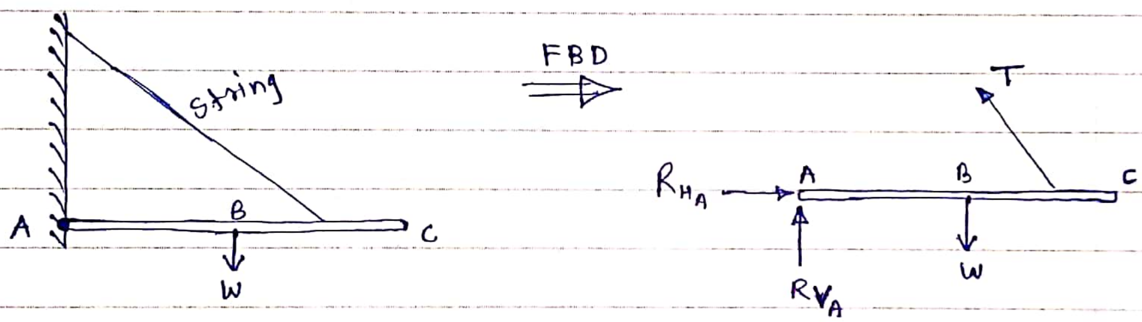
Equilibrium, Plane & Space Forces.
& BEAMS

* Free body diagram:-

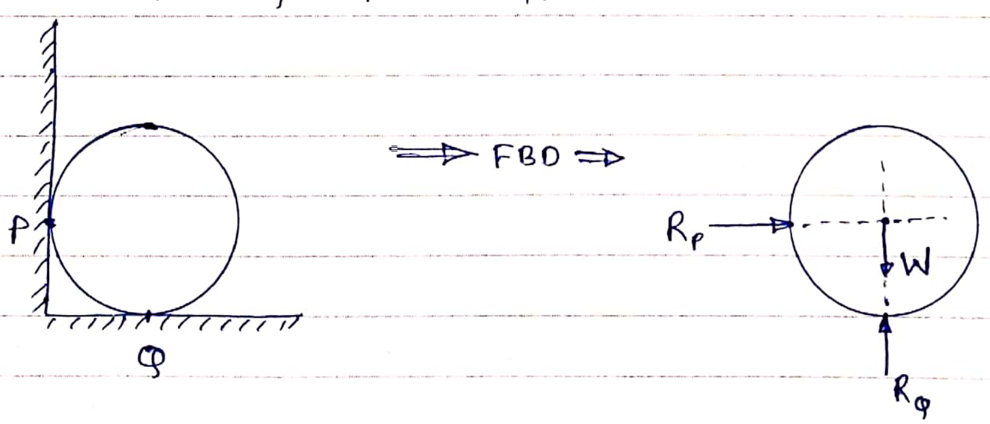
If a body is shown with all external forces acting on it so that the body is in equillibrium, such diagram is called as Free body diagram.

To draw the free body diagram, we have to remove all the restrictions like wall, floor, hinge, any other support and replace them by reactions which these support exterts on the body.

Ex① Draw the FBD of a bar supported and loaded as shown below.



② Draw FBD of sphere supported as show below.



* Equillibrium:-

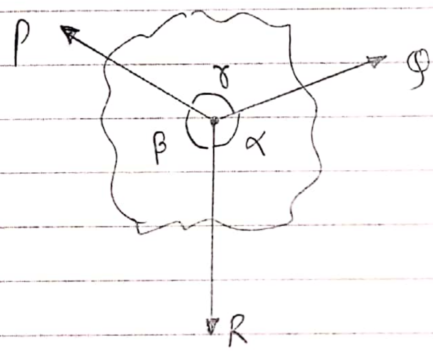
- A body is said to be in equillibrium if the resultant of all forces acting on it is zero.
- When the resultant of all the forces acting on the particle is zero, then, the particle is in equillibrium.
- If the resultant force acting on a particle is zero (or if particle is in equillibrium state) then, particle will remain at rest condition if it is originally at rest, or particle will move with constant speed in a straight line if it is originally in motion.

Conditions for Equillibrium :- $R = 0 \begin{Bmatrix} \Sigma f_x = 0 \\ \Sigma f_y = 0 \end{Bmatrix}$

$$\Sigma M = 0.$$

* Lami's Theorem:- (Body is subjected to three forces)

If three coplaner concurrent forces are in equillibrium then, the ratio of magnitude of any force to the sine of angle betn the other two is constant.



According to lami's Theorem:

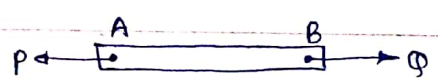
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Lami's theorem can also be written as;

If three coplaner concurrent forces acting at a point are in equillibrium, the each force is proportional to the sine of angle betn other two.

* Equilibrium of body subjected to two forces.

If an object is subjected to forces acting at two points, then body will be in equillibrium only when those two forces are equal & opposite.



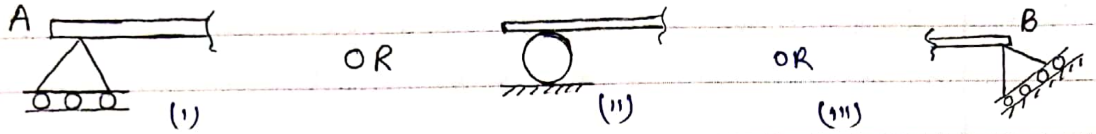
$P = Q$, Body is in equilibrium.

$P = Q$, but body is not in equilibrium.

* Types of Support

① Roller Support :-

Roller support is shown in two ways as shown.

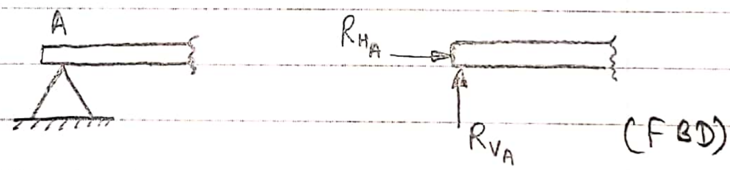


- A free body diagram of roller support is shown in the fig. given below.

FBD ⇒

- Roller support offers only one reaction, which is always perpendicular to the base or plane of roller.
- All the steel trusses of the bridges have one of their ends supported on rollers.
- Main advantage of the support is that beam can easily move towards left or right due to expansion & contraction.

② Hinge Support :-

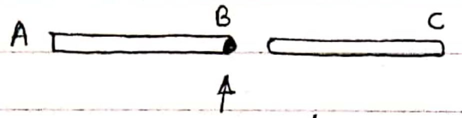


- In this case, end of the beam is hinged to the support as shown in the figure above.
- This type of support offers two reactions, one is parallel to base & other is perpendicular to the base.

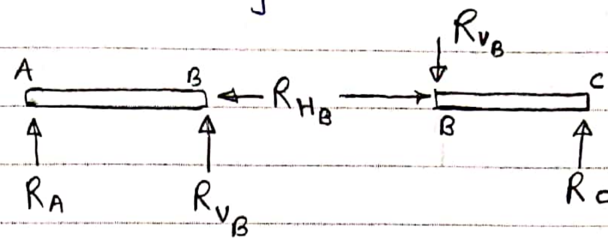
- All the steel trusses of the bridges have one of their end roller supported & other end is hinged.
- Main advantage of this support is that beam remains stable.
- If both ends of the beam are roller supported, then beam can not be stable. Thus one support of beam is made roller supported and other is hinged.

③ Internal Hinge :

when two members are connected by the hinge then this type of support is called as internal Hinge.



Internal Hinge.

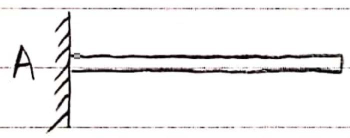


Reactions at internal Hinge.
(FBD)

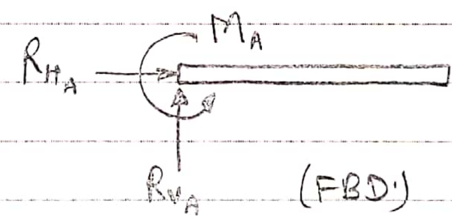
④ Fixed support :-

Fixed support offers three reactions as shown in fig. below.

- Reaction in Horizontal direction
- Reaction in vertical direction.
- Moment (fixing moment)



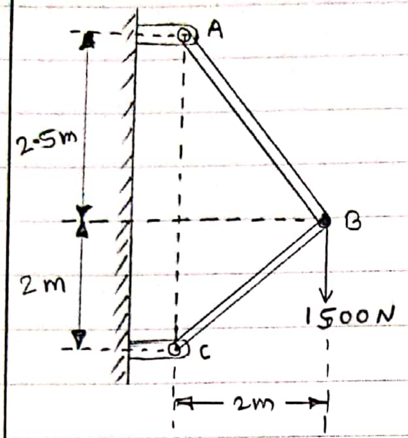
Fixed support

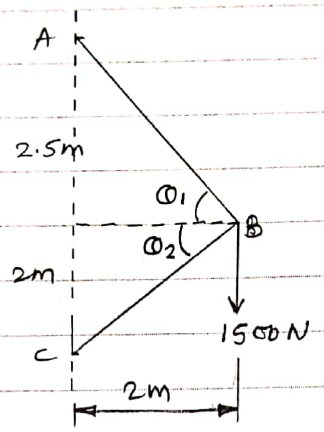


(FBD)

Numericals on Lami's Theorem.
(Equilibrium of three concurrent forces)

① Find the axial force in the bar AB & AC as shown in figure:



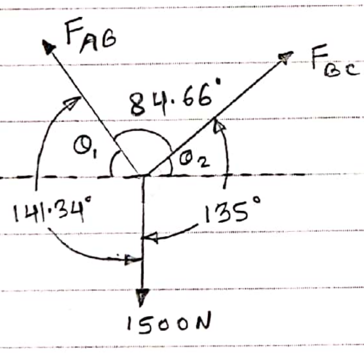


consider the free body diagram of all forces at B.

$\theta_1 = \tan^{-1}\left(\frac{2.5}{2}\right) = 51.34^\circ$

$\theta_2 = \tan^{-1}\left(\frac{2}{2}\right) = 45^\circ.$

Let the forces developed in the member AB & BC are F_{AB} & F_{BC} Respectively. The force diagram will be as follows at B.



Applying Lami's theorem;

$$\frac{F_{AB}}{\sin 135} = \frac{F_{BC}}{\sin 141.34} = \frac{1500}{\sin 83.66}$$

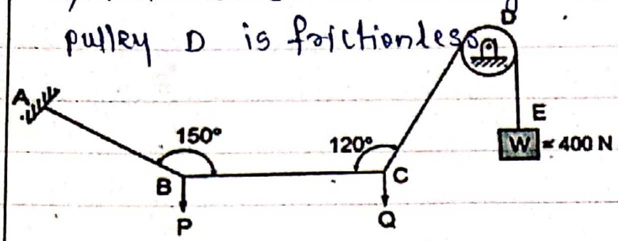
$$\therefore \frac{F_{AB}}{\sin 135} = \frac{1500}{\sin 83.66}$$

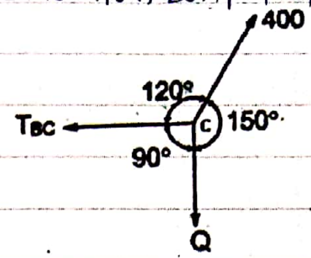
$$\therefore \boxed{F_{AB} = 1067.19 \text{ N}}$$

$$\frac{F_{BC}}{\sin 141.34} = \frac{1500}{\sin 83.66}$$

$$\therefore \boxed{F_{BC} = 942.81 \text{ N}}$$

Find the tensions in the string AB, BC, CD, DE of the given system as shown in figure. String BC is Horizontal & pulley D is frictionless.





FBD of Joint 'c'.

The string passes over the pulley & is attached to weight W = 400 N.

Thus $T_{CD} = T_{DE} = 400$ N.

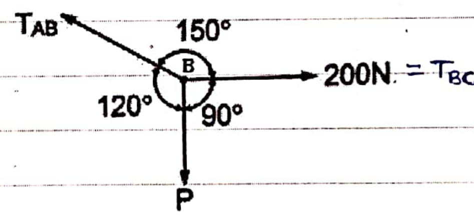
Consider Joint C & Apply Lami's theorem,

$$\therefore \frac{400}{\sin 90} = \frac{T_{BC}}{\sin 150} = \frac{Q}{\sin 120}$$

$$\therefore T_{BC} = \frac{400}{\sin 90} \times \sin 150 = \underline{\underline{200 \text{ N}}}$$

$$Q = \frac{400}{\sin 90} \times \sin 120 = \underline{346.41 \text{ N}}$$

Now, Consider FBD at Joint 'B' ⇒



FBD of Joint B

By using Lami's Theorem:

$$\frac{200}{\sin 120} = \frac{T_{AB}}{\sin 90} = \frac{P}{\sin 150}$$

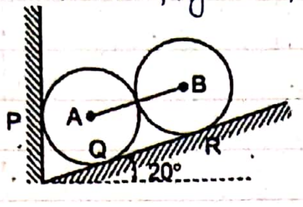
$$\therefore T_{AB} = \frac{200}{\sin 120} \times \sin 90$$

$$\boxed{T_{AB} = 230.94 \text{ N}}$$

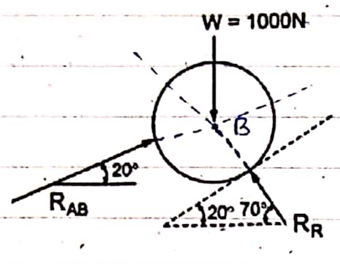
$$\therefore P = \frac{200}{\sin 120} \times \sin 150$$

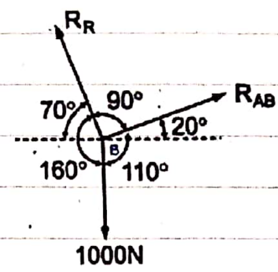
$$\boxed{P = 115.47 \text{ N}}$$

Two identical spheres having weight 1000 N are kept in a container as shown in fig. Find the Reactions at all contact surfaces. ⑦



⇒ consider the free body diagram of sphere B as shown below.





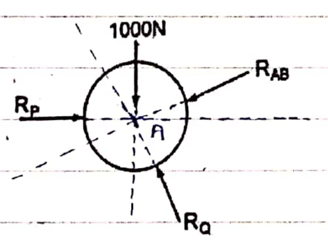
By Lami's theorem, $\frac{1000}{\sin 90} = \frac{R_R}{\sin 110} = \frac{R_{AB}}{\sin 160}$

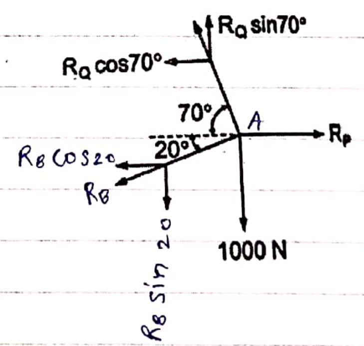
$\therefore R_R = \frac{1000}{\sin 90} \times \sin 110$

$\therefore \boxed{R_R = 939.69 \text{ N}}$

$\therefore R_{AB} = R_B = \frac{1000}{\sin 90} \times \sin 160 \qquad \therefore \boxed{R_B = 342.02 \text{ N}} = R_{AB}$

Now, consider FBD of sphere A,





Using the conditions of equilibrium,

$\Sigma F_x = 0$

$- R_B \cos 20 - R_Q \cos 70 + R_P = 0$

$- 342.02 \cos 20 - R_Q \sin 70 + R_P = 0.$

$\therefore R_P = 321.39 + 0.34 R_Q \quad \text{------} \quad (1)$

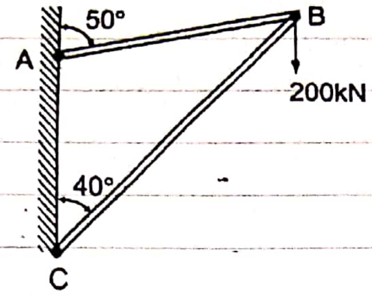
$\Sigma F_y = 0$

$R_Q \sin 70 - 342.02 \sin 20 - 1000 = 0.$

$\therefore \boxed{R_Q = 1188.66 \text{ N}}$ – put in eqⁿ ①

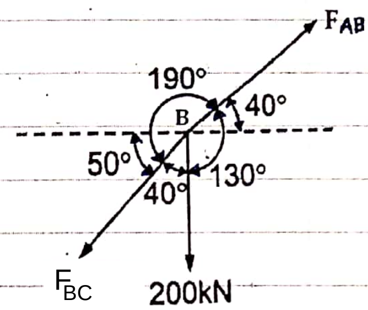
$\therefore \boxed{R_P = \cancel{1438.73 \text{ N}}} \quad 725.5 \text{ N}$

Two Members AB & BC are supporting a load of 200 KN as shown in figure. find the forces developed in the members AB & BC.



⟹

Consider the free Body diagram of Joint B as shown in fig ⟹



FBD of joint B

Assume that member BC is in compression & AB is in Tension.

By using Lami's theorem at B,

$$\frac{F_{AB}}{\sin 40} = \frac{F_{AC}}{\sin 130} = \frac{200}{\sin 190}$$

$$\therefore \frac{F_{AB}}{\sin 40} = \frac{200}{\sin 190}$$

$$\therefore F_{AB} = \frac{200}{\sin 190} \times \sin 40$$

$$\therefore \boxed{F_{AB} = -740.33 \text{ KN.}}$$

– ve sign indicates that member AB is not in Tension But is in compression.

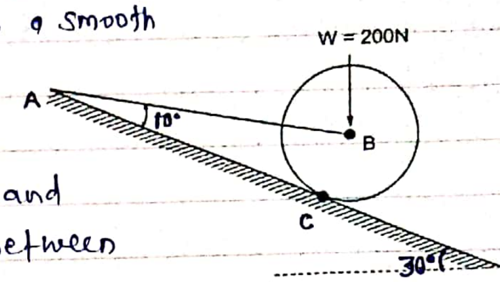
Now,

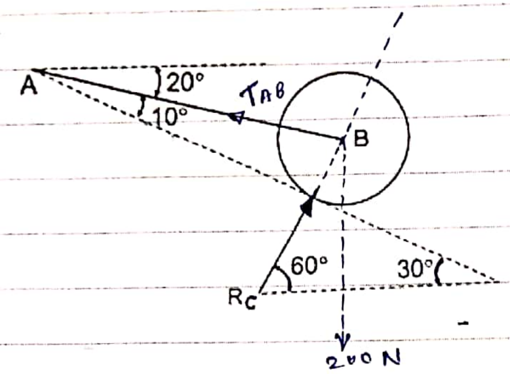
$$F_{AC} = \frac{200}{\sin 190} \times \sin 130$$

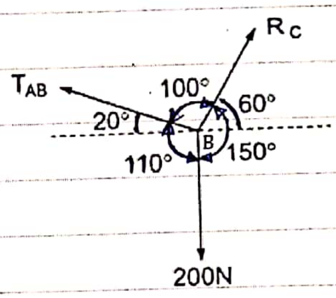
$$\boxed{F_{AC} = -882.29 \text{ KN}}$$

– ve sign indicates that member BC is not in Compression but it is in tension.

A sphere of weight 200 N rests on a smooth inclined plane. A sphere is kept in equilibrium by means of cable as shown. Find Tension in cable and reaction at the point of contact between surface & sphere.







Consider the FBD of sphere B as shown in fig. above.

Applying Lami's Theorem; we have,

$$\frac{R_C}{\sin 110} = \frac{200}{\sin 100} = \frac{T_{AB}}{\sin 150}$$

$$\therefore \frac{R_C}{\sin 110} = \frac{200}{\sin 100}$$

$$\therefore R_C = \frac{200 \times \sin 110}{\sin 100}$$

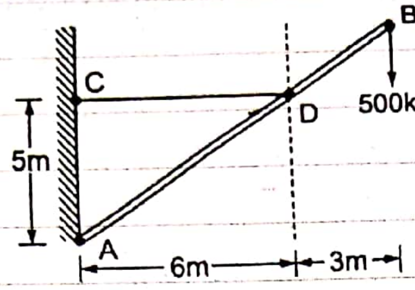
$$\therefore \boxed{R_C = 190.84 \text{ N}}$$

$$\frac{200}{\sin 100} = \frac{T_{AB}}{\sin 150}$$

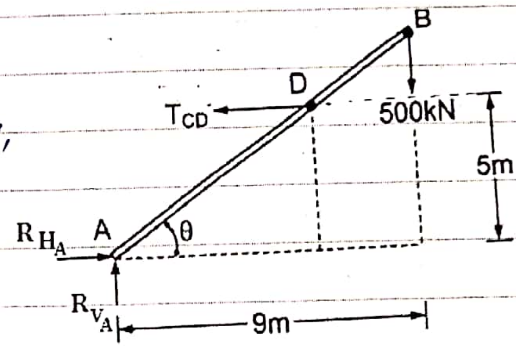
$$\therefore T_{AB} = \frac{200 \times \sin 150}{\sin 100}$$

$$\boxed{T_{AB} = 101.54 \text{ N}}$$

The bar AB supports a load of 500 KN as shown in figure. The bar is kept in equilibrium by means of horizontal rope. Find Tension in rope and reaction at A.



$\Rightarrow$ Consider the FBD as shown $\Rightarrow$



Applying conditions of Equillibrium,

$\Sigma f_x = 0$

$R_{HA} - T_{CD} = 0$

$\therefore R_{HA} = T_{CD}$ --- ①

$\Sigma f_y = 0$

$R_{VA} - 500 = 0.$

$\therefore \boxed{R_{VA} = 500 \text{ KN}}$

Taking moment about point A,

$\Sigma M_A = 0$

$-(T_{CD} \times 5) + (500 \times 9) = 0$

$-T_{CD} \times 5 = -4500$

$T_{CD} = \frac{-4500}{-5}$

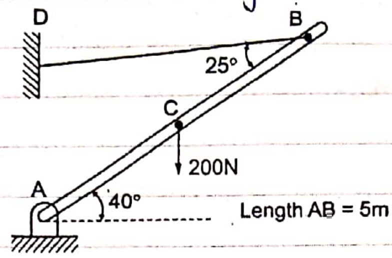
$\therefore \boxed{T_{CD} = 900 \text{ KN}}$

putting the value in eqⁿ ①

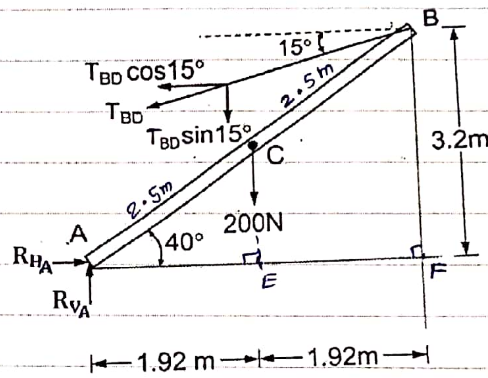
$R_{HA} = T_{CD}$

$\boxed{R_{HA} = 900 \text{ KN}}$

A bar of weight 200 N is hinged at A and pulled by the string attached at B as shown in fig. The length of bar is 5 m. Find the support reactions at A & corresponding Tension in String.



$\Rightarrow$



Consider Δ^{le} AEC

$\sin 40 = \dfrac{EC}{2.5}$

$\therefore EC = 2.5 \sin 40$

$\boxed{EC = 1.61\,m}$

Consider Δ^{le} A EC

$\cos 40 = \dfrac{AE}{AC} = \dfrac{AE}{2.5}$

$\therefore \boxed{AE = 1.92\ m}$

Applying conditions of equilibrium

$\Sigma F_x = 0$

$-T_{BD}\cos 15 + R_{HA} = 0.$ ---- (1)

$\Sigma F_y = 0$

$R_{VA} - 200 - T_{BD}\sin 15 = 0$

$R_{VA} - 200 - 0.26\,T_{BD} = 0$ ---- (2)

Taking moment @ A

$\Sigma M_A = 0$

$-(T_{BD}\cos 15 \times 3.2) + (T_{BD}\sin 15 \times 3.84) + (200 \times 1.92) = 0$

$-3.09\,T_{BD} + 0.99\,T_{BD} + 384 = 0.$

$-2.1\,T_{BD} = -384$

$\therefore \boxed{T_{BD} = 182.86\ N}$ — put in (1) & (11)

$\therefore R_{HA} = T_{BD}\cos 15 = 182.86 \cos 15$

$\therefore \boxed{R_{HA} = 176.62\ N}$

$R_{VA} = 200 + 0.26\,T_{BD} = 200 + (0.26 \times 182.86)$

$\boxed{R_{VA} = 247.54 N}$

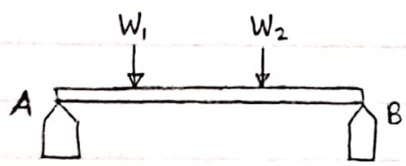
Beam

It is horizontal structural member on which different types of loads can be supported. It takes load from floor or roof & transfers to column.
The Beam may be sometimes constructed in the inclined position for architectural point of view.

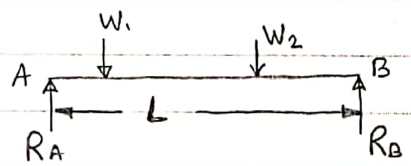
Types of Beams :-

① Simply Supported Beam :

A beam which is just resting on the supports at the ends without any connection is known as simply supported beam. It is generally used for vertical loading system.



Simply supported beam

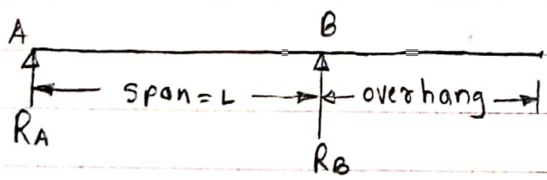


FBD of simply supported beam

② Overhanging Beam :

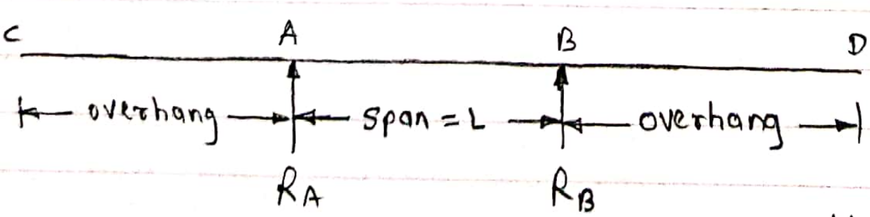
A beam which is supported at the intermediate point other than ends is called as overhanging beam. Here portion of beam is extended beyond the support.

a)



Singly overhanging Beam.

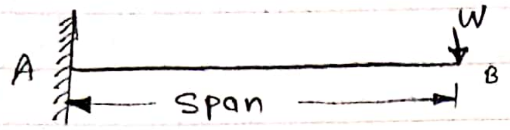
b)

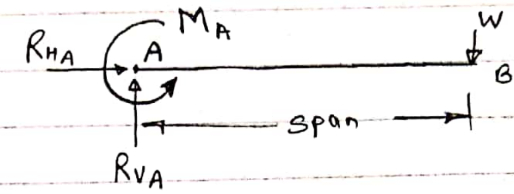


Doubly overhanging Beam

③ Cantilever Beam :-

A beam which is fixed at one end & free at other end is called as cantilever Beam.



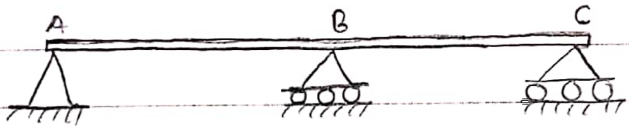


Here, there are three reaction components:
1) vertical reaction at A (R_{VA})
2) Horizontal reaction at A (R_{HA})
3) fixing moment at A (M_A).

We can assume any direction for above components.

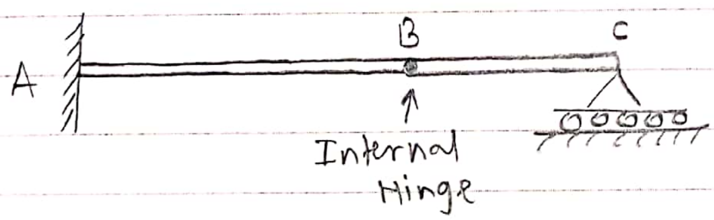
④ Continuous Beam:

A Beam having more than two support is called as continuous beam.

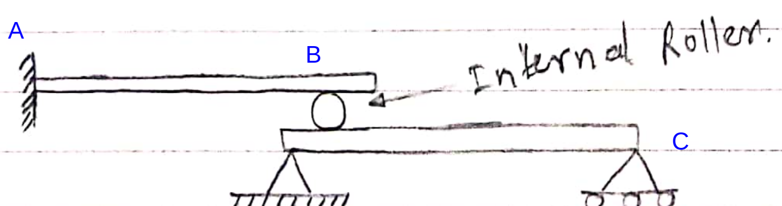


⑤ Compound Beam :-

When two or more beams are joined together by using internal Hinge; or when one beam rests over another beam by using internal roller, then such beam is called as compound Beam.



①

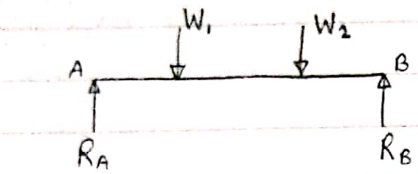


②

* Types of Loads on the Beam.

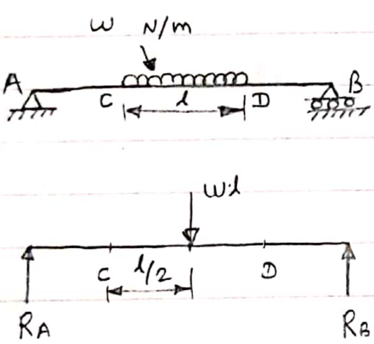
① Point Load

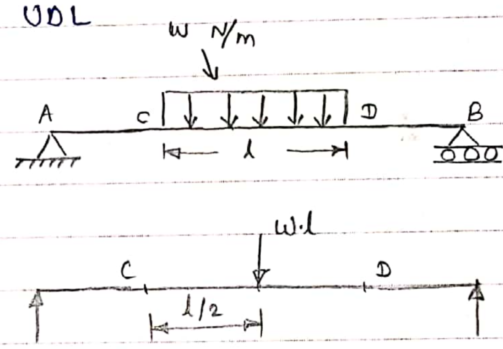
Load acting at a single point on the beam is known as point load. Its unit is N or kN.



② Uniformely Distributed load (UDL) or Rectangular Load.

A load which is spread over the beam or part of the beam uniformely is known as UDL





= Area of Rectangle = w.l

Conversion of UDL or Rectangular load into point Load = Intensity of UDL × Distance (Length of UDL)

$= w.l. \quad \ldots\ldots \quad N.$

③ Uniformely varying load or Triangular Load.

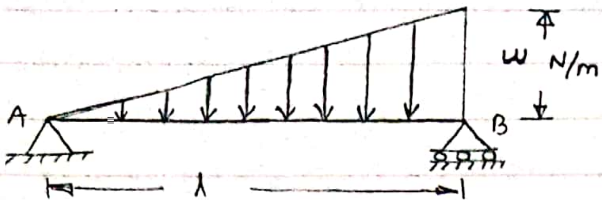
A load whose intensity is linearly varying betn the two points on the beam is known as UVL.

If An intensity of the load at one end is zero and and at other end is maximum then it is called as Uniformely varying load or Triangular load.

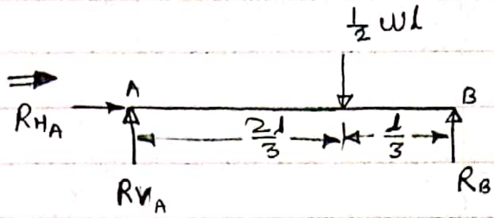
Following figures shows different types of UVL.

- To convert UVL into point load, Calculate the area of Uniformely Varying Load.
- For location of point load find x cordinate of centroid of the UVL area.

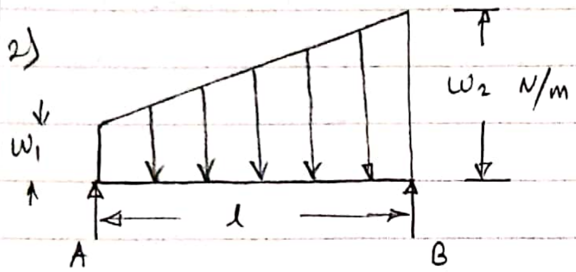
1)


A
B
w N/m
l

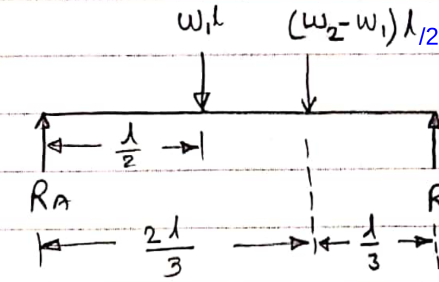
⇒


$\frac{1}{2} wl$
A
B
R_{H_A}
$\frac{2l}{3}$
$\frac{l}{3}$
R_{V_A}
R_B

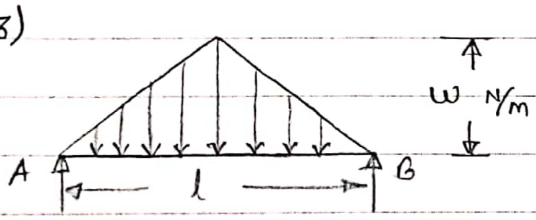
2)


w_1
w_2 N/m
l
A
B

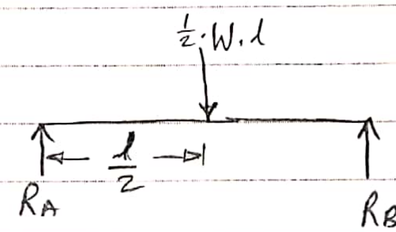
⇒


$w_1 l$
$(w_2 - w_1) l/2$
$\frac{l}{2}$
R_A
R_B
$\frac{2l}{3}$
$\frac{l}{3}$

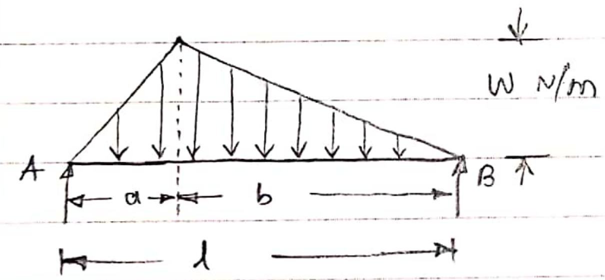
3)


w N/m
A
B
l

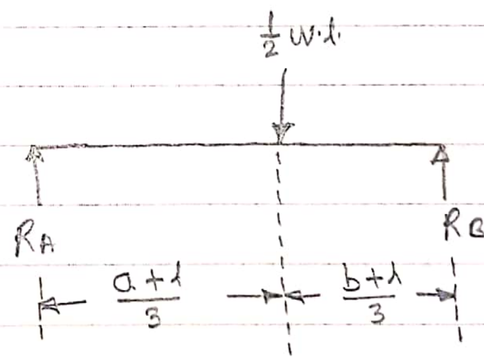
⇒


$\frac{1}{2} \cdot W \cdot l$
$\frac{l}{2}$
R_A
R_B

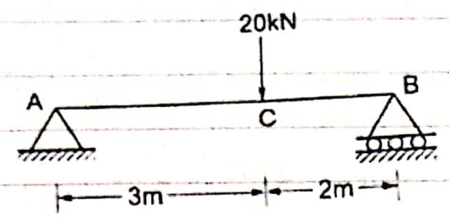
4)


W N/m
A
B
a
b
l

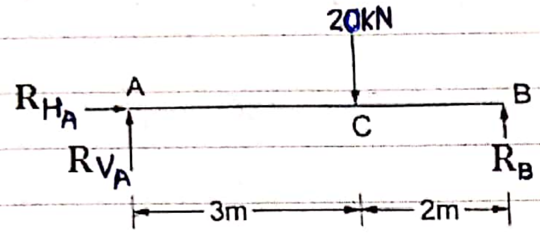
⇒


$\frac{1}{2} W \cdot l$
R_A
R_B
$\frac{a+l}{3}$
$\frac{b+l}{3}$

Find the support Reactions of given beam for Loading as shown below.



⟹



Draw FBD of given beam & Applying conditions of equilibrium,

$\Sigma F_x = 0$

$\boxed{R_{HA} = 0}$

$\Sigma F_{xy} = 0$

$R_{VA} - 20 + R_B = 0$

$R_{VA} + R_B = 20$ ------ ①

Taking moments @ A

$\Sigma M_A = 0$

$(20 \times 3) - (R_B \times 5) = 0$

$60 - 5R_B = 0$

$60 = 5R_B$

$\therefore R_B = \frac{60}{5}$

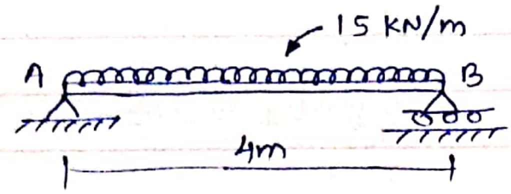
$\boxed{R_B = 12\ kN} \uparrow$ – put in eqⁿ ①

$\therefore R_{VA} + 12 = 20$

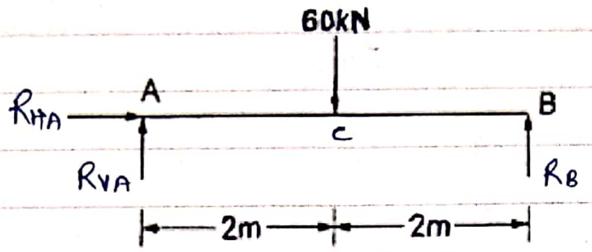
$\therefore R_{VA} = 20 - 12$

$\boxed{R_{VA} = 8\ kN} \uparrow$

find support Reaction for given Beam.



$\Rightarrow$



Resolving forces Horizontally,

$\Sigma F_x = 0$

$\boxed{R_{HA} = 0}$

Resolving forces vertically,

$\Sigma F_y = 0$

$R_{VA} - 60 + R_B = 0$

$R_{VA} + R_B = 60 \quad \text{---} \quad (1)$

Taking moments @ A

$\Sigma M_A = 0$

$(60 \times 2) - (R_B \times 4) = 0$

$\therefore \quad 120 - 4R_B = 0$

$\therefore \quad R_B = \frac{120}{4}$

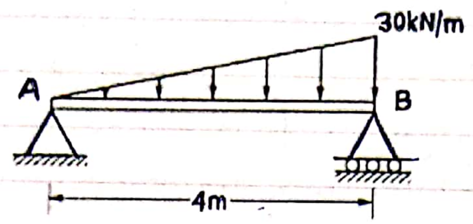
$\boxed{R_B = 30 \text{ KN}} \uparrow$ - put in eqn (1)

$\therefore \quad R_{VA} + R_B = 60$

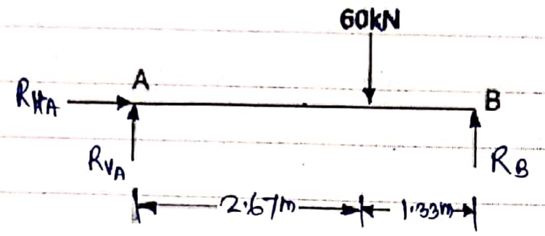
$R_{VA} = 60 - 30$

$\boxed{R_{VA} = 30 \text{ KN}} \uparrow$

Find the support Reactions of a given loading for Beam.



$\Rightarrow$



Resolving forces Horizontally

$\Sigma F_x = 0$

$\boxed{R_{HA} = 0}$

Resolving forces vertically

$\Sigma F_y = 0$

$R_{VA} - 60 + R_B = 0$

$R_{VA} + R_B = 60 \quad ---- \quad (1)$

Taking moments @ A

$\Sigma M_A = 0$

$(R_{VA} \times 0) + (60 \times 2.67) - 4R_B = 0$

$160.2 - 4R_B = 0$

$\therefore R_B = \frac{160.2}{4}$

$\boxed{R_B = 40\ kN}$ $(\uparrow)$ - put in eqn (1)

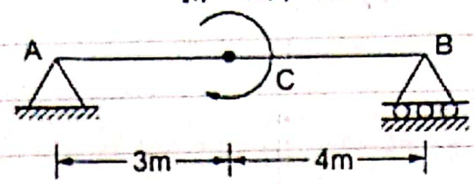
we get

$R_{VA} + 40 = 60$

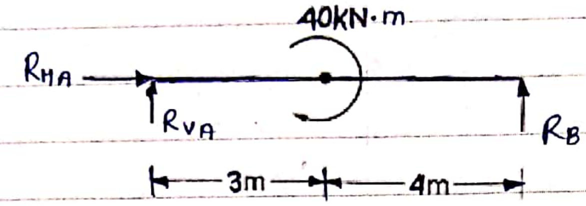
$R_{VA} = 60 - 40$

$\boxed{R_{VA} = 20\ kN}$ $(\uparrow)$

Find the support Reactions for given Beam.



$\Longrightarrow$



Resolving forces Horizontally,

$\Sigma F_x = 0$

$\boxed{R_{HA} = 0}$

Resolving forces vertically

$\Sigma F_y = 0$

$R_{VA} + R_B = 0 \quad \text{-- -- -- (1)}$

Taking moments @ A.

$\therefore \; 40 - 7R_B = 0$

$\therefore \; R_B = \frac{40}{7}$

$\boxed{R_B = 5.71 \text{ KN}} \; (\uparrow)$

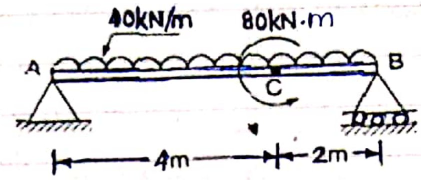
put above value in eq^n (1)

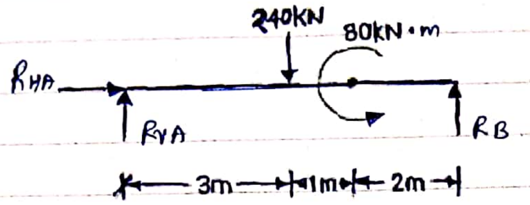
$\therefore \; R_{VA} + R_B = 0$

$R_{VA} + 5.71 = 0$

$R_{VA} = -5.71 \text{ KN}$

$\boxed{R_{VA} = 5.71 \text{ KN}} \; (\downarrow)$





Resolving forces Horizontally,

$\Sigma fx = 0$

$\boxed{R_{HA} = 0}$

Resolving forces vertically

$\Sigma fy = 0$

$R_{VA} - 240 + R_B = 0$

$R_{VA} + R_B = 240$ — (1)

Taking moments @ A

$\Sigma M_A = 0$

$(240 \times 3) - 80 - 6R_B = 0$

$640 - 6R_B = 0$

$R_B = \dfrac{640}{6}$

$\boxed{R_B = 106.67}$ kN (↑)

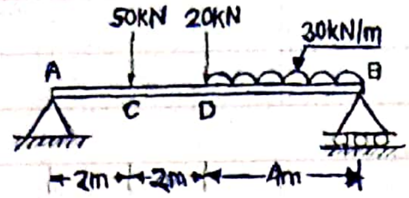
put this value in eqn (1)

$R_{VA} + R_B = 240$

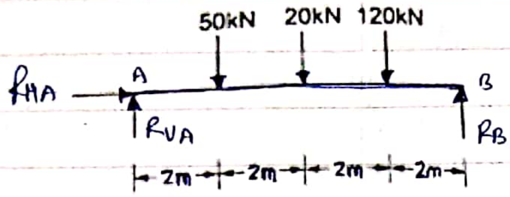
$R_{VA} = 240 - 106.67$

$\boxed{R_{VA} = 133.33 \text{ kN}}$ (↑)

find support reactions for the loading as (
shown Below.



$\Rightarrow$



Resolving forces vertically

$\Sigma F_y = 0$

$R_{VA} - 50 - 20 - 120 + R_B = 0$

$R_{VA} + R_B = 190$ --- (1)

Resolving forces Horizontally,

$\Sigma F_x = 0$

$\boxed{R_{HA} = 0}$

Taking moments @ A

$\Sigma M_A = 0$

$(50\times2) + (20\times4) + (120\times6) - 8R_B = 0$

$100 + 80 + 720 - 8R_B = 0$

$900 - 8R_B$

$\therefore R_B = \frac{900}{8}$

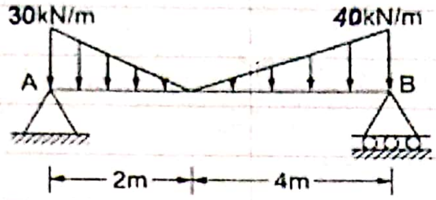
$\boxed{R_B = 112.5\ KN}$ $(\uparrow)$

Put Above value in eqn (1)

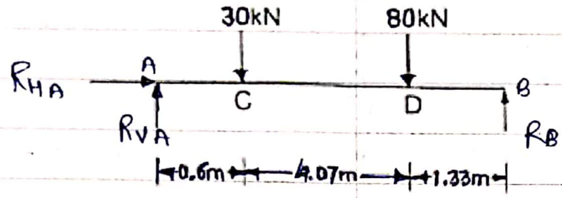
$R_{VA} + 112.50 = 190$

$\therefore \boxed{R_{VA} = 77.5\ KN}$ $(\uparrow)$

find the Reactions at support A & B.



$\Rightarrow$



Resolving forces Horizontally;

$\Sigma F_x = 0$

$\boxed{R_{HA} = 0}$

Resolving forces vertically;

$\Sigma F_y = 0$

$R_{VA} - 30 - 80 + R_B = 0$

$R_{VA} + R_B = 110 \quad \text{---} \ (1)$

Taking moments about point A,

$\Sigma M_A = 0$

$(30 \times 0.6) + [80 \times (4.07 + 0.6)] - (R_B \times 6) = 0$

$18 + 373.6 - 6R_B = 0$

$6R_B = 391.6$

$R_B = \frac{391.6}{6}$

$\boxed{R_B = 65.267 \text{ KN}}$

put this value in eqn (1)

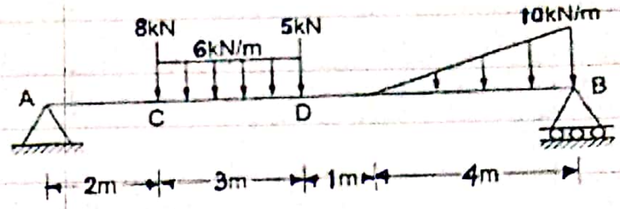
$\therefore$ $R_{VA} + R_B = 110$

$R_{VA} + 65.267 = 0110$

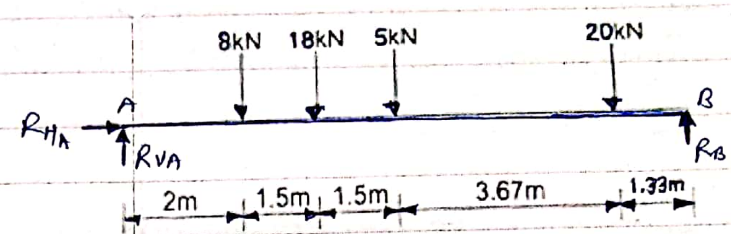
$R_{VA} = 110 - 65.267$

$A_y = \boxed{R_{VA} = 44.733 \text{ KN}}$

find support Reactions at A & B. (28)



$\Rightarrow$



Resolving forces Horizontally

$\Sigma f_x = 0$

$\boxed{R_{HA} = 0}$

Resolving forces vertically,

$\Sigma F_y = 0$

$R_{VA} - 8 - 18 - 5 - 20 + R_B = 0$

$R_{VA} + R_B = 8 + 18 + 5 + 20$

$R_{VA} + R_B = 51$ ------ (1)

Taking moments at point A.

$\Sigma M_A = 0$

$(8 \times 2) + (18 \times 3.5) + (5 \times 5) + (20 \times 8.67) - (10 \times R_B) = 0$

$16 + 63 + 25 + 173.4 - 10 R_B = 0$

$277.4 - 10 R_B = 0$

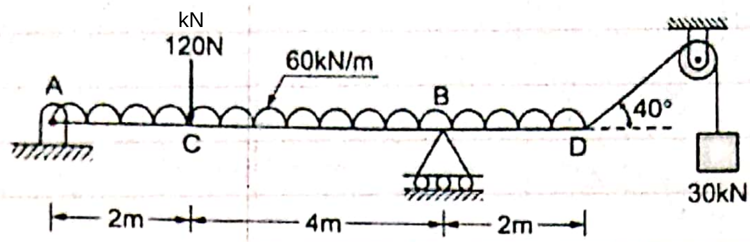
$R_B = \frac{277.4}{10}$

$\boxed{R_B = 27.74 \text{ kN}}$ — put in eqn (1)

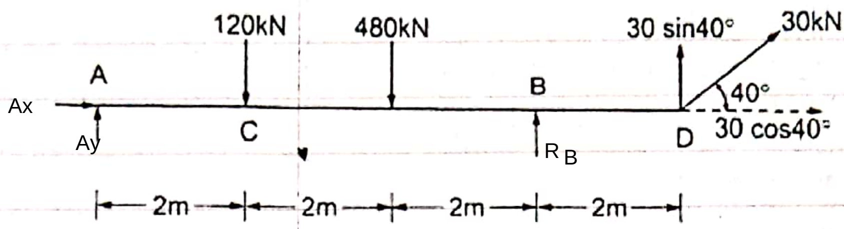
$R_{VA} + R_B = 51$

$R_{VA} = 51 - 27.74$

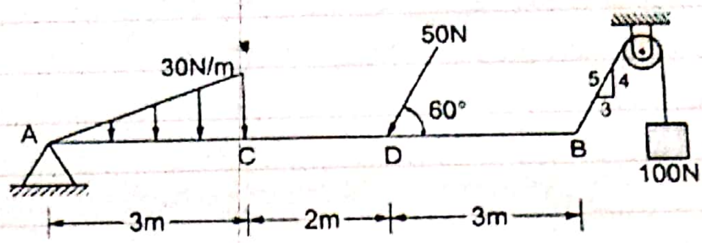
$\boxed{R_{VA} = 23.26 \text{ kN}}$

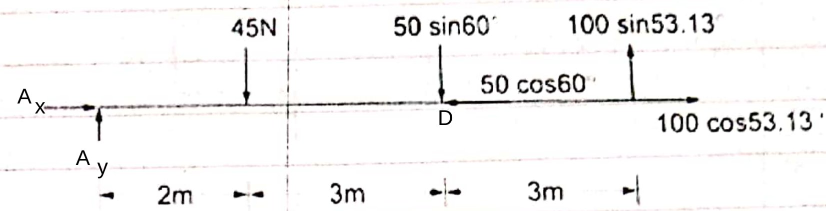


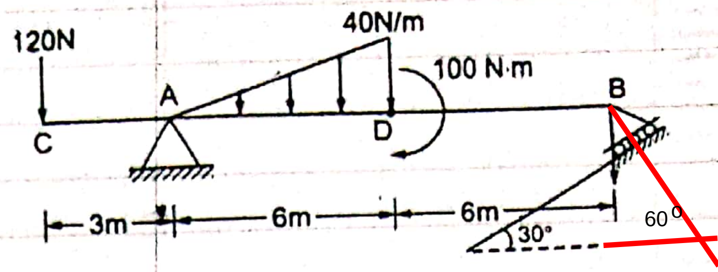
kN
120N
60kN/m
A
B
C
D
40°
30kN
2m
4m
2m

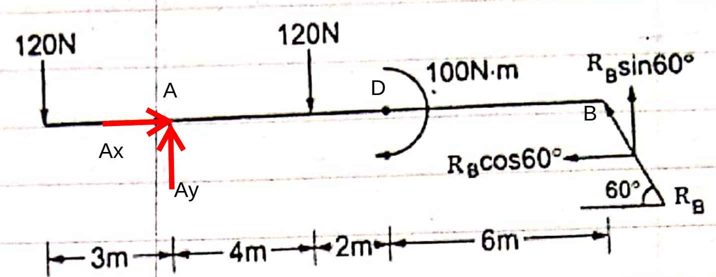


120kN
480kN
30 sin40°
30kN
A
B
40°
Ax
C
D
30 cos40°
Ay
R B
2m
2m
2m
2m


30N/m
50N
60°
5
4
3
A
C
D
B
100N
3m
2m
3m


45N
50 sin60°
100 sin53.13°
50 cos60°
Ax
D
100 cos53.13°
Ay
2m
3m
3m

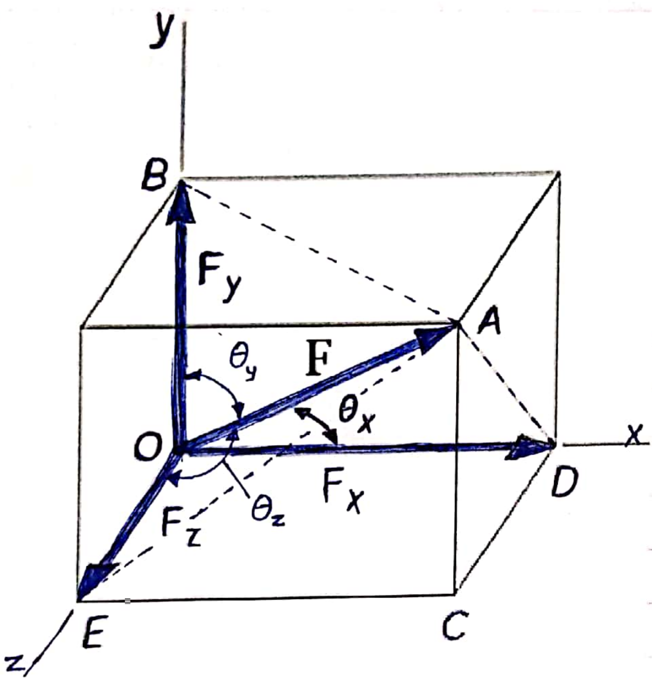

120N
40N/m
100 N·m
A
B
C
D
3m
6m
6m
30°
60°


120N
120N
100N·m
$R_B \sin 60°$
A
D
B
Ax
Ay
$R_B \cos 60°$
60°
R_B
3m
4m
2m
6m

Space forces

- The force system which is acting in different planes is called as Non-coplaner force sys. or space force.
- If a force is defined by three co-ordinates axis then it is called as space force.

* Rectangular Components of force in space:-



Let us consider the force F which is acting along the diagonal OA as shown above and making angles θ_x, θ_y, θ_z with x, y, z axis respectively.

The above force F in a three diamensional space can be resolved into rectangular components F_x, F_y, F_z.

Then

$$\boxed{\begin{aligned} F_x &= F\cos\theta_x \\ F_y &= F\cos\theta_y \\ F_z &= F\cos\theta_z \end{aligned}}$$

$$\therefore \quad \boxed{F = \sqrt{F_x^2 + F_y^2 + F_z^2}}$$

Here $\cos\theta_x$, $\cos\theta_y$, $\cos\theta_z$ are the cosines of θ_x, θ_y & θ_z and are known as direction cosines of force F.

The Force vector $\bar{F}$ can be expressed as;

$$\bar{F} = (F_x i + F_y j + F_z k)$$
$$\bar{F} = (F\cos\theta_x i + F\cos\theta_y j + F\cos\theta_z k)$$
$$\bar{F} = F(\cos\theta_x i + \cos\theta_y j + \cos\theta_z k) = F.\bar{e}$$

In above eqn,

F = magnitude of force

$\bar{e}$ = unit vector along the direction of force.

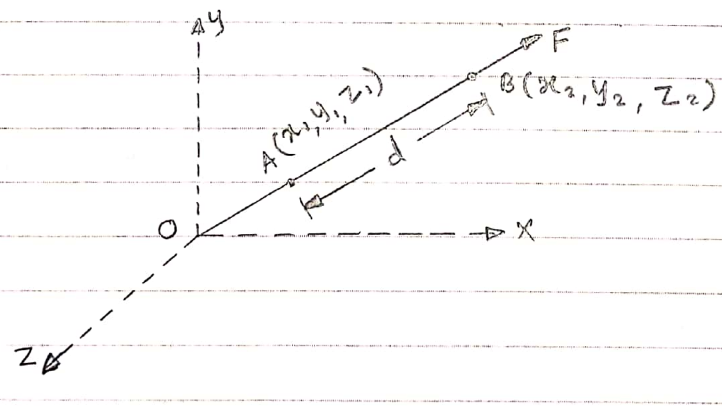
$\bar{e} = (\cos\theta_x i + \cos\theta_y j + \cos\theta_z k)$

Here, ∴ $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

Direction cosines can be found out by,

$\cos\theta_x = \frac{F_x}{F}$
$\cos\theta_y = \frac{F_y}{F}$
$\cos\theta_z = \frac{F_z}{F}$

* When Force F is defined in three diamensional space by using two points on its line of action, Then, Rectangular components can be abtained as follows.



Consider Force F acting at point O. Let $A(x_1, y_1, z_1)$ & $B(x_2, y_2, z_2)$ are the two points on the line of action of force F.

Let the distance betn two points (on the line of action of force) A & B is 'd'.

Now,

$\therefore$ Vector $\overline{AB} = \overline{d} = dx\,i + dy\,j + dz\,k.$

$\boxed{\text{magnitude of } AB = d = \sqrt{dx^2 + dy^2 + dz^2}}$

we know that Vector = magnitude x unit vector along the AB.

$$\therefore \quad \overline{d} = d \times \overline{e}_{AB}$$

$$\therefore \overline{e}_{AB} = \frac{\overline{d}}{d}$$

$$\therefore \overline{e}_{AB} = (dx\,i + dy\,j + dz\,k) \times \frac{1}{d}$$

we know that, Force $\overline{F}$ = magnitude x unit vector along line of action.

$$\therefore \quad \overline{F} = F \times e_{AB}$$

$$\overline{F} = \frac{F}{d}(dx\,i + dy\,j + dz\,k)$$

$$\overline{F} = \left(\frac{F\,dx}{d}\right)i + \left(\frac{F\,dy}{d}\right)j + \left(\frac{F\,dz}{d}\right)k.$$

from above eqn we can have three components of force,

$$\boxed{\begin{aligned} F_x &= \frac{F.dx}{d} \\ F_y &= \frac{F.dy}{d} \\ F_z &= \frac{F.dz}{d} \end{aligned}} \qquad \begin{aligned} \therefore \frac{F_x}{dx} &= \frac{F}{d} \\ \therefore \frac{F_y}{dy} &= \frac{F}{d} \\ \therefore \frac{F_z}{dz} &= \frac{F}{d}. \end{aligned}$$

Thus, $\boxed{\dfrac{F}{d} = \dfrac{F_x}{dx} = \dfrac{F_y}{dy} = \dfrac{F_z}{dz}}$

* unit vector along the direction of force

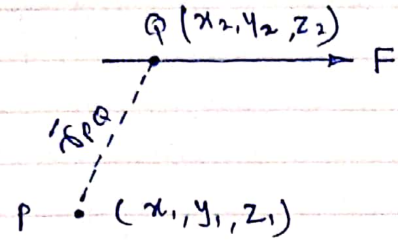
$$\overline{e}_{AB} = \frac{dx\,i + dy\,j + dz\,k}{d.}$$

$$e_{AB} = \frac{(x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k}{\sqrt{dx^2 + dy^2 + dz^2}}$$

$$\boxed{e_{AB} = \frac{(x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}} \rightarrow \text{unit vector.}$$

* Position vector *

- consider force F passing through point Q (x_2, y_2, z_2).
- Let P (x_1, y_1, z_1) is the point away from line of action.



Then

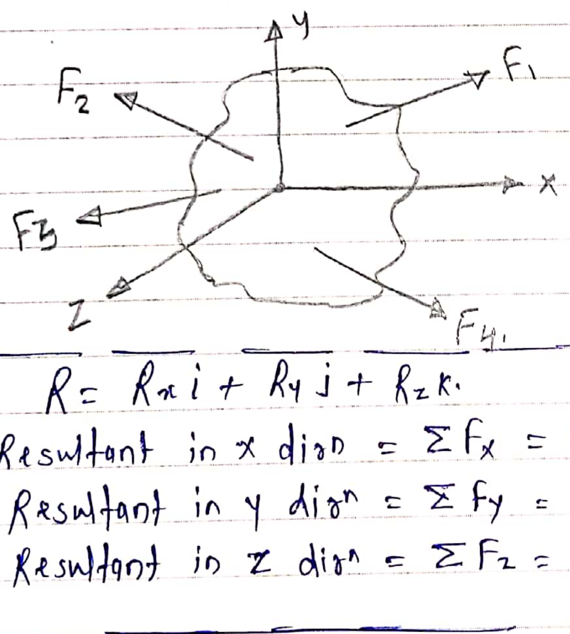
$\bar{r}_{PQ}$ = position vector of P w.r.t Q.

$= dx\, i + dy\, j + dz\, k.$

$\bar{r}_{PQ} = (x_2 - x_1)\, i + (y_2 - y_1)\, j + (z_2 - z_1)\, k.$

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

* Resultant of Several forces in space:-



Consider a body on which forces F_1, F_2, F_3, F_4 are acting as shown.

$$R = R_x i + R_y j + R_z k.$$

R_x = Resultant in x dirn $= \Sigma F_x = F_{x_1} + F_{x_2} + F_{x_3} + F_{x_4}$

R_y = Resultant in y dirn $= \Sigma F_y = F_{y_1} + F_{y_2} + F_{y_3} + F_{y_4}$

R_z = Resultant in z dirn $= \Sigma F_z = F_{z_1} + F_{z_2} + F_{z_3} + F_{z_4}$

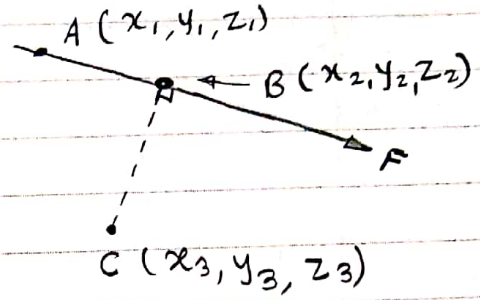
$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{\Sigma F_x^2 + \Sigma F_y^2 + \Sigma F_z^2}.$$

Direction of Resultant w.r.t. x, y, z axis is

$\cos \alpha_x$	$= \frac{R_x}{R}$	$= \frac{\Sigma F_x}{R}$
$\cos \alpha_y$	$= \frac{R_y}{R}$	$= \frac{\Sigma F_y}{R}$
$\cos \alpha_z$	$= \frac{R_z}{R}$	$= \frac{\Sigma F_z}{R}$

* Moment of force in space about point :

consider a force ($\bar{F} = F_x i + F_y j + F_z k$) is passing through two points $A(x_1, y_1, z_1)$ & $B(x_2, y_2, z_2)$ as shown. Then moment of this force about point $C(x_3, y_3, z_3)$ will be



$$M_C = \bar{r}_{CA} \times \bar{F}$$

$$M_C = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Here $[\, r_x = (x_1 - x_3)$, $r_y = y_1 - y_3$ & $r_z = z_1 - z_3 \,]$

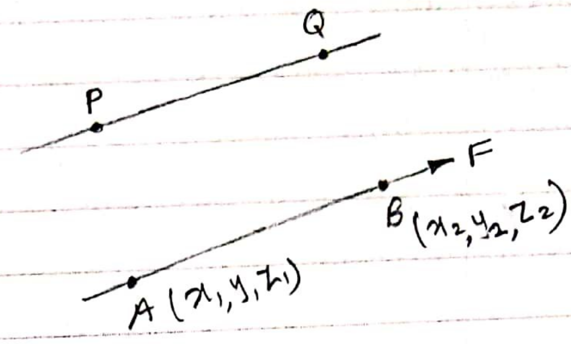
$$M_x = (r_y F_z - r_z F_y)$$
$$M_y = (r_z F_x - r_x F_z)$$
$$M_z = (r_x F_y - r_y F_x)$$

$$M_C = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

* Moment of force in space about a line.

Consider the force of magnitude F Passing through the points $A(x_1, y_1, z_1)$ & $B(x_2, y_2, z_2)$.

consider any line passing through the points $P(x_3, y_3, z_3)$ & $Q(x_4, y_4, z_4)$



Moment of this Force About line PQ is given by:-

$$\bar{M}_{PQ} = \bar{M}_P \cdot \bar{e}_{PQ}$$

Space forces
(summary)

* **Rectangular components of force in space.**

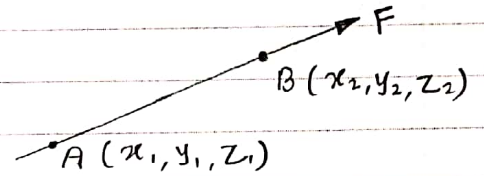
1) $F_x = F\cos\theta_x$ 2) $F_y = F\cos\theta_y$ 3) $F_z = F\cos\theta_z$

θ_x = Angle of force with x axis
θ_y = angle of force with y axis
θ_z = angle of force with z axis. &

$$\boxed{F = \sqrt{F_x^2 + F_y^2 + F_z^2}}$$

** **Force vector (in space):**

$$\bar{F} = F_x\, i + F_y\, j + F_z\, k$$



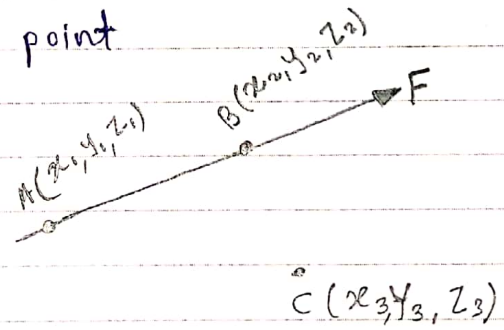
* $\bar{F} = F \times \bar{e}_{AB}$

where

$\bar{e}_{AB}$ = unit vector along force dirn.

$$\bar{e}_{AB} = \left[\frac{(x_2 - x_1)\, i + (y_2 - y_1)\, j + (z_2 - z_1)\, k}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}\right]$$

** **Moment of force in space about a point**



Moment of force F About Point C is given by,

$$\bar{M}_C = \bar{r}_{CA} \times \bar{F}$$

Solving above eqn we will get moment vector
Moment vector

$$\bar{M}_C = M_x\, i + M_y\, j + M_z\, k$$

$$M_C = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

** **Varignon's Theorem for forces in space:**

$$\Sigma M = M_R$$

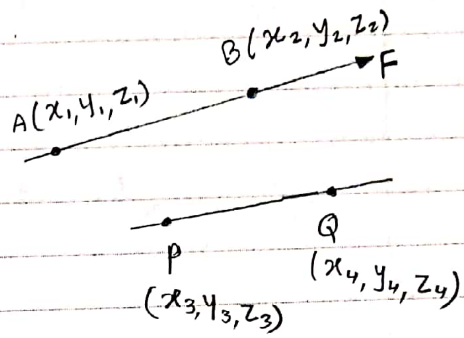
sum of moment of all forces about a point = Moment of Resultant about same point.

** Moment of Force About a line:

Key points :- moment of Force will be zero if

a) a line (about which moment is taken) is parallel to Force

b) if a force intersects the line about which the moment is to be taken.



The moment of force about a line PQ is

$$\bar{M}_{PQ} = \bar{M}_P \cdot \bar{e}_{PQ}$$

where

$$(\bar{M}_P = \bar{r}_{PA} \times \bar{F})$$

** Resultant of forces in space.

$$R_x = \Sigma F_x = F_{x_1} + F_{x_2} + \cdots$$

$$R_y = \Sigma F_y = F_{y_1} + F_{y_2} + F_{y_3} + \cdots$$

$$R_z = \Sigma F_z = F_{z_1} + F_{z_2} + F_{z_3} + \cdots$$

$$* \; R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

Direction of the Resultant is;

$$\cos\alpha_x = \frac{R_x}{R} = \frac{\Sigma F_x}{R}$$

$$\cos\alpha_y = \frac{R_y}{R} = \frac{\Sigma F_y}{R}$$

$$\cos\alpha_z = \frac{R_z}{R} = \frac{\Sigma F_z}{R}.$$

** Equilibrium of concurrent & Parallel forces in space:

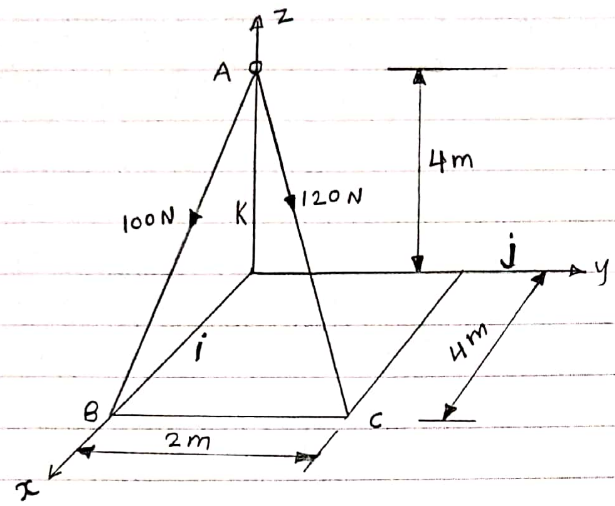
$$\Sigma F_x = 0 \qquad \Sigma M_x = 0$$

$$\Sigma F_y = 0 \qquad \Sigma M_y = 0$$

$$\Sigma F_z = 0 \qquad \Sigma M_z = 0$$

The cable exert a forces $F_{AB} = 100\,N$ & $F_{AC} = 120\,N$ on the ring at point A as shown in figure. Determine the magnitude of the resultant force acting at A.

A (0, 0, 4)
B (4, 0, 0)
C (4, 2, 0)



Express the forces interms of Force vector,

∴

$$\bar{F}_{AB} = F_{AB} \times \bar{e}_{AB}$$

$$= 100 \left[\frac{(4-0)\,i + (0-0)\,j + (0-4)\,k}{\sqrt{(4-0)^2 + (0-0)^2 + (0-4)^2}} \right]$$

$$= 100 \left[\frac{(4)\,i + (0)\,j + (-4)\,k}{\sqrt{4^2 + 0^2 + (-4)^2}} \right]$$

$$= 100 \left[\frac{4i + 0j + (-4)k}{\sqrt{32}} \right]$$

$$= 100\,(0.70\,i + 0j - 0.70\,k)$$

$$\therefore \bar{F}_{AB} = 70.7\,i + 0j - 70.7\,K \quad - (1)$$

Now, $\bar{F}_{AC} = F_{AC} \times \bar{e}_{AC}$

$$\therefore \bar{F}_{AC} = 120 \left[\frac{(4-0)\,i + (2-0)\,j + (0-4)\,k}{\sqrt{(4-0)^2 + (2-0)^2 + (0-4)^2}} \right]$$

$$= 120 \left[\frac{4i + 2j - 4k}{\sqrt{4^2 + 2^2 + (-4)^2}} \right]$$

$\therefore \overline{F_{AC}} = 120 \left[\frac{4i + 2j - 4k}{\sqrt{36}} \right]$

$\therefore \overline{F_{AC}} = 120 \, (0.667 i + 0.333 j - 0.667 k)$

$\therefore \overline{F_{AC}} = (80.04 i + 39.6 j - 80.04 k)$ — (II)

From eqn (I) & (II)

$R_x = \Sigma F_x = 70.7 + 80.04 = 150.11 \text{ N}$

$R_y = \Sigma F_y = 0 + 39.6 = 39.6 \text{ N}$

$R_z = \Sigma F_z = -70.7 - 80.04 = -150.11 \text{ N}$

$\therefore$ Resultant, $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

$R = \sqrt{(150.11)^2 + (39.6)^2 + (-150.11)^2}$

$\boxed{R = 216 \text{ N}}$

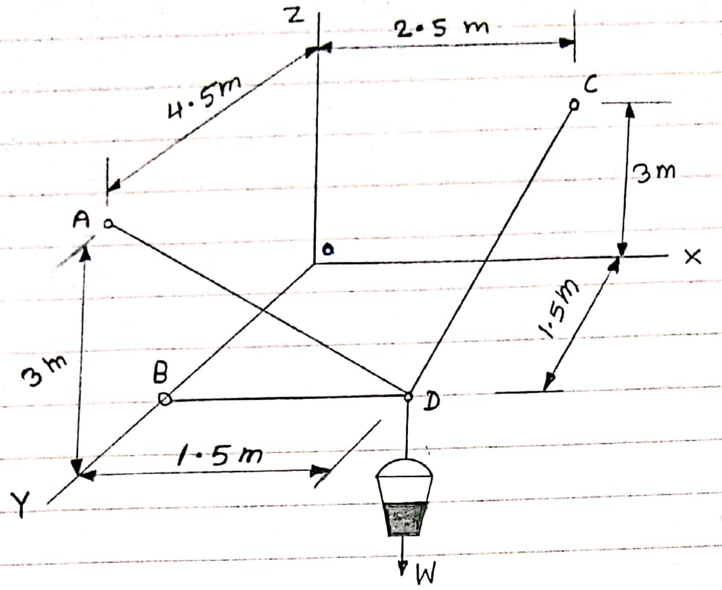
Direction of Resultant,

$\alpha_x = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{150.11}{216}\right) = \underline{45.97^\circ}$

$\alpha_y = \cos^{-1}\left(\frac{R_y}{R}\right) = \cos^{-1}\left(\frac{39.6}{216}\right) = \underline{79.43^\circ}$

$\alpha_z = \cos^{-1}\left(\frac{R_z}{R}\right) = \cos^{-1}\left(\frac{-150.11}{216}\right) = \underline{134.02^\circ}$

If each cable can sustain a maximum tension of 600N, Determine the greatest weight of the bucket and its contents that can be supported. Refer the figure.



A (0,4.5, 3)
B(0,1.5,0)
C (2.5,0,3)
D(1.5,1.5,0)

Force Developed in each cable will be tension, then, force vectors can be written as below,

(i)

$$\overline{T}_{DB} = T_{DB} \cdot \bar{e}_{DB} = T_{DB}\left[\frac{(0-1.5)i + (1.5-1.5)j + (0-0)k}{\sqrt{(0-1.5)^2 + (1.5-1.5)^2 + (0-0)^2}}\right]$$

$$\therefore \overline{T}_{DB} = T_{DB}\left[\frac{-1.5i + 0j + 0k}{1.5}\right]$$

$$\therefore \overline{T}_{DB} = T_{DB}(-1i + 0j + 0k)$$

$$\therefore \overline{T}_{DB} = -T_{DB}\,i + 0j + 0k$$

(ii) $$\overline{T}_{DA} = T_{DA} \cdot \bar{e}_{DA} = T_{DA}\left[\frac{(0-1.5)i + (4.5-1.5)j + (3-0)k}{\sqrt{(0-1.5)^2 + (4.5-1.5)^2 + (3-0)^2}}\right]$$

$$\therefore \overline{T}_{DA} = T_{DA}\left[\frac{-1.5i + 3j + 3k}{4.5}\right]$$

$$\therefore \overline{T}_{DA} = T_{DA}(-0.333i + 0.667j + 0.667k)$$

$$\therefore \overline{T}_{DA} = (-0.333\,T_{DA})i + (0.667\,T_{DA})j + (0.667\,T_{DA})k$$

(iii) $\overline{T_{DC}} = T_{DC} \cdot e_{DC} = T_{DC}\left[\dfrac{(2.5-1.5)i + (0-1.5)j + (3-0)k}{\sqrt{(2.5-1.5)^2 + (0-1.5)^2 + (3-0)^2}}\right]$

$\therefore \overline{T_{DC}} = T_{DC}\left(\dfrac{1i - 1.5j + 3K}{3.5}\right)$

$\therefore \overline{T_{DC}} = T_{DC}(0.286i - 0.428j + 0.857k)$

$\therefore \overline{T_{DC}} = (0.286\,T_{DC})i - (0.428\,T_{DC})j + (0.857\,T_{DC})k$

(iv) $\overline{W} = -W.k = 0i + 0j - wk$

Now Applying Equilibrium conditions.

$\Sigma f_x = 0$

$\therefore -T_{DB} - 0.33T_{DA} + 0.286\,T_{DC} + 0 = 0$ — (i)

$\Sigma f_y = 0$

$\therefore 0 + 0.667\,T_{DA} - 0.428\,T_{DC} + 0 = 0$ — (ii)

$\Sigma f_z = 0$

$\therefore 0 + 0.667\,T_{DA} + 0.857\,T_{DC} - W = 0$ — (iii)

Now it is given that each cable can sustain maximum Tension of 600 N, Thus our answer must be less than 600 or equal to 600 N.

(a) let, $T_{DA} = 600$ N, Then solving eqn (i), (ii), (iii), we get

$T_{DB} = 69.42$ N
$T_{DC} = 935$ N
$W = 1201$ N

Here Tension in one cable is more than 600N, which is not allowed.

(b) let $T_{DB} = 600$ N, then solving eqn (i), (ii), (iii), we get,

$T_{DA} = 5185$ N
$T_{DC} = 8081$ N
$W = 10384$ N

Here Tension in the cables is more than 600 N which is not allowed.

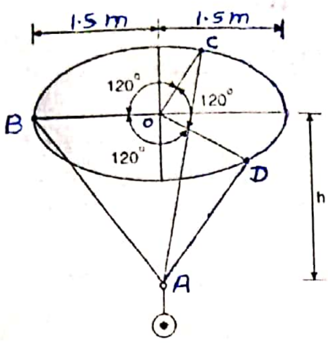
(c) when $T_{DC} = 600$ N, then solving eqn (i), (ii), (iii), we get,

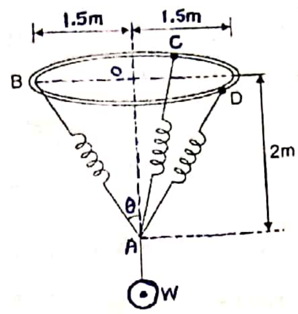
$\boxed{T_{DB} = 44.54 \text{ N}}$
$\boxed{T_{DA} = 385 \text{ N}}$
$\boxed{W = 771 \text{ N}}$

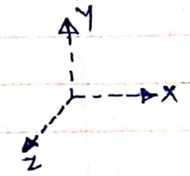
Here Tension in each cable is less than 600 N, Thus this answer is correct.

$\boxed{\text{weight of Bucket} = W = 771 \text{ N.}}$

The ball is suspended from the horizontal ring using three springs each having a stiffness of $k = 50$ N/m and an unstreched length of 1.5 m. If $h = 2$ m, determine the weight of ball. Refer the figure.



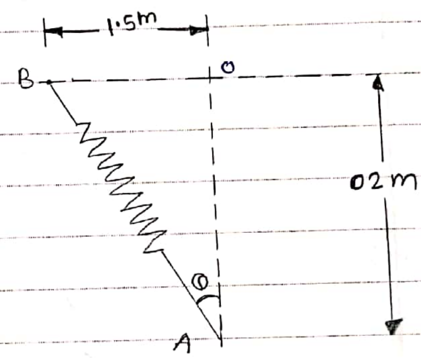




The length of each spring is

$$L = \sqrt{1.5^2 + 2^2}$$

$$L = 2.5 \text{ m.}$$



unstreched length of spring

$$L_0 = 1.5 \text{ m.}$$

$\therefore$ Difflection of spring

$$x = L - L_0 = 2.5 - 1.5$$

$$\boxed{x = 1 \text{ m.}}$$

Force developed in each spring $= kx = 50 \times 1 = 50$ N.

As shown in figure, each spring makes θ angle with vertical.

$$\therefore \sin\theta = \frac{1.5}{2.5}$$

$$\therefore \boxed{\theta = 36.87°}$$

y component of each spring force will be =

$$F_y = F\cos\theta = 50 \cos 36.87 = \underline{40 \text{ N.}}$$

for Equillibrium, $\Sigma f_y = 0$

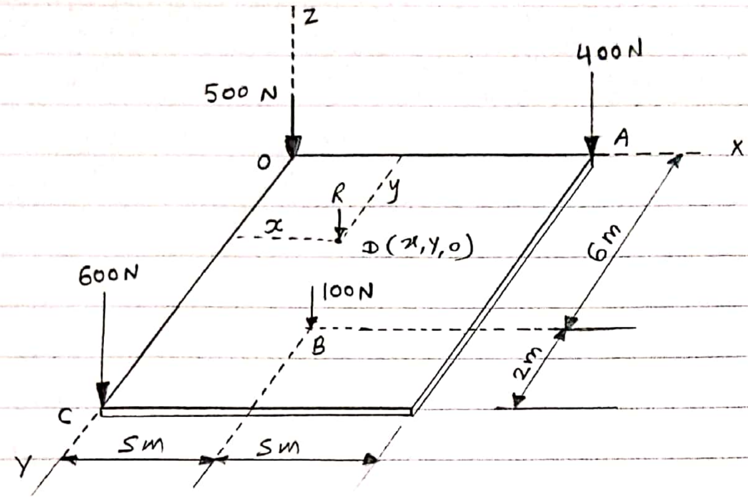
$$\therefore -W + 3(F_y) = 0$$

$$-W + 3 \times 40 = 0$$

$$\therefore \boxed{W = 120 \text{ N}}$$ — weight of ball.

Numericals - Parallel Forces in space (Resultant and Equillibrium)

Determine the magnitude and direction of Resultant Force of given force system as shown in figure & locate its point of application on the slab. (DEC-15)



Let Resultant acts at point D $(x, y, 0)$.

In Above case, we can observe that all forces are parallel to each other & are parallel to z axis. There fore

$R_x = \Sigma f_x = 0$ &

$R_y = \Sigma f_y = 0$. &

$R_z = \Sigma f_z = -500 - 400 - 100 - 600$

$\therefore R_z = \Sigma f_z = -1600$ N.

$\therefore$ Resultant $= R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

$\therefore \boxed{R = 1600 \text{ N}} (\downarrow)$

Downward.

Resultant is // to z axis.

Now to find the location of Resultant, assume that R is acting at point D on the slab.

D $(x, y, 0)$.

By using varignon's theorem & About X-X axis.

$\Sigma M_{xx} = M_{Rx}$

$\therefore (100 \times 6) + (600 \times 8) = (R \cdot y)$

$600 + 4800 = 1600 y$

$\therefore \boxed{y = 3.375 \text{ m}}$

By using varignon's theorem about y-y axis,

$\Sigma M_{yy} = M_{Ry}$

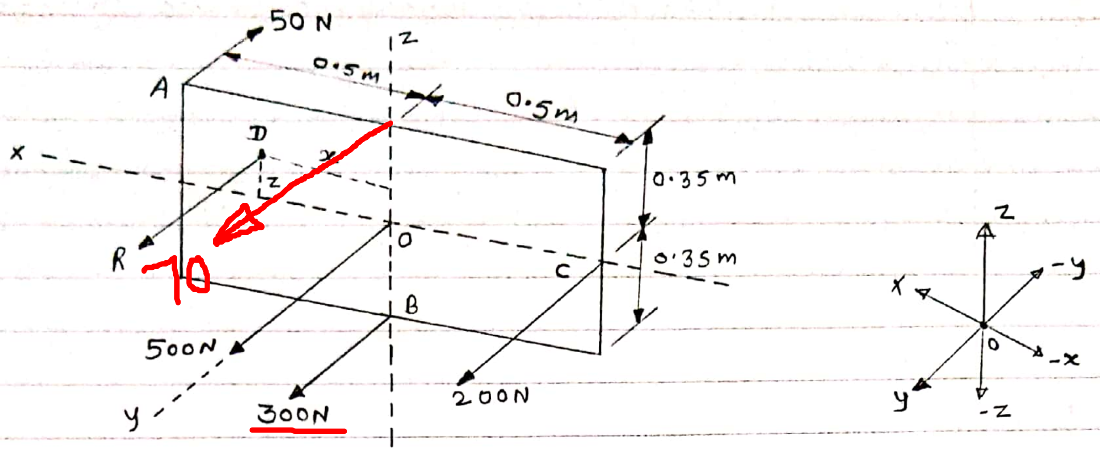
$-(400 \times 10) - (100 \times 5) = -(R \cdot x)$

$-4000 - 500 = -1600 x$

$\therefore \boxed{x = 2.81 \text{ m}}$

The resultant will act at point D(2.81, 3.375, 0).

Determine the Resultant of parallel Force system which acts on the plate as shown in figure.



Let us assume that Resultant is acting at point $D(x,0,z)$

Here all forces are parallel to y axis, so their x & z components are zero.

$\therefore R_x = \Sigma F_x = 0$

$\therefore R_z = \Sigma F_z = 0$

$\therefore R_y = \Sigma F_y = -50 + 500 + 300 + 200$

$\therefore R_y = \Sigma F_y = 950$ N.

Resultant $= R = \sqrt{0^2 + 0^2 + (950)^2}$

$\boxed{R = 950 \text{ N}}$

(Parallel to +ve y axis)

$\therefore$ To find the position of resultant, let us use varignon's theorem.

By using varignon's theorem about x-x axis; we have

$\Sigma M_{xx} = M_{Rx}$

$(300 \times 0.35) + (50 \times 0.35) = (-R \cdot z)$

$105 + 17.5 = -(950 \times z)$

$\therefore \boxed{z = -0.129 \text{ m}}$

By using varignon's theorem about z-z axis,

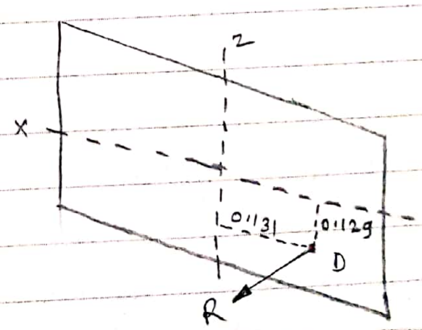
$\Sigma M_{zz} = M_{Rz}$

$\therefore -(50 \times 0.5) - (200 \times 0.5) = R \cdot x$

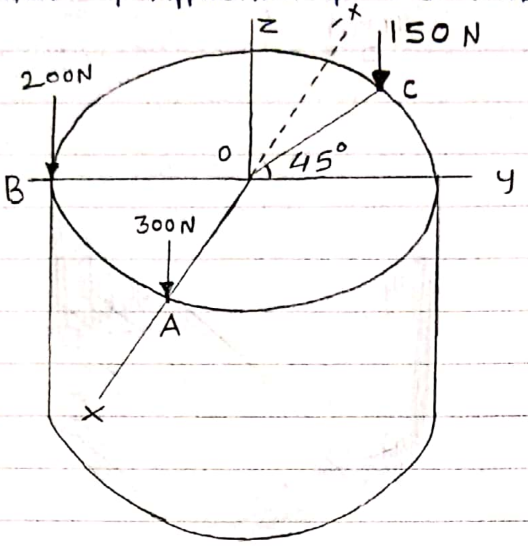
$-25 - 100 = 950\, x.$

$\therefore \boxed{x = -0.131 \text{ m}}$

Thus resultant will act as shown in the figure below.



Three parallel bolting forces act on the rim of the circular cover plate as shown in fig. Determine the magnitude & direction of Resultant, point of Application of resultant.



Assuming radius of cover plate as 1m.

As all forces are parallel to each other & to z axis, Resultant is given by,

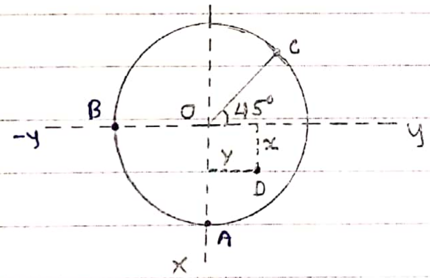
$R_x = \Sigma F_x = 0$

$R_y = \Sigma F_y = 0$

$R_z = \Sigma F_z = -200 - 150 - 300 = -650$ N.

$\therefore R = \sqrt{0^2 + 0^2 + (650)^2} = 650$ N (↓) parallel to z-axis

Now



Assume that Resultant is acting at point D(x, y, 0)

using varignon's theorem about xx axis

$\Sigma M_{xx} = M_{Rx}$

$(200 \times r) - (150 \times r\cos 45) = -650\,y.$

$\therefore \boxed{y = -0.145 \text{ m}}$

using varignon's theorem about y axis.

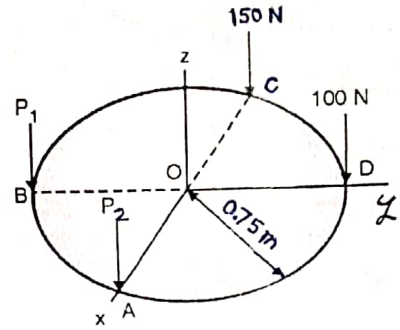
$\Sigma M_{yy} = M_{Ry}$

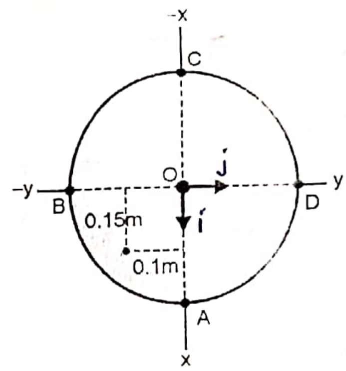
$(300 \times r) - (150\ r\sin 45) = 650 \cdot x$

$\therefore \boxed{x = 0.298 \text{ m}}$

$\therefore$ point of Application of Resultant, D (0.298, −0.145, 0)

Four parallel bolting forces act on the rim of the circular cover plate as shown in figure, If the resultant force 750 N is passing through (0.15m, -0.1 m) from the origin O, Determine the magnitude of forces P_1 & P_2.





As all forces are parallel to z-axis, the Resultant can be find out by,

$R = \Sigma F_z$

$-750 = -P_1 - P_2 - 150 - 100$

$\therefore -750 + 150 + 100 = -P_1 - P_2$

$-500 = -P_1 - P_2$

$\therefore P_1 + P_2 = 500 \quad -(1)$

Now using varignon's theorem about x-x axis

$\Sigma M_{xx} = M_{Rx}$

$(P_1 \times 0.75) - (100 \times 0.75) = -(R \times y)$

$0.75 P_1 - 75 = -[750 \times (-0.1)]$

$0.75 P_1 - 75 = 75$

$0.75 P_1 = 75 + 75 = 150$

$\boxed{P_1 = 200\ N}\ (\downarrow)$

from eqn (1)

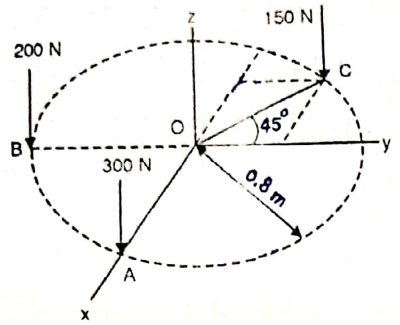
$P_1 + P_2 = 500$

$200 + P_2 = 500$

$\therefore P_2 = 500 - 200$

$\boxed{P_2 = 300\ N}\ (\downarrow)$

Three parallel bolting forces act on the rim of the circular cover plate as shown in fig. Determine the magnitude, nature and point of application of the Resultant force with respect to O.



As all forces are parallel to z axis, their x & y components are zero. Thus $\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma F_z = -200 - 150 - 300 = -650$ N

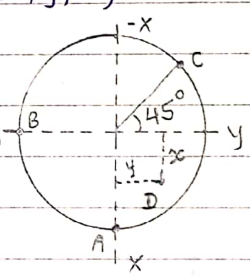
Resultant $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2 + \Sigma F_z^2}$

As $\Sigma F_x = \Sigma F_y = 0$

$R = \Sigma F_z = -650$ N

$\therefore \boxed{R = 650 \text{ N } (\downarrow)}$ parallel to z axis.

Assume that Resultant acts at point D $(x, y, 0)$.



using varignon's theorem about x-x axis;

$\Sigma M_{xx} = M_{Rx}$

$(200 \times 0.8) - (150 \times 0.8 \cos 45) = -R \cdot y$

$160 - 84.85 = -650 \cdot y$

$\therefore \boxed{y = -0.115 \text{ m}}$

using varignon's theorem about y-y axis,

$\Sigma M_{yy} = M_{Ry}$

$\therefore (300 \times 0.8) - (150 \times 0.8 \sin 45) = R \cdot x$

$240 - 84.85 = 650 \cdot x$

$\therefore \boxed{x = 0.206 \text{ m}}$

Resultant acts at point D $(0.206, -0.115, 0)$