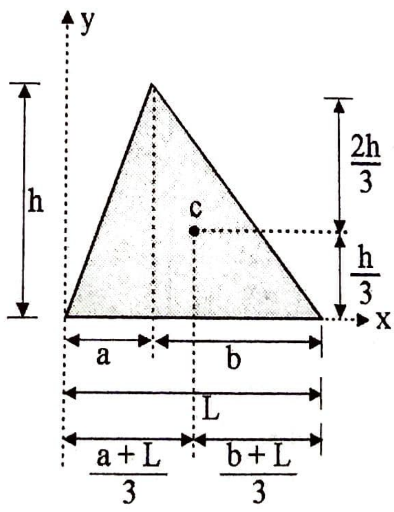
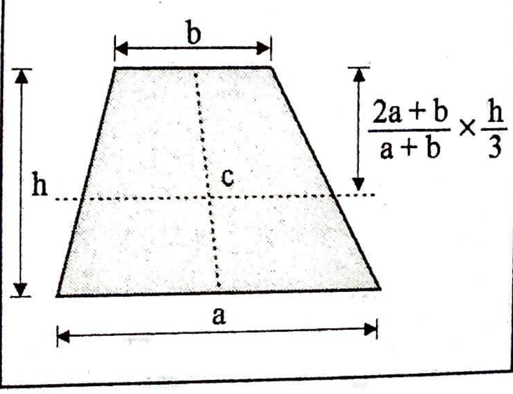
1

Centroid / Beams

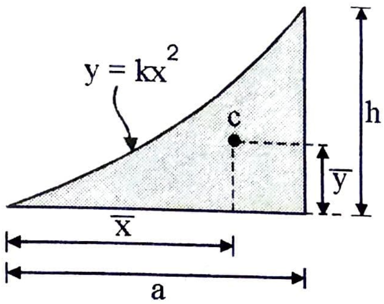
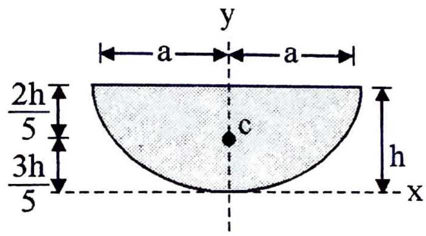
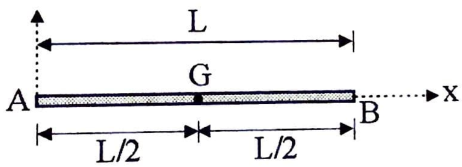
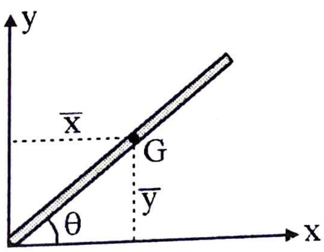
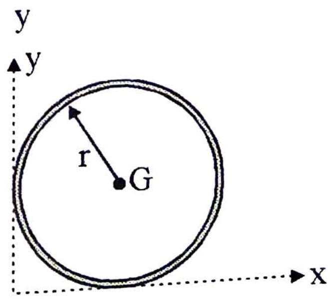
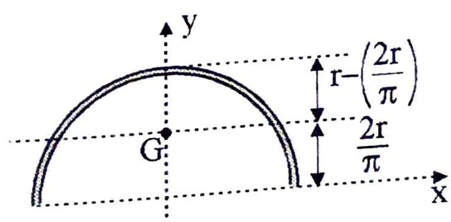
Important Concepts and Formulae

Centroid and Areas of Some Common Plane Areas :

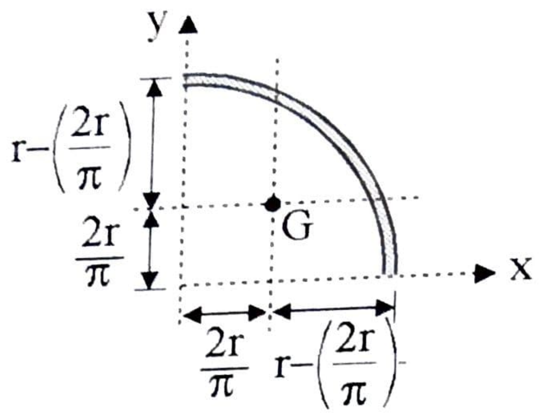
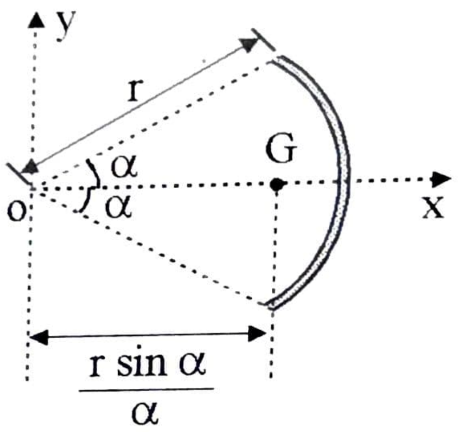
Sr. No.	Name	Shape	Area	$\overline{x}$	$\overline{y}$
1.	Rectangle		$b \times d$	$b/2$	$d/2$
2.	Square		a^2 OR $\frac{(\text{Diagonal})^2}{2}$	$\frac{a}{2}$	$\frac{a}{2}$
3.	Circle		πr^2 OR $\frac{\pi}{4}D^2$	r	r
4.	Semi.circle		$\frac{\pi r^2}{2}$	0	$\frac{4r}{3\pi}$
5.	Quarter circle		$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$

Sr. No.	Name	Shape	Area	$\bar{x}$	$\bar{y}$
6.	Sector of a circle		αr^2 (α in radian) Note : α is semiangle	$\frac{2 r \sin\alpha}{3\alpha}$ $\frac{(\alpha \text{ in degree})}{(\alpha \text{ in radian})}$	0
7.	Triangle				
(a)	Symmetrical triangle		$\frac{1}{2}\times b\times h$	0	h/3
(b)	Right angled triangle		$\frac{1}{2}\times b\times h$	b/3	h/3
(c)	Unsymmetrical triangle		$\frac{1}{2}\times L\times h$	$\frac{a+L}{3}$	$\frac{h}{3}$
8.	Trapezoid		$\left(\frac{a+b}{2}\right)h$	–	–

Some common line segments :

Sr. No.	Name	Shape	Area	$\overline{x}$	$\overline{y}$
9.	Parabolic spandrel		$\frac{ah}{3}$	$\frac{3a}{4}$	$\frac{3h}{10}$
10.	Parabolic area		$\frac{4ah}{3}$	0	$\frac{3h}{5}$
11.	A straight line		L	L/2	0
12.	An inclined line		L	$\frac{L}{2}\cos\theta$	$\frac{L}{2}\sin\theta$
13.	Circular Arc		$2\pi r$	r	r
14.	Semi. circular Arc		πr	0	$\frac{2r}{\pi}$

Some Common solid bodies : [only for reference]

Sr. No.	Name	Shape	Length	$\bar{x}$	$\bar{y}$
15.	Quarter circular Arc		$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$
16.	An arc of a circle		$2 r \alpha$	$\frac{r \sin \alpha}{\alpha}$	0

Multiple Choice Questions for Online Exam I

Q. 1 C. G of a body is the point thorough which, in whatever position the body is placed, passes the line of action of the

(a) Resultant of the forces exerted by the attraction of the earth upon its constituent parts

(b) Resultant of the forces acting on it.

(c) Both of the above.

(d) None of the above

Ans. : (a)

Q. 2 The centroid of a plane lamina will not be at its geometrical centre if it is a

(a) Circle (b) Equilateral triangle

(c) Square placed with one diagonal horizontal (d) Right angled triangle.

Ans. : (d)

Explanation :

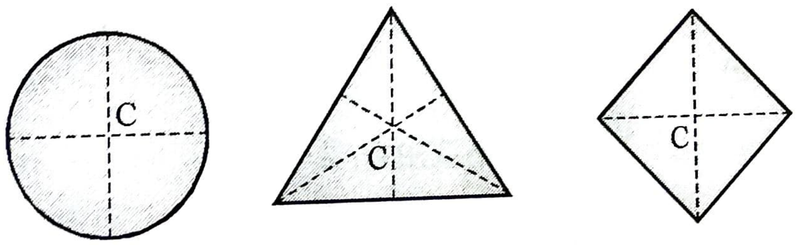


Fig. 1

For circle, equilateral triangle and for a square geometrical centre is centroid itself, but for a right angle triangle it is not so.

So, option (d) is correct.

Q. 3 Centroid of composite figure can be determined by

(a) Analytical method (b) Graphical method (c) Both (d) None.

Ans. : (c)

Explanation : There are two methods to find centroid

(a) Analytical method (b) graphical method

So, option (c) is correct.

Q. 4 Which of the following statement is correct

(a) An irregular body can have more than one C.G.

(b) The c.g. of triangle lies at a point where any two medians meet each other.

(c) The c.g. of triangle lies at a point where the bisector of all three angles meet.

(d) All above.

Ans. : (b)

Explanation :

Anybody whether regular or irregular it has only one c.g. The c.g. of triangle is the point where medians intersect.

So option (b) is correct.

Q. 5 The centroid of an isosceles triangle with base 'a' and sides 'b' is —— from its base.

(a) $\frac{1}{6}\sqrt{4b^2 - a^2}$ (b) $\frac{1}{6}\sqrt{4a^2 - b^2}$ (c) $\frac{a^2 - b^2}{4}$ (d) $\frac{a^2 + b^2}{4}$

Ans. : (a)

Explanation : Height of triangle

$$h = \sqrt{b^2 - \left(\frac{a}{2}\right)^2} = \sqrt{\frac{4b^2 - a^2}{4}} = \frac{1}{2}\sqrt{4b^2 - a^2}$$

Now distance of centroid from base is $\frac{1}{3} \times h$

So, $\frac{1}{3} \times \frac{1}{2}\sqrt{4b^2 - a^2} = \frac{1}{6}\sqrt{4b^2 - a^2}$

So, option (a) is correct.

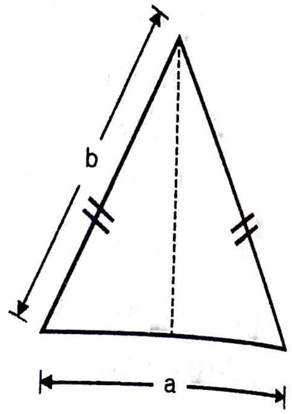


Fig. 2

Q. 6 The centroid of an equilateral triangle with each side a is —— from any of the three sides.

(a) $\frac{\sqrt{3}a}{2}$ (b) $2\sqrt{3a}$ (c) $\frac{a}{2\sqrt{3}}$ (d) $3\sqrt{2a}$

Ans. : (c)

Explanation : Height of Δ ABD $h = \frac{\sqrt{3}a}{2}$

$$\therefore \text{ centroid at } \frac{1}{3}h = \frac{\sqrt{3}a}{2 \times 3} = \frac{a}{2\sqrt{3}} \text{ from base}$$

So, option (c) is correct.

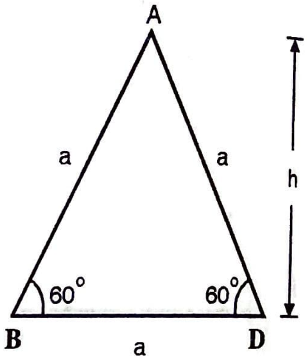


Fig. 3

Q. 7 A triangle of height r and base 2r is removed from a semicircular lamina of radius r. The distance of centroid of remaining area from base is,

(a) 0.5 r (b) 0.424 r (c) 0.584 r (d) 0.33 r

Ans. : (c)

Explanation :

$A_1 = \frac{\pi r^2}{2}$ | $y_1 = \frac{4r}{3\pi}$

$A_2 = \frac{1}{2} \times 2r \times r = r^2$ | $y_2 = \frac{r}{3}$

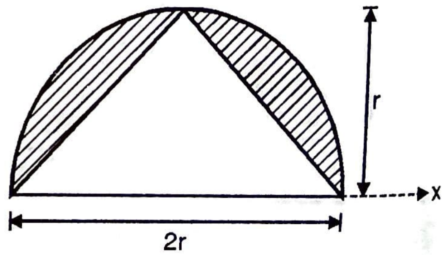


Fig. 4

$$\therefore \bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{\frac{\pi r^2}{2} \times \frac{4r}{3\pi} - r^2 \times \frac{r}{3}}{\frac{\pi r^2}{2} - r^2} = \frac{\frac{2r^3}{3} - \frac{r^3}{3}}{\frac{\pi r^2}{2} - r^2} = \frac{r\left(\frac{2}{3} - \frac{1}{3}\right)}{\left(\frac{\pi}{2} - 1\right)} = 0.584\,r;$$

So, option (c) is correct.

Q. 8 A circular hole of radius r is cut out from a circular disc of radius 2r in such a way that diameter of hole is radius of disc. The centroid lies at

(a) Centre of disc (b) Centre of hole

(c) Somewhere in the disc (d) Somewhere in the hole.

Ans. : (c)

Explanation : When we cut a circular hole from left side, centroid shifts to the right, so anywhere we cut the hole (at top, right or bottom) centroid lies within the plate only.

So, option (c) is correct.

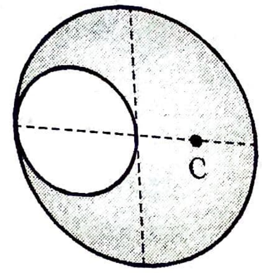


Fig. 5

Q. 9 The angle made by side of a square lamina with horizontal if suspended freely from a corner is,

(a) 30° (b) 45° (c) 90° (d) zero

Ans. : (b)

Explanation : When a square lamina a suspended, the line joining point of suspension and c.g. will be a vertical line. So, angle made by side with vertical and horizontal is 45°.

So, option (b) is correct.

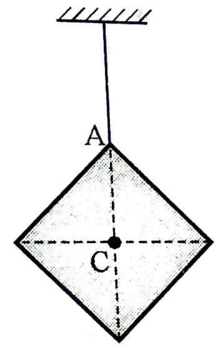


Fig. 6

Q. 10 The centroidal distanced of a quarter circular area along its line of symmetry is

(a) $\dfrac{4r}{3\pi}$ (b) $\dfrac{3r}{4\pi}$ (c) $\dfrac{4\pi}{3r}$ (d) $\sqrt{2}\cdot\dfrac{4r}{3\pi}$

Ans. : (d)

Explanation :

$$\therefore \quad \text{Distance, } OC = \sqrt{\left(\frac{4r}{3\pi}\right)^2 + \left(\frac{4r}{3\pi}\right)^2} = \sqrt{2}\,\frac{4r}{3\pi}$$

So, option (d) is correct.

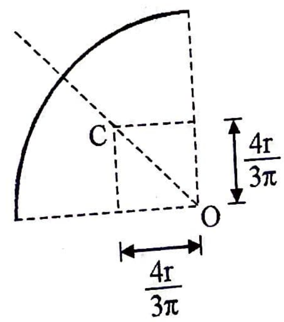


Fig. 7

Q. 11 The centroidal distance for the arc of a circle of rad r and total angle 2α from '0' is

(a) $\dfrac{2r\sin\alpha}{3\alpha}$ (b) $\dfrac{r\sin\alpha}{\alpha}$

(c) $\dfrac{4r\sin\alpha}{3\alpha}$ (d) $\dfrac{3r\sin\alpha}{4\alpha}$

Ans. : (b)

Explanation :

So, option (b) is correct.

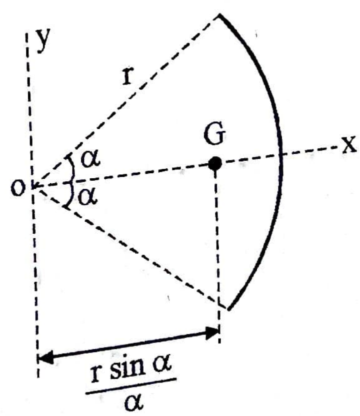


Fig. 8

Q. 12 The centroid of an equilateral triangle of side 'a' with one side parallel to x-axis is

(a) $\left(\frac{a}{2}, \frac{a}{\sqrt{6}}\right)$ (b) $\left(\frac{a}{2}, \frac{a}{\sqrt{12}}\right)$ (c) $\left(\frac{a}{2}, \frac{a}{\sqrt{24}}\right)$ (d) $\left(\frac{a}{2}, \frac{a}{3}\right)$

Ans. : (b)

Explanation : [Please refer Solution of Q. 6]

Q. 13 Assuming a square of side 'a' to be made up of two right angled triangles then the distance of centroid of each triangle with respect to diagonal is

(a) $\frac{a}{\sqrt{2}}$ (b) $\frac{a}{\sqrt{3}}$ (c) $\frac{\sqrt{2}\,a}{3}$ (d) $\frac{a}{\sqrt{18}}$

Ans. : (d)

Explanation : The height of any triangle is $\frac{a}{\sqrt{2}}$

$\therefore$ Centroid is at $\frac{1}{3} \times \frac{a}{\sqrt{2}} = \frac{a}{\sqrt{18}}$

So, option (d) is correct.

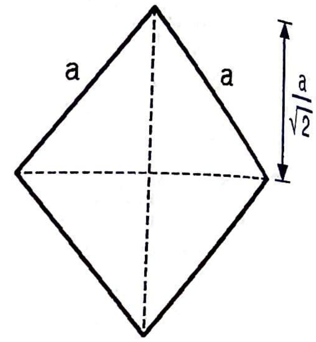


Fig. 9

Q. 14 If r is radius of a hemisphere then c.g. of solid hemisphere will lie on the line of symmetry at a distance of —— from plane base

(a) $\frac{3r}{4\pi}$ (b) $\frac{4r}{3\pi}$ (c) $\frac{3r}{8}$ (d) $\frac{5r}{8}$

Ans. : (c)

Explanation :

So, option (c) is correct.

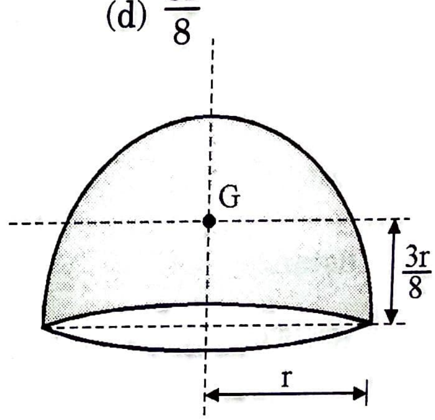


Fig. 10

Q. 15 A rectangular plate of 4m × 3m is suspended from one of its corners. In suspended position the angle made by its longer side with vertical is,

(a) 43.13° (b) 36.87° (c) 53.13° (d) 45°

Ans. : (b)

Explanation : When rectangular plate is suspended the line joining point of suspension and centroid is a vertical line.

$$\tan\theta = \frac{3}{4}$$

$$\therefore \theta = 36.87^\circ$$

So, option (b) is correct.

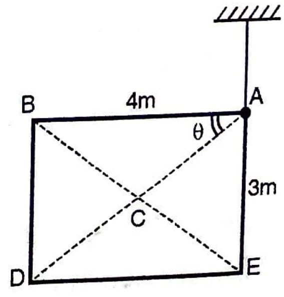


Fig. 11

Q. 16 c.g. of a thin hollow cone lies on the axis of symmetry at a height of

(a) One half of the total height

(b) One third of the total height

(c) One fourth of the total height

(d) None of these.

Ans. : (b)

Explanation :

So, option (a) is correct.

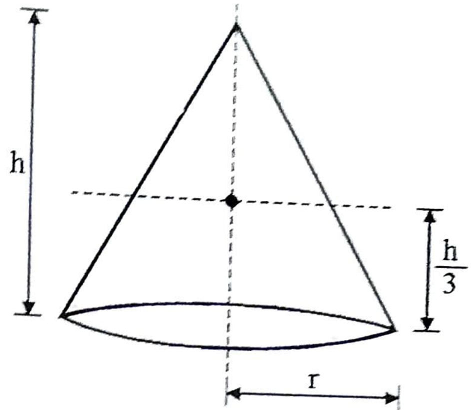


Fig. 12

Q. 17 The c.g. of a solid cone from its apex on line of symmetry is

(a) $\frac{3h}{4}$ (b) $\frac{4h}{3}$

(c) $\frac{2h}{3}$ (d) $\frac{h}{4}$

Ans. : (a)

Explanation :

So, option (a) is correct.

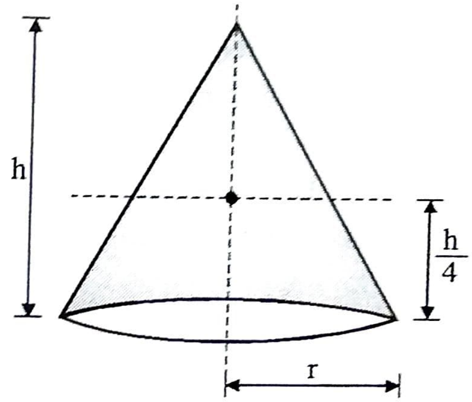


Fig. 13

Q. 18 The c.g. of hemisphere of rad r from its base along its line of symmetry is

(a) $\frac{r}{8}$ (b) $\frac{r}{2}$ (c) $\frac{3r}{8}$ (d) $\frac{4r}{3}$

Ans. : (c)

Explanation : [Please refer Q. 14]

Q. 19 A semicircular plate is suspended from one of the ends of its diameter. The angle made by diameter with vertical in suspended position is,

(a) 45° (b) 23° (c) 67° (d) 32.5°

Ans. : (b)

Explanation :

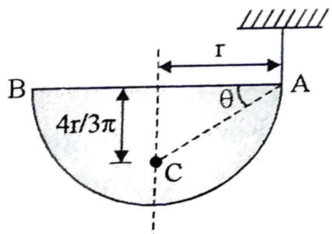


Fig. 14

$$\tan \theta = \frac{\frac{4r}{3\pi}}{r} = \frac{4}{3\pi}$$

$$\therefore \theta = 23° \text{ with vertical}$$

So, option (b) is correct.

Q. 20 A semicircular uniform rod is suspended from one of the ends . The angle made by diameter with horizontal in suspended position is :

(a) 45° (b) 23° (c) 57.5° (d) 32.5°

Ans. : (c)

Explanation : $\tan\theta = \dfrac{\frac{2r}{\pi}}{r} = \dfrac{2}{\pi}$

$\therefore\ \theta = 32.5°$ with vertical

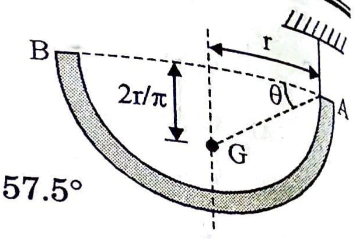


Fig. 15

$\therefore$ Angle made by diameter with horizontal will be $90 - \theta = 57.5°$

So, option (c) is correct.

Q. 21 Out of the following which are the examples of distributed forces

I) A load which is continuously along the length of a cable suspended between two supports

II) Water pressure acting against the face of the dam

III) self weight of a prismatic bar supported at its ends

IV) The sand piled along the beam with variable depth

(a) only I is correct (b) I and II are correct

(c) I,II and III are correct (d) All are correct

Ans. : (d)

Q. 22 For a plane figure having two axes of symmetry, the centroid lies on

(a) lies on horizontal axis (b) lies on vertical axis

(c) lies on intersection point of two axes (d) not on any axis

Ans. : (c)

Explanation : Centroid of any geometric figure always lies on line or lines of symmetry.

So, option (c) is correct.

Q. 23 The centroid 'c' is a point which defines the ________ of an object

(a) area (b) volume (c) geometric centre (d) all of the above

Ans. : (c)

Explanation : For any plane lamina centroid is defined as the geometrical centre of plane area.

For example,

(a) For a rectangle intersection point of two diagonals

(b) For a triangle intersection point of three medians

So, option (c) is correct.

Q. 24 A trapezoid having two parallel sides 'a' and 'b' and height 'h'. The Y centroidal distance from bottom side 'b' is,

(a) $0.5\,h\dfrac{(b+2a)}{b+a}$ (b) $0.5\,h\dfrac{(b-2a)}{b+a}$ (c) $\dfrac{h(b+2a)}{3(b+a)}$ (d) $\dfrac{h(b-2a)}{3(b-a)}$

Ans. : (c)

Explanation :

$$A_1 = ah \qquad y_1 = \frac{h}{2}$$

$$A_2 = \frac{1}{2} \times \left(\frac{b-a}{2}\right) \times h$$

$$y_2 = \frac{h}{3} = \frac{(b-a)\,h}{4}$$

Using $\bar{y} = \dfrac{A_1y_1 + 2A_2y_2}{A_1 + 2A_2}$

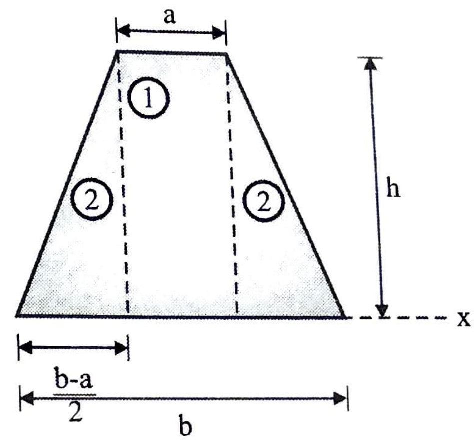


Fig. 16

$$= \frac{(ah)\left(\frac{h}{2}\right) + 2\left[\frac{(b-a)h}{4} \times \frac{h}{3}\right]}{ah + 2\frac{(b-a)h}{4}}$$

$$= \frac{\frac{ah^2}{2} + \frac{(b-a)h^2}{6}}{ah + (b-a)\frac{h}{2}} = \frac{\frac{ah^2}{2} - \frac{ah^2}{6} + \frac{bh^2}{6}}{\frac{ah}{2} + \frac{bh}{2}} = \frac{h(b+2a)}{3(a+b)}$$

So, option (c) is correct.

Q. 25 A parabolic lamina of base 10 cm and height 5 cm is given by equation $y = \dfrac{hx^2}{a^2}$. The y centroidal distance is

(a) 1.5 cm (b) 1.67 cm (c) 3.75 cm (d) 6.67 cm

Ans. : (a)

Explanation : For a parabolic spandrel $y = Kx^2$ centroidal distance is shown in Fig. 17.

$$\bar{x} = \frac{3a}{4}, \bar{y} = \frac{3h}{10}$$

Now as per given equation, $y = \dfrac{hx^2}{a^2}$

Where h = 5 cm, a = 10 cm given

So, option (a) is correct.

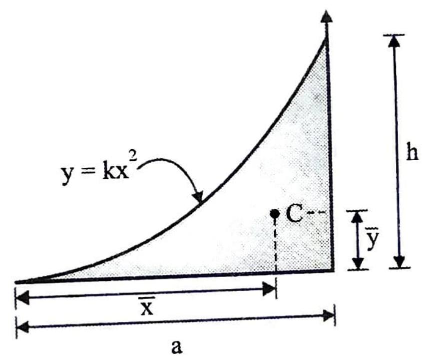


Fig. 17

$$\therefore y = \frac{5x^2}{(10)^2} = \frac{1}{20}x^2 \text{ which is similar to } y = Kx^2$$

$$\text{Hence,} \quad \bar{y} = \frac{3h}{10} = \frac{3 \times 5}{10} = 1.5 \text{ cm}$$

So, option (a) is correct.

Q. 26 If a parabolic area of height h is symmetric about y-axis, the centroidal x co-ordinate is

(a) $\dfrac{3h}{10}$ (b) zero (c) $\dfrac{3h}{5}$ (d) $\dfrac{3a}{8}$

Ans. : (b)

Explanation :

Explanation :

$$A_1 = ah \qquad y_1 = \frac{h}{2}$$

$$A_2 = \frac{1}{2} \times \left(\frac{b-a}{2}\right) \times h$$

$$y_2 = \frac{h}{3} = \frac{(b-a)\,h}{4}$$

Using $$\bar{y} = \frac{A_1y_1 + 2A_2y_2}{A_1 + 2A_2}$$

$$= \frac{(ah)\left(\frac{h}{2}\right) + 2\left[\frac{(b-a)h}{4} \times \frac{h}{3}\right]}{ah + 2\frac{(b-a)h}{4}}$$

$$= \frac{\frac{ah^2}{2} + \frac{(b-a)h^2}{6}}{ah + (b-a)\frac{h}{2}} = \frac{\frac{ah^2}{2} - \frac{ah^2}{6} + \frac{bh^2}{6}}{\frac{ah}{2} + \frac{bh}{2}} = \frac{h(b+2a)}{3(a+b)}$$

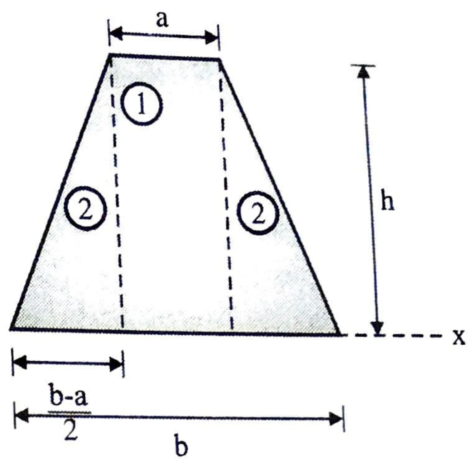


Fig. 16

So, option (c) is correct.

Q. 25 A parabolic lamina of base 10 cm and height 5 cm is given by equation $y = \frac{hx^2}{a^2}$. The y centroidal distance is

(a) 1.5 cm (b) 1.67 cm (c) 3.75 cm (d) 6.67 cm

Ans. : (a)

Explanation : For a parabolic spandrel $y = Kx^2$ centroidal distance is shown in Fig. 17.

$$\bar{x} = \frac{3a}{4}, \bar{y} = \frac{3h}{10}$$

Now as per given equation, $y = \frac{hx^2}{a^2}$

Where h = 5 cm, a = 10 cm given

So, option (a) is correct.

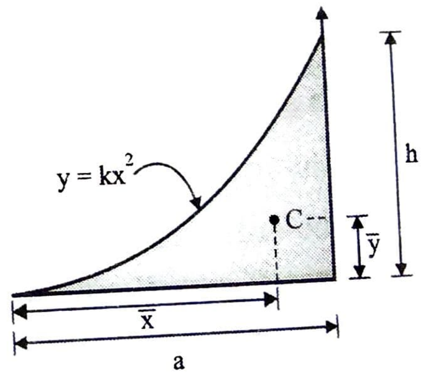


Fig. 17

$$\therefore y = \frac{5x^2}{(10)^2} = \frac{1}{20}x^2 \text{ which is similar to } y = Kx^2$$

Hence, $$\bar{y} = \frac{3h}{10} = \frac{3 \times 5}{10} = 1.5 \text{ cm}$$

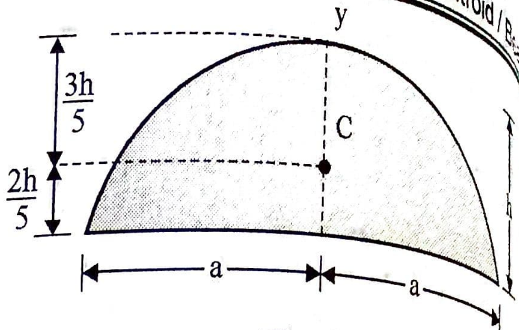
So, option (a) is correct.

Q. 26 If a parabolic area of height h is symmetric about y-axis, the centroidal x co-ordinate is

(a) $\frac{3h}{10}$ (b) zero (c) $\frac{3h}{5}$ (d) $\frac{3a}{8}$

Ans. : (b)

Explanation :



As Fig. 18 is symmetrical about y-axis

$$\therefore \quad \bar{x} = 0$$

$$\text{and } \bar{y} = \frac{2h}{5} \text{ from base}$$

So, option (b) is correct.

Fig. 18

Q. 27 Centroidal distance of an equilateral triangle with side 'a' from any of the three sides is,

(a) 0.866 a (b) 0.471 a (c) 0.288 a (d) 0.235 a

Ans. : (c)

Explanation : Height 'h' of an equilateral triangle

$$h = \frac{\sqrt{3}a}{2} = 0.866\ a$$

$$\therefore \text{ distance of centroid from base } = \frac{h}{3}$$

$$\therefore \frac{h}{3} = \frac{0.866a}{3} = 0.288\ a$$

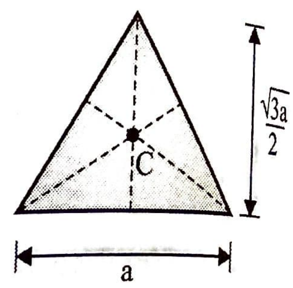


Fig. 19

So, option (c) is correct.

Q. 28 For a line of length 2.5 m passing through origin and inclination 60° with x-axis, centroid along x is

(a) 0.625 (b) 0.5 (c) 2.5 (d) 1

Ans. : (a)

Explanation :

x co-ordinate of centroid = 1.25 cos 60° = 0.625 m

y co-ordinate of centroid = 1.25 sin 60° = 1.082 m

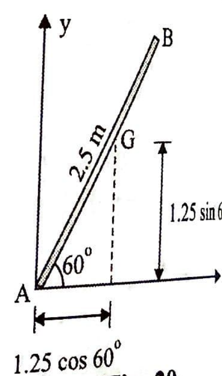


Fig. 20

So, option (a) is correct.

Q. 29 The y centroidal distance of an unequal I-section from its bottom having upper flange 15 cm × 5 cm, lower flange 10 cm × 5 cm and web 5 cm × 15 cm deep is

(a) 12.5 cm (b) 13.75 cm (c) 20 cm (d) 15 cm

Ans. : (b)

Explanation :

$A_1 = 15 \times 5 = 75\ cm^2$ $\quad y_1 = 5 + 15 + \frac{5}{2} = 22.5$ cm

$A_2 = 5 \times 15 = 75\ cm^2$ $\quad y_2 = 5 + \frac{15}{2} = 12.5$ cm

$A_3 = 10 \times 5 = 50\ cm^2$ $\quad y_3 = 2.5$ cm

$$\therefore \bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

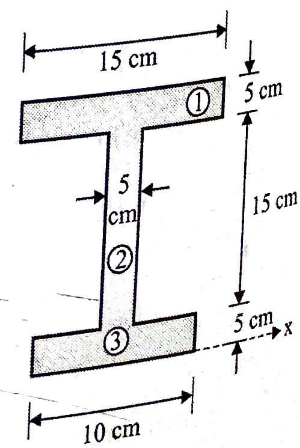


Fig. 21

$$= \frac{(75 \times 22.5) + (75 \times 12.5) + (50 \times 2.5)}{75 + 75 + 50}$$

$$\bar{y} = 13.75 \text{ cm}$$

So, option (b) is correct.

Q. 30 A triangular hole is cut from circular lamina of radius 10 cm such that the vertex of triangle is on y-axis and base coincides with horizontal diameter. If base of triangle is 20 cm and height is 10 cm. The C.G. of remaining lamina is,

(a) 2.22 cm (b) – 2.22 cm (c) 1.55 cm (d) –1.55 cm

Ans. : (d)

Explanation :

$$A_1 = \pi r^2 = \pi (10)^2 = 100\,\pi \text{ cm}^2 \qquad y_1 = 0$$

$$A_2 = \frac{1}{2} \times 20 \times 10 = 100 \text{ cm}^2 \qquad y_2 = \frac{10}{3} = 3.33 \text{ cm}$$

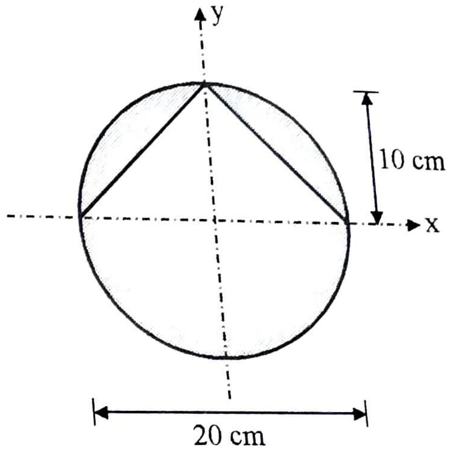


Fig. 22

Now, $$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{(100\,\pi \times 0) - (100 \times 3.33)}{100\,\pi - 100} = -1.55 \text{ cm}$$

So, option (d) is correct.

Q. 31 The C.G. of an isosceles triangle with base 10 cm and sides 20 cm is ______ from its base.

(a) 6.455 cm (b) 5 cm (c) 7 cm (d) 9 cm

Ans. : (a)

Explanation : Form Δ AMB,

$$h = \sqrt{(20)^2 - (5)^2}$$

$$\therefore \quad h = 19.36 \text{ cm}$$

$\therefore$ Distance of centroid from base is $\frac{h}{3} = \frac{19.36}{3} = 6.455$ cm

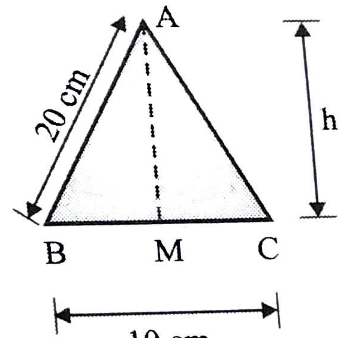


Fig. 23

So, option (a) is correct.