

## TRUSS :-

A Rigid structure formed by connecting various two force members to each other by using pin joint.

### Plane truss:

when all members of the truss lies in one plane, then truss is known as plane truss.

Rigid truss: A truss which do not collapse when external load is applied on it.

### Simple truss:

The structure formed by basic triangle made by connecting various members are called simple truss.

## \* Classification of Truss:

Perfect truss

(stable)

$(n = 2j - R)$

Imperfect (~~not~~ unstable)  
 $(n \neq 2j - R)$

overstable  
(Redundant)

$(n > 2j - R)$

Deficient  
Truss.  
 $(n < 2j - R)$

### \* perfect truss :

A truss which does not collapse under the action of load is called perfect truss.

Condition :  $n = 2j - R$

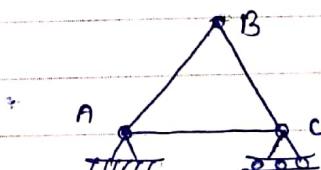
$n$  = no. of members

$j$  = no. of joints

$R$  = no. of Reaction

In truss ABC,  $n = 3$  &  $2j - R = 3$

so it is perfect truss.



$$n = 3$$

$$j = 3$$

$$R = 3$$

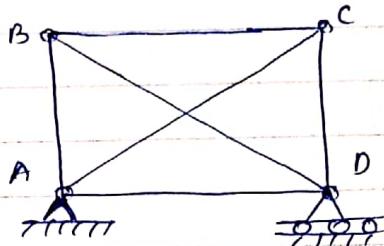
### \* Imperfect truss:

A truss which collapses under the load is called imperfect or unstable truss.

Here  $D \neq 2J - R$ .

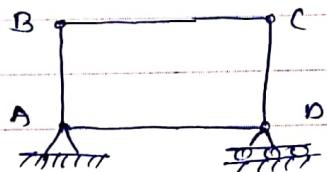
### Overstable (Redundant Truss)

A truss in which  $n > 2J - R$ ,  
Then it is overstable truss.



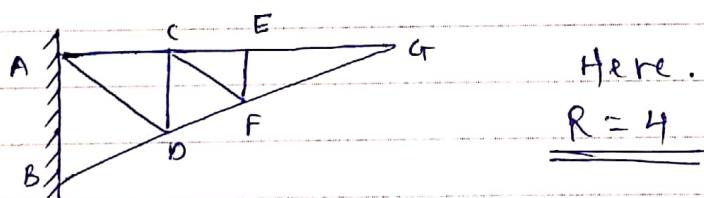
### Deficient Truss

It is a truss in which  
 $n < 2J - R$ .



### \* Cantilever Truss:

A truss which is fixed on one side & free at other end is called as cantilever truss.

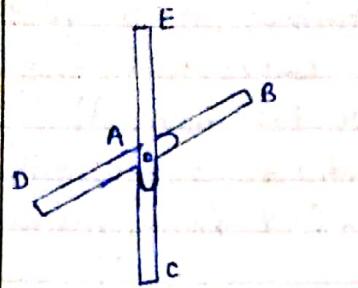


Here.

$$\underline{\underline{R = 4}}$$

### \* Assumptions made in the analysis:

- 1) Given truss is a perfect truss
- 2) The truss members are connected by joints only.
- 3) External loads are acting at the joints only.
- 4) All members are two force members.
- 5) The self-weight of members is neglected.



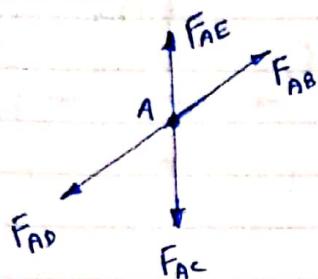
① IF

When four members are connected at single joint in such a way that opposite members lie in a single straight line  
if

there is no external load acting at the joint

Then

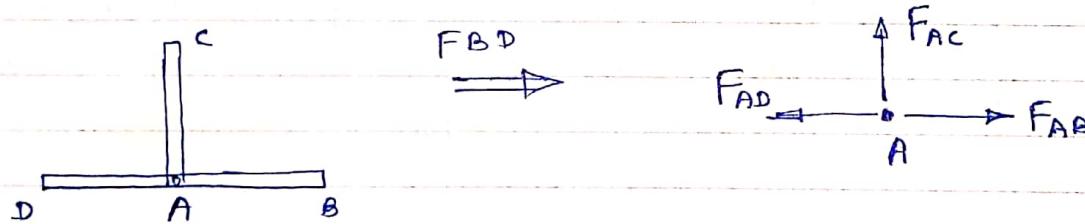
Forces in the opposite members are equal.



$$\therefore F_{AE} = F_{AC} \text{ & } F_{AB} = F_{AD}.$$

② IF

there are only three members at a joint and out of three, Two are colinear and one is inclined to first two or  $\perp$  to first two members,  
with no external load at joint,



Then,

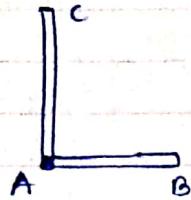
a) Forces in the two opposite (co-linear) members are equal. i.e.  $F_{AD} = F_{AB}$

b) The force in the inclined member is zero  
i.e.  $F_{AC} = 0$

Force in the perpendicular member is zero  
 $F_{AC} = 0$

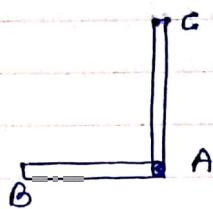
(4) IF

These are two members at a joint with one member horizontal & one vertical,  
& there is no external load at joint,



FBD  
→

↑  
F<sub>AC</sub>  
→ F<sub>AB</sub>



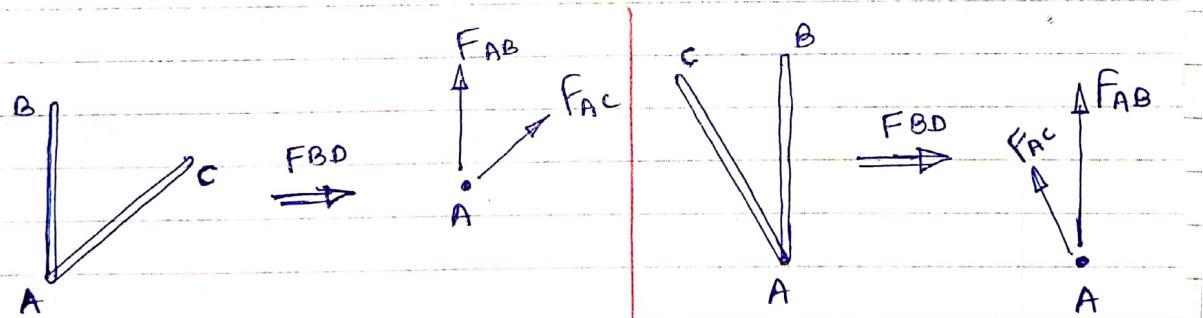
FBD  
→

↑  
F<sub>AC</sub>  
← F<sub>AB</sub>

Then, Both members are zero force member,

i.e.  $F_{AB} = 0$   
 $F_{AC} = 0$

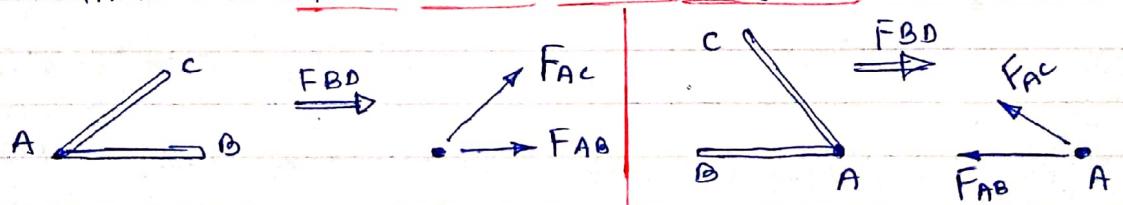
(5) IF there are only two members at a joint with one member vertical & other inclined  
& there is NO external load at joint,



Then Both members are zero force members.  
i.e.  $F_{AB} = 0$  &  $F_{AC} = 0$

(6)

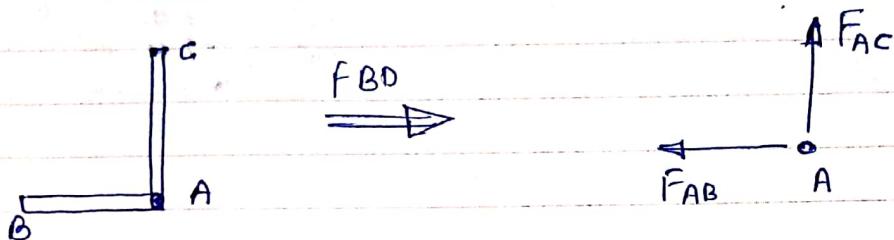
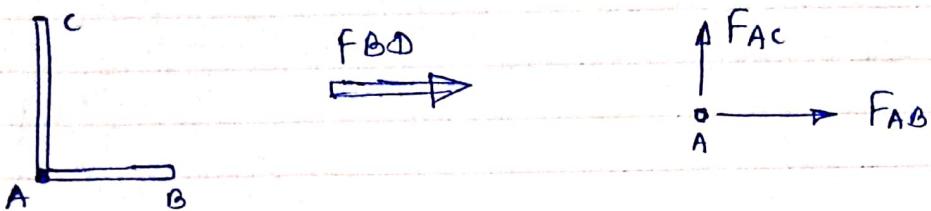
IF there are only two members at a joint with one member horizontal & other inclined and there is No External load at joint.



Then Both members are zero force members  
∴  $F_{AB} = 0$  and  $F_{AC} = 0$

④ IF

There are two members at a joint with one member horizontal & one vertical,  
& there is no external load at joint,

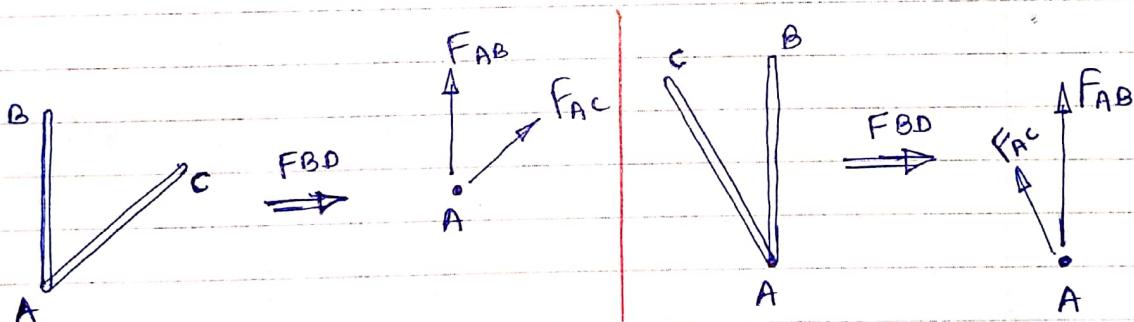


Then, Both members are zero force member,

$$\text{i.e. } F_{AB} = 0$$

$$F_{AC} = 0$$

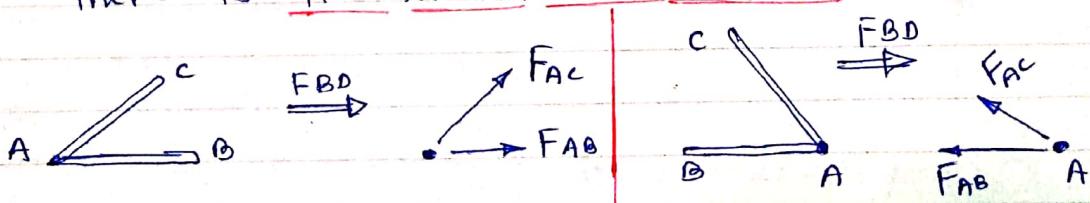
⑤ IF there are only two members at a joint with one member vertical & other inclined  
& there is no external load at joint,



Then Both members are zero force members.

$$\text{i.e. } F_{AB} = 0 \text{ & } F_{AC} = 0$$

⑥ If there are only two members at a joint with one member horizontal & other inclined and there is no external load at joint.



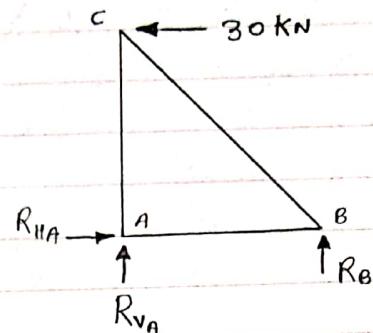
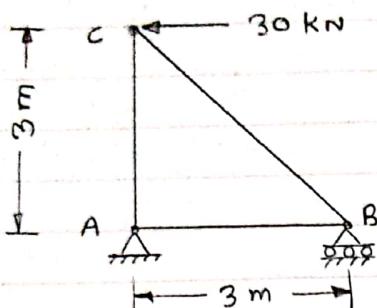
Then Both members are zero force members

$$\therefore F_{AB} = 0 \text{ and } F_{AC} = 0$$

## - 8 Numericals -

### \* Analysis of Truss By method of joints:-

- ① Determine the forces in all members of truss by joint method.



FBD of Truss

Consider FBD of Truss, Applying conditions of equilibrium,

$$\sum F_x = 0$$

$$R_{HA} = 30 \text{ kN}$$

$$\sum F_y = 0$$

$$R_{VA} + R_B = 0$$

Taking moment at point A,  $\sum M_A = 0$

$$-(R_B \times 3) - (30 \times 3) = 0$$

$$\therefore R_B = -30 \text{ kN}$$

$$R_B = 30 \text{ kN} \quad (\downarrow)$$

$$\therefore R_{VA} = 30 \text{ kN} \quad (\uparrow)$$

Consider joint C, Assuming forces in member AC & BC to be Tensile, Applying conditions of equilibrium,

$$\sum F_x = 0$$

$$-30 + F_{CB} \cos 45^\circ = 0$$

$$\therefore F_{CB} = \frac{30}{\cos 45^\circ} = 42.42 \text{ kN} \quad (\text{T})$$

$$\sum F_y = 0$$

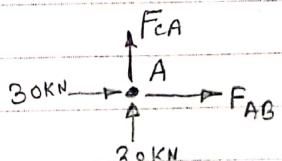
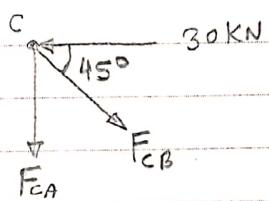
$$-F_{CA} - F_{CB} \sin 45^\circ = 0$$

$$-F_{CA} - 42.42 \sin 45^\circ = 0$$

$$-F_{CA} - 30 = 0$$

$$\therefore F_{CA} = -30 \text{ kN}$$

$$[F_{CA} = -30 \text{ kN}] \quad (\text{C})$$

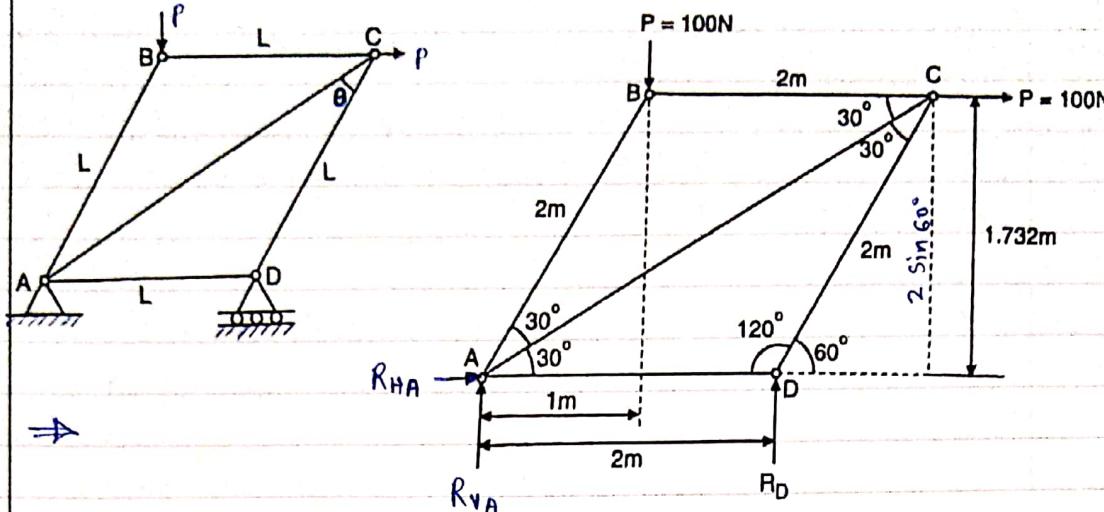


Consider joint A, Assuming all forces in member AB & AC to be Tensile, Apply condition of equilibrium.

$$\sum F_x = 0 \quad \therefore 30 + F_{AB} = 0$$

$$\therefore [F_{AB} = -30 \text{ kN}] \quad (\text{C})$$

Determine the forces in each member of the plane truss as shown in Fig. in terms of the external loading and state if the members are in tension or compression. Use  $\theta = 30^\circ$ ,  $L = 2 \text{ m}$  and  $P = 100 \text{ N}$ .



FBD of Truss.

consider FBD of Truss,  
for equilibrium,  $\Sigma F_x = 0$

$$R_{HA} + 100 = 0$$

$$\therefore \boxed{R_{HA} = -100 \text{ KN}} \quad (\leftarrow)$$

$$\Sigma F_y = 0$$

$$R_{VA} + R_D - 100 = 0$$

$$R_{VA} + R_D = 100 \quad \dots \quad (1)$$

$\Sigma M_A = 0$  Taking moment @ A

$$-(R_D \times 2) + (100 \times 1) + (100 \times 1.732) = 0$$

$$\therefore -2R_D + 100 + 173.2 = 0$$

$$\therefore \boxed{R_D = 136.6 \text{ N}} \quad (\uparrow)$$

from eqn (1)

$$R_{VA} = 100 - 136.6$$

$$R_{VA} = -36.6 \text{ N}$$

$$\boxed{R_{VA} = -36.6 \text{ N}} \quad (\downarrow)$$

Consider joint D, for equilibrium,

$$\Sigma F_x = 0$$

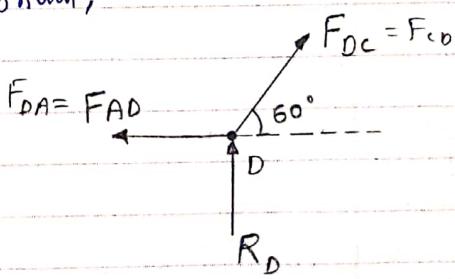
$$\therefore -F_{AD} + F_{CD} \cos 60^\circ = 0 \quad (II)$$

$$\Sigma F_y = 0$$

$$\therefore 136.6 + F_{CD} \sin 60^\circ = 0$$

$$\therefore \boxed{F_{CD} = -157.73 \text{ N.}} \quad (III)$$

$$\text{from eqn (II), } \boxed{F_{AD} = -78.87 \text{ N}} \quad (IV)$$



(5)

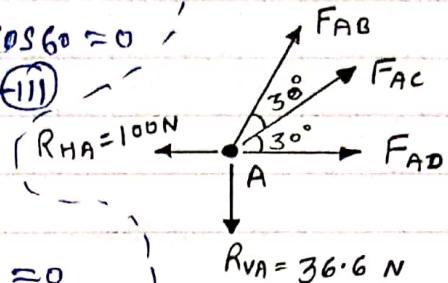
Consider joint A, for the equilibrium of joint A,

$$\sum F_x = 0$$

$$-100 + F_{AD} + F_{AC} \cos 30 + F_{AB} \cos 60 = 0$$

$$-100 + (-78.87) + F_{AC} \cos 30 + F_{AB} \cos 60 = 0$$

$$\therefore F_{AC} \cos 30 + F_{AB} \cos 60 = 178.87 \quad \text{--- (III)}$$



$$\sum F_y = 0$$

$$-36.6 + F_{AC} \sin 30 + F_{AB} \sin 60 = 0$$

$$\therefore F_{AC} \sin 30 + F_{AB} \sin 60 = 36.6$$

--- (IV)

Solving eqn (III) and (IV)

$$\boxed{F_{AC} = 273.21 \text{ N}} \quad (\text{T})$$

$$F_{AB} = -115.47 \text{ N}$$

$$\boxed{F_{AB} = 115.47 \text{ N}} \quad (\text{C})$$

Consider joint B, for the equilibrium of joint,

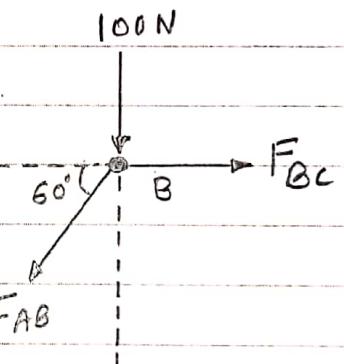
$$\sum F_x = 0$$

$$-F_{AB} \cos 60 + F_{BC} = 0$$

$$-[-115.47] \cos 60 + F_{BC} = 0$$

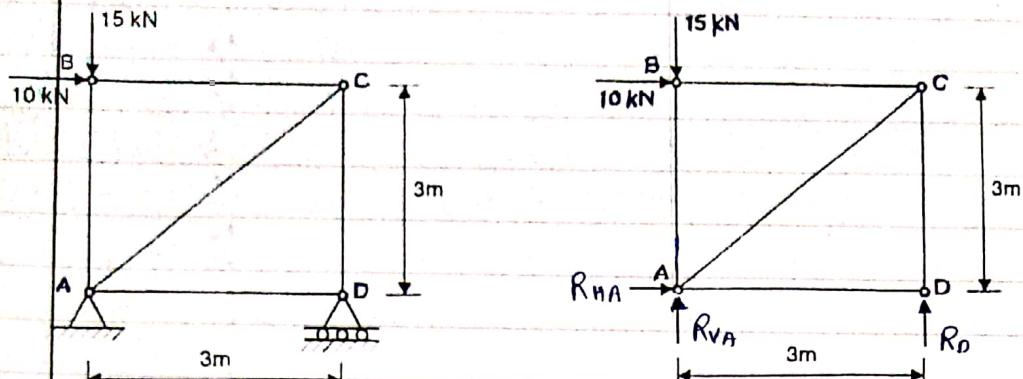
$$\therefore \boxed{F_{BC} = -57.73 \text{ N}}$$

$$\therefore \boxed{F_{BC} = 57.73 \text{ N}} \quad (\text{C})$$



Member	AB	BC	CD	AD	AC
force	115.47 N	57.73 N	157.73 N	78.87 N	273.21 N
Nature	C	C	C	C	T

Determine the axial forces in each member of the plane truss as shown in Figure.



FBD of Truss.



Consider FBD of Truss,  
for the equilibrium of Truss,  $\sum F_x = 0$

$$\therefore R_{HA} + 10 = 0$$

$$R_{HA} = -10 \text{ kN}$$

$$R_{HA} = 10 \text{ kN} (\leftarrow)$$

Resolving forces vertically,

$$\sum F_y = 0$$

$$R_{VA} + R_D - 15 = 0$$

$$\therefore R_{VA} + R_D = 15 \quad \dots \textcircled{1}$$

Taking moment about point A,

$$\sum M_A = 0$$

$$(10 \times 3) - 3 R_D = 0$$

$$30 = 3 R_D$$

$$\therefore R_D = 10 \text{ kN} \quad (\uparrow)$$

$$\therefore R_{VA} = 5 \text{ kN} \quad (\uparrow)$$

Now consider joint B, FBD of joint B is shown below.

Assuming forces developed in all members to be Tensile,

for the equilibrium of joint  
we have,

$$\sum F_x = 0$$

$$10 + F_{BC} = 0$$

$$\therefore F_{BC} = -10 \text{ kN}$$

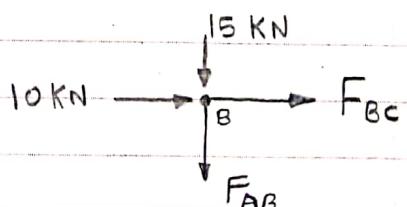
$$F_{BC} = 10 \text{ kN} \quad (\text{C})$$

$$\sum F_y = 0$$

$$-15 - F_{AB} = 0$$

$$\therefore F_{AB} = -15 \text{ kN}$$

$$F_{AB} = 15 \text{ kN} \quad (\text{C})$$



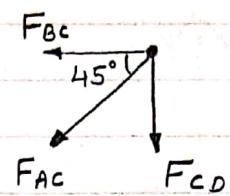
Now consider joint C,  
for the equilibrium of joint,

$$\sum F_x = 0$$

$$-F_{BC} - F_{AC} \cos 45^\circ = 0$$

$$-(-10) - F_{AC} \cos 45^\circ = 0$$

$$10 = F_{AC} \cos 45^\circ$$



$$\therefore F_{AC} = \frac{10}{\cos 45^\circ}$$

$$F_{AC} = 14.14 \text{ kN (T)}$$

$$\sum F_y = 0$$

$$-F_{AC} \sin 45^\circ - F_{CD} = 0$$

$$-14.14 \sin 45^\circ = F_{CD}$$

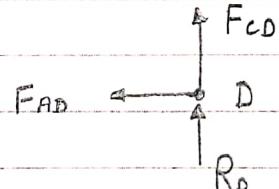
∴

$$F_{CD} = -10 \text{ kN}$$

$$F_{CD} = 10 \text{ kN (C)}$$

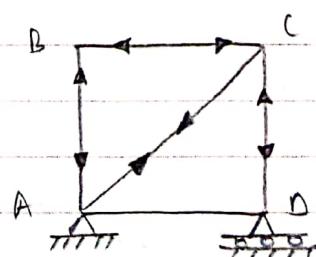
consider joint D,

By observation,



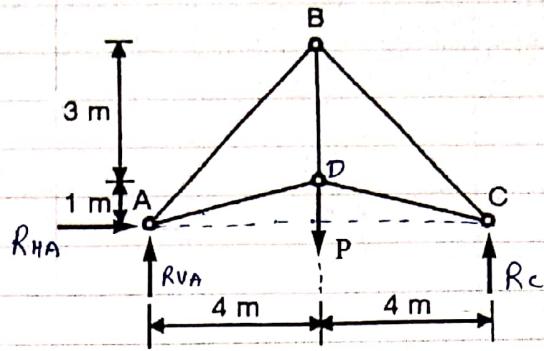
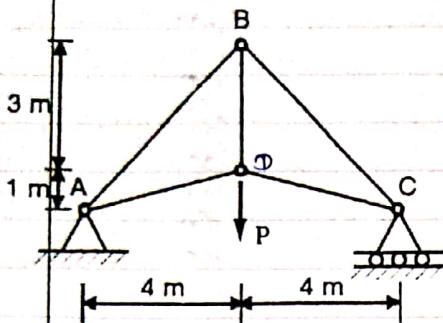
$$F_{AD} = 0$$

sr.no.	Member	Force	Nature
1	AB	15 kN	C
2	BC	10 kN	C
3	CD	10 kN	C
4	DA	0	-
5	AC	14.14	T



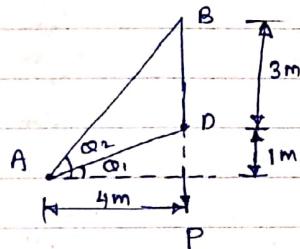
(54)

Member AB & BC can support a maximum compressive force of 800 N & members AD, DC, BD can support a max. tensile force of 2000 N. Determine the greatest load P that the truss can support.



FBD of Truss

Consider following geometry of the figure.



$$\tan \alpha_1 = \frac{1}{4}$$

$$\therefore \alpha_1 = \tan^{-1} \left( \frac{1}{4} \right) = 14.04^\circ$$

$$\alpha_2 = \tan^{-1} \left( \frac{4}{1} \right) = 45^\circ$$

Consider

$$\sum F_x = 0 \therefore R_{HA} = 0$$

$$\sum F_y = 0 \therefore R_{VA} + R_C = P \quad \text{--- (1)}$$

$$\sum M_A = 0$$

$$\therefore H_P - 8R_C = 0$$

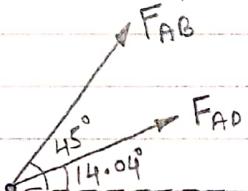
$$\therefore R_C = \frac{P}{2} \text{ N } (\uparrow) \quad \therefore R_{VA} = \left( \frac{P}{2} \right) \text{ N } (\uparrow)$$

Consider joint A. (Assuming all forces as Tensile)

$$\sum F_x = 0$$

$$\therefore F_{AD} \cos 14.04^\circ + F_{AB} \cos 45^\circ = 0 \quad \text{--- (II)}$$

$$\sum F_y = 0 \therefore \frac{P}{2} + F_{AD} \sin 14.04^\circ + F_{AB} \sin 45^\circ = 0 \quad \text{--- (III)}$$



Let  $F_{AD} = 2000 \text{ N (T)}$  Then from eqn (II) & (III)

$$F_{AB} = -2743.9 \text{ N} > 800 \text{ N (Not allowed)}$$

Let  $F_{AB} = -800 \text{ N (C)}$ , Then put this in eqn (II) & (III), we get

$$\therefore F_{AD} = 583 \text{ N} < 2000 \text{ N (Allowed)}$$

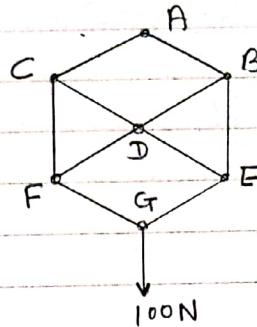
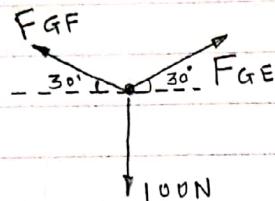
$\therefore$  from eqn (I)

$$P = 848.9 \text{ N} .$$

Determine the forces in members BE and BD of the truss which supports the load as shown in figure. All interior angles are  $60^\circ$  &  $120^\circ$



Consider joint G.



$$\sum F_x = 0 \quad \therefore F_{GE} \cos 30^\circ - F_{GF} \cos 30^\circ = 0$$

$$\therefore F_{GE} \cos 30^\circ = F_{GF} \cos 30^\circ$$

$$\therefore \boxed{F_{GE} = F_{GF}}$$

$$\therefore \sum F_y = 0 \quad \therefore F_{GE} \sin 30^\circ + F_{GF} \sin 30^\circ - 100 = 0$$

$$\therefore F_{GE} \sin 30^\circ + F_{GE} \sin 30^\circ - 100 = 0$$

$$\therefore \boxed{F_{GE} = 100 \text{ N (T)}}$$

Consider joint E

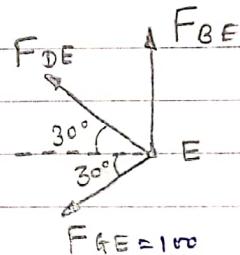
$$\sum F_x = 0$$

$$\therefore -F_{DE} \cos 30^\circ - F_{CE} \cos 30^\circ = 0$$

$$\therefore F_{DE} \cos 30^\circ = F_{CE} \cos 30^\circ = 100 \cos 30^\circ = 0$$

$$\therefore F_{DE} = -100 \text{ N}$$

$$\boxed{F_{DE} = 100 \text{ N (C)}}$$



$$\sum F_y = 0$$

$$\therefore F_{DE} \sin 30^\circ + F_{BE} \sin 30^\circ - 100 \sin 30^\circ = 0$$

$$(-100) \sin 30^\circ + F_{BE} \sin 30^\circ - 100 \sin 30^\circ = 0$$

$$\therefore \boxed{F_{BE} = 100 \text{ N (T)}}$$

Consider joint B.

$$\sum F_x = 0 \quad \therefore -F_{AB} \cos 30^\circ - F_{BD} \cos 30^\circ = 0$$

$$\therefore -F_{AB} - F_{BD} = 0 \quad \text{--- (1)}$$

$$\sum F_y = 0$$

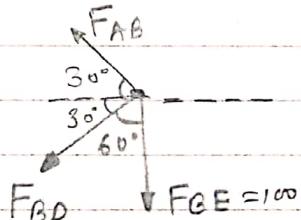
$$\therefore F_{AB} \sin 30^\circ - F_{BD} \sin 30^\circ - F_{BE} = 0$$

$$F_{AB} \sin 30^\circ - F_{BD} \sin 30^\circ = 100$$

$$(F_{AB} - F_{BD}) = 200 \quad \text{--- (2)}$$

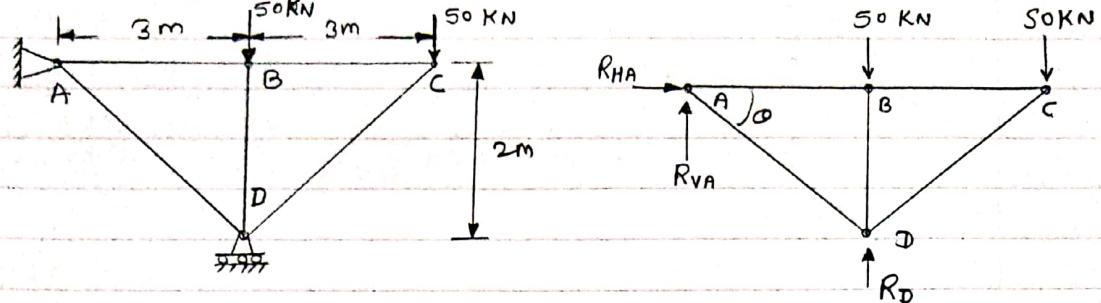
Solving (1) & (2)

$$\boxed{\begin{aligned} F_{AB} &= 100 \text{ N (T)} \\ F_{BD} &= 100 \text{ N (C)} \end{aligned}}$$



(S6)

Determine the force in each member of the truss as shown in fig. & tabulate the result with magnitude & nature of force in the members.



FBD of Truss

consider FBD of Truss,  
for equilibrium,  $\sum F_x = 0$

$$\therefore \boxed{R_{HA} = 0}$$

$$\therefore \sum F_y = 0$$

$$R_{VA} - 50 - 50 + R_D = 0$$

$$R_{VA} + R_D = 100$$

Taking moment @ point A,

$$\sum M_A = 0$$

$$-(R_D \times 3) + (50 \times 3) + (50 \times 6) = 0$$

$$-3R_D + 150 + 300 = 0$$

$$\therefore -3R_D = -450$$

$$\boxed{R_D = 150 \text{ KN}} \dagger$$

In  $\triangle ABD$

$$\tan \theta = \frac{2}{3}$$

$$\boxed{\theta = 33.7^\circ}$$

$$\therefore R_{VA} + R_D = 100$$

$$R_{VA} + 150 = 100$$

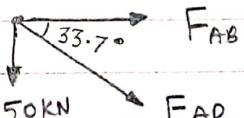
$$R_{VA} = -50$$

$$\boxed{R_{VA} = 50 \text{ KN}} \dagger$$

Consider joint A,

$$\sum F_x = 0,$$

$$F_{AB} + F_{AD} \cos 33.7^\circ = 0$$



$$\sum F_y = 0$$

$$\therefore -50 - F_{AD} \sin 33.7^\circ = 0$$

$$\therefore F_{AD} = \frac{50}{\sin 33.7^\circ}$$

$$\therefore F_{AD} = -90.11 \text{ KN} \quad (\dagger)$$

$$\therefore F_{AB} + (-90.11) \cos 33.7^\circ = 0 \quad \therefore F_{AB} = 74.96 \text{ KN} \quad (\text{T})$$

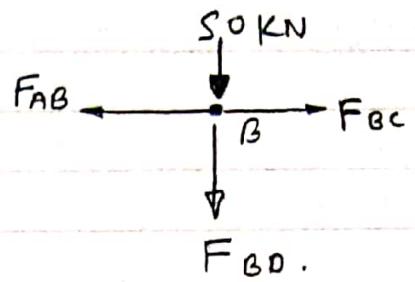
Consider joint B.

$$\sum F_x = 0$$

$$-F_{AB} + F_{BC} = 0$$

$$F_{BC} = F_{AB}$$

$$\therefore \boxed{F_{BC} = 74.96 \text{ KN}} \text{ (T)}$$



$$\sum F_y = 0$$

$$-50 - F_{BD} = 0$$

$$\therefore F_{BD} = -50 \text{ KN}$$

$$\boxed{F_{BD} = 50 \text{ KN}} \text{ (C)} .$$

Consider joint C :-

$$\sum F_x = 0$$

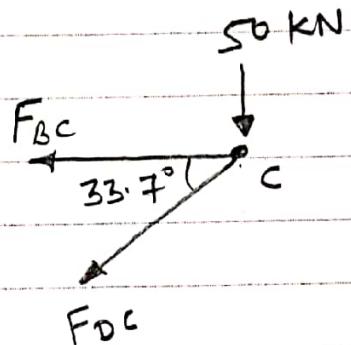
$$-F_{BC} - F_{DC} \cos 33.7^\circ = 0$$

$$-74.96 - F_{DC} \cos 33.7^\circ = 0$$

$$\therefore F_{DC} \cos 33.7^\circ = -74.96$$

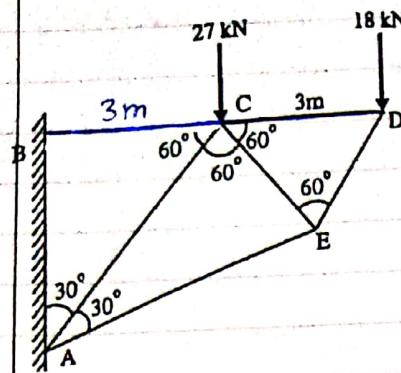
$$\therefore F_{DC} = -90.11 \text{ KN}$$

$$\boxed{F_{DC} = 90.11 \text{ KN}} \text{ (C)} .$$



Sl. No	member	Force	Nature
1	AB	74.96 KN	T
2	BC	74.96 KN	T
3	CD	90.11 KN	C
4	AD	90.11 KN	C
5	BD	50 KN	C

For given truss find forces in the members BC, AC and AE.



Cantilever Truss

For cantilever truss,  
Reactions can be  
calculated only after  
the calculation of forces  
in each member.

∴ consider joint D, for Equilibrium of joint.

$$\sum F_x = 0$$

$$-F_{CD} - F_{DE} \cos 60^\circ = 0 \quad \dots \textcircled{1}$$

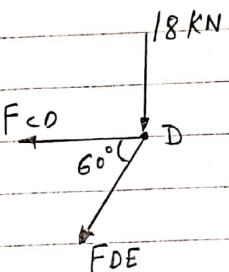
$$\sum F_y = 0$$

$$-18 - F_{DE} \sin 60^\circ = 0$$

$$\therefore -F_{DE} \sin 60^\circ = 18$$

$$F_{DE} = -20.78 \text{ KN}$$

$$\boxed{F_{DE} = 20.78 \text{ KN}} \quad (\text{C})$$



from eqn ①

$$-F_{CD} - (-20.78) \cos 60^\circ = 0$$

$$\therefore \boxed{F_{CD} = 10.39 \text{ KN}} \quad (\text{T})$$

consider joint E, for equilibrium of joint,

$$\sum F_x = 0$$

$$\therefore -F_{CE} \cos 60^\circ - F_{AE} \cos 30^\circ + F_{DE} \cos 60^\circ = 0$$

$$\therefore -0.5 F_{CE} - 0.866 F_{AE} + (-20.78 \times 0.5) = 0$$

$$\therefore -0.5 F_{CE} - 0.866 F_{AE} = 10.39 \quad \dots \textcircled{II}$$

$$\sum F_y = 0$$

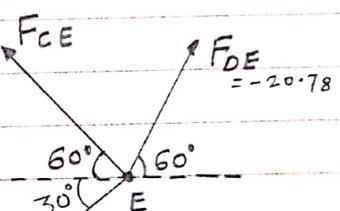
$$(-20.78 \sin 60^\circ) + F_{CE} \sin 60^\circ - F_{AE} \sin 30^\circ = 0$$

$$0.866 F_{CE} - 0.5 F_{AE} = 17.99 \quad \textcircled{III}$$

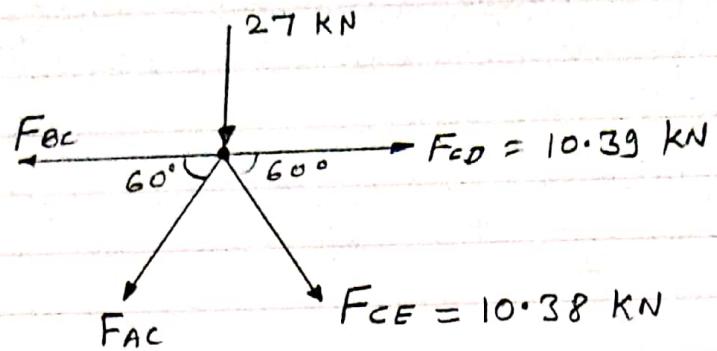
solving eqn ② & ③

$$\boxed{F_{CE} = 10.38 \text{ KN} \quad (\text{T})}$$

$$\boxed{F_{AE} = -17.99 \text{ KN} = 17.99 \text{ KN} \quad (\text{C})}$$



Consider joint C, for Equilibrium of the joint,



$$\sum F_x = 0$$

$$\therefore -F_{BC} + F_{CD} + F_{CE} \cos 60^\circ - F_{AC} \cos 60^\circ = 0$$

$$-F_{BC} + 10.39 + 10.38 \cos 60^\circ - F_{AC} \cos 60^\circ = 0$$

$$\therefore -F_{BC} - F_{AC} \cos 60^\circ = -15.58 \quad \text{--- (IV)}$$

$$\sum F_y = 0$$

$$-27 - F_{AC} \sin 60^\circ - F_{CE} \sin 60^\circ = 0$$

$$-27 - F_{AC} \sin 60^\circ - 10.38 \sin 60^\circ = 0$$

$$-F_{AC} \sin 60^\circ = 35.99$$

$$\therefore F_{AC} = -41.56 \text{ kN}$$

$$\boxed{F_{AC} = 41.56 \text{ kN} \quad (\text{C})}$$

from eqn (IV),

$$-F_{BC} - (-41.56 \cos 60^\circ) = -15.58$$

$$-F_{BC} = -15.58 - 20.78$$

$$-F_{BC} = -36.36 \text{ kN}$$

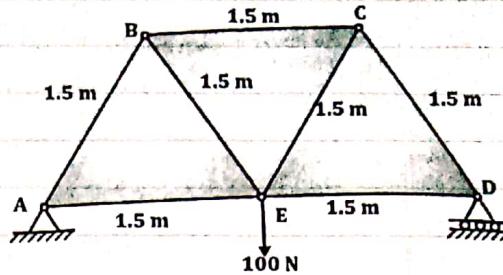
$$\therefore \boxed{F_{BC} = 36.36 \text{ kN}} \quad (\text{T}).$$

Member	BC	AC	AE
Force	36.36 kN	41.56 kN	17.99 kN
Nature	T	C	C

Determine the forces in the members of the truss loaded and supported as shown the Fig. Tabulate the result with magnitude and nature of force in the members.

Consider the FBD of Truss as shown below.

for equilibrium of Truss



$$\sum F_x = 0$$

$$\therefore R_{HA} = 0$$

Taking moment @ A

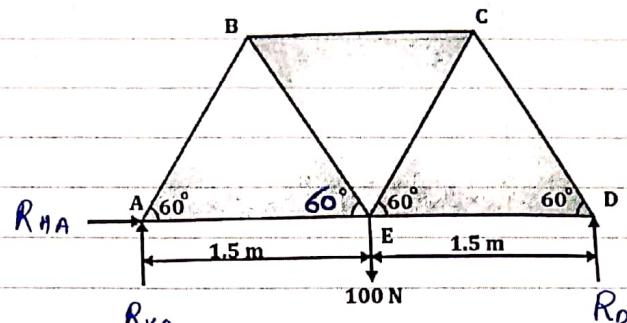
$$\sum M_A = 0$$

$$\therefore (100 \times 1.5) - (3 \times R_D) = 0$$

$$\therefore 150 - 3R_D = 0$$

$$\therefore 3R_D = 150$$

$$\therefore R_D = 50 \text{ N} \quad (\uparrow)$$



FBD of Truss

$$\sum F_y = 0$$

$$\therefore R_{VA} + R_D - 100 = 0$$

$$\therefore R_{VA} + 50 - 100 = 0$$

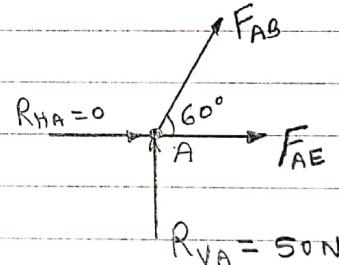
$$\therefore R_{VA} = 50 \text{ N} \quad (\uparrow)$$

Consider joint A,

for equilibrium of joint,

$$\sum F_x = 0$$

$$\therefore F_{AB} \cos 60 + F_{AE} = 0 \quad \dots \textcircled{1}$$



$$R_{VA} = 50 \text{ N}$$

$$\sum F_y = 0$$

$$\therefore R_{VA} + F_{AB} \sin 60 = 0$$

$$50 + F_{AB} \sin 60 = 0$$

$$F_{AB} = \frac{-50}{\sin 60}$$

$$F_{AB} = -57.73 \text{ N}$$

$$\boxed{F_{AB} = 57.73 \text{ N (C)}}$$

From eqn \textcircled{1}

$$-57.73 \cos 60 + F_{AE} = 0$$

$$\boxed{F_{AE} = 28.86 \text{ N}} \quad (\text{T})$$

(6)

Consider joint B, for the equilibrium of joint,

$$\sum F_x = 0$$

$$\therefore F_{BC} + F_{BE} \cos 60^\circ - F_{AB} \cos 60^\circ = 0$$

$$\therefore F_{BC} + F_{BE} \cos 60^\circ - (-57.73 \cos 60^\circ) = 0$$

$$\therefore F_{BC} + F_{BE} \cos 60^\circ = -28.86 \quad \text{--- (i)}$$

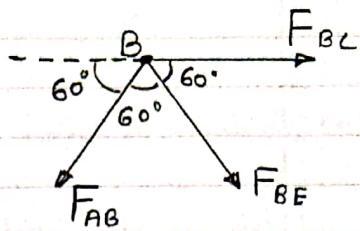
$$\sum F_y = 0$$

$$-F_{AB} \sin 60^\circ - F_{BE} \sin 60^\circ = 0$$

$$-(-57.73 \sin 60^\circ) = F_{BE} \sin 60^\circ$$

$$57.73 \sin 60^\circ = F_{BE} \sin 60^\circ$$

$$\therefore \boxed{F_{BE} = 57.73 \text{ N}} \quad (\text{T})$$



from eqn (i),

$$F_{BC} + 57.73 \cos 60^\circ = -28.86$$

$$F_{BC} + 28.86 = -28.86$$

$$F_{BC} = -57.73 \text{ N}$$

$$\boxed{F_{BC} = 57.73 \text{ N}} \quad (\text{C})$$

Consider joint C, for equilibrium of joint

$$\sum F_x = 0$$

$$\therefore -57.73 - F_{CE} \cos 60^\circ + F_{CD} \cos 60^\circ = 0$$

$$\therefore -F_{CE} \cos 60^\circ + F_{CD} \cos 60^\circ = 57.73 \quad \text{--- (ii)}$$

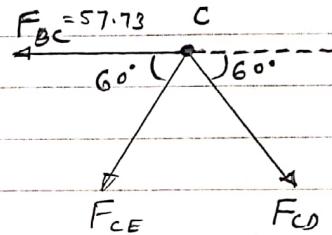
$$\sum F_y = 0$$

$$-F_{CE} \sin 60^\circ - F_{CD} \sin 60^\circ = 0 \quad \text{--- (iii)}$$

solving eqn (ii) and (iii)

$$\therefore F_{CE} = -57.73 \text{ N}$$

$$\therefore \boxed{F_{CE} = 57.73 \text{ N} \quad (\text{C})} \quad \& \quad \boxed{F_{CD} = 57.73 \text{ N} \quad (\text{T})}$$



Consider joint D, for equilibrium of joint,

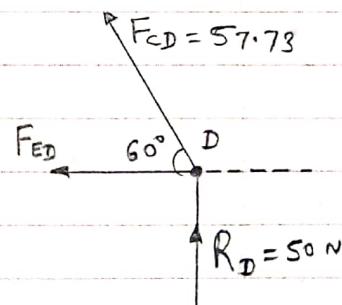
$$\sum F_x = 0$$

$$\therefore -F_{ED} - F_{CD} \cos 60^\circ = 0$$

$$\therefore -F_{ED} - 57.73 \cos 60^\circ = 0$$

$$\therefore F_{ED} = -28.86 \text{ N}$$

$$\boxed{F_{ED} = 28.86 \text{ N} \quad (\text{T})}$$



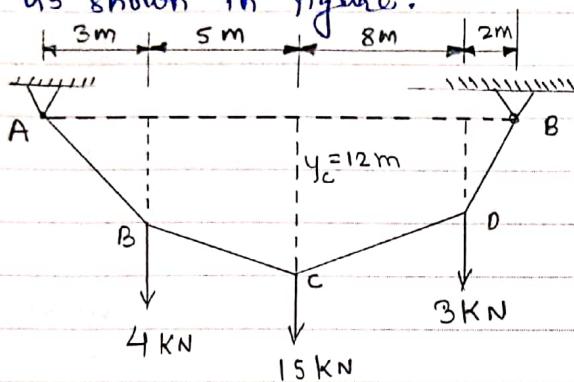
member	AB	BC	CD	DE	AE	BE	CE
force	57.73 N	57.73 N	57.73 N	28.86 N	28.86 N	57.73 N	57.73 N
nature	C	C	T	T	T	T	C

## \* Assumptions in cable Analysis :-

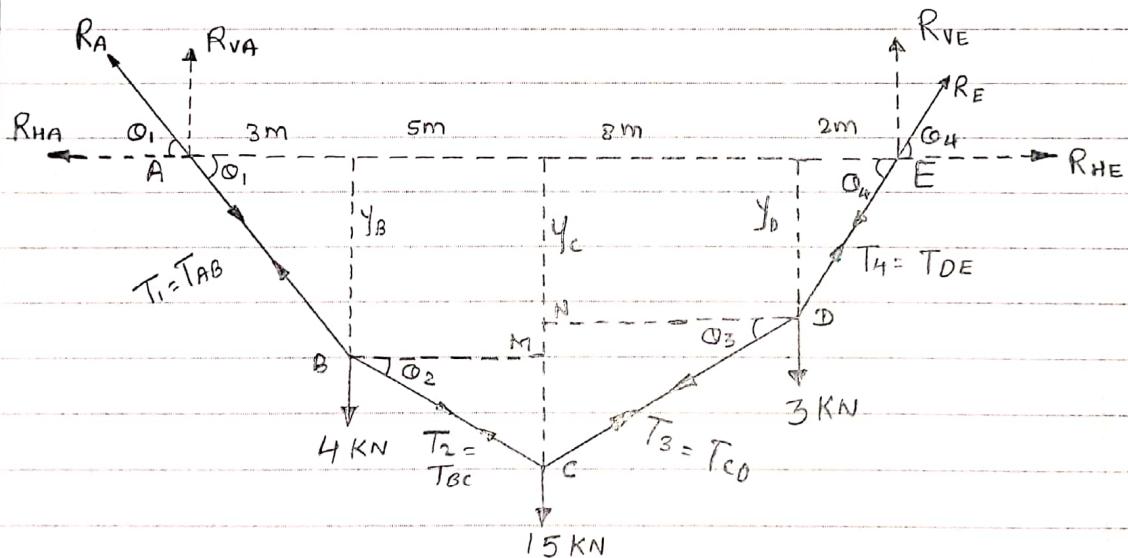
- 1) cables are considered as inextensible.
- 2) self weight of cable is neglected.
- 3) loads acting on cables are concentrated (point loads) load.
- 4) supports are hinged support.
- 5) deflected shape of cables b/w the two load points is a straight line.

## Numericals - Cables

- ① Determine the tensions in each segment of cable as shown in figure.



⇒ Draw FBD of cable system.



For the equilibrium of entire cable system,  $\sum M_A = 0$ .

∴ Taking moment about point A,

$$(4 \times 3) + [15 \times 8] + (3 \times 16) - (R_{VE} \times 18) = 0$$

$$12 + 120 + 48 - 18 R_{VE} = 0$$

$$180 = 18 R_{VE}$$

$$\therefore R_{VE} = 10 \text{ kN} \quad (1)$$

Using,  $\Sigma F_y = 0$

$$\therefore R_{VA} - 4 - 15 - 3 + R_{VE} = 0$$

$$R_{VA} + R_{VE} = 22 \quad \therefore \boxed{R_{VA} = 12 \text{ KN}} \quad (\uparrow)$$

$\therefore \Sigma F_x = 0$

$$R_{HE} - R_{HA} = 0$$

$$\therefore \boxed{R_{HE} = R_{HA}} \quad \dots \quad \textcircled{1}$$

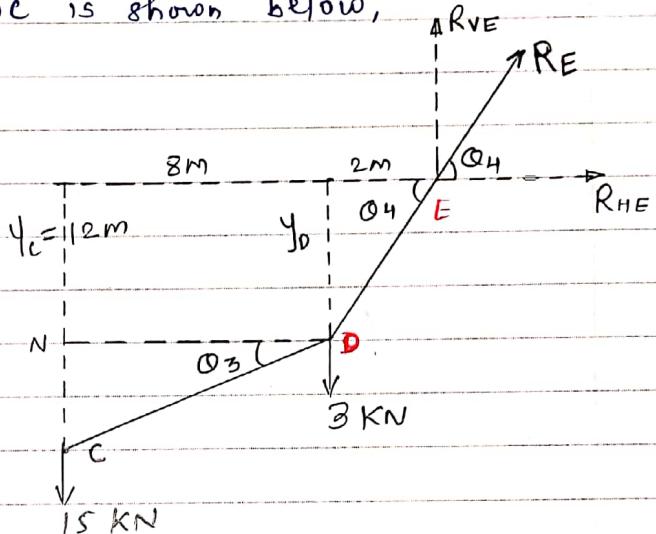
- Now As we have taken the moments about point A, Let us consider cable system EDC, because the sag of point C is known.

FBD of cable EDC is shown below,

for the equilibrium,

$$\sum M_C = 0$$

$\therefore$  Taking moments  
about point C,



$$(3 \times 8) - (R_{VE} \times 10) + (R_{HE} \times 12) = 0$$

$$24 - (10 \times 10) + 12 R_{HE} = 0$$

$$\therefore 12 R_{HE} = 76$$

$$\therefore \boxed{R_{HE} = 6.33 \text{ KN}} \quad (\rightarrow)$$

from eqn \textcircled{1}

$$\boxed{R_{HA} = 6.33 \text{ KN}} \quad (\leftarrow)$$

- Now let us find out the angles made by cable with Horizontal. i.e.  $\theta_1, \theta_2, \theta_3, \theta_4$ , & sag of each point.

$$\therefore \tan \theta_1 = \frac{R_{VA}}{R_{HA}} = \frac{Y_B}{3}$$

$$\therefore \theta_1 = \tan^{-1} \left( \frac{R_{VA}}{R_{HA}} \right) = \tan^{-1} \left( \frac{12}{6.33} \right) = 62.19^\circ$$

$$\therefore \tan \theta_1 = \frac{Y_B}{3}$$

$$\therefore \tan 62.19^\circ = \frac{Y_B}{3} \quad \therefore \boxed{Y_B = 5.69 \text{ m}}$$

$$\tan \Theta_2 = \frac{cm}{Bm} = \frac{y_c - y_B}{Bm} = \frac{12 - 5.69}{5} = \frac{6.31}{5}$$

$$\therefore \Theta_2 = \tan^{-1}\left(\frac{6.31}{5}\right) \quad \therefore \boxed{\Theta_2 = 51.61^\circ}$$

$$\text{Now, } \tan \Theta_4 = \frac{R_{VE}}{R_{HE}} = \frac{y_D}{2}$$

$$\therefore \tan \Theta_4 = \frac{R_{VE}}{R_{HE}} = \frac{10}{6.33}$$

$$\Theta_4 = \tan^{-1}\left(\frac{10}{6.33}\right) = 57.67^\circ \quad \therefore \boxed{\Theta_4 = 57.67^\circ}$$

$$\therefore \tan 57.67 = \frac{y_D}{2}$$

$$\therefore \boxed{y_D = 3.15 \text{ m}}$$

$$\text{Also, } \tan \Theta_3 = \frac{CN}{DN} = \frac{y_c - y_D}{DN} = \frac{12 - 3.15}{8} = \frac{8.85}{8}$$

$$\therefore \Theta_3 = \tan^{-1}\left(\frac{8.85}{8}\right) \quad \therefore \boxed{\Theta_3 = 47.9^\circ}$$

Now let us find Tensions in each segment of cables.

$$\therefore T_1 = T_{AB} = R_A = \sqrt{(R_{HA})^2 + (R_{VA})^2}$$

$$\therefore T_1 = T_{AB} = R_A = \sqrt{6.33^2 + 12^2}$$

$$\therefore \boxed{T_1 = T_{AB} = R_A = 13.57 \text{ KN}}$$

Consider the equilibrium of point C,

using Lami's Theorem,

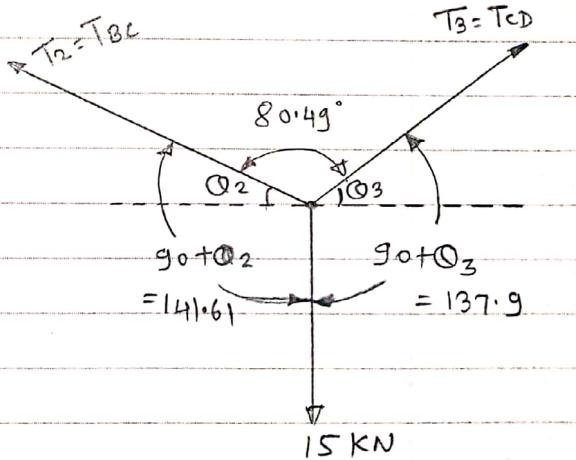
$$\frac{15}{\sin 80.49} = \frac{T_2}{\sin 137.9} = \frac{T_3}{\sin 141.61}$$

$$\therefore T_2 = \frac{15 \sin 137.9}{\sin 80.49}$$

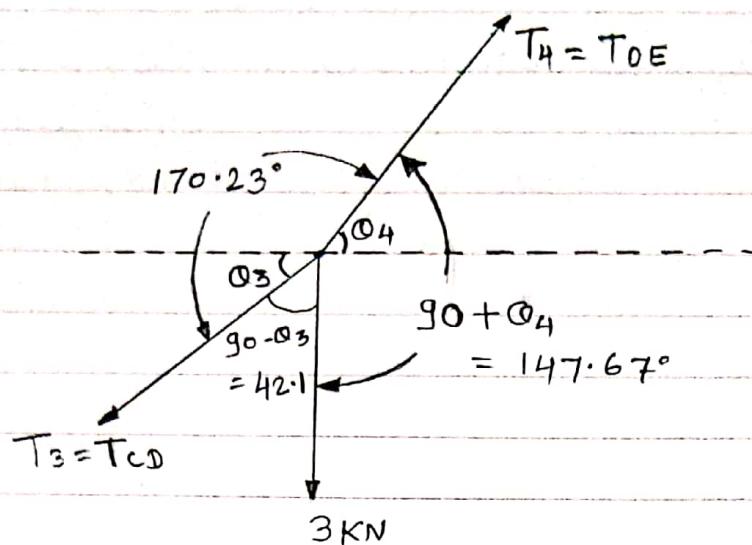
$$\therefore \boxed{T_2 = 10.2 \text{ KN}}$$

$$T_3 = \frac{15 \sin 141.61}{\sin 80.49}$$

$$\therefore \boxed{T_3 = 9.45 \text{ KN}}$$



Consider equilibrium of point D,



Using Lami's Theorem,

$$\frac{3}{\sin 170.23} = \frac{T_3}{\sin 147.67} = \frac{T_4}{\sin 42.1}$$

$$\therefore T_3 = \frac{3 \sin 147.67}{\sin 170.23} = 9.45$$

$$\therefore [T_3 = 9.45 \text{ kN}]$$

$$\therefore T_4 = \frac{3 \sin 42.1}{\sin 170.23}$$

$$\therefore [T_4 = 11.85 \text{ kN}]$$

$$\therefore T_1 = T_{AB} = 13.57 \text{ kN.}$$

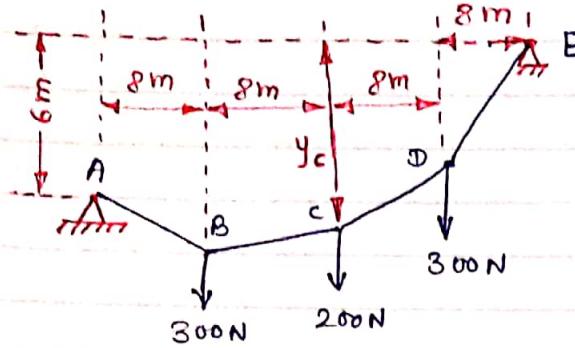
$$T_2 = T_{BC} = 10.2 \text{ kN}$$

$$T_3 = T_{DC} = 9.45 \text{ kN}$$

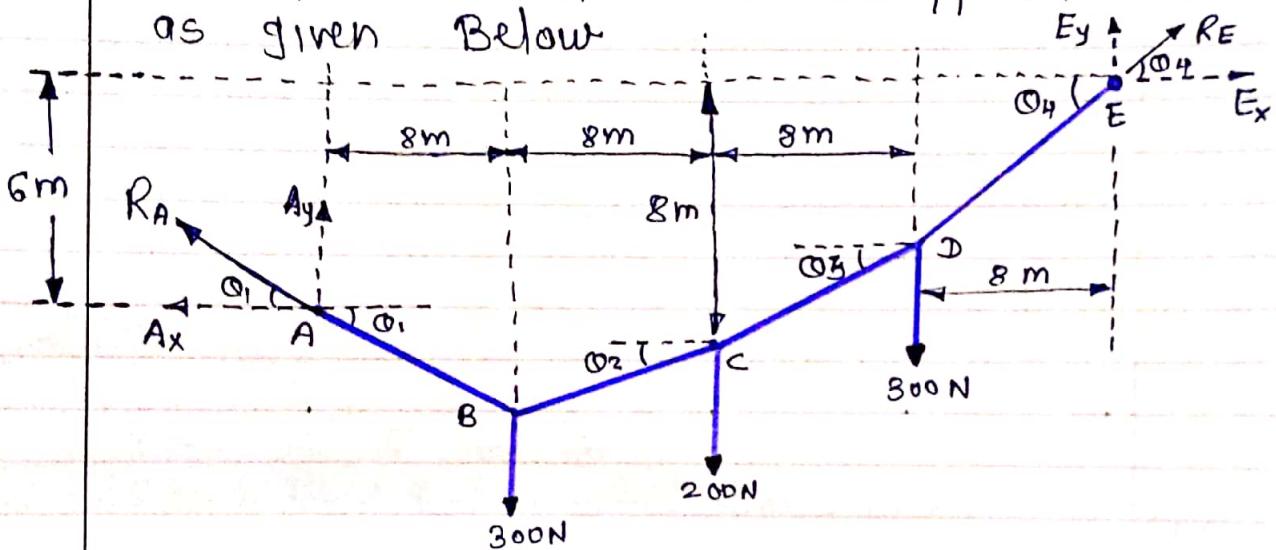
$$T_4 = T_{DE} = 11.85 \text{ kN}$$

IF  $y_c = 8\text{ m}$ , determine Reaction at A & E &

Max. Tension in  
the cable.



FBD for the cable loaded & supported will be  
as given Below



For equilibrium of cable,

$$\sum M_A = 0$$

$$\therefore (300 \times 8) + (200 \times 16) + (300 \times 24) - (32 E_y) + (6 E_x) = 0$$

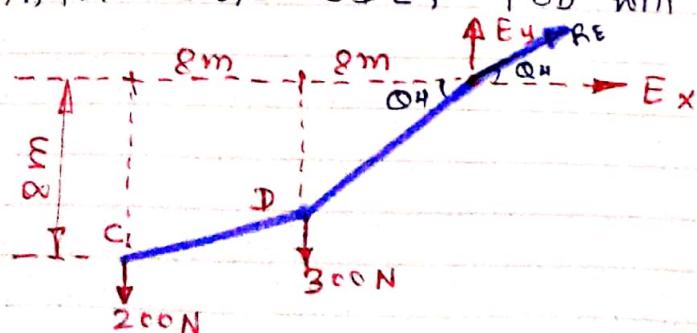
$$\therefore 6 E_x - 32 E_y = -12800 \quad \dots \dots \textcircled{1}$$

$$\therefore \sum F_y = 0 \quad \therefore A_y + E_y - 300 - 200 - 300 = 0$$

$$\therefore A_y + E_y = 800 \quad \dots \dots \textcircled{2}$$

$$\therefore \sum F_x = 0 \quad \therefore -A_x + E_x = 0 \quad \therefore E_x = A_x \quad \dots \dots \textcircled{3}$$

As distance of point C is known, let us  
consider cable CDE, FBD will be as below



For equilibrium of cable [CDE],

$$\therefore \sum M_C = 0$$

$$(300 \times 8) + (8E_x) - (16E_y) = 0$$

$$8E_x - 16E_y = -2400 \quad \dots \quad (4)$$

$$\therefore 6E_x - 32E_y = -12800 \quad \dots \quad (5)$$

$$\therefore 8E_x - 16E_y = -2400 \quad \dots \quad (4)$$

Solving (5) & (4) simultaneously.

$$\begin{aligned} \therefore E_x &= 800 \text{ N} \rightarrow \\ E_y &= 550 \text{ N} \uparrow \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Ans}$$

$\therefore R_E$  = Reaction at E.

$$\therefore R_E = \sqrt{E_x^2 + E_y^2}$$

$$\therefore \boxed{R_E = 970.82 \text{ N}}$$

Direction of  $R_E$ ,

$$\Theta_4 = \tan^{-1} \left[ \frac{E_y}{E_x} \right]$$

$$\therefore \boxed{\Theta_4 = 34.5^\circ}$$

From eqn (2) and (3)

$$\begin{aligned} \therefore A_y + E_y &= 800 \quad \therefore A_y = 250 \text{ N} \uparrow \\ &\text{ &} \\ \therefore A_x &= E_x \quad \therefore A_x = 800 \text{ N} \leftarrow \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Ans}$$

$\therefore$  Reaction at A,  $R_A = \sqrt{A_x^2 + A_y^2}$

$$\therefore \boxed{R_A = 838.15 \text{ N}}$$

Direction of  $R_A$ ,

$$\Theta_1 = \tan^{-1} \left[ \frac{A_y}{A_x} \right]$$

$$\therefore \boxed{\Theta_1 = 17.35^\circ}$$

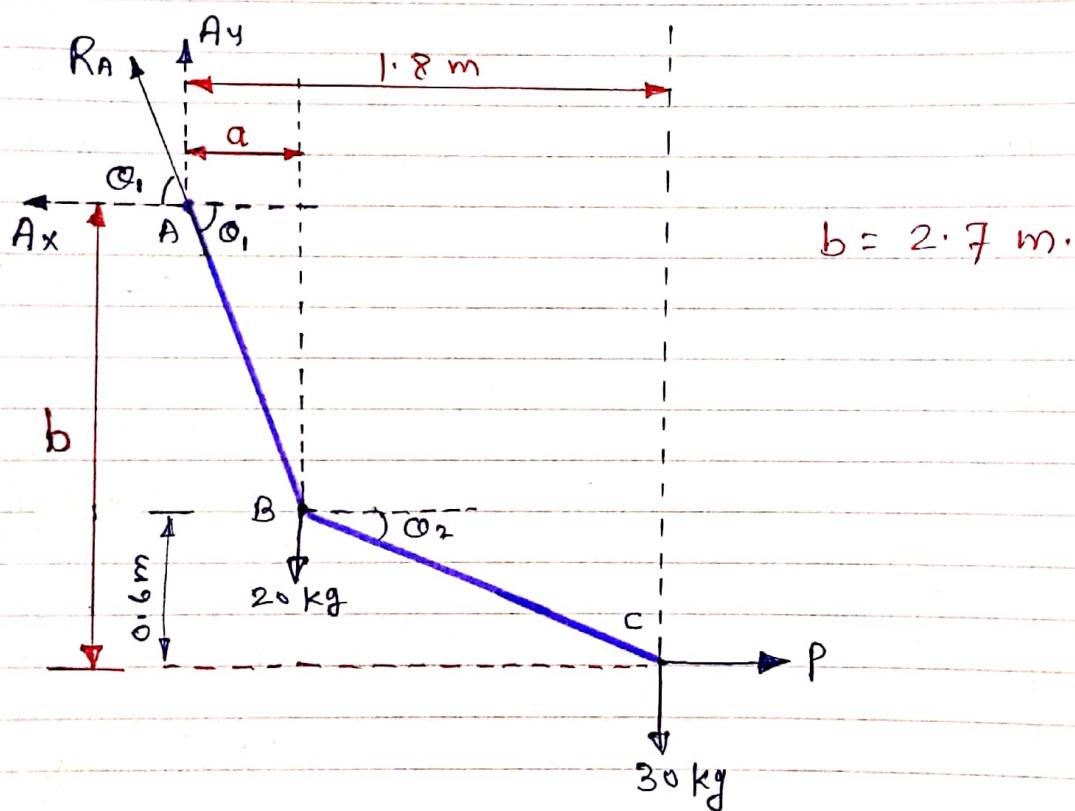
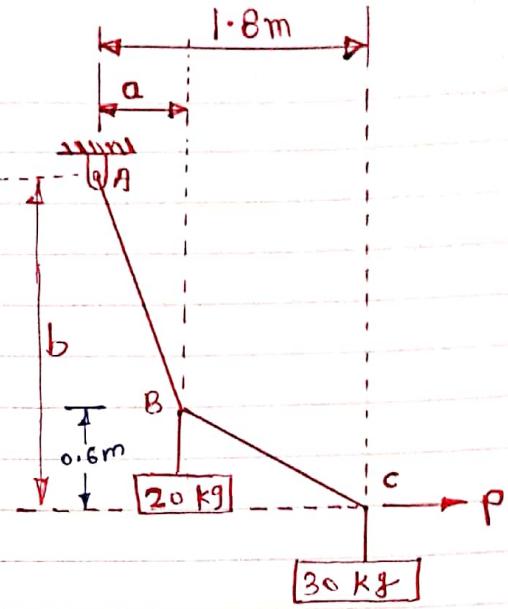
As,  $E_y > A_y$ , max. tension will occur in cable part DE.

$$\therefore \boxed{T_{DE} = R_E = 970.82 \text{ N}} \quad \text{- max. Tension.}$$

CABLE ABC supports two boxes as shown in figure. knowing that  $b = 2.7\text{ m}$ , find the required magnitude of Horizontal force  $P$  at corresponding distance 'a'.



FBD of cable system is given below



for equilibrium of cable,

$$\therefore \sum M_A = 0$$

$$\therefore (20 \times 9.81 \times a) + (30 \times 9.81 \times 1.8) - (P \times 2.7) = 0$$

$$\therefore 196.2a - 2.7P = -529.74 \quad \dots \textcircled{1}$$

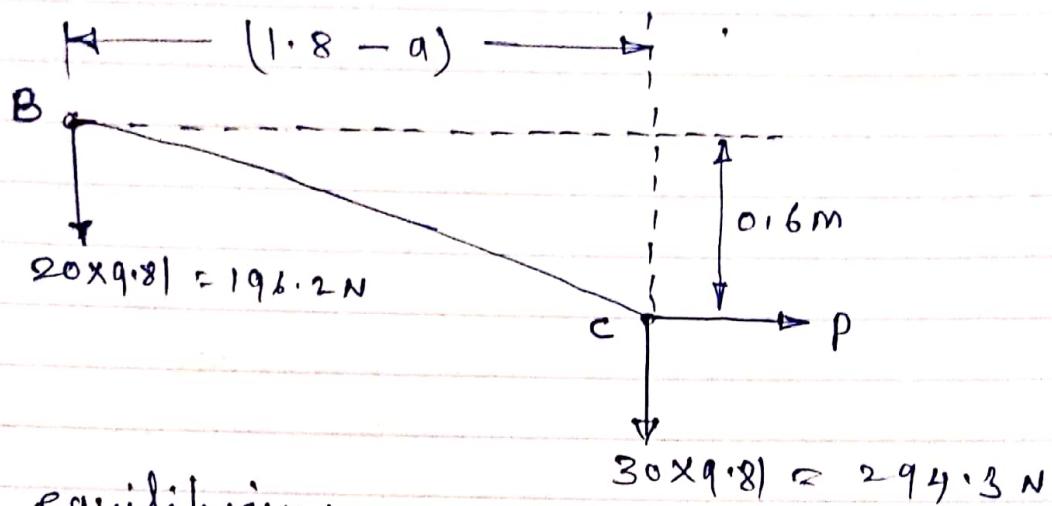
$$\sum F_y = 0 \quad \therefore A_y - (20 \times 9.81) - (30 \times 9.81) = 0$$

$$\therefore \boxed{A_y = 490.5 \text{ N} \uparrow}$$

$$\sum F_x = 0 \quad \therefore -A_x + P = 0$$

$$\therefore \boxed{A_x = P}$$

Now, let us consider cable part BC.  
FBD is shown below.



for equilibrium,

$$\sum M_B = 0$$

$$\therefore (196.2 \times 0) + [294.3 \times (1.8 - a)] - 0.6 P = 0$$

$$\therefore 529.74 - 294.3a - 0.6 P = 0$$

$$\therefore -294.3a - 0.6 P = -529.74$$

$$\therefore 294.3a + 0.6 P = 529.74 \quad \text{--- (2)}$$

Solving eqn ① & ②

$$196.2a - 2.7P = -529.74$$

$$294.3a + 0.6P = 529.74$$

Solving; we get,

$$a = 1.22 \text{ m}$$

$$P = 284.81 \text{ N}$$

## Frames

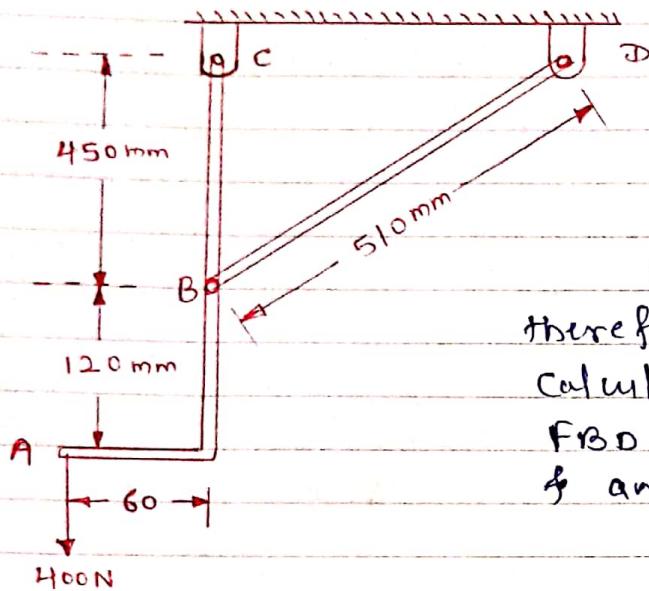
### \* Analysis of Frames \*

- Frame is a structure which consists of two or more members connected to each other by using pin joint. In the frame at least one member is multiforce member.
- Frames are stationary structure used to carry the load only.
- Unlike the truss, forces can be applied anywhere on the members of frame.
- In truss forces are applied at the joint only.
- Members of frame are subjected to bending as well as tension & compression.
- In the frame, forces acting on the members may not be co-linear with the axis of members.

### \* Difference between Frame and Truss:-

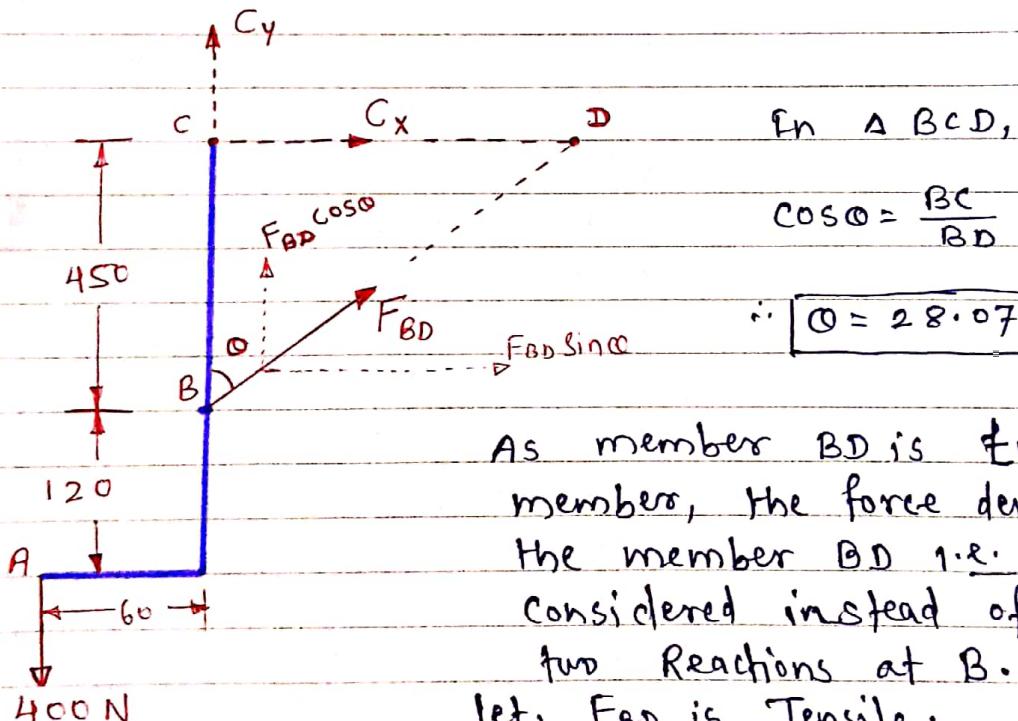
Truss	Frame
1. Stationary structure which consist of only two-force members.	stationary structure which consist of various members out of which at least one must be multiforce member
2. All joints are pin joints.	Here also all joints are pins.
3. Loads are applied at the joint only.	Loads are applied anywhere on the member.
4. for analysis, FBD of each joint is considered	for analysis, FBD of each member is considered

Determine the forces in member BD of the frame as shown in Figure. Also find Reaction at C.



As in this frame, there are 2 hinge supports therefore, 4 reactions can not be calculated. So, consider the FBD of each member directly & analyze the frame.

$\Rightarrow$  FBD of member ABC will be as given below.



As member BD is two force member, the force developed in the member BD i.e.  $F_{BD}$  is considered instead of two Reactions at B.

Let,  $F_{BD}$  is Tensile.

For equilibrium,

$$\therefore \sum M_c = 0 \quad \therefore -[(F_{BD} \sin \theta) \times 450] - (400 \times 60) = 0$$

$$\therefore -(F_{BD} \sin 28.07 \times 450) - 24000 = 0$$

$$-211.75 F_{BD} = 24000$$

$$\therefore F_{BD} = -113.34 \text{ N}$$

$$\therefore \sum F_y = 0 \quad \therefore -400 + C_y + F_{BD} \cos 28.07 = 0$$

$$\therefore -400 + C_y + (-113.34 \cos 28.07) = 0$$

$$\therefore C_y = 500 \text{ N}$$

$$\therefore \Sigma F_x = 0$$

$$\therefore F_{BD} \sin \alpha + C_x = 0$$

$$\therefore (-113.34 \sin 28.07) + C_x = 0$$

$$\therefore [C_x = 53.33 \text{ N}]$$

As  $F_{BD}$  is negative, our assumption is wrong about the nature of  $F_{BD}$  force.

Thus  $F_{BD}$  is not tensile but it is compressive force.

i. Ans.

$$F_{BD} = 113.34 \text{ N (c)}$$

$$C_y = 500 \text{ N } (\uparrow)$$

$$C_x = 53.33 \text{ N } (\rightarrow)$$

Reaction at C,

$$R_c = \sqrt{C_x^2 + C_y^2}$$

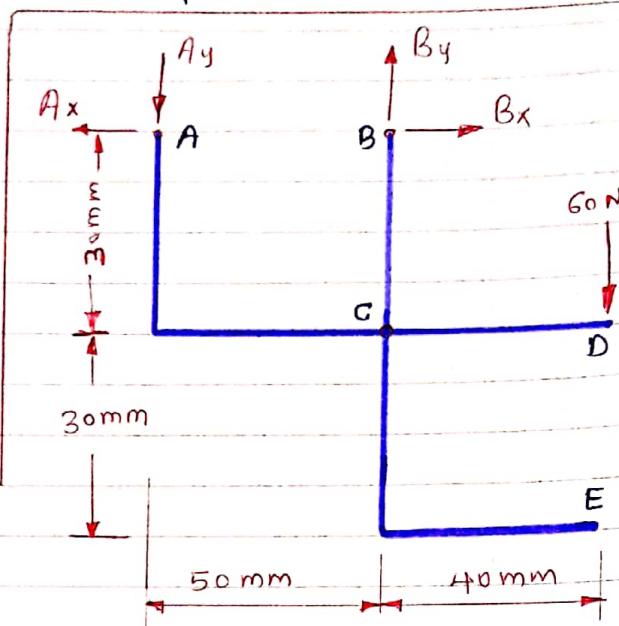
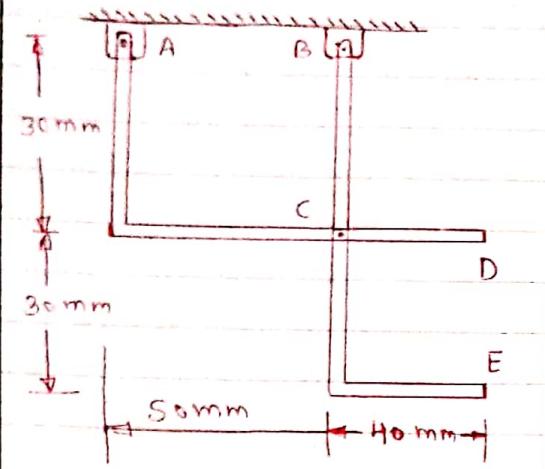
$$\therefore [R_c = 502.84 \text{ N}]$$

Angle of Reaction  $R_c$

$$\alpha = \tan^{-1} \left( \frac{C_y}{C_x} \right)$$

$$[\alpha = 83.91^\circ]$$

Determine the components of Reactions at A and B if 60N load is applied at point D.



FBD of Frame  $\Rightarrow$

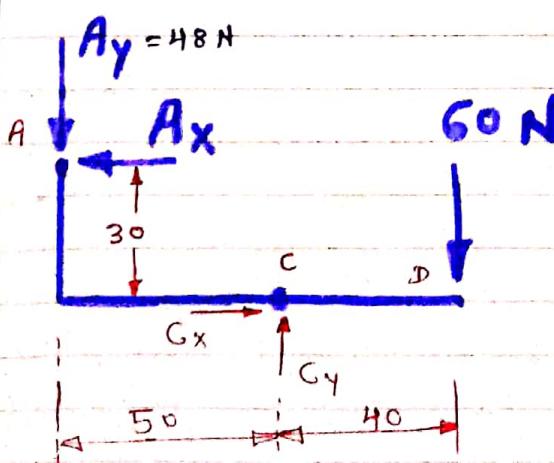
For the equilibrium of entire frame,  
applying conditions of equilibrium,

$$\therefore \sum M_A = 0 \quad \therefore -(50 B_y) + (60 \times 90) = 0$$

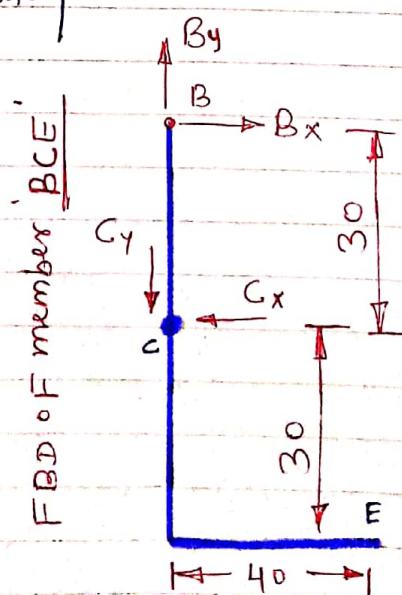
$$\therefore B_y = 108 \text{ N} \uparrow$$

$$\begin{aligned} \therefore \sum F_y &= 0 \quad \therefore -A_y + B_y - 60 = 0 \\ &\therefore -A_y + 108 - 60 = 0 \\ &\therefore -A_y + 48 = 0 \\ &\therefore A_y = 48 \text{ N} \downarrow \end{aligned} \quad \begin{aligned} \therefore \sum F_x &= 0 \\ &\therefore -A_x + B_x = 0 \\ &\therefore B_x = A_x \end{aligned}$$

Now let us separate the frame members and draw FBD of each member separately.



FBD of member ACD.



FBD of member BCE

FBD of members ACD & BCE are shown above.  
forces acting (Reactions) at A, B, C are also shown.

Let us apply conditions of equilibrium to member 'ACD'.

$$\therefore \sum M_C = 0$$

$$\therefore -(A_y \times 50) - (A_{gc} \times 30) + (60 \times 40) = 0$$

$$\therefore - (48 \times 50) - (30 A_x) + 2400 = 0$$

$$\therefore - 2400 - 30 A_x + 2400 = 0$$

$$\therefore \boxed{A_x = 0}$$

$$\therefore \sum F_y = 0$$

$$\therefore -A_y + C_y - 60 = 0$$

$$\therefore -48 + C_y - 60 = 0$$

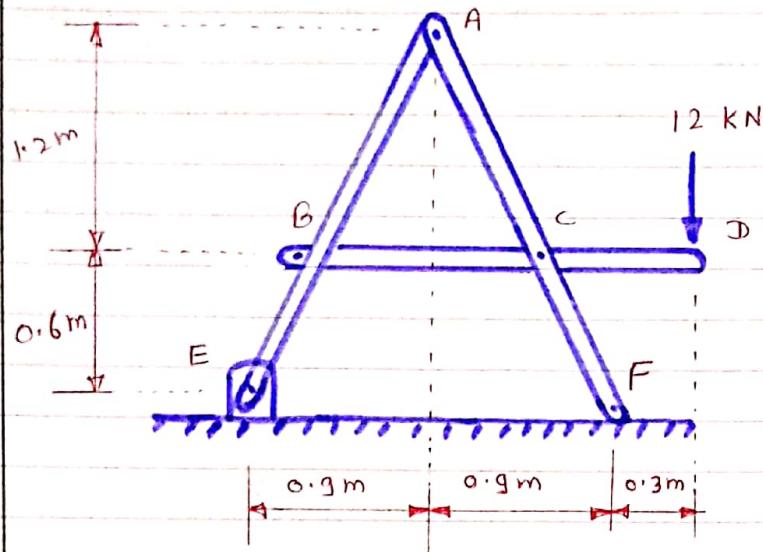
$$\therefore \boxed{C_y = 108 \text{ N } \uparrow}$$

$$\therefore \sum F_x = 0$$

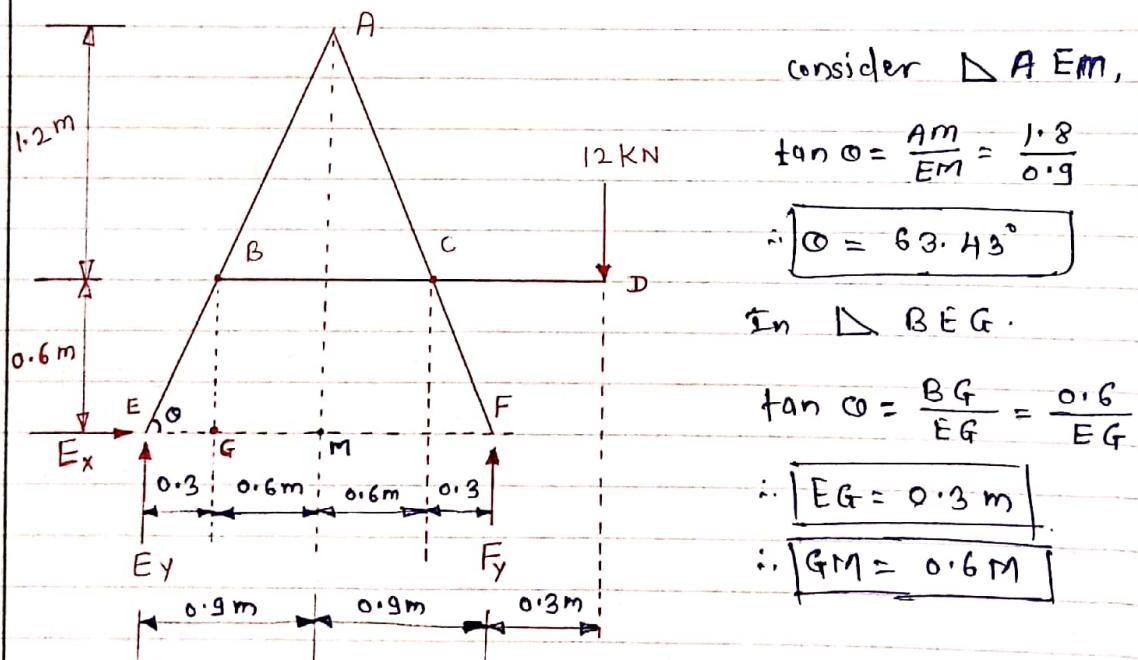
$$\therefore -A_x + C_x = 0$$

$$\therefore \boxed{C_x = 0}$$

For the Frame loaded and supported as shown in figure, Determine the components of all forces acting on frame member ABE.



→ consider FBD of the given frame as shown below



for equilibrium of frame,

$$\therefore \sum M_E = 0 \quad \therefore -(1.8 F_y) + (12 \times 2.1) = 0$$

$$\therefore F_y = 14 \text{ kN } \uparrow$$

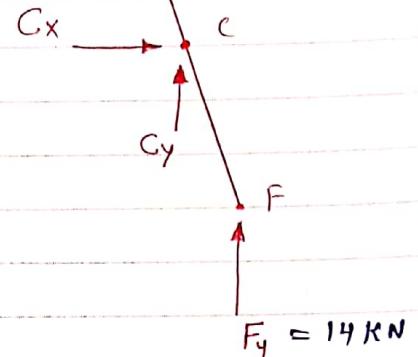
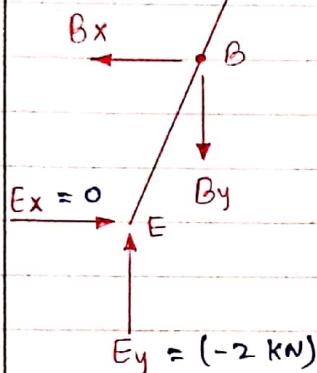
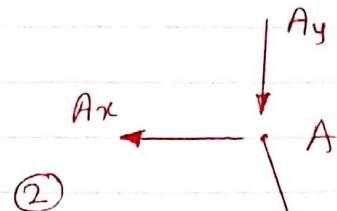
$$\therefore \sum F_y = 0$$

$$\therefore E_y + F_y - 12 = 0$$

$$\therefore E_y + 14 - 12 = 0$$

$$\therefore E_y = -2 \text{ kN} = 2 \text{ kN } \downarrow$$

$$\therefore \sum F_x = 0 \quad \therefore \boxed{E_x = 0}$$



FBD of member ABE

FBD of member

ACF

③



FBD of member BCD

Consider FBD of member BCD,

for equilibrium,

$$\therefore \sum M_B = 0$$

$$\therefore 1.2 C_y + (12 \times 1.8) = 0$$

$$\therefore \boxed{C_y = -18 \text{ kN} = 18 \text{ kN} \uparrow}$$

$$EF_y = 0 \quad \therefore B_y - C_y - 12 = 0$$

$$\therefore B_y - (-18) - 12 = 0$$

$$\therefore B_y + 6 = 0$$

$$\therefore \boxed{B_y = -6 \text{ kN} = 6 \text{ kN} \downarrow}$$

$$EF_x = 0 \quad \therefore B_x - C_x = 0 \quad \therefore \boxed{B_x = C_x} - ①$$

\* Now consider FBD of member ABE,  
for equilibrium,

$$\therefore \sum M_A = 0$$

$$\therefore - (B_y \times 0.6) + (E_y \times 0.9) + (B_x \times 1.2) - (E_x \times 1.8) = 0$$

$$\therefore - 0.6 B_y + 0.9 E_y + 1.2 B_x = 0$$

$$\therefore - 0.6 (-6) + 0.9 (-2) + 1.2 B_x = 0$$

$$\therefore \boxed{B_x = -1.5 \text{ KN} \rightarrow}$$

$$\sum F_y = 0 \quad \therefore E_y - B_y + A_y = 0$$

$$\therefore (-2) - (-6) + A_y = 0$$

$$\therefore -2 + 6 + A_y = 0$$

$$\therefore \boxed{A_y = -4 \text{ KN} = 4 \text{ KN} \downarrow}$$

$$\sum F_x = 0 \quad \therefore A_x - B_x = 0$$

$$\therefore \boxed{A_x = -1.5 \text{ KN} = 1.5 \text{ KN} \rightarrow}$$

$$\therefore \boxed{C_x = -1.5 \text{ KN} = 1.5 \text{ KN} \rightarrow} \quad \text{from eq ⑤}$$

\* Consider FBD of member ACF.  
(just for check)

$$\therefore \sum F_x = 0$$

$$\therefore -A_x + C_x = 0$$

$$\therefore -(-1.5) + (-1.5) = 0$$

$$\therefore 0 = 0 \quad \therefore \text{LHS} = \text{RHS.}$$

$$\sum F_y = 0$$

$$\therefore C_y - A_y + F_y = 0$$

$$\therefore -18 - (-4) + 14 = 0$$

$$\therefore -18 + 18 = 0$$

$$0 = 0 \quad \therefore \text{LHS} = \text{RHS.}$$

Thus, in member ACF,  $\sum F_x = 0$   
 $\sum F_y = 0$

$\therefore$  It is in equilibrium.