

Clocks

The topic of Clocks in aptitude tests involves understanding the mechanics of time, how the hands of a clock move, and how to solve related problems. This topic is often encountered in competitive exams, especially in sections testing logical reasoning and quantitative aptitude.

Basic Concepts

1. Structure of a Clock:

- A clock has 12 hours marked from 1 to 12.
- The hour hand completes one full rotation in 12 hours.
- The minute hand completes one full rotation in 60 minutes.
- The second hand (if considered) completes one full rotation in 60 seconds.

2. Angular Movement:

- The total angle in a circle is 360 degrees.
- Each hour mark on the clock is 30 degrees apart because $360/12 = 30$ degrees.
- The minute hand moves 6 degrees per minute because $360/60 = 6$ degrees.
- The hour hand moves 0.5 degrees per minute because $30/60 = 0.5$.

3. Relative Speed:

- The minute hand moves faster than the hour hand.
- In one hour, the hour hand moves 30 degrees, and the minute hand moves 360 degrees.

Key Formulas

1. Angle Between the Hour and Minute Hand:

-The angle between the hour and minute hand at a given time can be calculated using the formula:

$$\text{Angle} = \left| (30 \times \text{Hour}) - \left(\frac{11}{2} \times \text{Minutes} \right) \right|$$

- If the calculated angle is greater than 180 degrees, subtract it from 360 degrees to find the smaller angle.

2. Time Calculation When the Hands Are at a Right Angle (90 Degrees):

- The hands of a clock are at a right angle 22 times in 12 hours.
- The times can be found by solving the equation:

$$\left| (30 \times \text{Hour}) - \left(\frac{11}{2} \times \text{Minutes} \right) \right| = 90^\circ$$

3. Time Calculation When the Hands Are in a Straight Line (180 Degrees):

- The hands are in a straight line 11 times in 12 hours.
- The times can be found by solving the equation:

$$\left| (30 \times \text{Hour}) - \left(\frac{11}{2} \times \text{Minutes} \right) \right| = 180^\circ$$

4. Coincidence of Hands:

- The hands of a clock coincide (overlap) 11 times in 12 hours.
- The time when they coincide can be calculated using:

$$\text{Time of Coincidence} = \frac{12}{11} \times \text{Hour}$$

Example Problems and Solutions

Example 1: Finding the Angle at a Given Time

Problem: What is the angle between the hour hand and the minute hand at 3:30?

Solution:

- The hour hand at 3:30 will be between 3 and 4.
- The minute hand at 3:30 will be at 6.

Using the formula:

$$\text{Angle} = \left| (30 \times 3) - \left(\frac{11}{2} \times 30 \right) \right|$$
$$\text{Angle} = |90^\circ - 165^\circ| = |-75^\circ| = 75^\circ$$

The angle between the hour hand and the minute hand at 3:30 is 75 degrees.

Example 2: Time When the Hands Are Perpendicular

Problem: At what time between 4:00 and 5:00 will the hour and minute hands be at a right angle?

Solution:

- Use the equation for a right angle:

$$\left| (30 \times 4) - \left(\frac{11}{2} \times \text{Minutes} \right) \right| = 90^\circ$$

$$120 - \frac{11}{2} \times \text{Minutes} = 90$$

$$\frac{11}{2} \times \text{Minutes} = 30$$

$$\text{Minutes} = \frac{30 \times 2}{11} = \frac{60}{11} \approx 5.45$$

So, the hands will be at a right angle at approximately 4:05 and 27 seconds.

Practice Problems

1. Problem: Find the angle between the hour and minute hands at 7:20.

- Solution: Apply the angle formula and calculate.

2. Problem: Determine the time between 2:00 and 3:00 when the hands will be coincident.

- Solution: Use the coincidence formula to find the exact time.

3. Problem: What are the times between 8:00 and 9:00 when the hands of the clock are in a straight line?

- Solution: Solve the equation for a 180-degree angle.

Conclusion

Understanding clocks in aptitude requires grasping the basic mechanics of how clock hands move and how to calculate angles and specific times based on that movement. By mastering these concepts and formulas, you'll be well-prepared to solve a wide variety of clock-related problems in competitive exams.