20110029 Ashutosh-Goyal ME-639

Assignment-3

(1) We know that mapping of end-effector velocities and joint variables are done by using Jacobian.

$$\boxed{\dot{x} = J(q)\dot{q}}$$

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = J\dot{q}$$

For n-link manipulator Jacobian is basically $(6 \times n)$ matrix.

Since, the Jacobian is a function of the configuration q, those configuration for which the rank of J decrease are called as singular configuration.

To check the singular configuration we will find the rank of Jacobian and if Jacobian is found to be rank-deficient then it will be singular configuration

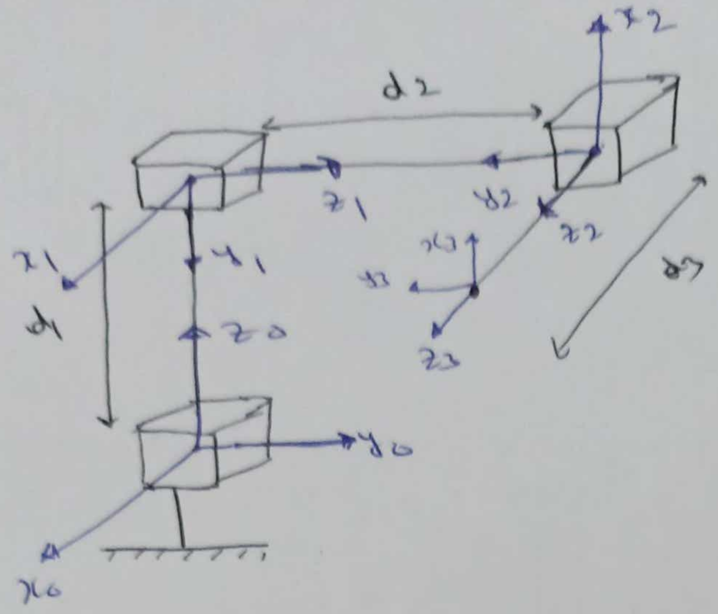
For square Jacobian matrices, we can also check the Determinant of the Jacobian and if $\det(J) = 0$ ⇒ configuration is singular.

For experimental purposes :-

$$\boxed{dx = J(q)\,dq}$$

If change in dq will not resulting in a change of dx ⇒ configuration is singular and hence we can not go in arbitary direction our workspace is limited.

(5)



⇒ DH - Parameters table :-

Link	a_i	α_i	d_i	θ_i
1	0	$-90°$	d_1	0
2	0	-90	d_2	-90
3	0	0	d_3	0

$\boxed{d_1, d_2 \text{ \& } d_3 \text{ are the variables}}$

General - D-H matrix $= A_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Hence, $A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Similarly:- $A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Now, $T_0^3 = A_1 A_2 A_3$

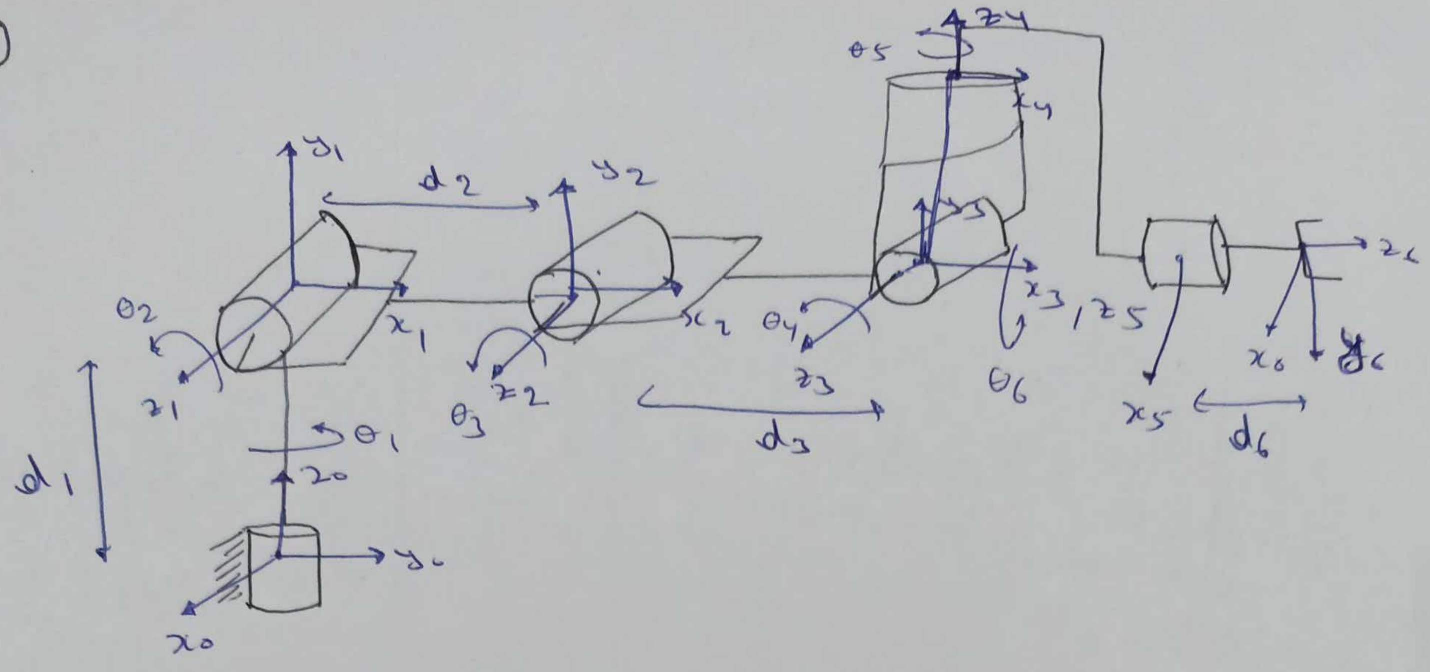
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow T_0^3 = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ 0 & -1 & 0 & d_2 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence, $\boxed{\begin{aligned} x &= d_3 \\ y &= d_2 \\ z &= d_1 \end{aligned}}$ → co-ordinates of the end-effector.

Also, orientation of end-effector is given by $\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(6)



$\Rightarrow$

	Link	a_i	α_i	d_i	θ_i
	1	0	90°	d_1	θ_1^*
	2	d_2	0	0	θ_2^*
	3	d_3	0	0	θ_3^*
wrist	4	0	−90°	0	θ_4^*
	5	0	−90°	0	θ_5^*
	6	0	0	d_6	θ_6^*

As, we know that

$$A_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & d_2C_2 \\ S_2 & C_2 & 0 & d_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & d_3C_3 \\ S_3 & C_3 & 0 & d_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_5 = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence, $T_0^3 = \underbrace{A_1 A_2}\, A_3$

$$= \begin{bmatrix} C_1C_2 & -C_1S_2 & S_1 & d_2C_2C_1 \\ S_1C_2 & -S_1S_2 & -C_1 & S_1d_2S_2 \\ S_2 & C_2 & 0 & d_1+d_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & d_3S_3 \\ S_3 & C_3 & 0 & d_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} C_1C_{23} & -C_1S_{23} & S_1 & d_3C_1C_{23} + d_2C_1C_2 \\ S_1C_{23} & -S_1S_{23} & -C_1 & d_3S_1C_{23} + d_2S_1S_2 \\ S_{23} & C_{23} & 0 & d_3S_2C_3 + d_3C_2S_3 + d_1 + d_2S_2 \\ 0 & 0 & 1 & d_3S_{23} + d_1 + d_2S_2 \end{bmatrix}}_{T_0^3}$$

Similarly, $T_3^6 = A_4 A_5 A_6$

$$\Rightarrow \boxed{T_0^6 = T_0^3 \, T_3^6}$$

Hence, $T_3^6 = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & C_4S_5 & C_4S_5d_6 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & S_4S_5 & S_4S_5d_6 \\ -S_5C_6 & S_5S_6 & C_5 & C_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Therefore, $T_0^6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & dx \\ r_{21} & r_{22} & r_{23} & dy \\ r_{31} & r_{32} & r_{33} & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Now, we know that $T_0^3 * T_3^6$

Hence, $\boxed{T_0^6 = T_0^3 \, T_3^6}$

Therefore, we get

$$r_{11} = C_1C_{23}(C_4C_5C_6 - S_4S_6) - C_1S_{23}(S_4C_5C_6 + C_4S_6) - S_1S_5C_6$$

$$r_{12} = C_1C_{23}(-C_4C_5S_6 - S_4S_6) - C_1S_{23}(-S_4C_5C_6 + C_4S_6) + S_1S_5S_6$$

$$r_{13} = C_1C_{23}C_4S_5 - C_1S_{23}S_4S_6 + S_1C_5$$

$$r_{21} = S_1C_{23}(C_4C_5C_6 - S_4S_6) - S_1S_{23}(S_4C_5C_6 + C_4S_6) + C_1S_5C_6$$

$$r_{22} = S_1C_{23}(-C_4C_5S_6 - S_4S_6) - S_1S_{23}(-S_4C_5S_6 + C_4C_6) - C_1S_5S_6$$

$$r_{23} = S_1 C_{23} C_4 S_5 - S_1 S_{23} S_4 S_5 - C_1 C_5$$

$$r_{31} = S_{23}(C_4 C_5 C_6 - S_4 S_6) + C_{23}(S_4 C_5 C_6 + C_4 S_6)$$

$$r_{32} = S_{23}(-C_4 C_5 S_6 - S_4 C_6) + C_{23}(-S_4 C_5 S_6 + C_4 C_6)$$

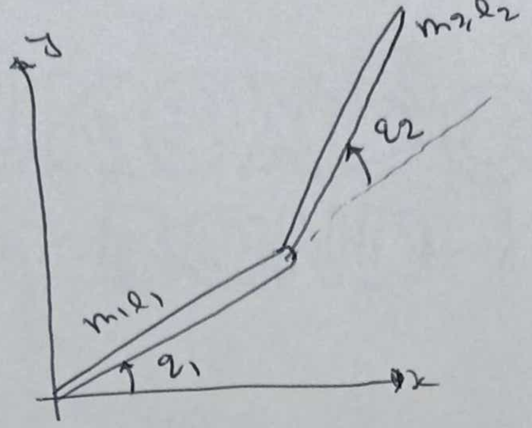
$$r_{33} = S_{23} C_4 S_5 + C_{23} S_4 S_5$$

Also, $d_x = C_1 C_{23} C_4 S_5 d_6 - C_1 S_{23} S_4 S_5 d_6 + S_1 C_5 d_6 + d_3 C_1 C_{23} + d_2 C_1 C_2$

$$d_y = S_1 C_{23} C_4 S_5 d_6 - S_1 S_{23} S_4 S_5 d_6 - C_1 C_5 d_6 + d_3 S_1 C_{23} + d_2 S_1 S_2$$

$$d_z = S_{23} C_4 S_5 d_6 + C_{23} C_4 S_5 d_6 + d_3 S_{23} + d_1 + d_2 S_2$$

(7) 1) Direct drive :-



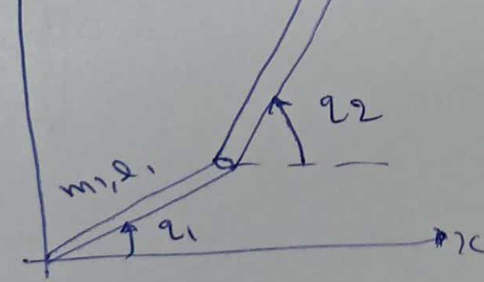
⇒ Dynamic eqn of motion is given by:

Ref:- From textbook:-

$$\begin{cases} d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{121}\dot{q}_1\dot{q}_2 + c_{211}\dot{q}_2\dot{q}_1 + c_{221}\dot{q}_2^2 + \phi_1 = \tau_1 \\ d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \phi_2 = \tau_2 \end{cases}$$

Here, Dynamic eqn contains both $(\dot{q}_i \dot{q}_j)$ & $(\dot{q}_i)^2$ terms which implies that centrifugal terms and coriolis effect are present in this case.

2) Planar elbow manipulator with remotely driven link :-



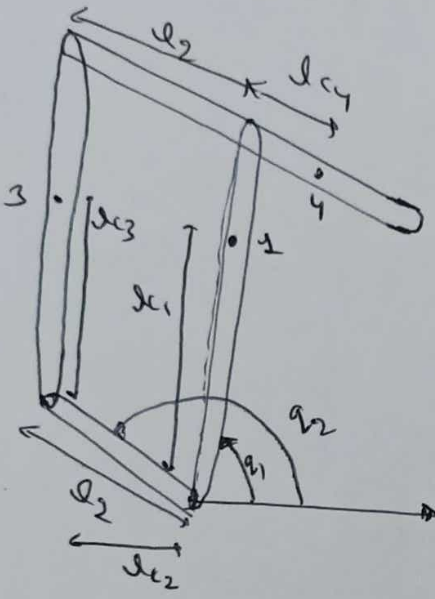
⇒ Here, dynamic eqn of motion is given by (ref + from book)

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{221}\dot{q}_2^2 + \phi_1 = \tau_1$$

$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_2^2 + \phi_2 = \tau_2$$

Note:- The dynamics eqn of remotely driven link contains only $(\dot{q}_2)^2$ term which is centrifugal term, we have eliminated the coriolis force in this case.

3) **Five-Bar-Parallelogram Arrangement:-**



Here, Dynamics eqns are given by

$$d_{11}\ddot{q}_1 + \phi_1(q_1) = \tau_1$$

$$d_{22}\ddot{q}_2 + \phi_2(q_2) = \tau_2$$

Here:- The dynamics eqn contains neither the centrifugal terms nor the Coriolis term. Therefore, In this arrangement, we have eliminated the coriolis as well as centrifugal term.

⇒ **Advantages of Various arrangements:-**

The first one direct drive is a general case which has both centrifugal as well as corioliss term and therefore torque required is calculated by considering the effect of both forces.

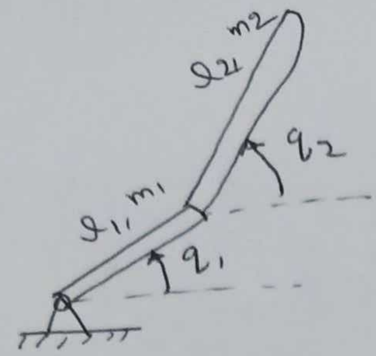
In remotely driven we had removed the coriolisis term, but we had still the centripital force which couple the two links. Therefore, the required torque will be less than what is required in the case of direct drive.

Also, in further modification in 5-Bar parallelogram, we find that there is no centripital as well as coriolis force due to which the required torque

will be less than what is required in the case of direct drive as well as in remotely driven. This helps to explain the increasing popularity of the parallelogram configuration in industrial robot.

If the relationship ($m_3 l_2 l_{c3} = m_4 l_1 l_{c4}$) is satisfied then we can adjust the q_1 & q_2 independently without worrying about interaction b/w the angles.

(8)



⇒ we know that

$$V_{c_1} = \left(-\frac{l_1}{2}\sin q_1\, \dot{q}_1\right)\hat{i} + \left(\frac{l_1}{2}\cos q_1\, \dot{q}_1\right)\hat{j}$$

$$\Rightarrow V_{c_1} = \begin{bmatrix} -\frac{l_1}{2}\sin q_1 & 0 \\ \frac{l_1}{2}\cos q_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

Similarly, $$V_{c_2} = \begin{bmatrix} -l_1\sin q_1 & -\frac{l_2}{2}\sin q_2 \\ l_1\cos q_1 & \frac{l_2}{2}\cos q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

Now, $\omega_1 = \dot{q}_1 \hat{k}$ & $\omega_2 = \dot{q}_2 \hat{k}$

Also, potential energy is given by = $$\boxed{V = m_1 g \frac{l_1}{2}\sin q_1 + m_2 g\left(l_1 \sin q_1 + \frac{l_2}{2}\sin q_2\right)}$$

Now, we know that the translational K.E is given by

$$\frac{1}{2}m_1 v_{c_1}^T v_{c_1} + \frac{1}{2}m_2 v_{c_2}^T v_{c_2}$$

As, $v_{c_1} = J_{v_{c_1}}\dot{q}$ & $v_{c_2} = J_{v_{c_2}}\dot{q}$

$\Rightarrow (v_{c_1})^T = (\dot{q})^T (J_{v_{c_1}})^T$ & $(v_{c_2})^T = (\dot{q})^T (J_{v_{c_2}})^T$

Hence, $$\boxed{\frac{1}{2}m_1 v_{c_1}^T v_{c_1} + \frac{1}{2}m_2 v_{c_2}^T v_{c_2} = \frac{1}{2}\dot{q}\left\{m_1 J_{v_{c_1}}^T J_{v_{c_1}} + m_2 J_{v_{c_2}}^T J_{v_{c_2}}\right\}\dot{q}}$$

Also, the rotational K.E term is given by

$$\sum_{i}^{2} \frac{1}{2} w_i^T I_i w$$

$$\Rightarrow \quad \frac{1}{2} \dot{q}^T \left\{ I_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + I_2 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\} \dot{q}$$

Therefore, the $D(q)$ can be given as

$$D(q) = m_1 J_{vc_1}^T J_{vc_1} + m_2 J_{vc_2}^T J_{vc_2} + \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix}$$

Hence,

$$D(q) = \begin{bmatrix} m_1 l_{c_1}^2 + m_2 l_1^2 + I_1 & m_2 l_1 l_{c_2} \cos(q_2 - q_1) \\ m_2 l_1 l_{c_2} \cos(q_2 - q_1) & m_2 l_{c_2}^2 + I_2 \end{bmatrix}$$

where, $\boxed{l_{c_1} = \frac{l_1}{2} \ \& \ l_{c_2} = \frac{l_2}{2}}$ Also, $V(q) = m_1 g l_{c_1} \sin q_1 + m_2 g (l_1 \sin q_1 + l_{c_2} \sin q_2)$

Now, the Christoffel symbols of first kind are given as!

$$C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

Hence,

$$C_{111} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_1} = 0$$

$$C_{121} = C_{211} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_2} = 0$$

$$C_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2} \frac{\partial d_{22}}{\partial q_1} = -m_2 l_1 l_{c_2} \sin(q_2 - q_1)$$

$$C_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2} \frac{\partial d_{11}}{\partial q_2} = m_2 l_1 l_{c_2} \sin(q_2 - q_1)$$

$$C_{212} = C_{122} = \frac{1}{2} \frac{\partial d_{22}}{\partial q_1} = 0 \quad \& \quad C_{222} = \frac{1}{2} \frac{\partial d_{22}}{\partial q_2} = 0$$

Now, $\phi_k(q) = \frac{\partial V}{\partial q_k}$

Hence, $\phi_1(q) = (m_1 l_{c_1} + m_2 l_1) g \cos q_1$

$\phi_2(q) = m_2 l_{c_2} g \cos q_2$

Hence, the Dynamic eqn are

$$\boxed{\sum_j d_{kj} \ddot{q}_j + \sum_{i,j} c_{ijk}(q) \dot{q}_i \dot{q}_j + \phi_k(q) = \tau_k}$$

Therefore, in our case, it is written as

$$\boxed{\begin{aligned} d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{221}(\dot{q}_2)^2 + \phi_1 &= \tau_1 \\ d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}(\dot{q}_1)^2 + \phi_2 &= \tau_2 \end{aligned}}$$ — Ans

In case of mini-project we also have the terms like $(\dot{q}_1 \dot{q}_2)$ which means that there was Co-riolisis effect which increases the torque. We have elimininated the Coriolish term here completely by using remotely-driven configuration.

(10) If we are provided $D(q)$ & $V(q)$

Then, eqn of motion can be written as

$$\boxed{\sum_j d_{kj} \ddot{q}_j + \sum_{i,j} C_{ijk}(q)\, \dot{q}_i \dot{q}_j + \phi_k(q) = \tau_k}$$

where k - signify that for which link we want to write the equation of motion.

Also,

$$\boxed{\phi_k(q) = \frac{\partial V}{\partial q_k}}$$

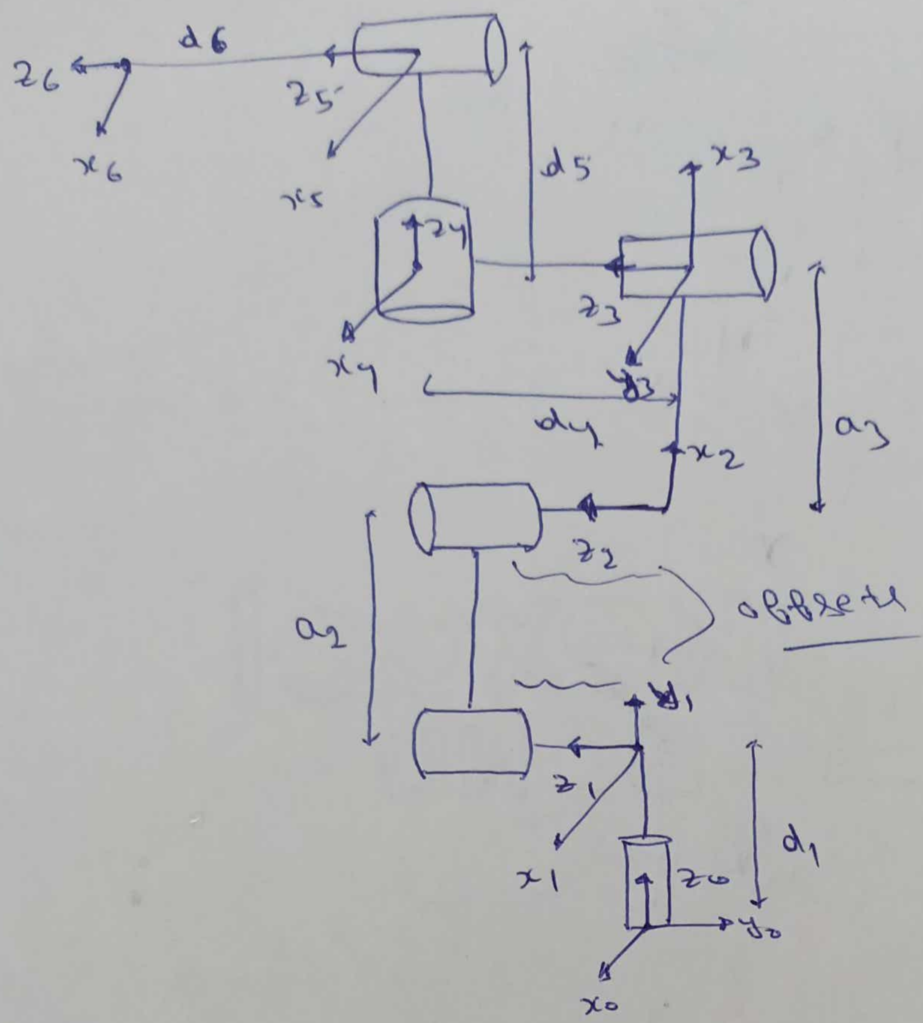
Also,

$$\boxed{C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]}$$

where, $[d_{11}, d_{12}, d_{21}, \cdots d_{nn}]$ are the elements of $D(q)$ matrix.

Therefore, if we know $D(q)$ & $V(q)$. we can easily derive the equation of motion.

(12) UR-5 robot



1) Total number of links = 6

2) Total number of joints = 6

3) All joints are revolute

D-H Parameters :-

link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	d_1	θ_1^*
2	a_2	0	0	θ_2^*
3	a_3	0	0	θ_3^*
4	0	$-\pi/2$	d_4	θ_4^*
5	0	$+\pi/2$	d_5	
6	0	0	d_6	θ_6^*

Ans