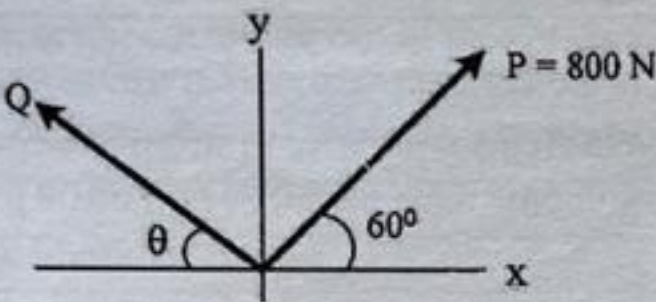
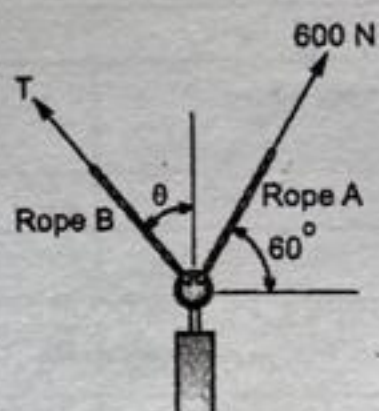
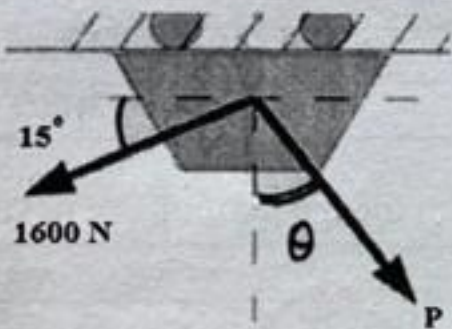
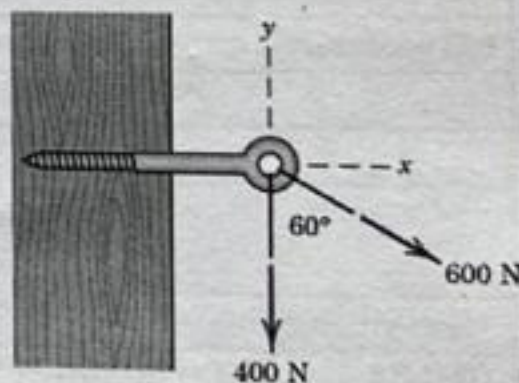
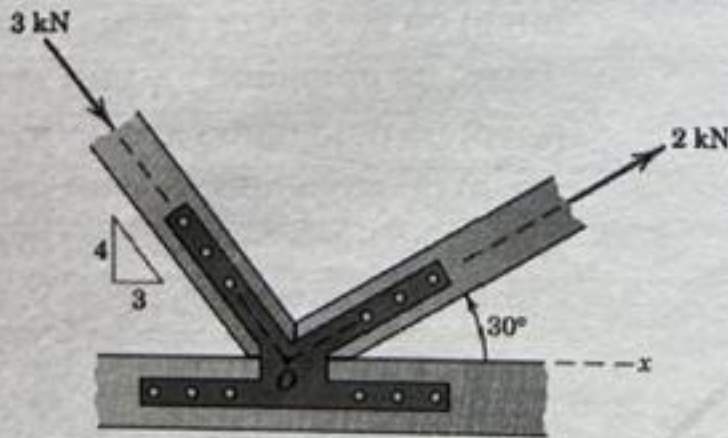
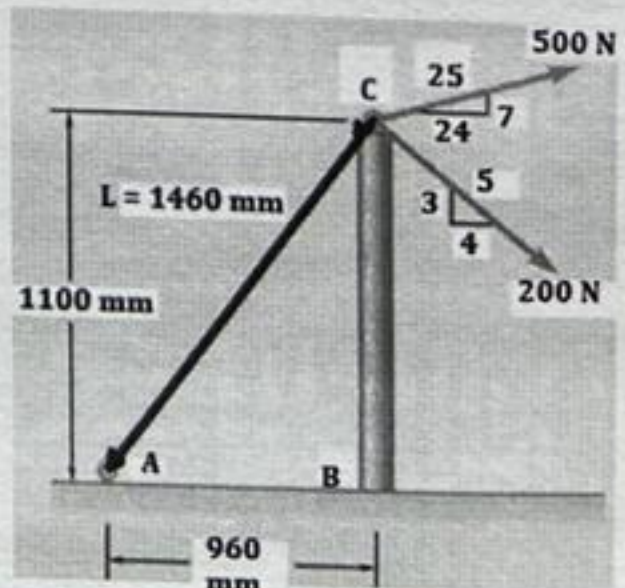


Unit wise Question Bank

Subject: EM

Unit No. 1: Resolution and composition of forces

Class: F.E.

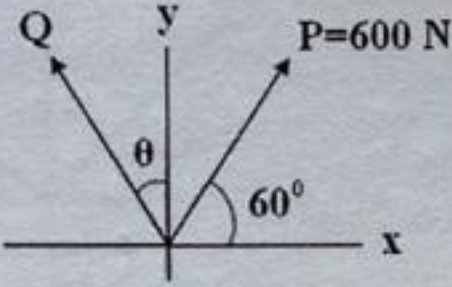
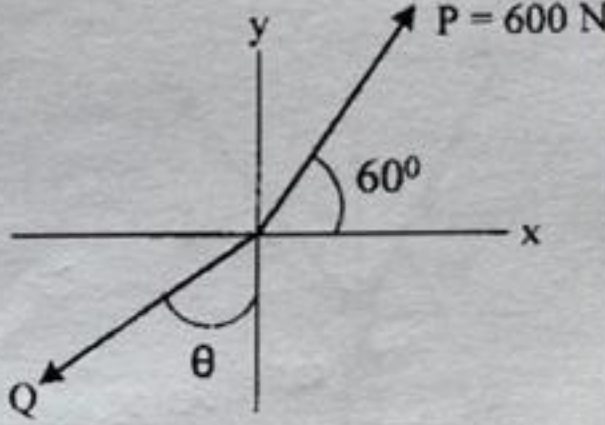
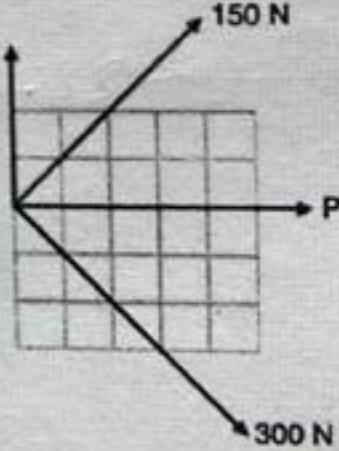
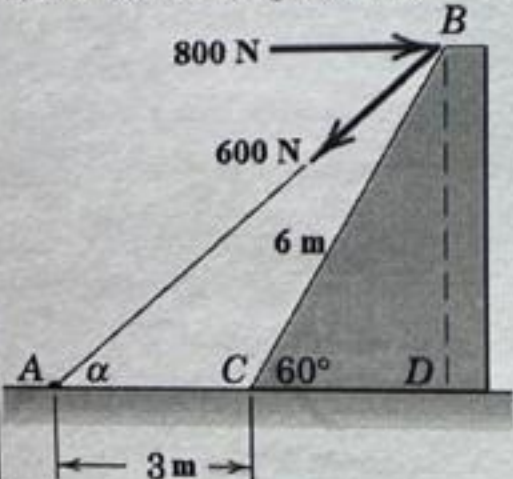
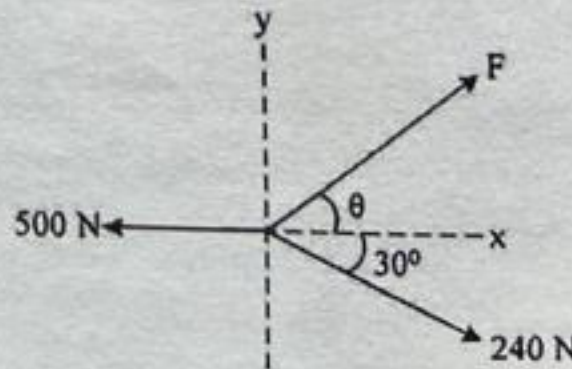
Que. No	Question Statement	Que. No	Question Statement
7	<p>The resultant of two forces P and Q is 1400 N vertical. Determine the force Q and the corresponding angle θ for the system of forces as shown in Fig.</p> 	8	<p>The post is to be pulled out of the ground using two ropes A and B. Rope A is subjected to a force of 600 N and is directed at 60° from the horizontal. If the resultant force acting on the post is 1200 N vertically upward, determine force T in rope B and corresponding angle θ.</p> <p>Ans: $\theta = 23.79^\circ$ $R = 743.7 \text{ N}$</p> 
9	<p>A trolley is acted upon by two forces as shown Fig. If $\theta = 25^\circ$ and the resultant R of the two forces is vertical, then determine the magnitude of the force P and resultant R.</p> <p>Ans:</p> 	10	<p>Determine the resultant R of the two forces shown by summing scalar components.</p> <p>Ans:</p> 
11	<p>The two structural members, one of which is in tension and the other in compression, exert the indicated forces on joint O. Determine the magnitude of the resultant R of the two forces and the angle which R makes with the positive x-axis.</p> 	12	<p>Knowing that the tension in rope AC is 365 N, determine the resultant of the three forces exerted at point C of post BC.</p> <p>Ans:</p> 

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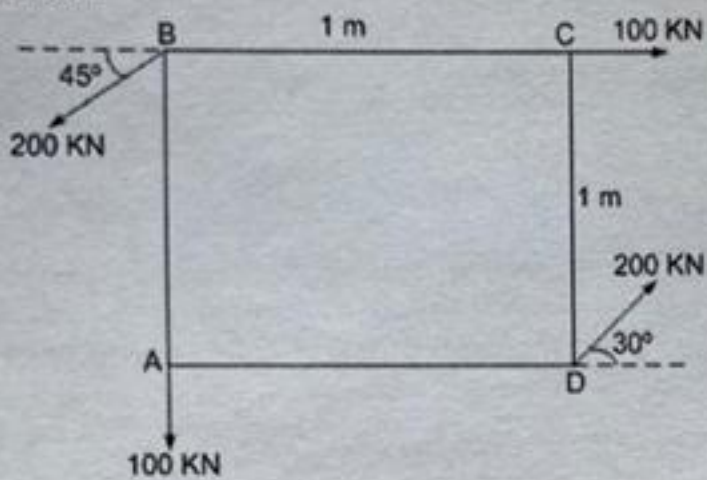
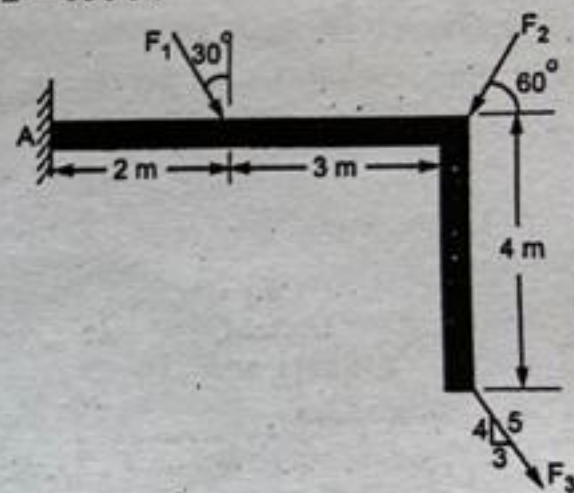
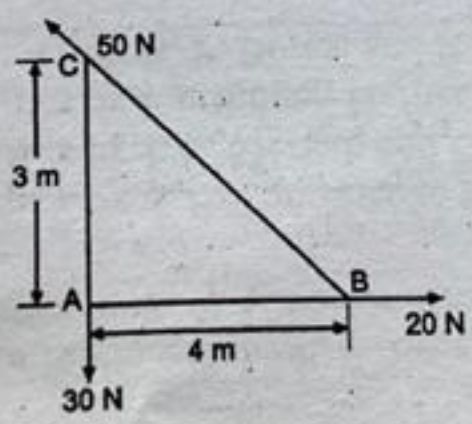
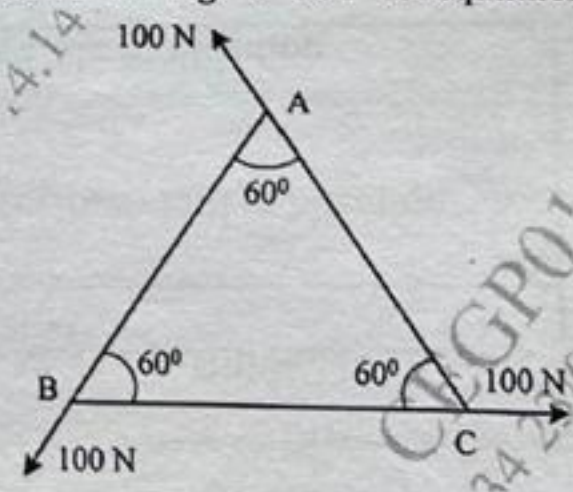
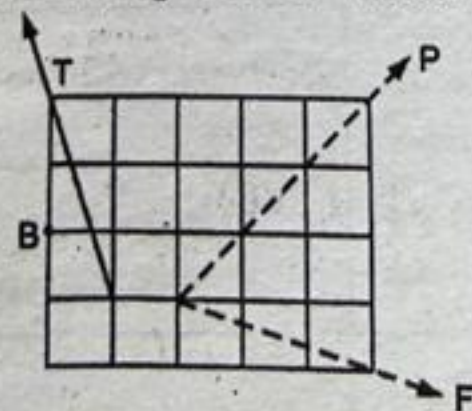
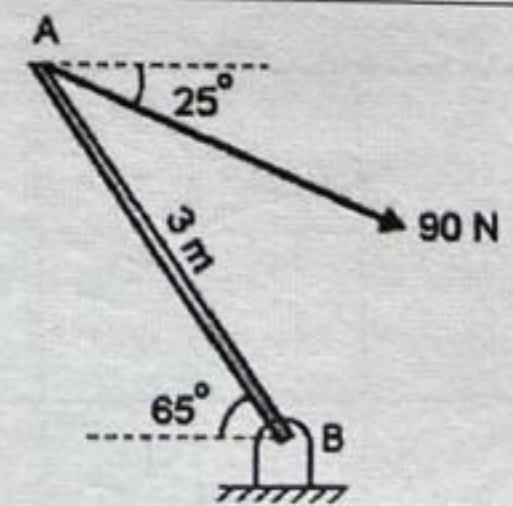
Que. No	Question Statement	Que. No	Question Statement
Resolution and composition - Resultant of Concurrent Forces			
1	<p>The resultant of two forces P and Q is 1200 N vertical. Determine the force Q and the corresponding angle θ for the system of forces as shown in Figure.</p> <p>Ans: $\theta = 23.79^\circ$, $Q = 743.59 \text{ N}$</p> 	2	<p>The resultant of two forces P and Q is 1200 N horizontally leftward. Determine the force Q and the corresponding angle θ for the system of force s as shown in Figure.</p> <p>Ans: $Q = 1587.45 \text{ N}$ $\theta = 70.89^\circ$</p> 
3	<p>The angle between the two concurrent forces is 90° and their resultant is 2500 N. The resultant makes an angle of 46° with one of the force; determine the magnitude of each force.</p> <p>Ans: $P = 1737 \text{ N}$ $Q = 1798 \text{ N}$</p>	4	<p>Determine the magnitude of force P so that the resultant of the force system as shown in Fig. is vertical and hence find magnitude of resultant force.</p> <p>Ans: $P = -318.2 \text{ N} = 318.2 \text{ N} \leftarrow$ $R = 106.07 \text{ N} \downarrow$</p> 
5	<p>Combine the two forces 800 N and 600 N, which act on the fixed dam structure at B, into a single equivalent force R if $AC = 3 \text{ m}$, $BC = 6 \text{ m}$ and angle $BCD = 60^\circ$. Refer given Fig.</p>  <p>Ans: $\alpha = 40.91^\circ$ $R = 523.94 \text{ N}$</p> <p>Direction of resultant $= 48.58^\circ$ in 4th Quadrant</p>	6	<p>The force system shown in Fig. have a resultant of 200 N along positive Y-axis, determine the magnitude and position θ of force F.</p> <p>Ans: $\theta = 47.6^\circ$ $R = 433.31 \text{ N}$</p> 

Unit wise Question Bank

Subject: EM

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Que. No	Question Statement	Que. No	Question Statement
5	<p>A rigid plate ABCD is subjected to forces as shown in figure. Calculate the Magnitude, Direction and line of action of the resultant force system with ref. to point A.</p> 	6	<p>If the resultant moment about point A is 4800 Nm clockwise, determine the magnitude of F_3 if $F_1=300$ N and $F_2 = 400$ N.</p> <p>Ans: $F_3 = 1592.7$ N</p> 
7	<p>Find the resultant and its point of application on y-axis for the force system acting on triangular plate as shown in Fig.</p> <p>Ans: $R = 20$ N ← $x = 6$ m from B</p> 	8	<p>Determine the magnitude and direction of resultant with reference to point A for the force system as shown in Fig. if side of equilateral triangle is 1 m.</p> <p>Ans:</p> 
9	<p>The three forces shown in Fig create a vertical resultant acting through point B. If $P = 361$ N, compute the values of T and F.</p> <p>Ans: $F = -581.716$ N $T = -929.282$ N</p> 	10	<p>90 N force is applied to control the rod AB. Determine the moment of this force about point B.</p> <p>Ans: $M_B = 173.53$ N.m</p> 

Ex. 4.5.36 : Determine the resultant of four forces tangential to the circle of radius 4 cm as shown in Fig. P. 4.5.36(a). What will be the location of the resultant with respect to the centre of the circle.

SPPU : April 94, 8 Marks

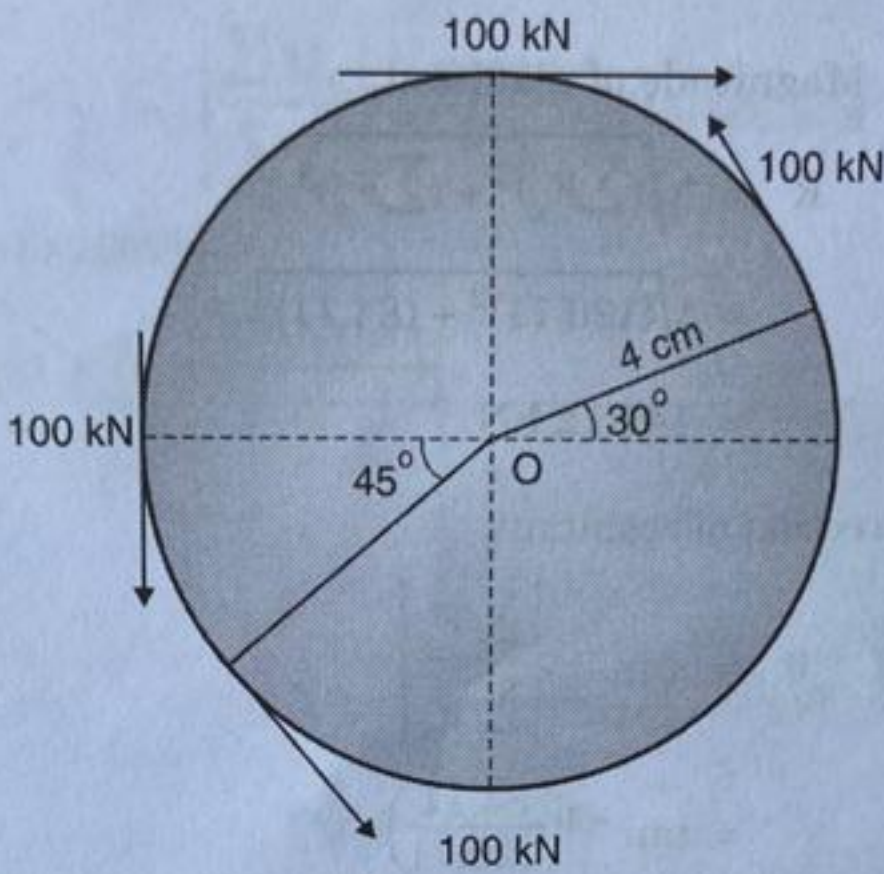


Fig. P. 4.5.36(a)

Soln. :

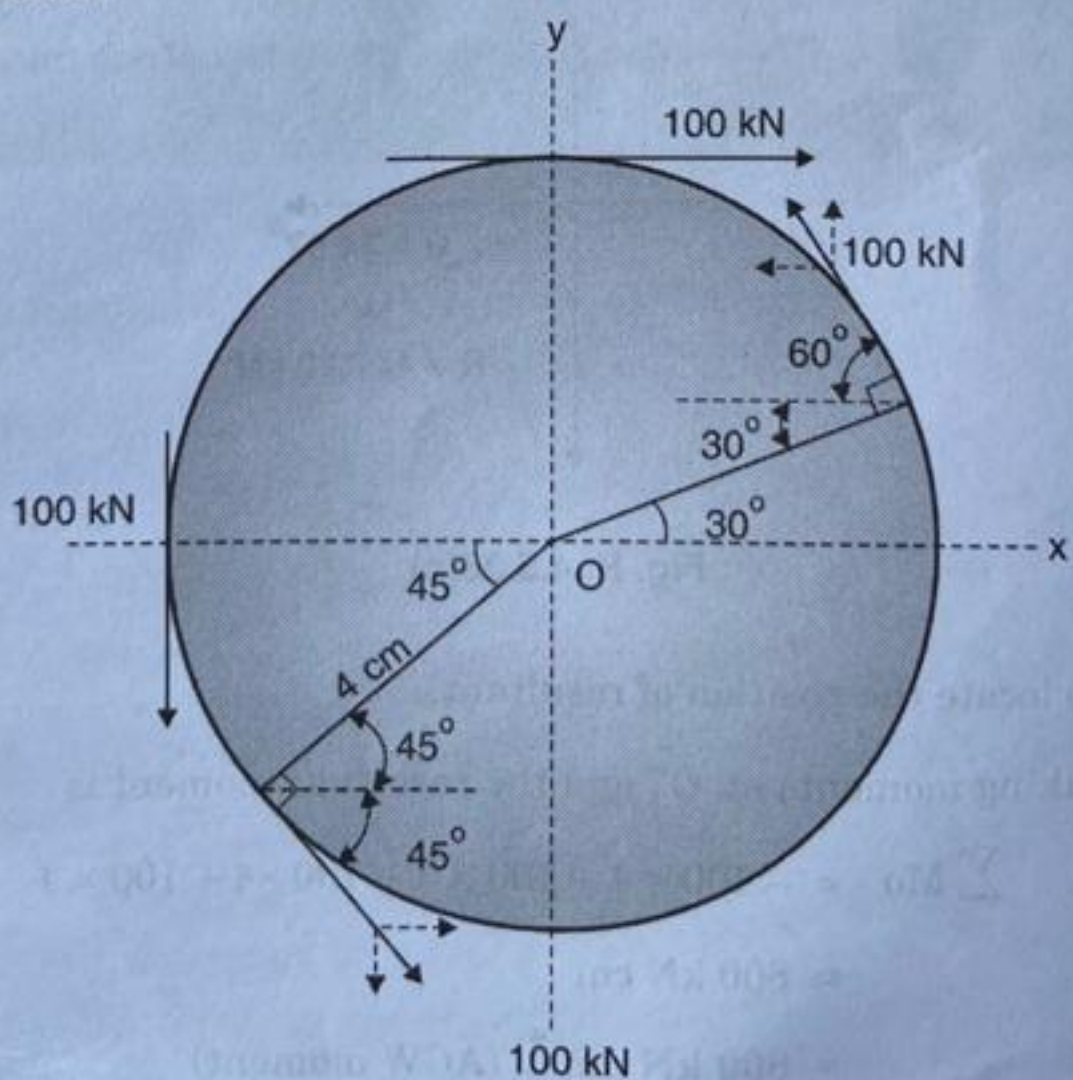


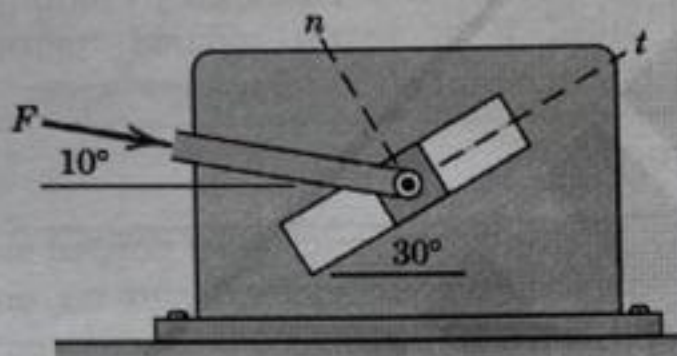
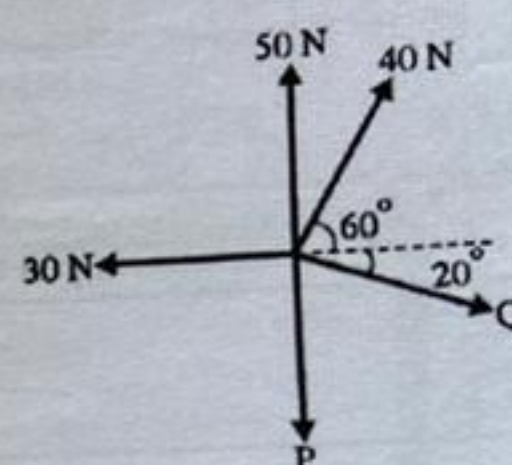
Fig. P. 4.5.36(b)

Unit wise Question Bank

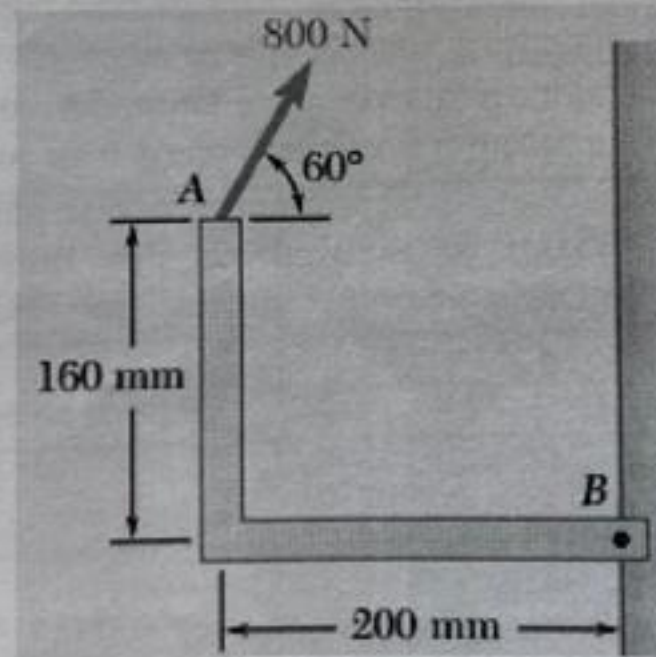
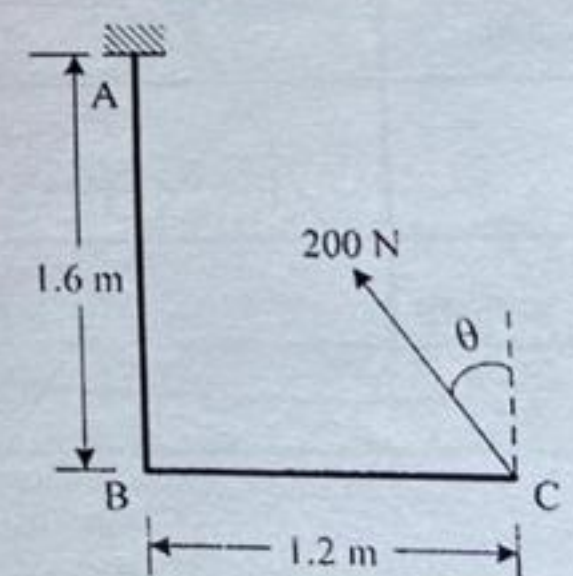
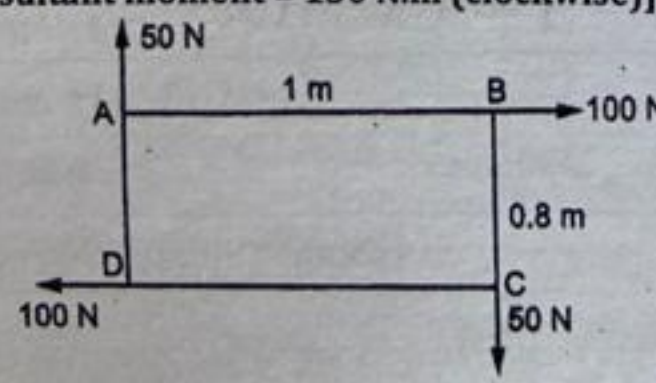
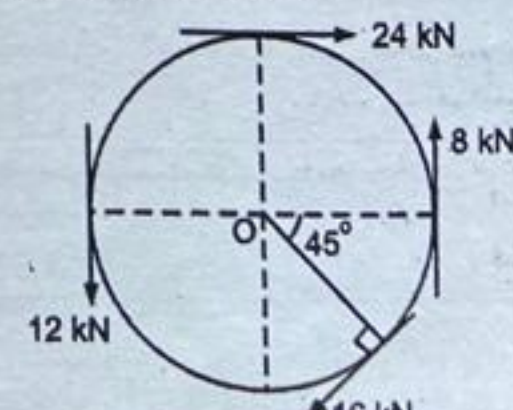
Subject: EM

Unit No. 1: Resolution and composition of forces

Class: F.E.

Que. No	Question Statement	Que. No	Question Statement
18	<p>The t-component of the force F is known to be 75 N. Determine the n-component and the magnitude of force F.</p> <p>Ans:</p> 	19	<p>Five concurrent coplanar forces act on a body as shown in Fig. Find forces P and Q such that resultant of the five forces is zero.</p> <p>Ans: $P = 81 \text{ N}$ $Q = 10.64 \text{ N}$</p> 

Resultant of Non Concurrent Forces, Moment, Varignon's Theorem

1	<p>A force of 800 N acts on a bracket as shown. Determine the moment of the force about B.</p> <p>Ans:</p> 	2	<p>The lever ABC fixed at A shown in Figure is subjected to a 200 N force at C at $\theta = 30^\circ$. Find the moment of this force about A. Also find the value of θ for which the moment about A is Zero.</p> <p>Ans:</p> 
3	<p>Find resultant moment of two couples for the loading as shown in Fig.</p> <p>[Ans: Resultant moment = 130 N.m (clockwise)]</p> 	4	<p>Determine the resultant of four forces tangent to the circle of radius 1.5 m as shown in figure. Also find the location of resultant of w.r.t. 'O'.</p> <p>Ans: $R = 19.88 \text{ N}$, $\alpha = 50.35^\circ$ $x = 1.51 \text{ m}$ from O.</p> 

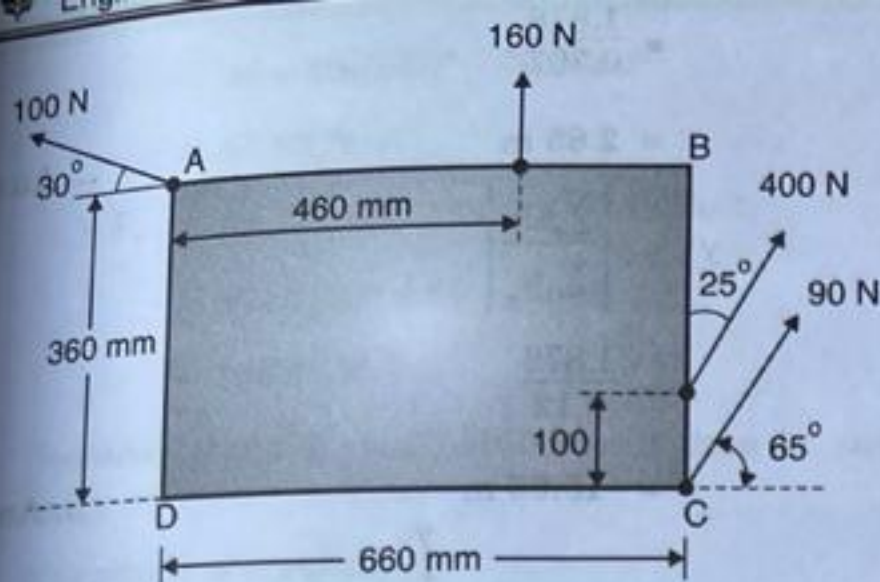


Fig. P. 4.5.39(a)

Soln. : Solving each force in to x and y components and taking algebraic sum;

$$\begin{aligned}\sum F_x &= R_x \text{ (x - component of resultant)} \\ &= 90 \cos 65^\circ + 400 \sin 25^\circ - 100 \cos 30^\circ \\ &= 120.48 \text{ N} \rightarrow\end{aligned}$$

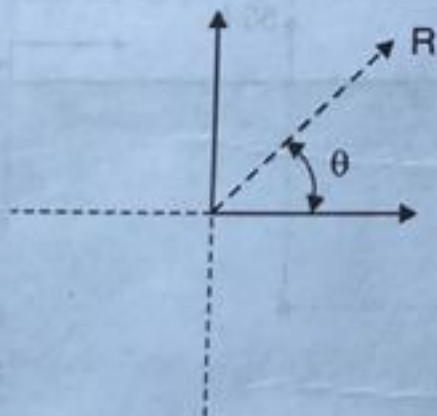
$$\begin{aligned}\sum F_y &= R_y \text{ (y - component of resultant)} \\ &= 90 \sin 65^\circ + 400 \cos 25^\circ + 160 \\ &\quad + 100 \sin 30^\circ = 654.10 \text{ N} \uparrow\end{aligned}$$

Magnitude of resultant,

$$\begin{aligned}R &= \sqrt{(\sum F_x)^2 + (\sum F_y)^2} \\ &= \sqrt{(120.48)^2 + (654.10)^2} \\ &= 665.103 \text{ N} \quad \dots \text{Ans.}\end{aligned}$$

Direction of resultant,

$$\begin{aligned}\theta &= \tan^{-1} \left| \frac{\sum F_y}{\sum F_x} \right| = \tan^{-1} \left(\frac{654.10}{120.48} \right) \\ &= 79.56^\circ \text{ w.r.t. x - axis} \quad \dots \text{Ans.}\end{aligned}$$



Taking moment at point 'A'

$$\begin{aligned}\sum M_A &= 160 \times 460 + 400 \cos 25^\circ \times 660 \\ &\quad + 400 \sin 25^\circ \times 260 + 90 \sin 65^\circ \times 660 \\ &\quad + 90 \cos 65^\circ \times 360 \\ &= 4.24 \times 10^5 \text{ Nmm} \curvearrowleft \text{ (ACW)}\end{aligned}$$

With respect to point 'A', the line of action of resultant intersects the edge AB at a distance,

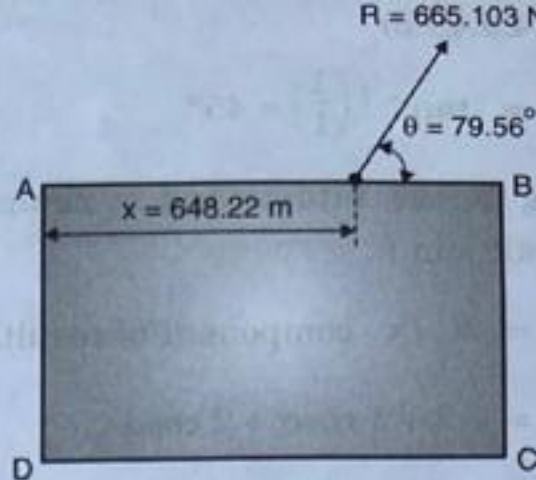


Fig. P. 4.5.39(b)

$$x = \left| \frac{\sum M_A}{\sum F_y} \right| = \left| \frac{4.24 \times 10^5}{654.10} \right|$$

$$= 648.22 \text{ mm}$$

... Ans.

Ex. 4.5.40 : The forces of magnitude 1 kN, 2 kN and 2 kN act along the sides of a rigid triangular frame formed by AB, BC and CD as shown in Fig. P. 4.5.40(a). Find the resultant and its intersection on x and y axes.

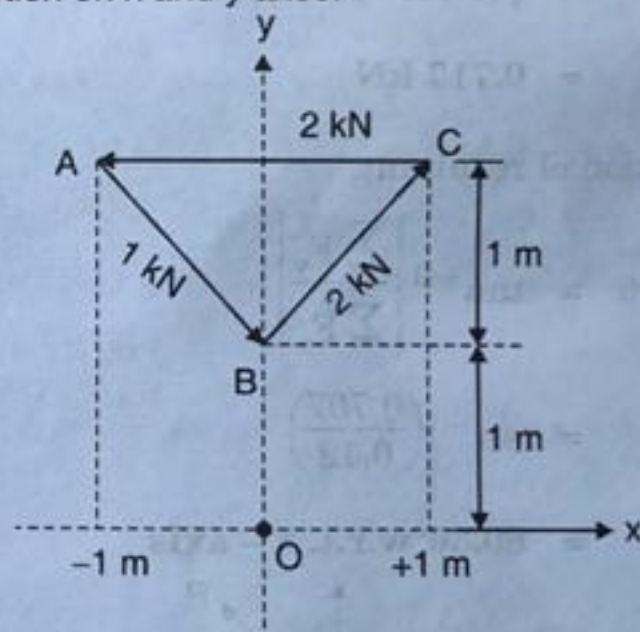


Fig. P. 4.5.40(a)

Soln. :

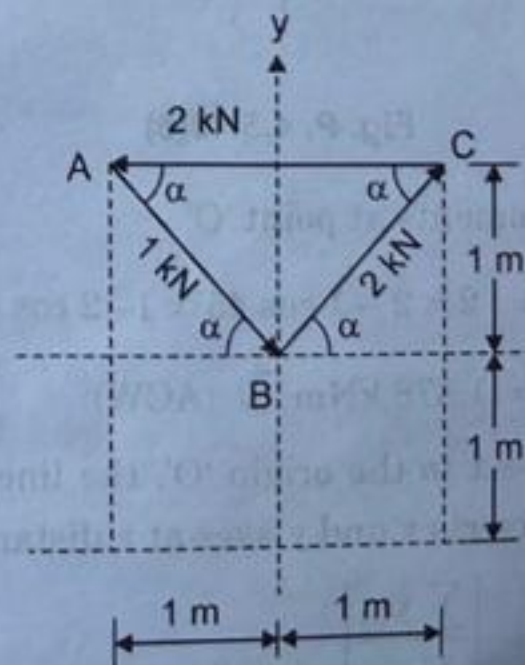


Fig. P. 4.5.40(b)

Resolving forces into x and y components and taking algebraic sum;

$$\sum F_x = R_x \quad (\text{x - component of resultant})$$

$$\sum F_x = 100 - 100\cos 60^\circ + 100\cos 45^\circ$$

$$= 120.71 \text{ kN} \rightarrow$$

$$\sum F_y = R_y \quad (\text{y - component of resultant})$$

$$\sum F_y = 100 \sin 60^\circ - 100 - 100 \sin 45^\circ$$

$$= -84.11 \text{ kN}$$

$$= 84.11 \text{ kN} \downarrow$$

\therefore Magnitude of resultant,

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$= \sqrt{(120.71)^2 + (84.11)^2}$$

$$= 147.12 \text{ kN}$$

... Ans.

Direction of resultant,

$$\theta = \tan^{-1} \left| \frac{\sum F_y}{\sum F_x} \right|$$

$$= \tan^{-1} \left(\frac{84.11}{120.71} \right)$$

$$= 34.87^\circ \text{ w.r.t. x - axis}$$

... Ans.

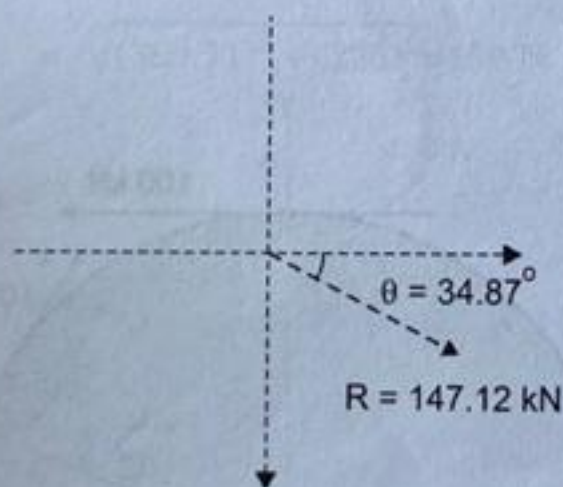


Fig. P. 4.5.36(c)

To locate the position of resultant,

Taking moments at 'O', and the resultant moment is

$$\sum M_o = -100 \times 4 + 100 \times 4 + 100 \times 4 + 100 \times 4$$

$$= 800 \text{ kN cm}$$

$$= 800 \text{ kN cm} \curvearrowright (\text{ACW moment})$$

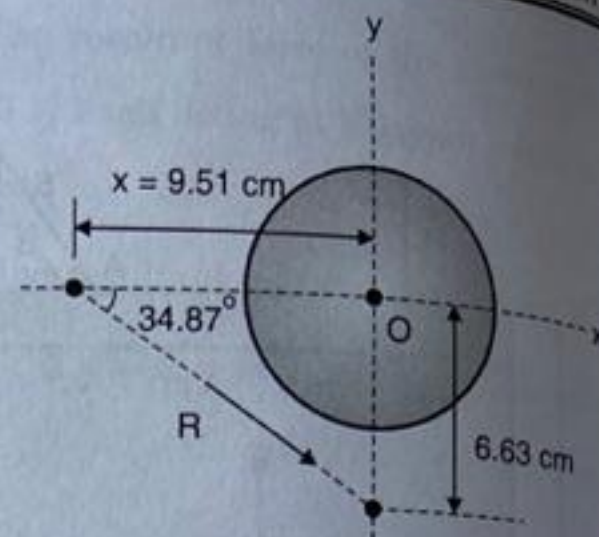


Fig. P. 4.5.36(d)

With respect to point 'O',

$$x = \left| \frac{\sum M_o}{\sum F_y} \right|$$

$$= \frac{800}{84.11}$$

$$= 9.51 \text{ cm}$$

$$y = \left| \frac{\sum M_o}{\sum F_x} \right|$$

$$= \frac{800}{120.71}$$

$$= 6.63 \text{ cm}$$

Note : 'R' is forming ACW moment at the centre of the circle 'O'.

Ex. 4.5.37 : A 150×300 mm plate is subjected to four forces. Find the resultant of the four forces 200 N and the two 250 N forces. At which the line of action of the resultant intersects the bottom edge of the plate.

SPPU : May 98, 8

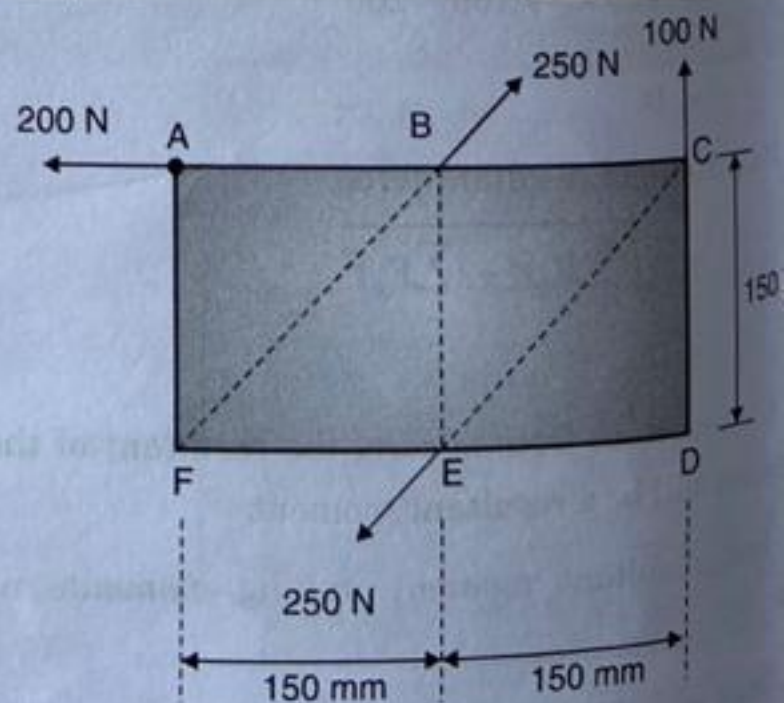


Fig. P. 4.5.37(a)

$$= 40 + 80\cos 30^\circ - 60\cos 40^\circ$$

$$= 63.32 \text{ N} \rightarrow$$

$$\sum F_y = R_y \text{ (y - component of resultant)}$$

$$= 80 \sin 30^\circ + 50 + 60 \sin 40^\circ$$

$$= 128.57 \text{ N} \uparrow$$

(Resultant force a couple 140 Nm is zero in any direction)

Magnitude of resultant,

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$= \sqrt{(63.32)^2 + (128.57)^2}$$

$$= 143.31 \text{ N} \quad \dots \text{Ans.}$$

Direction of resultant,

$$\theta = \tan^{-1} \left| \frac{\sum F_y}{\sum F_x} \right|$$

$$= \tan^{-1} \left(\frac{128.57}{63.32} \right)$$

$$= 63.78^\circ \text{ w.r.t. x - axis}$$

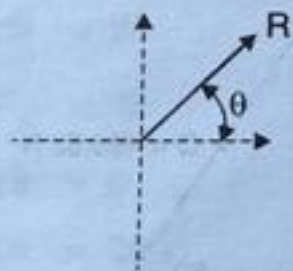


Fig. P. 4.5.41(b)

Taking moment at 'O',

$$\sum M_o = 140 + 60 \cos 40^\circ \times 4 - 60 \sin 40^\circ \times 7$$

$$- 50 \times 5$$

$$= -196.12 \text{ Nm}$$

$$= 196.12 \text{ Nm} \curvearrowleft \text{ (CW moment)}$$

\therefore with respect to point 'O', the line of action of resultant intersects x and y axes at a distance;

$$x = \left| \frac{\sum M_o}{\sum F_y} \right|$$

$$= \frac{196.12}{128.57}$$

$$= 1.525 \text{ m}$$

... Ans.

$$y = \left| \frac{\sum M_o}{\sum F_x} \right|$$

$$= \frac{196.12}{63.32}$$

$$= 3.097 \text{ m}$$

... Ans.

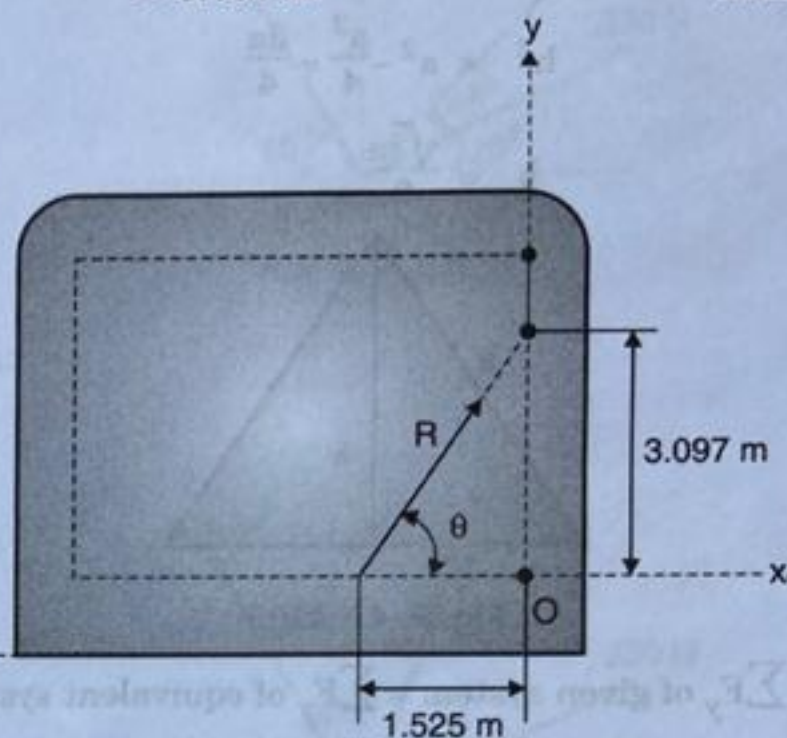


Fig. P. 4.5.41(c)

Ex. 4.5.42 : Replace the force $F_1 = 200 \text{ N}$, $F_2 = 150 \text{ N}$, $F_3 = 300 \text{ N}$ by an equivalent system of forces along the sides AB, BC and CA if the equilateral triangle shown in Fig. P. 4.5.42(a).

SPPU : Dec. 98, 6 Marks

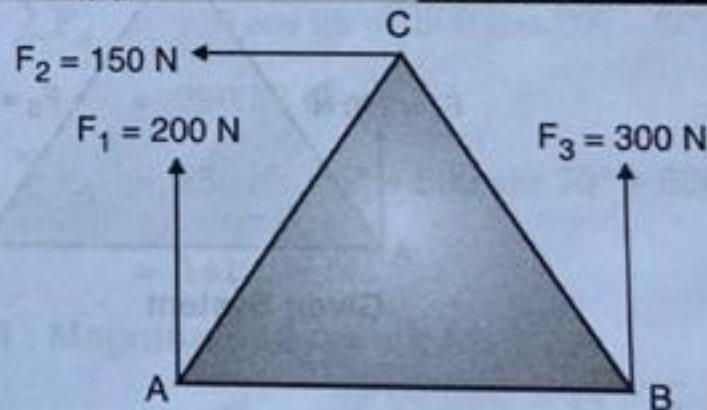


Fig. P. 4.5.42(a)

Soln. :

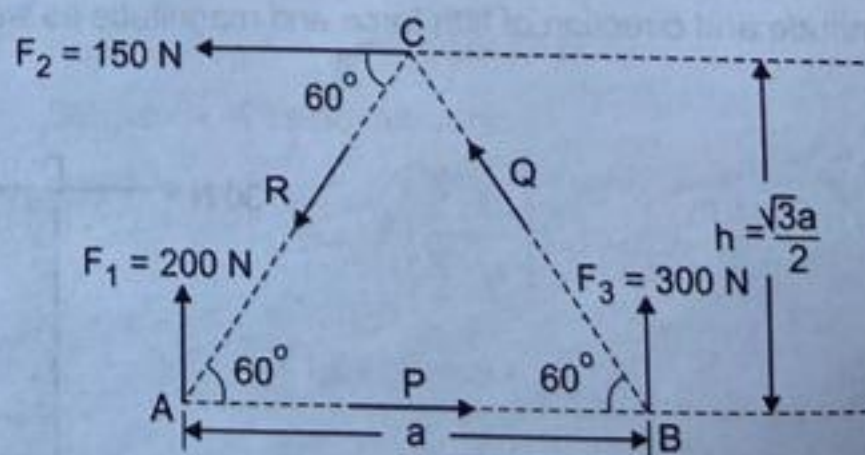


Fig. P. 4.5.42(b)

Let us consider an equivalent system formed by three forces, P, Q and R along the sides AB, BC and CA as shown in Fig. P. 4.5.42(b).

Fig. P. 4.5.40(d)

Note : Resultant is forming ACW moment at 'O'.

Ex. 4.5.41 : Determine the resultant of the four forces and one couple that act on the plate shown in Fig. P. 4.5.41(a). Locate it from point 'O'.

SPPU : May 98, 6 Marks

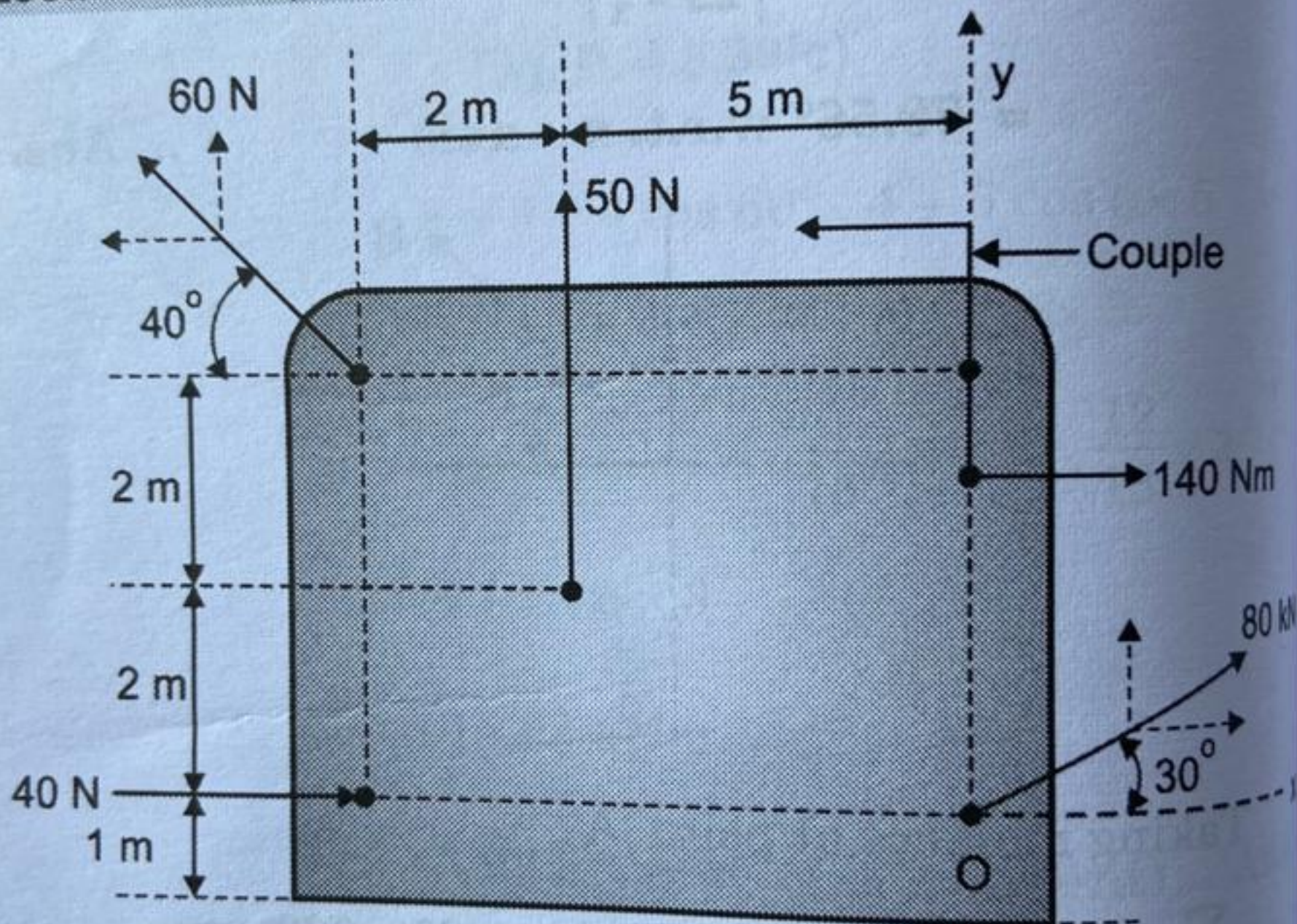


Fig. P. 4.5.41(a)

Soln. :

Resolving forces into x and y components and taking algebraic sum;

$$\sum F_x = R_x \text{ (x - component of resultant)}$$

∴ Line of action of resultant force of the system intersects line AB at a distance of 84.16 cm to the left of B and line BC at a distance of 4.17 cm downwards.

Note : The moment of resultant at B is having CW sense.

Ex. 4.5.33 : Find the magnitude, direction and line of action of the resultant of the plane force system shown in Fig. P. 4.5.33(a). **SPPU : Nov. 04, 8 Marks**

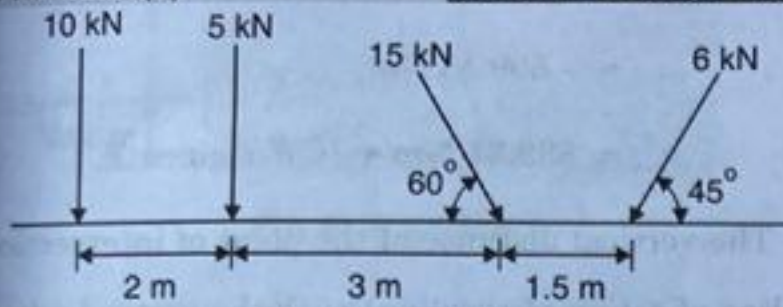


Fig. P. 4.5.33(a)

Soln. :

Let the forces are acting at the points shown in Fig. P. 4.5.33(b)

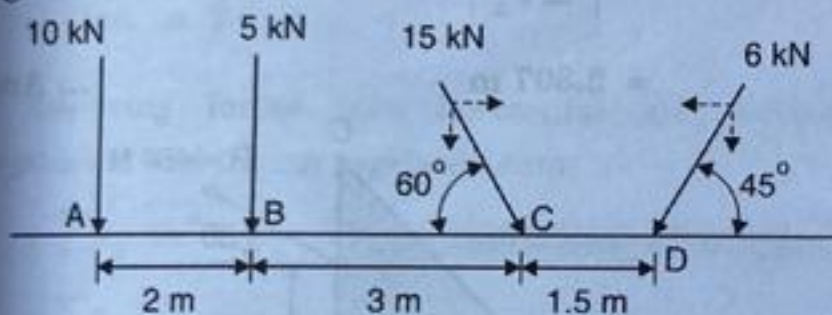


Fig. P. 4.5.33(b)

Resolving each force into x and y components and taking algebraic sum;

$$\sum F_x = R_x \text{ (x - components of resultant)}$$

$$\sum F_x = 15 \cos 60^\circ - 6 \cos 45^\circ = 3.26 \text{ kN} \rightarrow$$

$$\sum F_y = R_y \text{ (y - component of resultant)}$$

$$\sum F_y = -10 - 5 - 15 \sin 60^\circ - 6 \sin 45^\circ$$

$$= -32.23 \text{ kN} = 32.23 \text{ kN} \downarrow$$

Magnitude of resultant,

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$= \sqrt{(3.26)^2 + (32.23)^2}$$

$$= 32.40 \text{ kN}$$

... Ans.

Direction of resultant,

$$\theta = \tan^{-1} \left| \frac{\sum F_y}{\sum F_x} \right|$$

$$= \tan^{-1} \left(\frac{32.23}{3.26} \right)$$

$$= 84.22^\circ \text{ w.r.t. x - axis} \quad \dots \text{Ans.}$$

To find line of action of resultant,

Taking algebraic sum of the moments of all forces at point 'A',

Resultant moment,

$$\sum M_A = -5 \times 2 - 15 \sin 60^\circ \times 5 - 6 \sin 45^\circ \times 6.5$$

$$= -10 - 64.95 - 27.57$$

$$= -102.53 \text{ kNm}$$

$$= 102.53 \text{ kNm} \curvearrowright \text{ (CW moment)}$$

The given system can be replaced with single force (Resultant force) and single moment (Resultant moment) at point 'A' as shown in Fig.

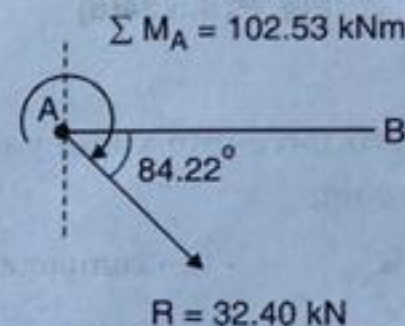


Fig. P. 4.5.33(c)

When the resultant force and resultant moment are combined, resultant force will be displaced horizontally by a distance,

$$x = \left| \frac{\sum M_A}{\sum F_y} \right|$$

$$= \left| \frac{102.53}{32.23} \right|$$

$$= 3.18 \text{ m}$$

... Ans.

Resultant will be displaced towards right as it forms CW moment about 'A'.

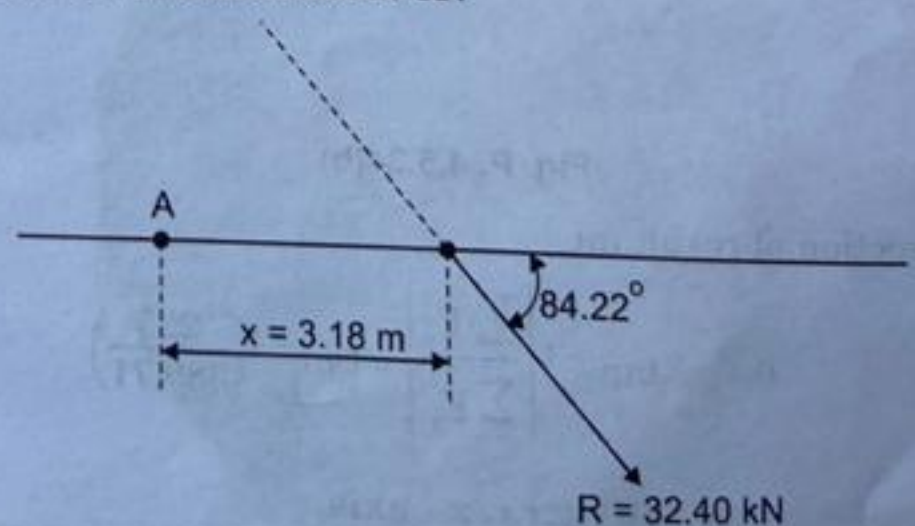


Fig. P. 4.5.33(d)

∴ The line of action of resultant is at a distance $x = 3.18 \text{ m}$ horizontally towards right from point 'A'.

Ex. 4.5.23 : Two like parallel forces of 25 N and 60 N are acting at the ends of a rod 300 mm long. Find the magnitude and direction of resultant force and the point where it acts.

Soln. :

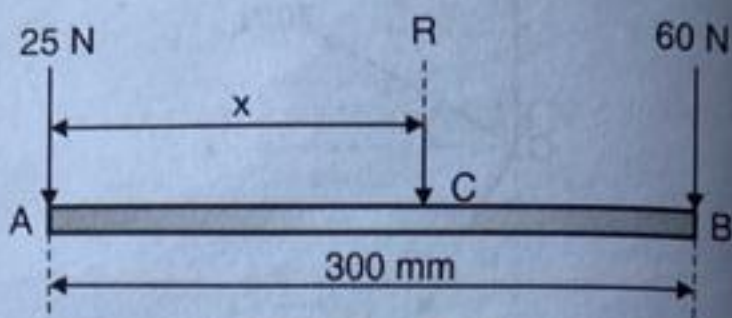


Fig. P. 4.5.23

Let the forces be act in y-direction

$$\sum F_y = R$$

$$\therefore R = -25 - 60$$

$$= -85 \text{ N}$$

$$= 85 \text{ N} \downarrow$$

...Ans.

Let the resultant acts at a distance 'x' mm from point 'A'.

Taking moments at 'A' and using varignon's theorem;

$$\sum M_A = \text{Moment of } R \text{ at 'A'}$$

$$25 \times 0 - 60 \times 300 = -R \times x$$

$$-60 \times 300 = -85 \times x$$

$$\therefore x = 211.76 \text{ mm}$$

...Ans.

Ex. 4.5.24 : Two unlike parallel forces of magnitude 450 N and 150 N are acting in such a way that their lines of action are 250 mm apart. Determine the magnitude of resultant force and the point where it acts.

Soln. :

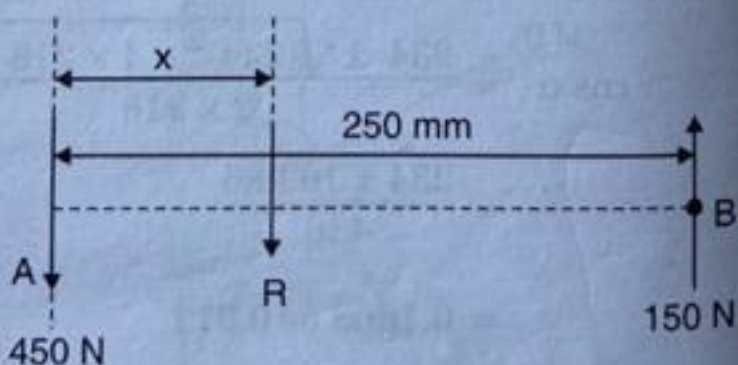


Fig. P. 4.5.24(a)

Let the forces be act in y-direction.

$$\therefore \sum F_y = R$$

$$R = -450 + 150$$

$$= -300 \text{ N} = 300 \text{ N} \downarrow$$

...Ans.

Ex. 4.5.32 : Three forces and a couple are applied to an angle bracket.

- (a) Find the resultant of this system of forces.
 (b) Locate the points where the line of action of resultant intersects line AB and line BC

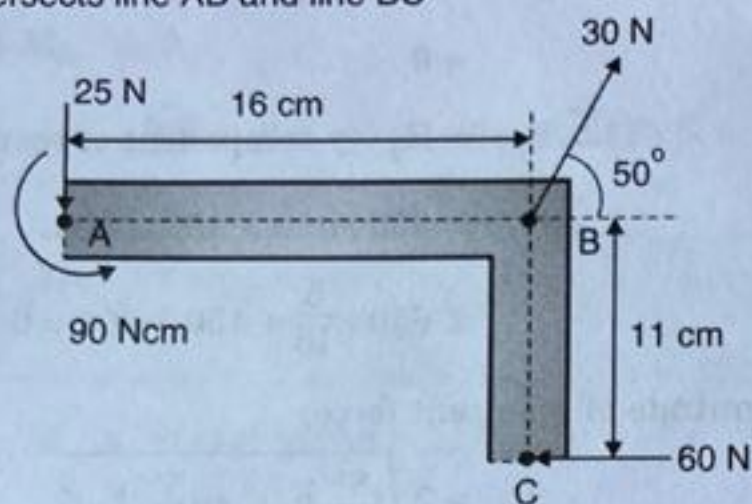


Fig. P. 4.5.32(a)

Soln. :

Resolving forces into x and y components and taking algebraic sum;

$$\begin{aligned}\sum F_x &= R_x \quad (\text{x - component of resultant}) \\ &= 30 \cos 50^\circ - 60 = -40.72 \text{ N} \\ &= 40.72 \text{ N} \leftarrow\end{aligned}$$

$$\begin{aligned}\sum F_y &= R_y \quad (\text{y - component of resultant}) \\ &= -25 + 30 \sin 50^\circ \\ &= -2.02 \\ &= 2.02 \text{ N} \downarrow\end{aligned}$$

Note : Resultant force of the couple 90 Ncm in any direction is zero

Magnitude of resultant force,

$$\begin{aligned}R &= \sqrt{(\sum F_x)^2 + (\sum F_y)^2} \\ &= \sqrt{(40.72)^2 + (2.02)^2} \\ &= 40.77 \text{ N}\end{aligned}$$

...Ans.

Direction of resultant force,

$$\begin{aligned}\theta &= \tan^{-1} \left| \frac{\sum F_y}{\sum F_x} \right| \\ &= \tan^{-1} \left(\frac{2.02}{40.72} \right) \\ &= 2.84^\circ \text{ w.r.t. x-axis}\end{aligned}$$

...Ans.

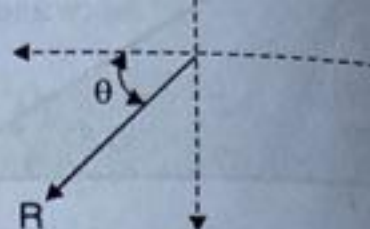


Fig. P. 4.5.32(b)

Taking moments at 'B',

Resultant moment,

$$\begin{aligned}\sum M_B &= 90 + 25 \times 16 - 60 \times 11 \\ &= -170 \text{ Ncm} = 170 \text{ Ncm} \curvearrowright (\text{CW moment})\end{aligned}$$

Note : Moment couple 90 Ncm is constant at any point.

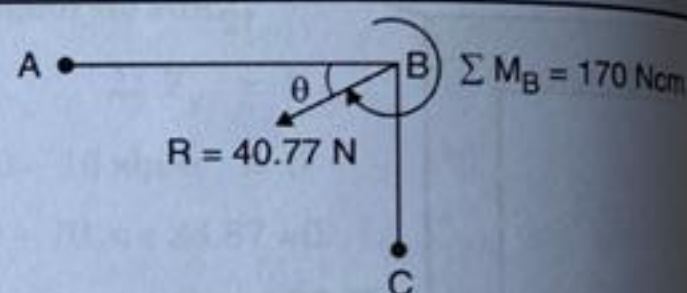


Fig. P. 4.5.32(c)

When force and moment are combined, the action of force intersects x and y axes at a distance

$$\begin{aligned}x &= \left| \frac{\sum M_B}{\sum F_y} \right| = \frac{170}{2.02} \\ &= 84.16 \text{ cm} \\ y &= \left| \frac{\sum M_B}{\sum F_x} \right| = \frac{170}{40.72} \\ &= 4.17 \text{ cm}\end{aligned}$$

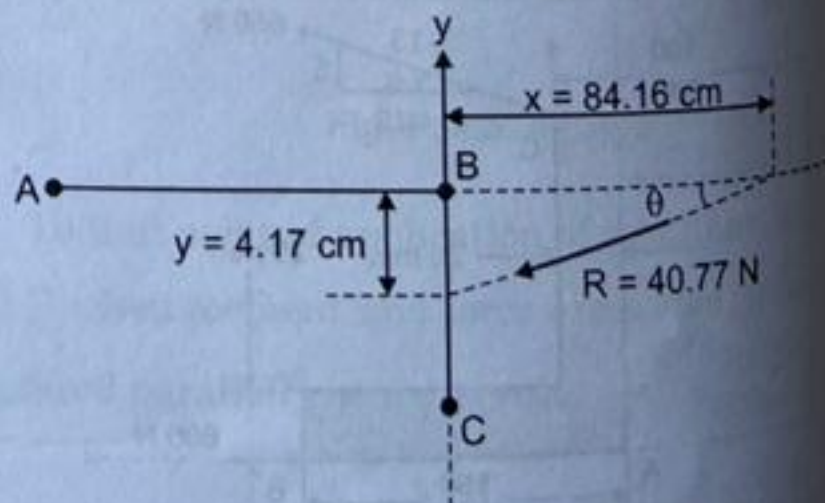


Fig. P. 4.5.32(d)

Let the position of resultant be at a distance 'x' from 450 N force.

Taking moments at 'A' and using varignon's theorem

$$\sum M_A = \text{Moment of 'R' at 'A'}$$

$$0 + 150 \times 250 = -R \times x$$

$$150 \times 250 = -300 \times x$$

$$\therefore x = -125 \text{ mm}$$

$$= 125 \text{ mm to the left of 'A'} \quad \dots \text{Ans.}$$

\therefore Resultant is acting at a distance of 125 mm to the left of 'A'.

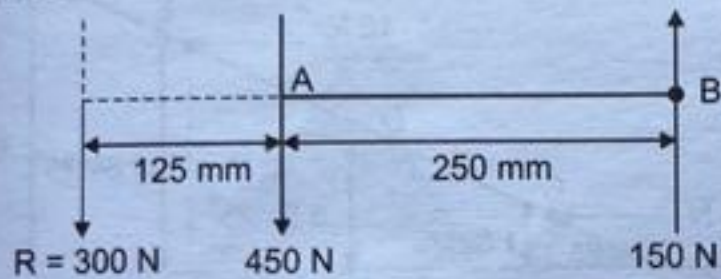


Fig. P. 4.5.24(b)

Ex. 4.5.25 : A force and a couple are applied to a beam as shown in Fig. P. 4.5.25(a). Replace this system with a single force applied at 'G' and determine distance 'd'.

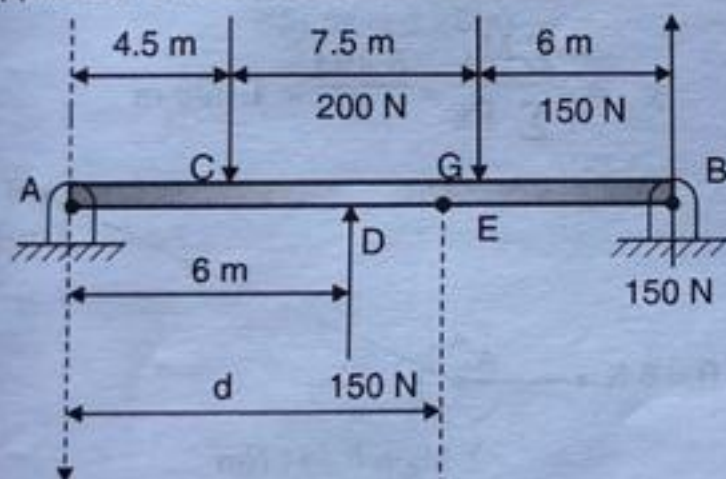


Fig. P. 4.5.25(a)

Soln. :

Let us first replace the given system to a single force and single moment at point 'A'.

Single force is nothing but resultant force and the single moment is resultant moment.

As all forces are acting in y-direction, the resultant force is,

$$\begin{aligned} R &= \sum F_y \\ &= -200 - 150 + 150 \\ &= -200 \text{ N} = 200 \text{ N} \downarrow \end{aligned}$$

Resultant moment at A is,

$$\begin{aligned} \sum M_A &= -200 \times 4.5 - 150 \times 12 + 150 \times 6 \\ &= -1800 \text{ Nm} \\ &= 1800 \text{ Nm} \curvearrowleft (\text{ACW moment}) \end{aligned}$$



$$\sum M_A = 1800 \text{ Nm}$$

Fig. P. 4.5.25(b)

When force and couple are added, force will displace parallelly by a distance,

$$\begin{aligned} d &= \left| \frac{\sum M_A}{R} \right| \\ &= \frac{1800}{200} = 9 \text{ m} \quad \dots \text{Ans.} \end{aligned}$$

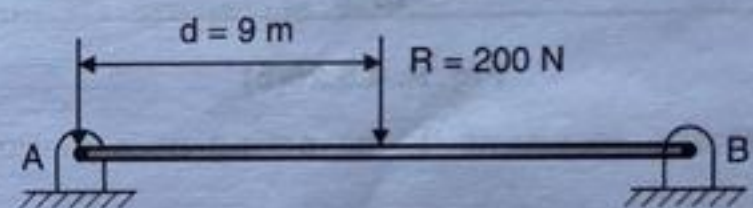


Fig. P. 4.5.25(c)

\therefore The resultant (single) force of the given system is 200 N acting downwards at a distance 9 m from point 'A'.

Ex. 4.5.26 : Three hikers are shown crossing footbridge. Knowing that the weights of the hikers at points C, D and E are 200N, 175N and 135N, respectively, determine the horizontal distance from 'A' to the line of action of the resultant of the three weights when $a = 3.3 \text{ m}$.

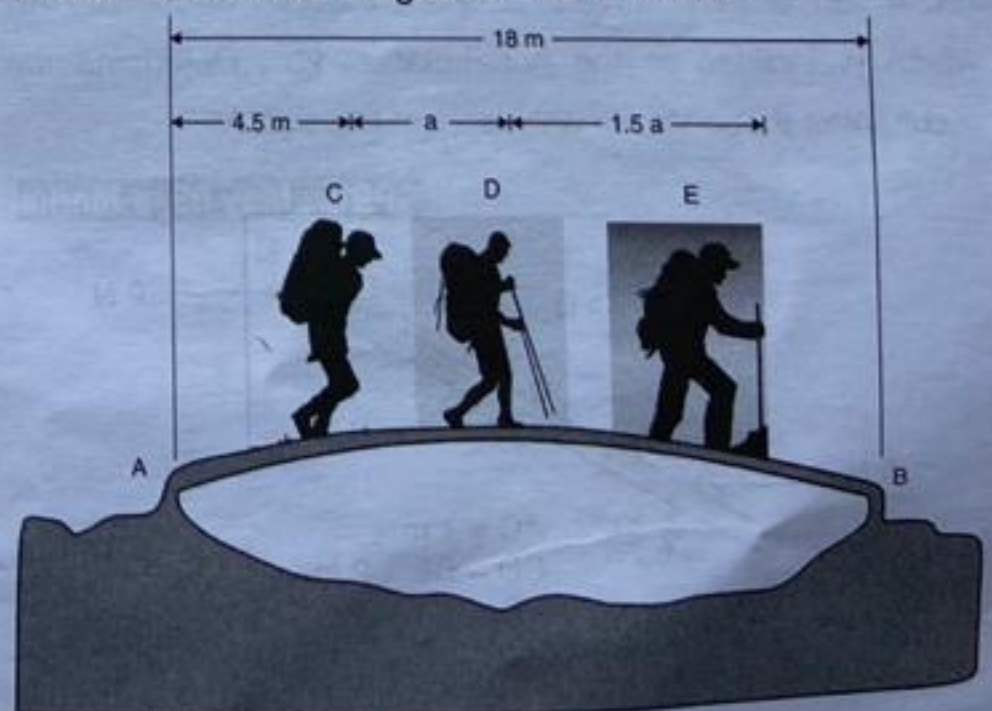


Fig. P. 4.5.26(a)

- Let the forces P and Q are acting at point 'O' at angle ' θ ' between them. These forces are represented by the two adjacent sides OA and OB of a parallelogram $OACB$.

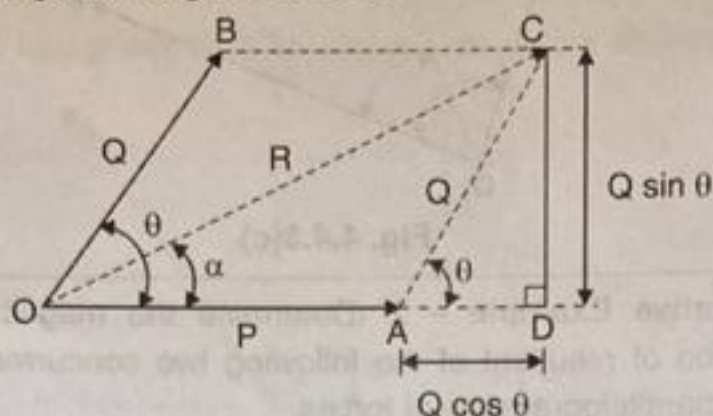


Fig. 4.4.3(b)

- Draw perpendicular CD on the line OA produced. The diagonal of the parallelogram OC represents the resultant of P and Q in magnitude and direction.
- ' α ' is the direction of resultant w.r.t. force ' P '.

From the $\triangle ADC$;

$$\sin \theta = \frac{CD}{AC} = \frac{CD}{Q}$$

$$\therefore CD = Q \sin \theta$$

$$\cos \theta = \frac{AD}{AC} = \frac{AD}{Q}$$

$$\therefore AD = Q \cos \theta$$

From the $\triangle ODC$;

$$OC^2 = OD^2 + CD^2$$

$$R^2 = (P + Q \cos \theta)^2 + (Q \sin \theta)^2$$

$$(OD = OA + AD = P + Q \cos \theta)$$

$$\therefore R^2 = P^2 + Q^2 \cos^2 \theta + 2 PQ \cos \theta + Q^2 \sin^2 \theta$$

$$R^2 = P^2 + Q^2 + 2 PQ \cos \theta$$

$$(\because \sin^2 \theta + \cos^2 \theta = 1)$$

$$\therefore \text{Magnitude of resultant, } R = \sqrt{P^2 + Q^2 + 2 PQ \cos \theta}$$

From the $\triangle ODC$;

$$\tan \alpha = \frac{CD}{OD} = \frac{CD}{OA + AD} = \frac{Q \sin \theta}{P + Q \cos \theta}$$

\therefore Direction of resultant,

$$\alpha = \tan^{-1} \left(\frac{Q \sin \theta}{P + Q \cos \theta} \right) \text{ w.r.t. force 'P'.$$

Note : Force ' P ' is not necessarily always horizontal. It can have any direction. But α is the direction of resultant w.r.t. force ' P '.

4.4.2 Parallelogram Law of forces :

- “If two coplanar concurrent forces are acting simultaneously at a point which are represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant may be represented in magnitude and direction by the diagonal of the parallelogram which passes through their point of intersection.”

Expression for resultant :

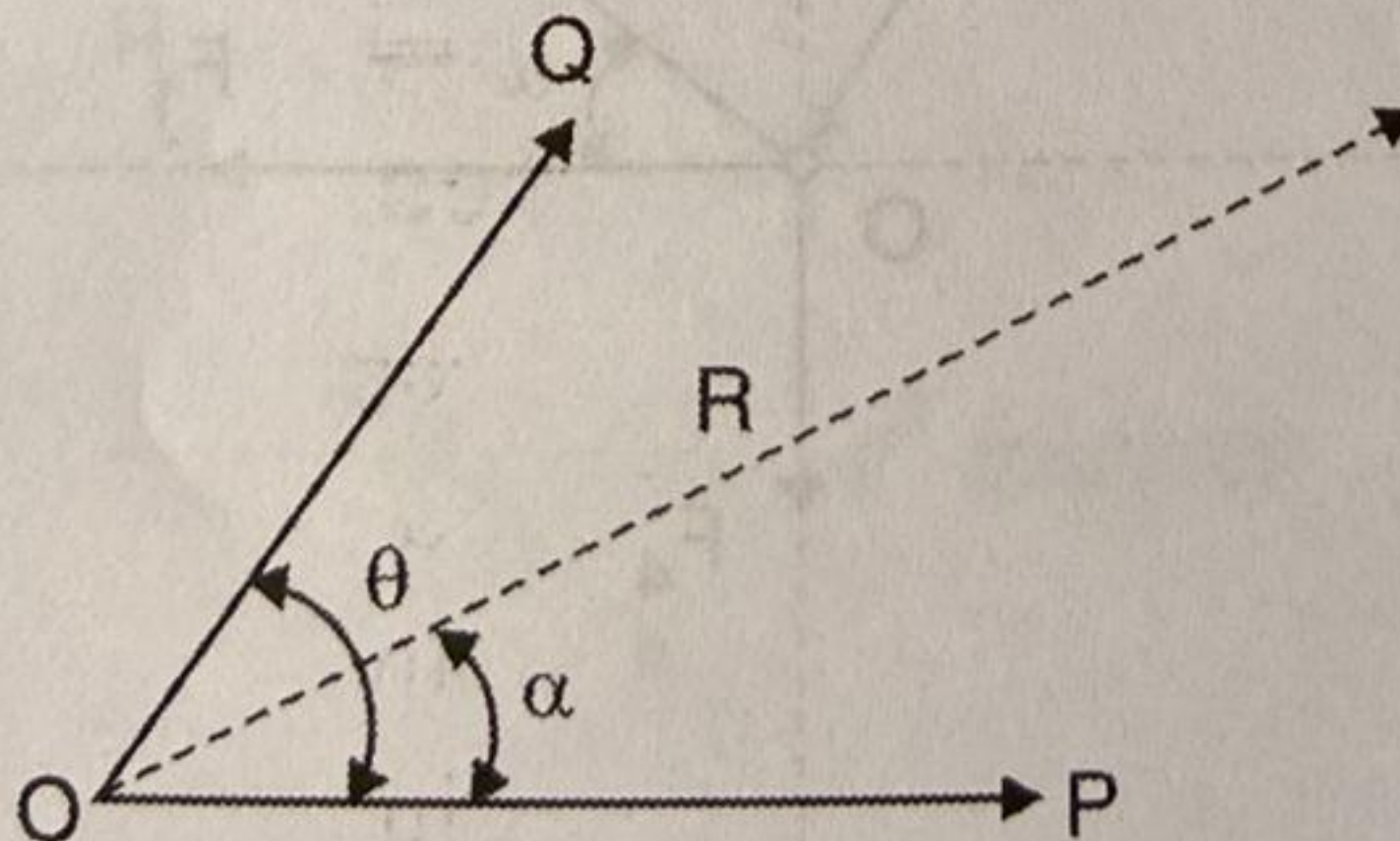


Fig. 4.4.3(a)

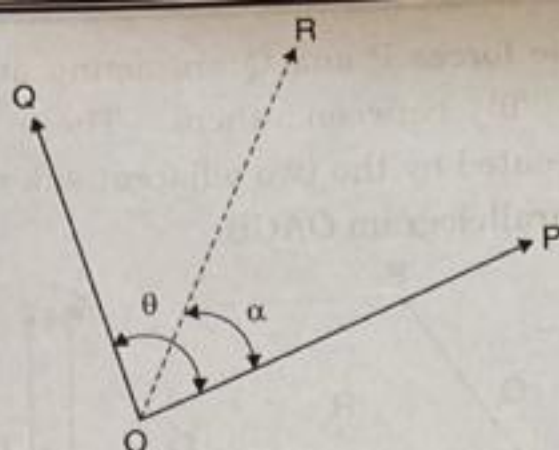


Fig. 4.4.3(c)

Illustrative Example – 2 : Determine the magnitude and direction of resultant of the following two concurrent forces using parallelogram law of forces.

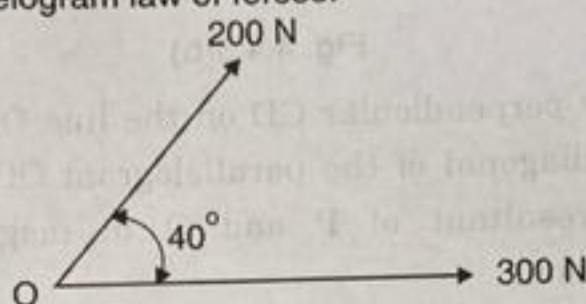


Fig. 4.4.4(a)

Soln. :

$$\text{Let } P = 300 \text{ N}$$

$$Q = 200 \text{ N}$$

$$\theta = 40^\circ \quad (\text{angle between } P \text{ and } Q)$$

Using parallelogram law of forces,

Magnitude of resultant is given by,

$$\begin{aligned} R &= \sqrt{P^2 + Q^2 + 2 PQ \cos \theta} \\ &= \sqrt{300^2 + 200^2 + 2 \times 300 \times 200 \times \cos 40^\circ} \\ &= 471.089 \text{ N} \end{aligned}$$

Direction of resultant is given by,

$$\begin{aligned} \alpha &= \tan^{-1} \left(\frac{Q \sin \theta}{P + Q \cos \theta} \right) \quad \text{w.r.t. force 'P'} \\ &= \tan^{-1} \left(\frac{200 \sin 40^\circ}{300 + 200 \cos 40^\circ} \right) \\ &= 15.84^\circ \quad \text{w.r.t. force 'P'} \end{aligned}$$

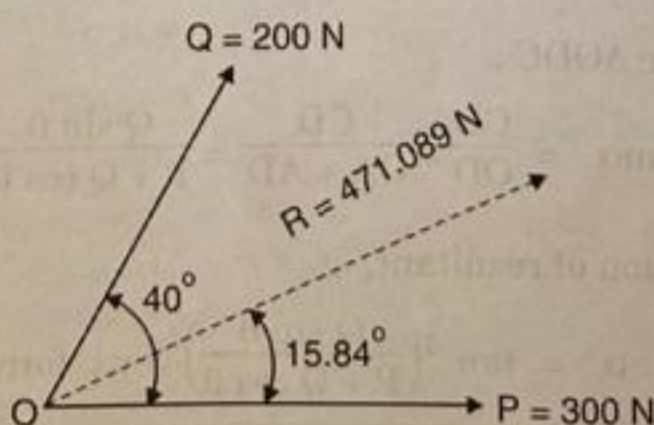


Fig. 4.4.4(b)

Ex. 4.5.6 :Determine the magnitude of force P so that the resultant of the force system as shown in Fig.P. 4.5.6(a) is vertical and hence find magnitude of resultant force.

SPPU-Dec. 14, 4 Marks

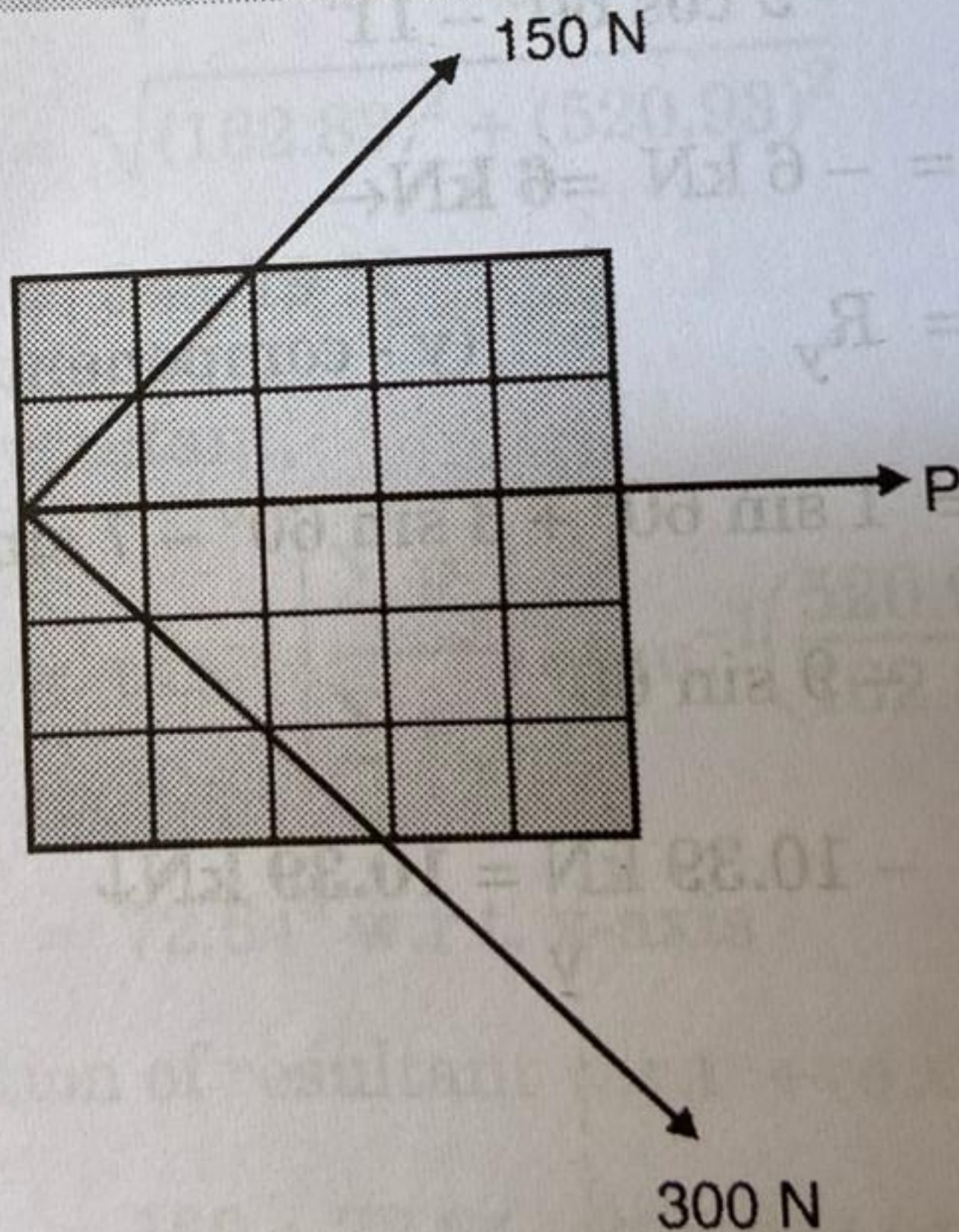


Fig. P. 4.5.6(a)

Ex.4.5.12 : The force system shown in Fig. P. 4.5.12(a) have a resultant of 200 N ALONG positive Y - axis, determine the magnitude and position θ of force F.

SPPU : Dec. 15, 4 Marks

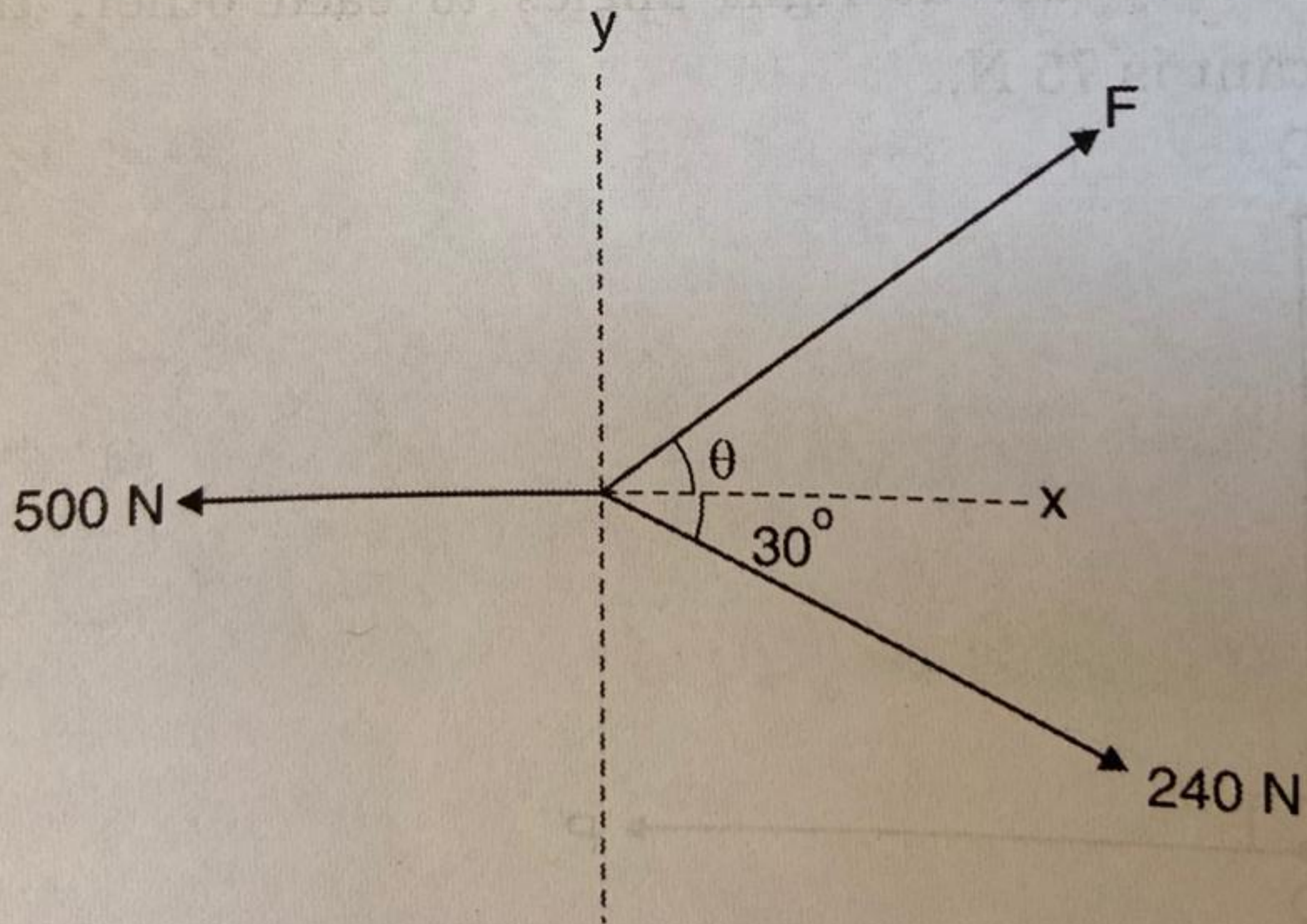


Fig. P. 4.5.12(a)

Ex. 4.5.3 : Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x-axis as shown in Fig. P. 4.5.3(a)

SPPU : May 13, May 08, 6 Marks

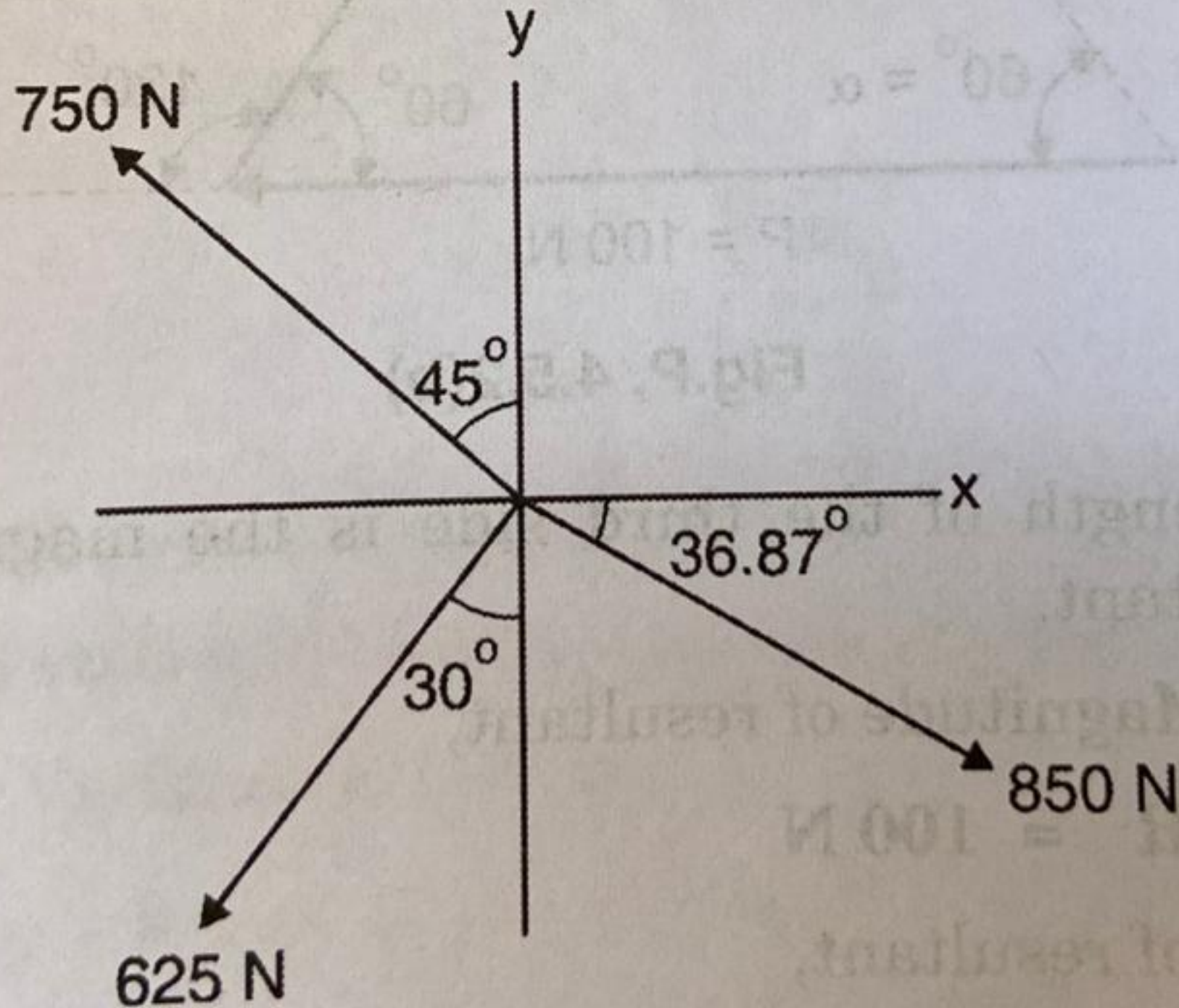


Fig. P. 4.5.3(a)