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Assignment – 3

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① We know that Jacobian relates the joint variables to the end-effector velocity as follows

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = J(q)\dot{q}$$

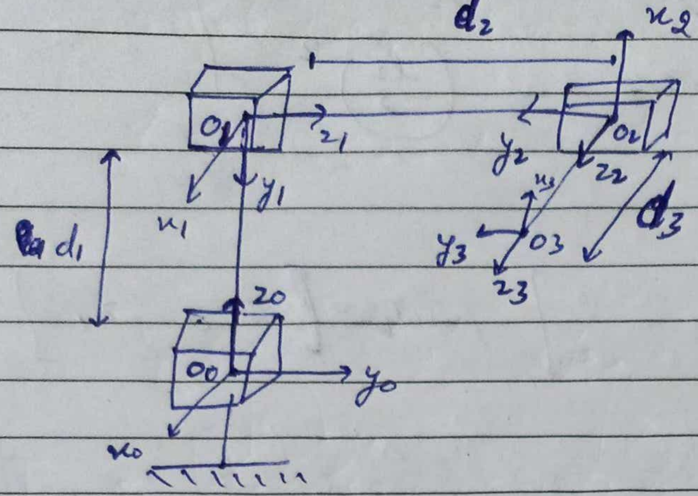
$J(q) \to (6 \times n)$, $n \to$ no. of link.

↳ Since the Jacobian is a function of Configuration (q), Thus, the configuration for ~~…~~ which the rank of Jacobian decreases are called Singular Configuration.
Rank decreases means that our manipulator loses one or more degrees-of-freedom & therefore cannot move in the corresponding direction.

↳ In order to check the singular configuration, we will find the rank of Jacobian & if it comes out to be rank deficient then the configuration will be singular configuration.

↳ Near singularities there will not exist a unique solution to the inverse kinematics problem. In such cases there may be no solution or there may be infinitely many solutions.

(5)



DH - parameter table.

Link	a_i	α_i	d_i	θ_i
1	0	-90	d_1	0
2	0	-90	d_2	-90
3	0	0	d_3	0

where d_1, d_2 & d_3 are variable

$$A_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^{\,3} = A_1 A_2 A_3$$

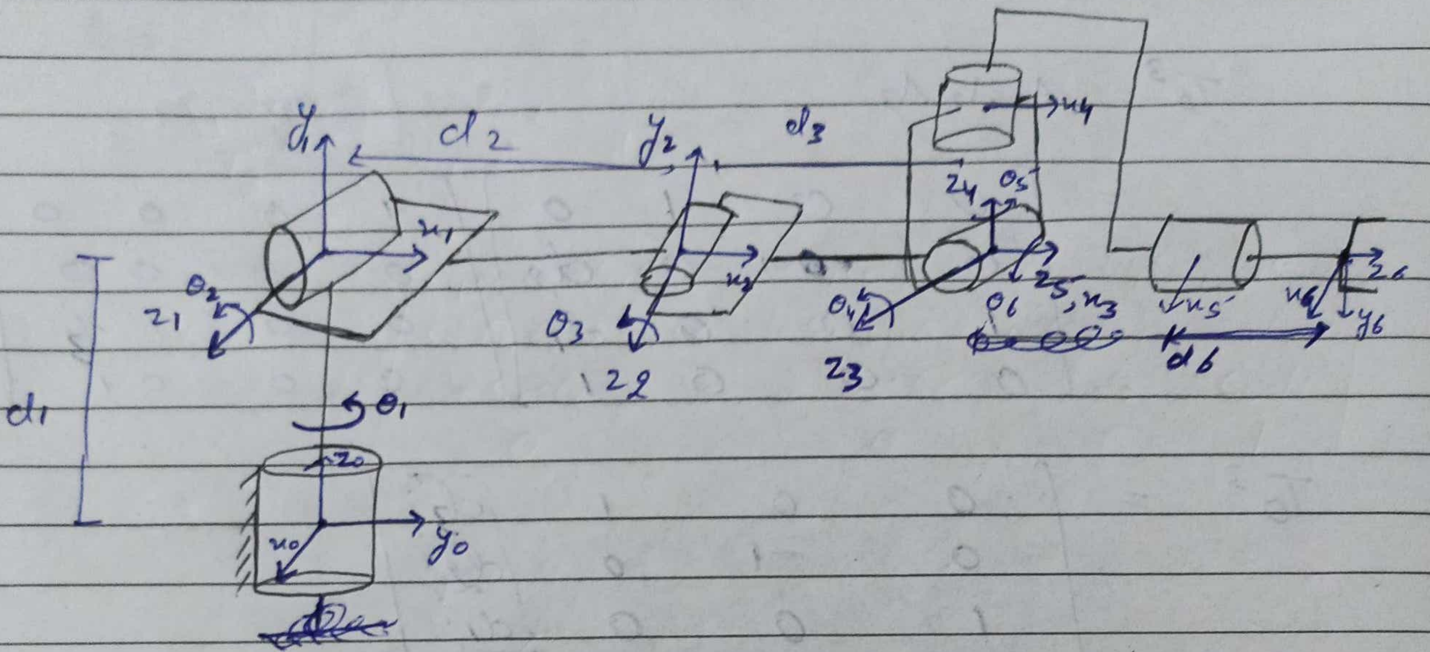
$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^{\,3} = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ 0 & -1 & 0 & d_2 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$x = d_3$
$y = d_2$
$z = d_1$ → these are the coordinates of end effector.

& orientation of endeffector is given by = $\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(b)



Link	a_i	α_i	d_i	θ_i
1	0	90	d_1	θ_1
2	d_2	0	0	θ_2
3	d_3	0	0	θ_3
4	0	-90	0	θ_4
5	0	-90	0	θ_5
6	0	0	d_6	θ_6

for spherical wrist (links 4–6)

$$A_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & d_2c_2 \\ s_2 & c_2 & 0 & d_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & d_3c_3 \\ s_3 & c_3 & 0 & d_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = \underbrace{A_1 A_2} A_3$$

$$= \begin{bmatrix} c_1c_2 & -c_1s_2 & s_1 & d_2c_2c_1 \\ s_1c_2 & -s_1s_2 & -c_1 & s_1d_2s_2 \\ s_2 & c_2 & 0 & d_2s_2+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & d_3c_3 \\ s_3 & c_3 & 0 & d_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c_1c_{23} $\quad\quad$ $-c_1s_{23}$

$$= \begin{bmatrix} c_1c_2c_3 - c_1s_2s_3 & -c_1c_2s_3 - c_1s_2c_3 & s_1 & \alpha \\ s_1c_2c_3 - s_1s_2s_3 & -s_1c_2s_3 - s_1s_2c_3 & -c_1 & \beta \\ s_2c_3 + c_2s_3 & -s_2s_3 + c_2c_3 & 0 & \gamma \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

s_1c_{23}

s_{23}

c_{23} $\quad\quad$ $-s_1s_{23}$

$$\alpha = d_3c_1c_2c_3 - d_3c_1s_2s_3 + d_2c_2c_1$$
$$= d_3c_1(c_{23}) + d_2c_2c_1$$

$$\beta = d_3s_1c_2c_3 - d_3s_1s_2s_3 + s_1d_2s_2$$
$$= d_3s_1c_{23} + d_2s_1s_2$$

$$r = d_3 s_2 c_3 + d_3 c_2 s_3 + d_1 + d_2 s_2$$

$$r = d_3 s_{23} + d_1 + d_2 s_2$$

$$T_3^6 = A_4 A_5 A_6$$

$$= \begin{bmatrix} c_4 c_5 c_6 - s_4 s_6 & -c_4 c_5 s_6 - s_4 c_6 & c_4 s_5 & c_4 s_5 d_6 \\ s_4 c_5 c_6 + c_4 s_6 & -s_4 c_5 s_6 + c_4 c_6 & s_4 s_5 & s_4 s_5 d_6 \\ -s_5 c_6 & s_5 s_6 & c_5 & c_5 d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^6 = T_0^3 \, T_3^6$$

$$= \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & d_3 c_1 c_{23} + d_2 c_1 c_2 \\ s_1 c_{23} & -s_1 s_{23} & -c_1 & d_3 s_1 c_{23} + d_2 s_1 c_2 \\ s_{23} & c_{23} & 0 & d_3 s_{23} + d_1 + d_2 s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r_{11} = c_1 c_{23}(c_4 c_5 c_6 - s_4 s_6) - c_1 s_{23}(s_4 c_5 c_6 + c_4 s_6) - s_1 s_5 c_6$$
$$r_{12} = c_1 c_{23}(-c_4 c_5 s_6 - s_4 s_6) - c_1 s_{23}(-s_4 c_5 s_6 + c_4 c_6) + s_1 s_5 s_6$$

$$r_{13} = c_1 c_{23} c_4 s_5 - c_1 s_{23} s_4 s_5 + s_1 c_5$$

$$\cancel{r_{21} = s_1 c_{23} (s_4 c_5 c_6 + c_4 s_6)}$$

$$r_{21} = s_1 c_{23} (c_4 c_5 c_6 - s_4 s_6) - s_1 s_{23} (s_4 c_5 c_6 + c_4 s_6) + c_1 s_5 c_6$$

$$r_{22} = s_1 c_{23} (-c_4 c_5 s_6 - s_4 c_6) - s_1 s_{23} (-s_4 c_5 s_6 + c_4 c_6) - c_1 s_5 s_6$$

$$r_{23} = s_1 c_{23} c_4 s_5 - s_1 s_{23} s_4 s_5 - c_1 c_5$$

$$r_{31} = s_{23} (c_4 c_5 c_6 - s_4 s_6) + c_{23} (s_4 c_5 c_6 + c_4 s_6)$$

$$r_{32} = s_{23} (-c_4 c_5 s_6 - s_4 c_6) + c_{23} (-s_4 c_5 s_6 + c_4 c_6)$$

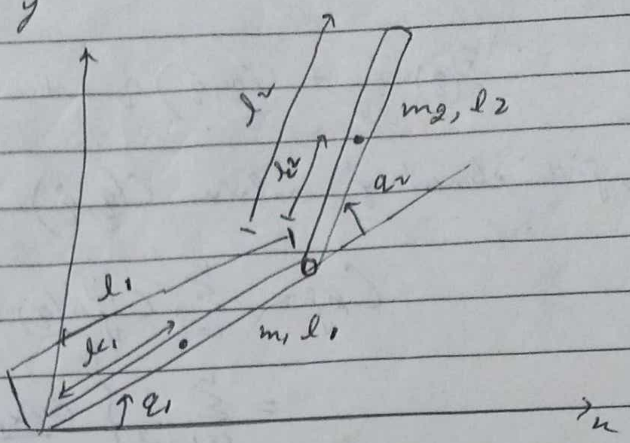
$$r_{33} = s_{23} c_4 s_5 + c_{23} s_4 s_5$$

$$d_x = c_1 c_{23} c_4 s_5 d_6 - c_1 s_{23} s_4 s_5 d_6 + s_1 c_5 d_6 + d_3 c_1 c_{23} + d_2 c_1 c_2$$

$$d_y = s_1 c_{23} c_4 s_5 d_6 - s_1 s_{23} s_4 s_5 d_6 - c_1 c_5 d_6 + d_3 s_1 c_{23} + d_2 s_1 s_2$$

$$d_z = s_{23} c_4 s_5 d_6 + c_{23} c_4 s_5 d_6 + d_3 s_{23} + d_1 + d_2 s_2$$

(7) Direct drive



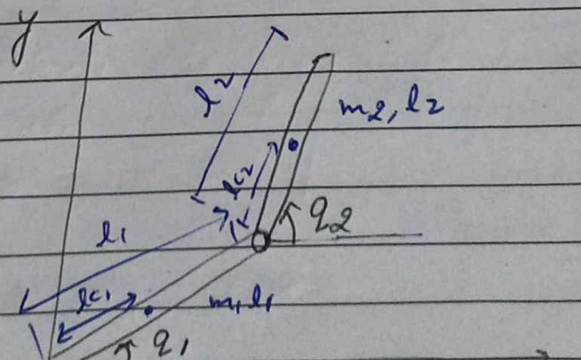
dynamic eq^n of motion is given by.

from book

$$\begin{cases} d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{121}\dot{q}_1\dot{q}_2 + c_{211}\dot{q}_2\dot{q}_1 + c_{221}\dot{q}_2^2 + \phi_1 = \tau_1 \\ d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \phi_2 = \tau_2 \end{cases}$$

* Note that dynamics eq^n of direct drive ~~mani~~ arrangement contains both $\dot{q}_i\dot{q}_j$ $(i \neq j)$ & $(\dot{q}_i)^2$ terms which means that Coriolis & Centrifugal terms respectively are present in this case.

→ Planar Elbow Manipulator with Remotely Driven link.



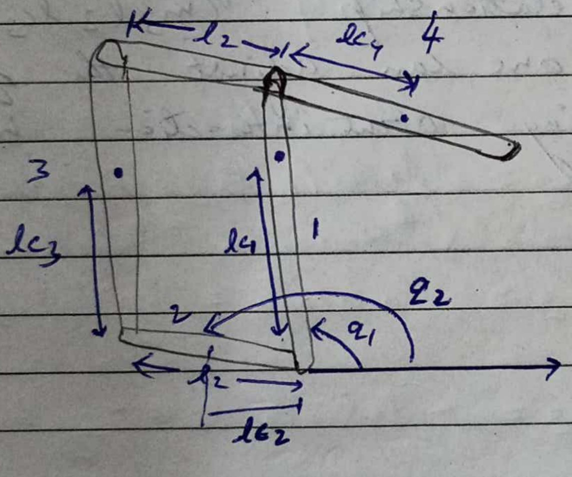
dynamic eq^n of motion is given by

from book
$$\begin{cases} d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{221}\dot{q}_2^2 + \phi_1 = \tau_1 \\ d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_2^2 + \phi_2 = \tau_2 \end{cases}$$

Note → the dynamics eq^n of remotely drive link ~~arrangement~~ contains only $(\dot{q}_2)^2$ term which is centrifugal terms,

we have eliminated the Coriolis forces in this case.

→ Five-Bar Parallelogram Arrangement



Dynamic eq^n of motion is given by

$$d_{11}\ddot{q}_1 + \phi_1(q_1) = \tau_1$$

$$d_{22}\ddot{q}_2 + \phi_2(q_2) = \tau_2$$

Note → this dynamic eq^n contains neither Coriolis nor centripetal terms. In this case, we have eliminated the Coriolis as well as centripetal forces both.

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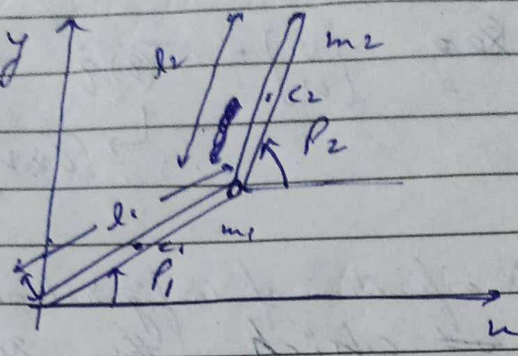
Advantages

1) We saw that by driving the second joint remotely from the base in case of ~~dis~~ remotely driven link, we have eliminated the coriolis forces, but we had still have the centripetal forces coupling the two link. Due to which the required torque will be less than the what is required in the case of Direct drive.

(2) We also saw that in the case of five-bar parallelogram dynamic equation contains neither coriolis nor centripetal terms. Due to which the require torque will be less than the what is required in the case of Direct drive as well as remotely driven link. This helps to explain the increasing popularity of the parallegram configuration in industrial robots. If the relationship ($m_3 l_2 l_{c_3} = m_4 l_1 l_{c_4}$) is satisfied then one can adjust the q_1 & q_2 independently without worrying about interaction b/w the two angles.

(8) Planar Elbow Manipulator with Remotely driven

↳ this was discussed in class.



$$x_{c_1} = \frac{l_1}{2}\cos\theta_1 \quad , \quad y_{c_1} = \frac{l_1}{2}\sin\theta_1$$

$$\dot{x}_{c_1} = -\frac{l_1}{2}\sin\theta_1\,\dot{\theta}_1 \quad , \quad \dot{y}_{c_1} = \frac{l_1}{2}\cos\theta_1\,\dot{\theta}_1$$

$$\therefore \quad v_{c_1} = \begin{bmatrix} -\frac{l_1}{2}\sin\theta_1 & 0 \\ \frac{l_1}{2}\cos\theta_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$x_{c_2} = l_1\cos\theta_1 + \frac{l_2}{2}\cos\theta_2$$

$$y_{c_2} = l_1\sin\theta_1 + \frac{l_2}{2}\sin\theta_2$$

$$\dot{x}_{c_2} = -l_1\sin\theta_1\,\dot{\theta}_1 - \frac{l_2}{2}\sin\theta_2\,\dot{\theta}_2$$

$$\dot{y}_{c_2} = l_1\cos\theta_1\,\dot{\theta}_1 + \frac{l_2}{2}\cos\theta_2\,\dot{\theta}_2$$

$$v_{c_2} = \begin{bmatrix} -l_1\sin\theta_1 & -\frac{l_2}{2}\sin\theta_2 \\ l_1\cos\theta_1 & \frac{l_2}{2}\cos\theta_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\omega_1 = \dot{\theta}_1 k \quad , \quad \omega_2 = \dot{\theta}_2 \hat{k}$$

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here kinetic energy of manipulator equals.

$$K = \dot{P}^T D(P) \dot{P}$$

where $D(P) = \begin{bmatrix} m_1 \frac{l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 l_1 \frac{l_2}{2} \cos(P_2 - P_1) \\ m_2 l_1 \frac{l_2}{2} \cos(P_2 - P_1) & m_2 \frac{l_2^2}{4} + I_2 \end{bmatrix}$

D(P) comes from]

$$D(P) = m_1 J_{vc_1}^T J_{vc_1} + m_2 J_{vc_2}^T J_{vc_2} + \underbrace{\begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix}}_{J_{\omega_1}^T I_1 J_{\omega_1} + J_{\omega_2}^T I_2 J_{\omega_2}}$$

Now

$$d_{11} = m_1 \frac{l_1^2}{4} + m_2 l_1^2 + I_1$$

$$d_{12} = d_{21} = m_2 l_1 \frac{l_2}{2} \cos(P_2 - P_1)$$

$$d_{22} = m_2 \frac{l_2^2}{4} + I_2.$$

Computing Christoffel Symbols.

$$c_{111} = \frac{1}{2} \frac{\partial d_{11}}{\partial P_1} = 0$$

$$c_{121} = c_{211} = \frac{1}{2} \frac{\partial d_{11}}{\partial P_2} = 0$$

$$c_{221} = \frac{\partial d_{12}}{\partial P_2} - \frac{1}{2} \frac{\partial d_{22}}{\partial P_1} = -m_2 l_1 \frac{l_2}{2} \sin(P_2 - P_1)$$

$$c_{112} = \frac{\partial d_{21}}{\partial P_1} - \frac{1}{2} \frac{\partial d_{11}}{\partial P_2} = m_2 l_1 \frac{l_2}{2} \sin(P_2 - P_1)$$

$$c_{212} = c_{122} = \frac{1}{2} \frac{\partial d_{22}}{\partial P_1} = 0$$

$$c_{222} = \frac{1}{2} \frac{\partial d_{22}}{\partial P_2} = 0$$

Now, P.E in terms of P_1 & P_2 is given by.

$$V = m_1 g \frac{l_1}{2} \sin P_1 + m_2 g \left(l_1 \sin P_1 + \frac{l_2}{2} \sin P_2 \right).$$

So, $\phi_k = \dfrac{\partial V}{\partial P_k}$

$$\phi_1 = \left(m_1 \frac{l_1}{2} + m_2 l_1 \right) g \cos P_1$$

$$\phi_2 = m_2 \frac{l_2}{2} g \cos P_2.$$

So, dynamic equations are

$$d_{11} \ddot{P}_1 + d_{12} \ddot{P}_2 + C_{221} \dot{P}_2^2 + \phi_1 = \tau_1$$

$$d_{21} \ddot{P}_1 + d_{22} \ddot{P}_2 + C_{112} \dot{P}_1^2 + \phi_2 = \tau_2.$$

↳ Comparing this eqⁿ with the mini-project we are not getting the coriolis term in this case but we have the coriolis term in the mini-project, dynamic derivation. In the above case by driving the second joint remotely from the base we have eliminated the coriolis forces thus reduces the required torque for motor.

(10) We are provided with $D(q)$ & $V(q)$ & eq^n of motion can be written as.

$$\sum_j d_{kj}(q)\ddot{q}_j + \sum_{i,j} c_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k \quad -(1)$$

$k = 1, 2, \ldots n$

n is the no. of link present

k tells the link for which we are writing the eq^n of motion.

$$\boxed{\phi_k(q) = \frac{\partial V(q)}{\partial q_k}}$$

1st step

2nd step →

$$\boxed{c_{ijk} = \frac{1}{2}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right\}}$$

Christoffel symbols of first kind

we know d_{ij} is the i^{th} row & j^{th} column element of the $D(q)$

Now once we have find c_{ijk} & $\phi_k(q)$ then putting the values of these in eq(1) we can easily derive the eq^n of motion for k^{th} link.

(8) Also, final eq^n can be written in matrix form

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

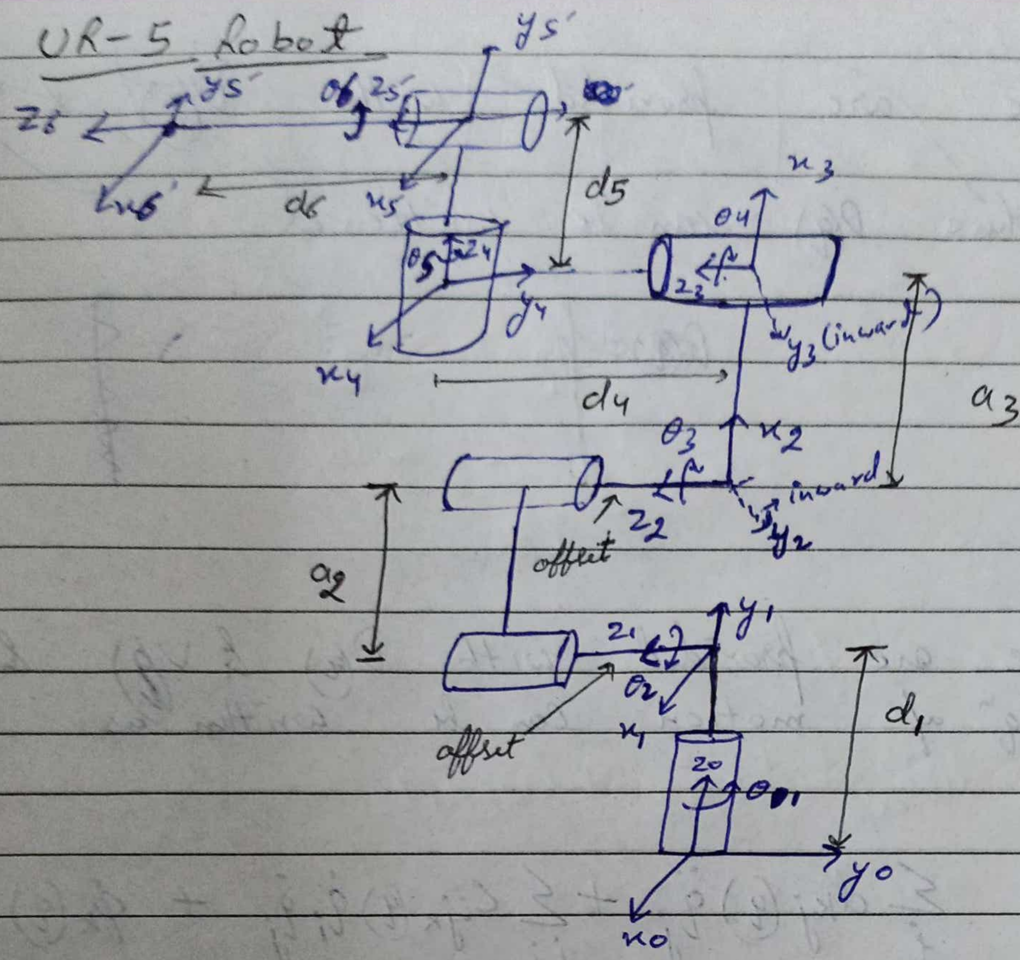
where k, j-th element of matrix $C(q,\dot{q})$ is defined as

$$C_{kj} = \sum_{i=1}^{n} C_{ijk}(q)\,\dot{q}_i$$

$$= \sum_{i=1}^{n} \frac{1}{2}\left\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right\}\dot{q}_i$$

& $g(q) = \begin{bmatrix} \frac{\partial V}{\partial q_1} \\ \frac{\partial V}{\partial q_2} \\ \vdots \\ \frac{\partial V}{\partial q_n} \end{bmatrix}$ & $\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_n \end{bmatrix}$

(12) UR-5 Robot



D-H Parameter

Link	a_i	α_i	d_i	θ_i
1	0	90	d_1	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3
4	0	-90	d_4	θ_4
5	0	90	d_5	θ_5
6	0	0	d_6	θ_6

- Total number of links = 6
- Total no. of joints = 6
- All 6 joints are revolute joints.