Lab Assignment 1: Algorithm Foundations

Course: Design and Analysis of Algorithms Lab (ENCA351)

Program: BCA (AI & DS), Semester: V

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Session: 2025-26

Assignment Overview

This document contains the completed solutions and experimental analysis for Lab Assignment 1
—
Algorithm Foundations. The implementation covers recursive and dynamic programming approaches
to the Fibonacci problem, various sorting algorithms, and binary search, including profiling and visualization of time and space complexities.

Task Algorithm Selection and Design

The following algorithms were implemented in Python and analyzed:
1. Fibonacci (Naïve Recursive and Dynamic Programming)
2. Merge Sort
3. Quick Sort
4. Insertion Sort
5. Bubble Sort
6. Selection Sort
7. Binary Search

1. (i) Recursive Fibonacci function

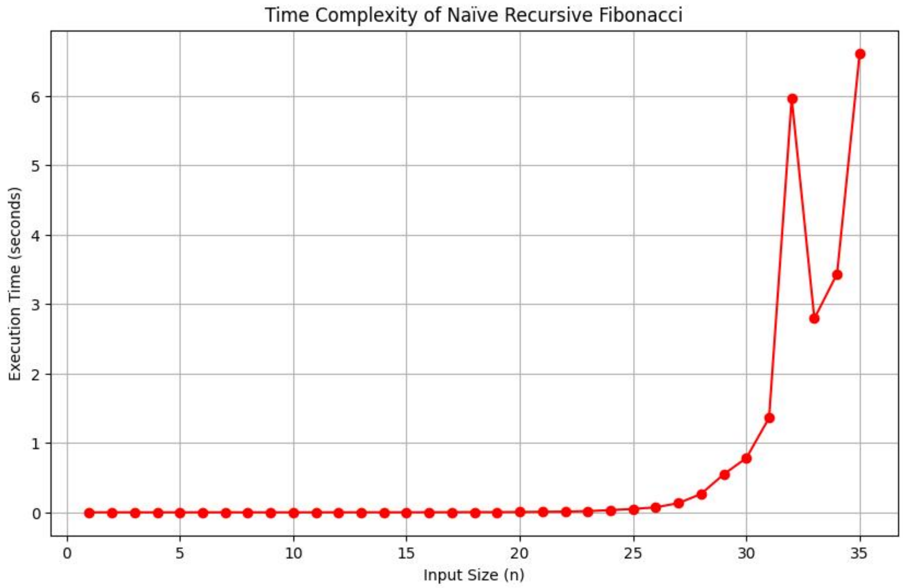
```
def fibonacci_recursion(n):

    if n <= 1:
        return n
    else:
        return fibonacci_recursion(n-1) + fibonacci_recursion(n-2)

print(f"The 14th Fibonacci number is: {fibonacci_recursion(14)}")
```

```
The 14th Fibonacci number is: 377
```

time complexity

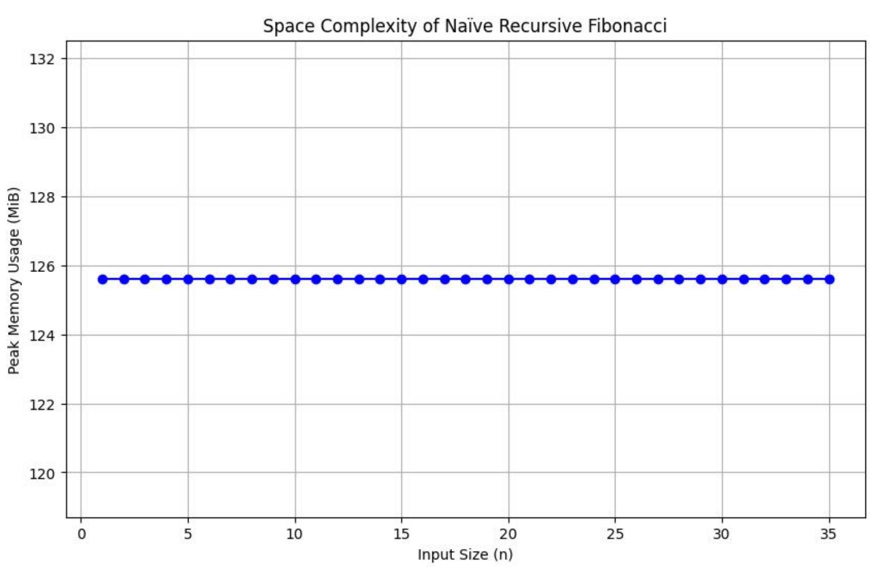
```
# Defining range of input size upto 36 to display Execution Time against n
n_values = np.arange(1, 36)
times = []

# Measuring execution time for each n
for n in n_values:
    start_time = time.time()
    fibonacci_recursion(n)
    end_time = time.time()
    times.append(end_time - start_time)

# Plotting the results
plt.figure(figsize=(10, 6))
plt.plot(n_values, times, marker='o', linestyle='-', color='r')
plt.xlabel("Input Size (n)")
plt.ylabel("Execution Time (seconds)")
plt.title("Time Complexity of Naive Recursive Fibonacci")
plt.grid(True)
plt.show()
```

Output:



space complexity

```
n_values = np.arange(1, 36)
memory_usages = []

# Measure peak memory usage for each n
for n in n_values:
    # Returns a list of memory samples; we take the max as the peak usage.
    usage = memory_usage((fibonacci_recursion, (n,)), interval=0.01)
    memory_usages.append(max(usage))

# Plotting the results
plt.figure(figsize=(10, 6))
plt.plot(n_values, memory_usages, marker='o', linestyle='-', color='b')
plt.xlabel("Input Size (n)")
plt.ylabel("Peak Memory Usage (MiB)")
plt.title("Space Complexity of Naive Recursive Fibonacci")
plt.grid(True)
plt.show()
```



(ii) dynamic programming Fibonacci function

```
# A cache to store computed Fibonacci values
fib_dict = {}

def fibonacci_dp(n):

    if n in fib_dict:
        return fib_dict[n]

    if n <= 1:
        result = n
    else:
        result = fibonacci_dp(n-1) + fibonacci_dp(n-2)

    fib_dict[n] = result
    return result

fib_dict = {}
print(f"The 14th Fibonacci number is: {fibonacci_dp(14)}")

# Can handle much larger numbers efficiently
fib_dict = {}
print(f"The 100th Fibonacci number is: {fibonacci_dp(100)}")
```

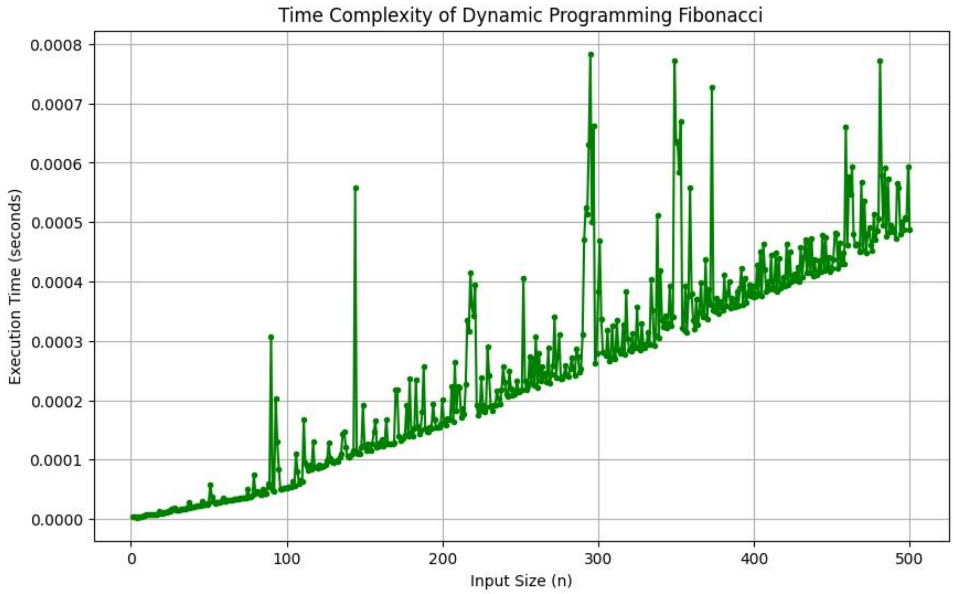
Output:

```
The 14th Fibonacci number is: 377
The 100th Fibonacci number is: 354224848179261915075
```

dynamic programming Fibonacci time complexity:

```
# Time Complexity Plotting
# Using a larger value of n
n_values = np.arange(1, 501)
times = []

for n in n_values:

    fib_dict = {}
    start_time = time.time()
    fibonacci_dp(n)
    end_time = time.time()
    times.append(end_time - start_time)

# --- Plotting the Time Complexity ---
plt.figure(figsize=(10, 6))
plt.plot(n_values, times, marker='.', linestyle='-', color='g')
plt.xlabel("Input Size (n)")
plt.ylabel("Execution Time (seconds)")
plt.title("Time Complexity of Dynamic Programming Fibonacci")
plt.grid(True)
plt.show()
```

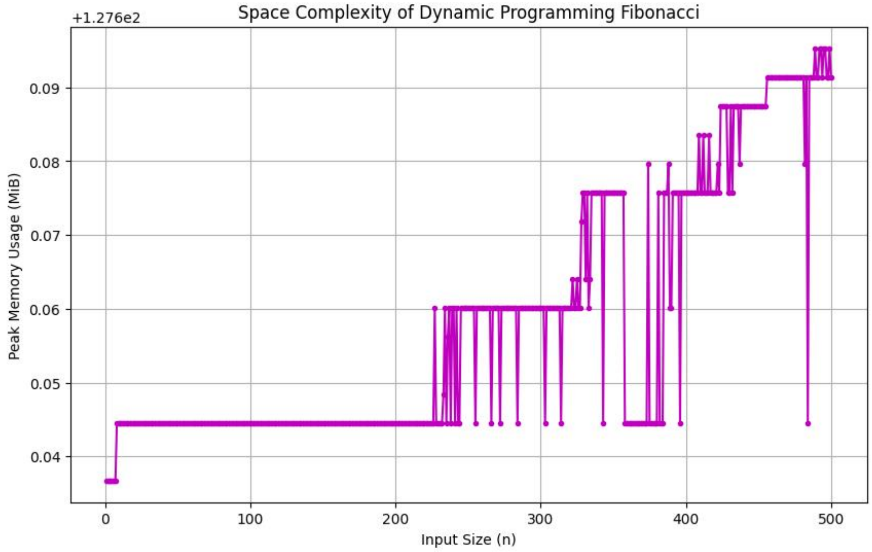
Output:



dynamic programming Fibonacci space complexity

```
# Space Complexity Plotting
n_values = np.arange(1, 501)
memory_usages = []

# This wrapper function ensures the cache is reset for each profiling run
def profile_fib_dp(n):
    global fib_dict
    fib_dict = {} # Reset cache for this specific run
    return fibonacci_dp(n)

for n in n_values:
    # memory_usage runs the function in a separate process
    usage = memory_usage((profile_fib_dp, (n,)), interval=0.01)
    memory_usages.append(max(usage))

# --- Plotting the Space Complexity ---
plt.figure(figsize=(10, 6))
plt.plot(n_values, memory_usages, marker='.', linestyle='-', color='m')
plt.xlabel("Input Size (n)")
plt.ylabel("Peak Memory Usage (MiB)")
plt.title("Space Complexity of Dynamic Programming Fibonacci")
plt.grid(True)
plt.show()
```

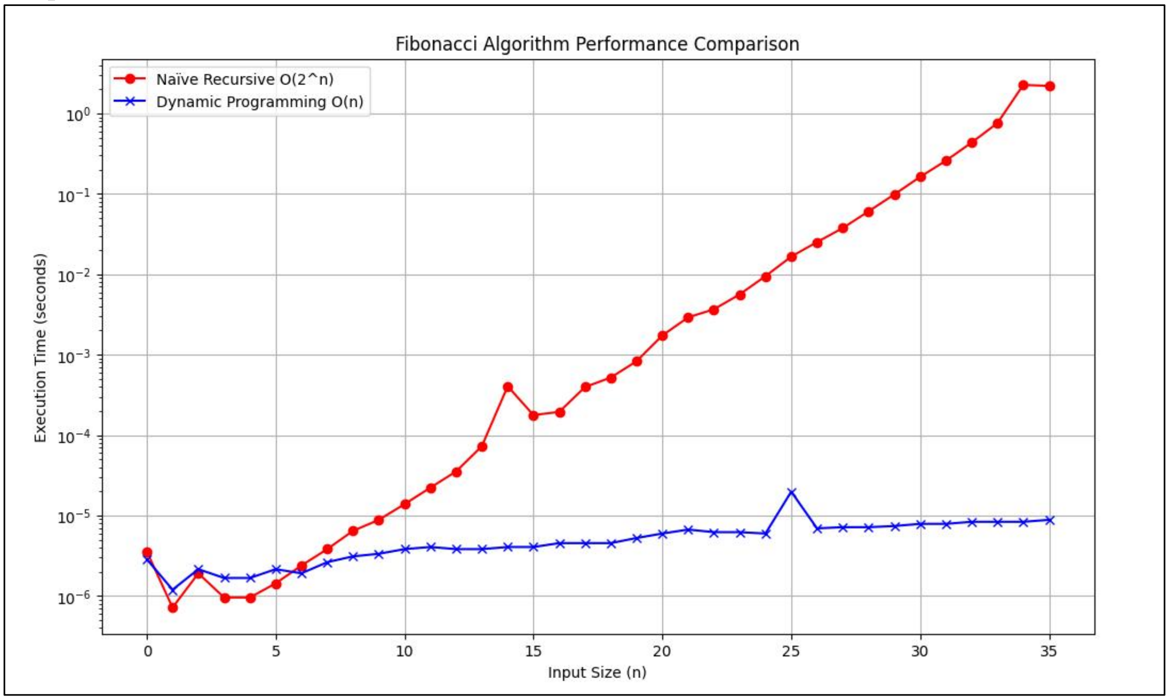
Output:



(iii) Fibonacci comparison

```
# --- Performance Measurement ---
# Fibonacci using Recursion vs Fibonacci using DP

recursive_n_values = range(36)
dp_n_values = range(36) # Using same range for a direct comparison on the plot

# Lists to store execution times
recursive_times = []
dp_times = []

# Measure time for the naive recursive version
for n in recursive_n_values:
    start_time = time.time()
    fibonacci_recursion(n)
    end_time = time.time()
    recursive_times.append(end_time - start_time)

# Measure time for the dynamic programming version
for n in dp_n_values:
    fib_dict = {} # Clear cache for each independent run
    start_time = time.time()
    fibonacci_dp(n)
    end_time = time.time()
    dp_times.append(end_time - start_time)

# --- Plotting the Results ---
plt.figure(figsize=(12, 7))
plt.plot(recursive_n_values, recursive_times, label='Naive Recursive O(2^n)', color='red', marker='o')
plt.plot(dp_n_values, dp_times, label='Dynamic Programming O(n)', color='blue', marker='x')

plt.xlabel('Input Size (n)')
plt.ylabel('Execution Time (seconds)')
plt.title('Fibonacci Algorithm Performance Comparison')
plt.legend()
plt.grid(True)
# Using a logarithmic scale for the y-axis to better visualize the massive difference
plt.yscale('log')
plt.show()
```

Output:



2. Binary Search function

```python
def binary_search(arr, x):

    low = 0
    high = len(arr) - 1

    while low <= high:
        mid = (high + low) // 2

        # If x is greater, ignore left half
        if arr[mid] < x:
            low = mid + 1
        # If x is smaller, ignore right half
        elif arr[mid] > x:
            high = mid - 1
        # means x is present at mid
        else:
            return mid

    # If we reach here, then the element was not present
    return -1

# --- Test ---
sorted_array = [2, 3, 4, 10, 40, 55, 67, 80]
target = 10

result = binary_search(sorted_array, target)

if result != -1:
    print(f"Element is present at index {result}")
else:
    print("Element is not present in array")
```

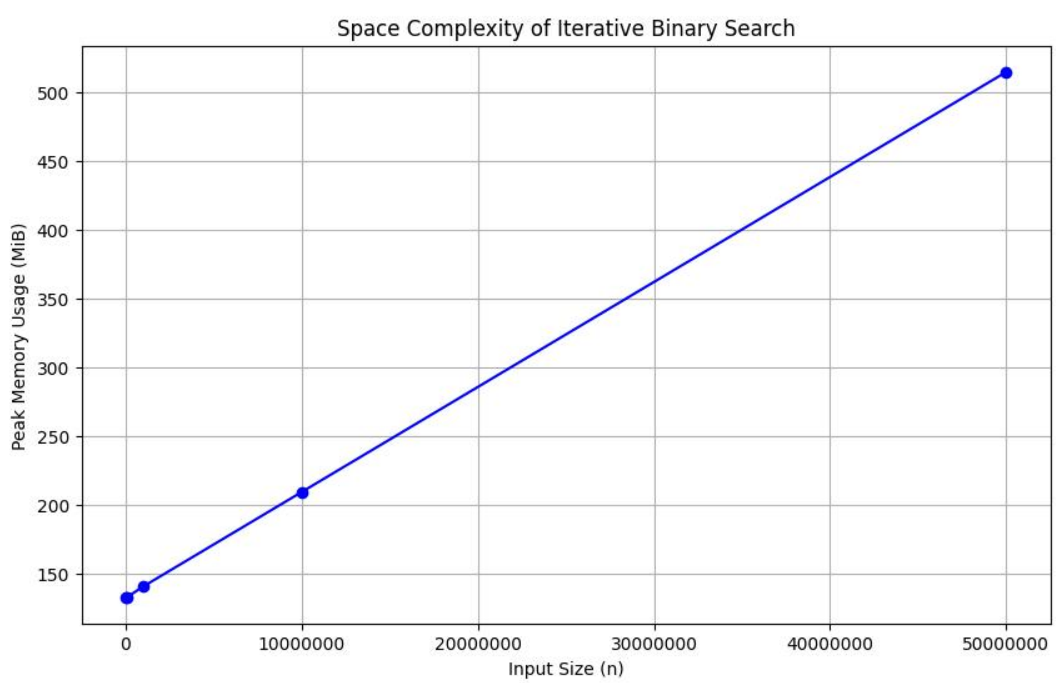
Output:

```
Element is present at index 3
```

Binary Search space complexity

```python
# --- Space Complexity Profiling ---
# Use very large input sizes to demonstrate constant space usage
input_sizes = [10000, 100000, 1000000, 10000000, 50000000]
memory_usages = []

# This loop measures the peak memory for each input size [cite: 60]
for size in input_sizes:
    # Create a large sorted array
    sorted_array = np.arange(size)
    # Target is -1, a value not in the array
    target = -1

    # Run the function through memory_profiler and get the peak memory
    usage = memory_usage((binary_search, (sorted_array, target)))
    memory_usages.append(max(usage))

# --- Plotting the Space Complexity ---
plt.figure(figsize=(10, 6))
plt.plot(input_sizes, memory_usages, marker='o', linestyle='-', color='b')
plt.xlabel("Input Size (n)")
plt.ylabel("Peak Memory Usage (MiB)")
plt.title("Space Complexity of Iterative Binary Search")
plt.grid(True)
# Using a non-logarithmic y-axis to clearly show the flat line
plt.ticklabel_format(style='plain', axis='x')
plt.show()
```

Output:



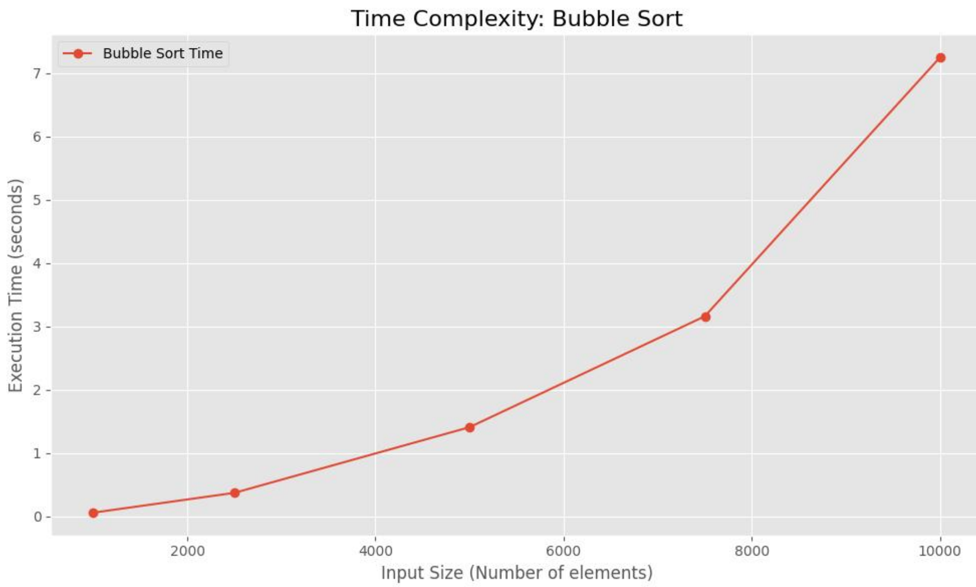
3. Bubble Sort

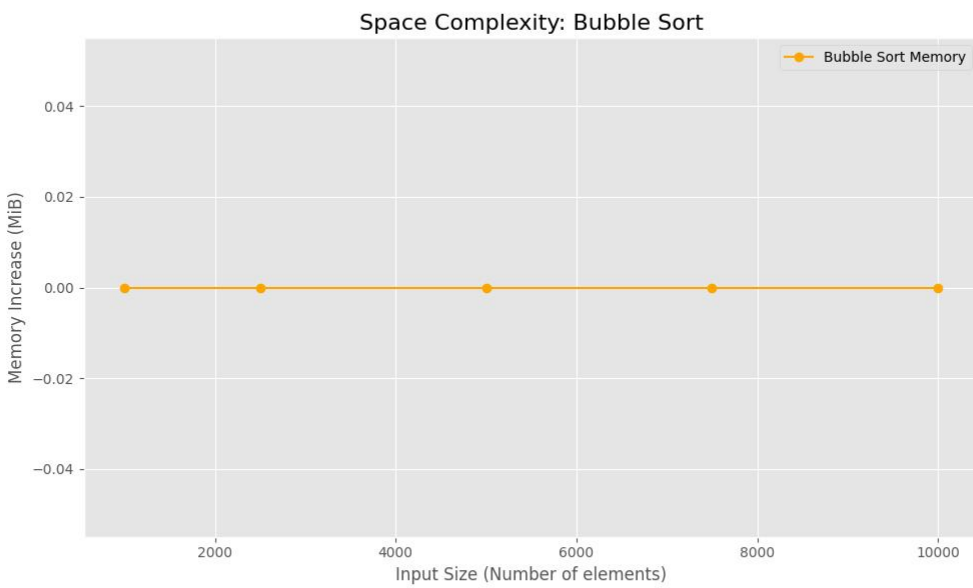
```
def bubble_sort(arr):

    n = len(arr)
    for i in range(n):
        swapped = False
        for j in range(0, n - i - 1):
            if arr[j] > arr[j + 1]:
                arr[j], arr[j + 1] = arr[j + 1], arr[j]
                swapped = True
        if not swapped:
            break
```

Bubble Sort analysis

```
def run_single_test_and_plot(algo, name, input_sizes):

    time_results = []
    memory_results = []

    for size in input_sizes:
        data = [random.randint(0, 1000000) for _ in range(size)]

        # --- Time Measurement ---
        start_time = time.time()
        algo(data)
        end_time = time.time()
        duration = end_time - start_time
        time_results.append(duration)

        # --- Memory Measurement using memory_profiler ---

        usage = memory_usage((algo, (data,)), interval=0.01)

        mem_used = max(usage) - min(usage)
        memory_results.append(mem_used)


    # Plotting Time Complexity
    plt.style.use('ggplot')
    plt.figure(figsize=(10, 6))
    plt.plot(input_sizes, time_results, marker='o', linestyle='-', label=f'{name} Time')
    plt.title(f'Time Complexity: {name}', fontsize=16)
    plt.xlabel('Input Size (Number of elements)', fontsize=12)
    plt.ylabel('Execution Time (seconds)', fontsize=12)
    plt.legend(fontsize=10)
    plt.grid(True)
    plt.tight_layout()
    plt.show()

    # Plotting Space Complexity
    plt.figure(figsize=(10, 6))
    plt.plot(input_sizes, memory_results, marker='o', linestyle='-', label=f'{name} Memory', color='orange'
    plt.title(f'Space Complexity: {name}', fontsize=16)
    plt.xlabel('Input Size (Number of elements)', fontsize=12)
    plt.ylabel('Memory Increase (MiB)', fontsize=12)
    plt.legend(fontsize=10)
    plt.grid(True)
    plt.tight_layout()
    plt.show()

if __name__ == "__main__":
    input_sizes = [1000, 2500, 5000, 7500, 10000]
    run_single_test_and_plot(bubble_sort, 'Bubble Sort', input_sizes)
```

Output:





4. Selection Sort

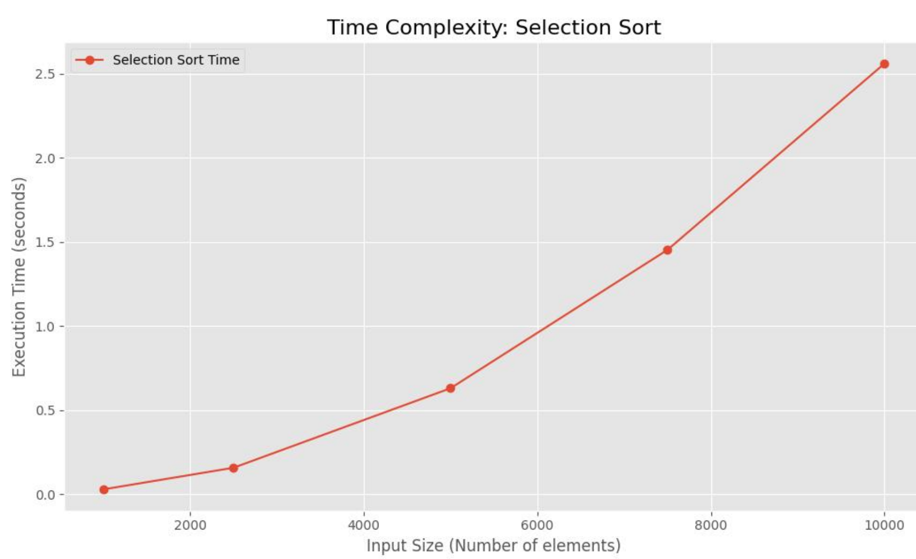
```
def selection_sort(arr):

    n = len(arr)
    for i in range(n):
        min_idx = i
        for j in range(i + 1, n):
            if arr[j] < arr[min_idx]:
                min_idx = j
        arr[i], arr[min_idx] = arr[min_idx], arr[i]
```

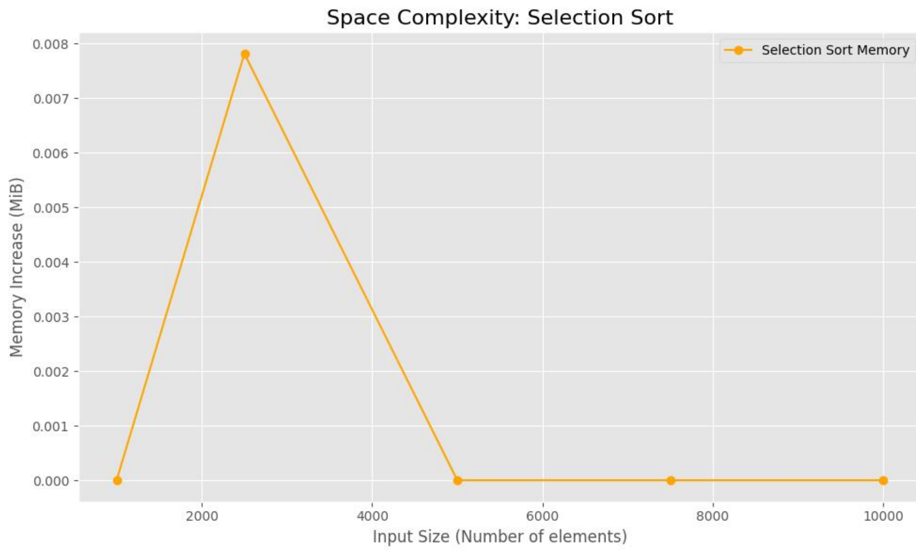
Selection Sort analysis

```
def run_single_test_and_plot(algo, name, input_sizes):

    time_results = []
    memory_results = []

    for size in input_sizes:
        data = [random.randint(0, 1000000) for _ in range(size)]

        # --- Time Measurement ---
        start_time = time.time()
        algo(data)
        end_time = time.time()
        duration = end_time - start_time
        time_results.append(duration)

        # --- Memory Measurement
        usage = memory_usage((algo, (data,)), interval=0.01)

        mem_used = max(usage) - min(usage)
        memory_results.append(mem_used)

    # Plotting Time Complexity [cite: 61]
    plt.style.use('ggplot')
    plt.figure(figsize=(10, 6))
    plt.plot(input_sizes, time_results, marker='o', linestyle='-', label=f'{name} Time')
    plt.title(f'Time Complexity: {name}', fontsize=16)
    plt.xlabel('Input Size (Number of elements)', fontsize=12)
    plt.ylabel('Execution Time (seconds)', fontsize=12)
    plt.legend(fontsize=10)
    plt.grid(True)
    plt.tight_layout()
    plt.show()

    # Plotting Space Complexity [cite: 61]
    plt.figure(figsize=(10, 6))
    plt.plot(input_sizes, memory_results, marker='o', linestyle='-', label=f'{name} Memory', color='orange'
    plt.title(f'Space Complexity: {name}', fontsize=16)
    plt.xlabel('Input Size (Number of elements)', fontsize=12)
    plt.ylabel('Memory Increase (MiB)', fontsize=12)
    plt.legend(fontsize=10)
    plt.grid(True)
    plt.tight_layout()
    plt.show()
```

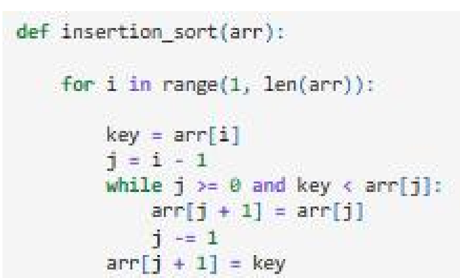
```
if __name__ == "__main__":
    input_sizes = [1000, 2500, 5000, 7500, 10000]
    run_single_test_and_plot(selection_sort, 'Selection Sort', input_sizes)
```

Output:





5. Insertion Sort

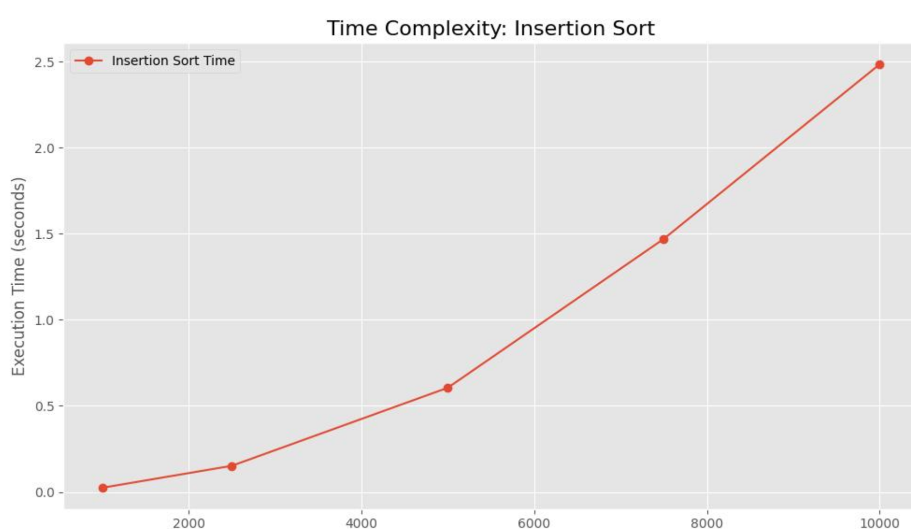


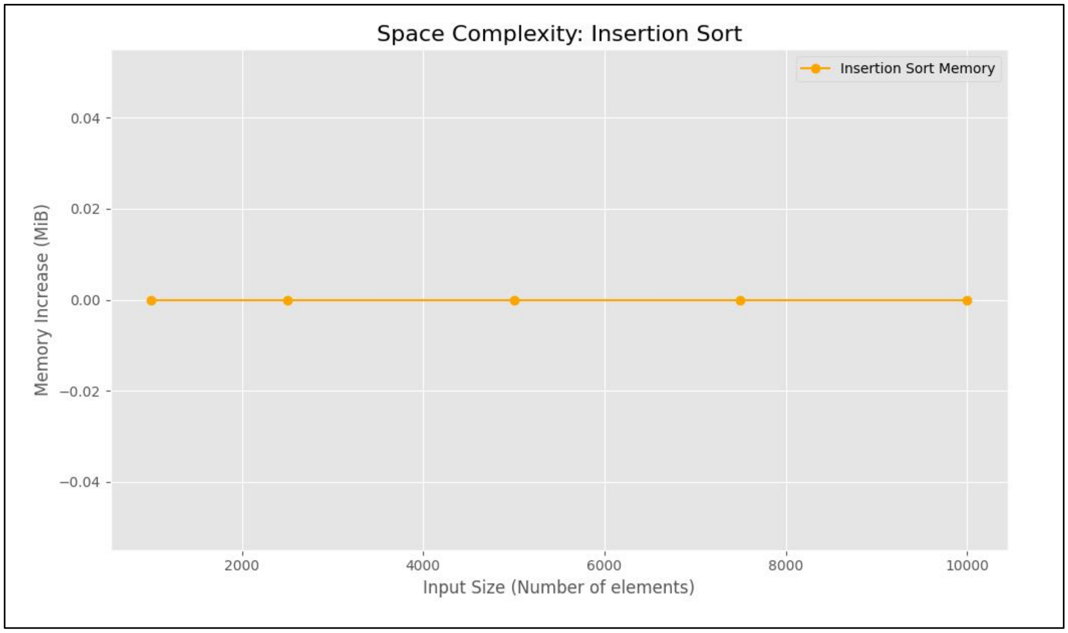
```
def insertion_sort(arr):

    for i in range(1, len(arr)):

        key = arr[i]
        j = i - 1
        while j >= 0 and key < arr[j]:
            arr[j + 1] = arr[j]
            j -= 1
        arr[j + 1] = key
```

Insertion Sort analysis

```python
def run_single_test_and_plot(algo, name, input_sizes):

    time_results = []
    memory_results = []

    for size in input_sizes:
        # Generate random data for sorting
        data = [random.randint(0, 1000000) for _ in range(size)]

        # --- Time Measurement ---
        start_time = time.time()
        algo(data)
        end_time = time.time()
        duration = end_time - start_time
        time_results.append(duration)

        # Memory Measurement
        usage = memory_usage((algo, (data,)), interval=0.01)

        mem_used = max(usage) - min(usage)
        memory_results.append(mem_used)


    # Plotting Time Complexity
    plt.style.use('ggplot')
    plt.figure(figsize=(10, 6))
    plt.plot(input_sizes, time_results, marker='o', linestyle='-', label=f'{name} Time')
    plt.title(f'Time Complexity: {name}', fontsize=16)
    plt.xlabel('Input Size (Number of elements)', fontsize=12)
    plt.ylabel('Execution Time (seconds)', fontsize=12)
    plt.legend(fontsize=10)
    plt.grid(True)
    plt.tight_layout()
    plt.show()

    # Plotting Space Complexity
    plt.figure(figsize=(10, 6))
    plt.plot(input_sizes, memory_results, marker='o', linestyle='-', label=f'{name} Memory', color='orange'
    plt.title(f'Space Complexity: {name}', fontsize=16)
    plt.xlabel('Input Size (Number of elements)', fontsize=12)
    plt.ylabel('Memory Increase (MiB)', fontsize=12)
    plt.legend(fontsize=10)
    plt.grid(True)
    plt.tight_layout()
    plt.show()

if __name__ == "__main__":
    input_sizes = [1000, 2500, 5000, 7500, 10000]
    run_single_test_and_plot(insertion_sort, 'Insertion Sort', input_sizes)
```

Output:





6. Merge Sort

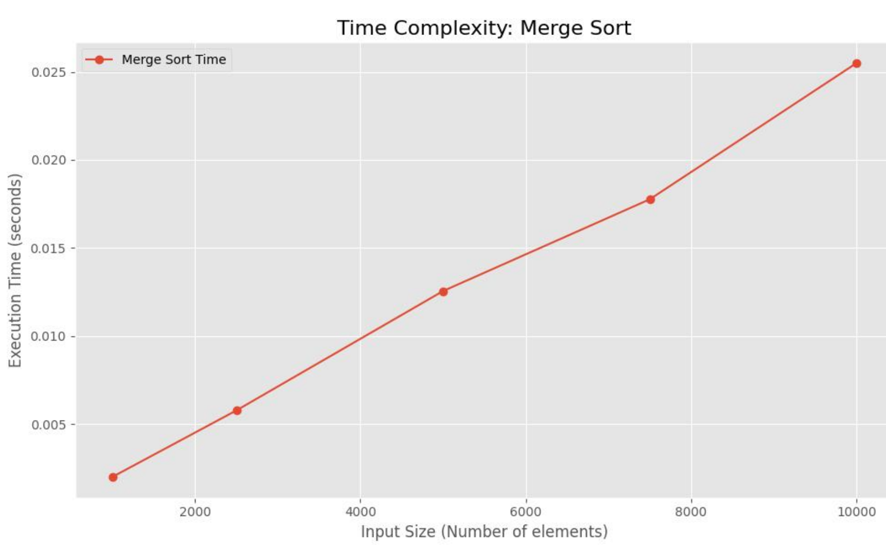
```
def merge_sort(arr):

    if len(arr) > 1:
        mid = len(arr) // 2
        L = arr[:mid]
        R = arr[mid:]

        merge_sort(L)
        merge_sort(R)

        i = j = k = 0

        while i < len(L) and j < len(R):
            if L[i] < R[j]:
                arr[k] = L[i]
                i += 1
            else:
                arr[k] = R[j]
                j += 1
            k += 1

        while i < len(L):
            arr[k] = L[i]
            i += 1
            k += 1

        while j < len(R):
            arr[k] = R[j]
            j += 1
            k += 1
```

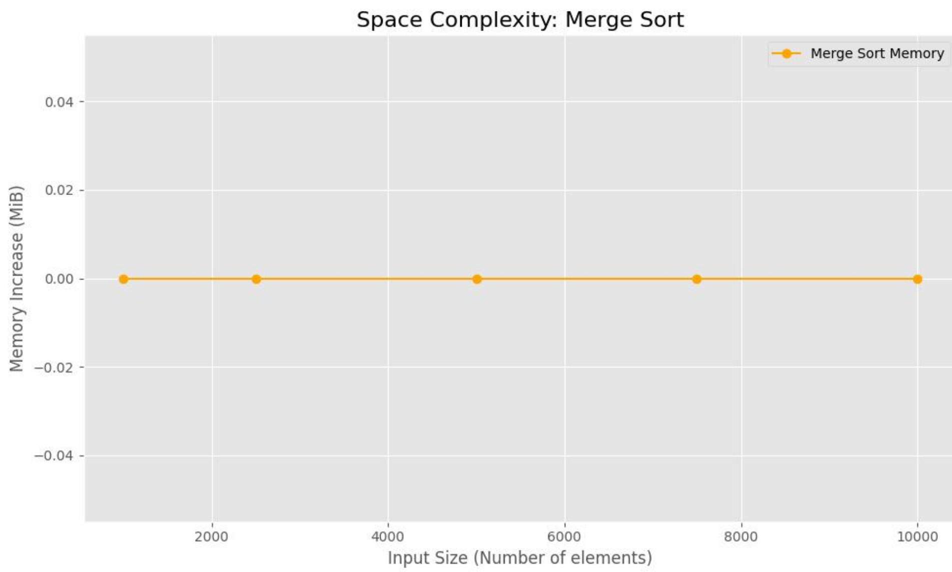
Merge Sort analysis

```
def run_single_test_and_plot(algo, name, input_sizes):
    time_results = []
    memory_results = []

    for size in input_sizes:
        data = [random.randint(0, 1000000) for _ in range(size)]

        # --- Time Measurement ---
        start_time = time.time()
        algo(data)
        end_time = time.time()
        duration = end_time - start_time
        time_results.append(duration)

        # --- Memory Measurement
        usage = memory_usage((algo, (data,)), interval=0.01)

        mem_used = max(usage) - min(usage)
        memory_results.append(mem_used)


    # Plotting Time Complexity
    plt.style.use('ggplot')
    plt.figure(figsize=(10, 6))
    plt.plot(input_sizes, time_results, marker='o', linestyle='-', label=f'{name} Time')
    plt.title(f'Time Complexity: {name}', fontsize=16)
    plt.xlabel('Input Size (Number of elements)', fontsize=12)
    plt.ylabel('Execution Time (seconds)', fontsize=12)
    plt.legend(fontsize=10)
    plt.grid(True)
    plt.tight_layout()
    plt.show()

    # Plotting Space Complexity
    plt.figure(figsize=(10, 6))
    plt.plot(input_sizes, memory_results, marker='o', linestyle='-', label=f'{name} Memory', color='orange'
    plt.title(f'Space Complexity: {name}', fontsize=16)
    plt.xlabel('Input Size (Number of elements)', fontsize=12)
    plt.ylabel('Memory Increase (MiB)', fontsize=12)
    plt.legend(fontsize=10)
    plt.grid(True)
    plt.tight_layout()
    plt.show()
```

```
if __name__ == "__main__":
    input_sizes = [1000, 2500, 5000, 7500, 10000]
    run_single_test_and_plot(merge_sort, 'Merge Sort', input_sizes)
```

Output:





7. Quick Sort

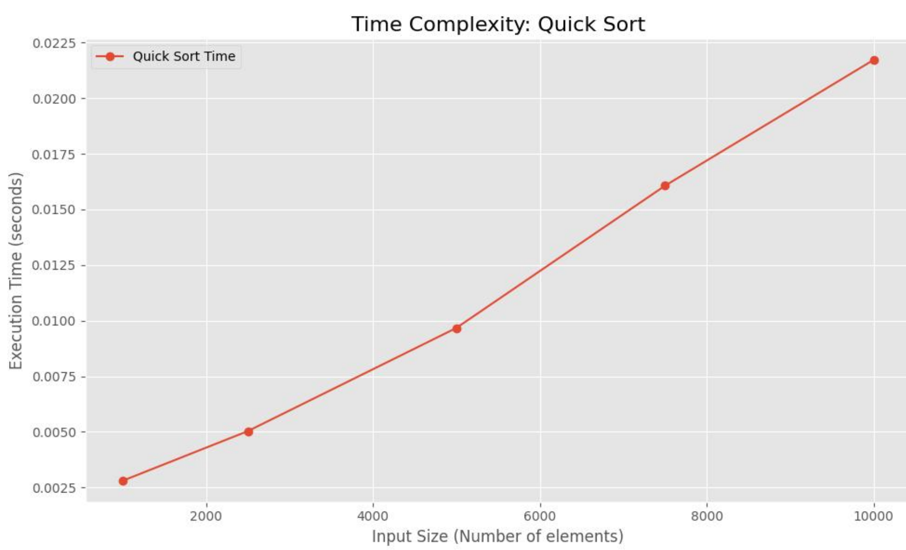
```
def quick_sort(arr):

    if len(arr) <= 1:
        return arr
    else:
        pivot = arr[len(arr) // 2]
        left = [x for x in arr if x < pivot]
        middle = [x for x in arr if x == pivot]
        right = [x for x in arr if x > pivot]
        return quick_sort(left) + middle + quick_sort(right)
```

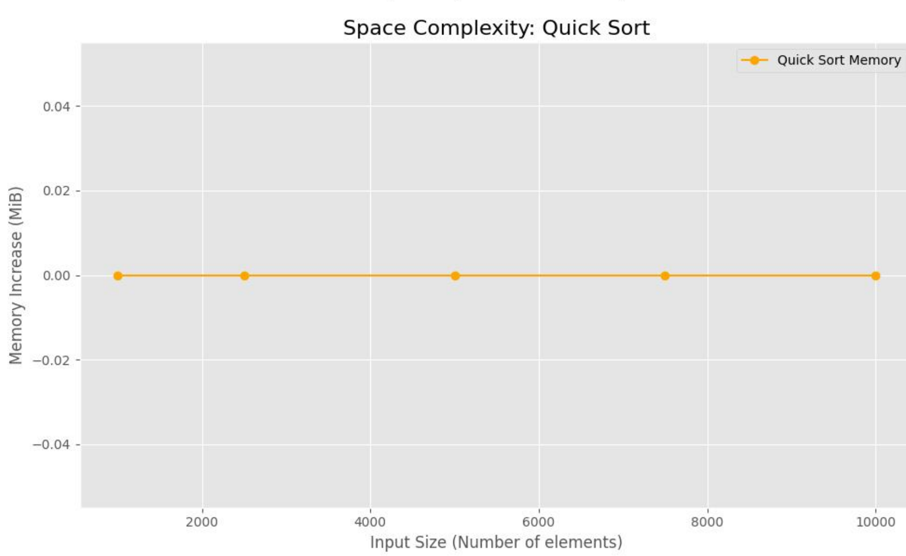
Quick Sort analysis

```
def run_single_test_and_plot(algo, name, input_sizes):

    time_results = []
    memory_results = []

    for size in input_sizes:
        data = [random.randint(0, 1000000) for _ in range(size)]

        # --- Time Measurement ---
        start_time = time.time()
        # Your quick_sort returns a new list, so we capture the result
        sorted_data = algo(data)
        end_time = time.time()
        duration = end_time - start_time
        time_results.append(duration)

        #  Memory Measurement
        usage = memory_usage((algo, (data,)), interval=0.01)

        mem_used = max(usage) - min(usage)
        memory_results.append(mem_used)

    # Plotting Time Complexity
    plt.style.use('ggplot')
    plt.figure(figsize=(10, 6))
```

```
    plt.plot(input_sizes, time_results, marker='o', linestyle='-', label=f'{name} Time')
    plt.title(f'Time Complexity: {name}', fontsize=16)
    plt.xlabel('Input Size (Number of elements)', fontsize=12)
    plt.ylabel('Execution Time (seconds)', fontsize=12)
    plt.legend(fontsize=10)
    plt.grid(True)
    plt.tight_layout()
    plt.show()

    # Plotting Space Complexity
    plt.figure(figsize=(10, 6))
    plt.plot(input_sizes, memory_results, marker='o', linestyle='-', label=f'{name} Memory', color='orange')
    plt.title(f'Space Complexity: {name}', fontsize=16)
    plt.xlabel('Input Size (Number of elements)', fontsize=12)
    plt.ylabel('Memory Increase (MiB)', fontsize=12)
    plt.legend(fontsize=10)
    plt.grid(True)
    plt.tight_layout()
    plt.show()
    print("Plots generated. \n")

if __name__ == "__main__":
    input_sizes = [1000, 2500, 5000, 7500, 10000]
    run_single_test_and_plot(quick_sort, 'Quick Sort', input_sizes)
```

Output:

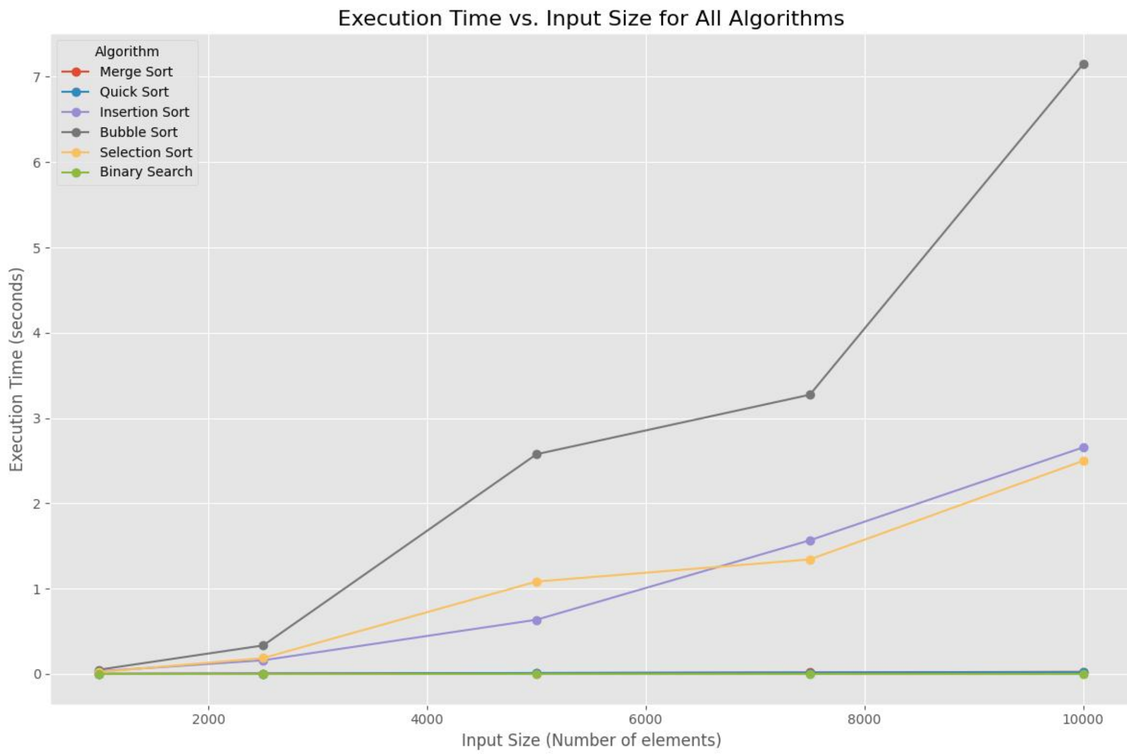




8. Comprehensive algorithm comparison Final Task

```python
def run_tests(algorithms, input_sizes):

    results = {name: [] for name in algorithms.keys()}

    for size in input_sizes:
        for name, algo in algorithms.items():

            if name == 'Binary Search':
                data = list(range(size)) # Create a simple sorted list
                target = -1 # Search for an element not in the list (worst case)

                start_time = time.process_time()
                algo(data, target) # Call with list and target
                end_time = time.process_time()
            else:
                data = [random.randint(0, 1000000) for _ in range(size)]

                start_time = time.process_time()
                if name == 'Quick Sort':
                    _ = algo(data)
                else:
                    algo(data)
                end_time = time.process_time()

            duration = end_time - start_time
            results[name].append(duration)

    return results

def plot_results(results, input_sizes):

    plt.style.use('ggplot')
    plt.figure(figsize=(12, 8))

    for name, times in results.items():
        plt.plot(input_sizes, times, marker='o', linestyle='-', label=name)

    plt.title('Execution Time vs. Input Size for All Algorithms', fontsize=16)
    plt.xlabel('Input Size (Number of elements)', fontsize=12)
    plt.ylabel('Execution Time (seconds)', fontsize=12)
    plt.legend(title='Algorithm', fontsize=10)
    plt.grid(True)
    plt.tight_layout()
    plt.show()

if __name__ == "__main__":
    # algorithms to test
    algorithms = {
        'Merge Sort': merge_sort,
        'Quick Sort': quick_sort,
        'Insertion Sort': insertion_sort,
        'Bubble Sort': bubble_sort,
        'Selection Sort': selection_sort,
        'Binary Search': binary_search
    }

    input_sizes = [1000, 2500, 5000, 7500, 10000]

    performance_results = run_tests(algorithms, input_sizes)

    # Plot the results
    plot_results(performance_results, input_sizes)
```

Output:



Task 4: Final Summary and Documentation

The following table summarizes time and space complexities of all implemented algorithms.

Algorithm	Time Complexity	Space Complexity	Remarks
Fibonacci (Recursive)	O(2^n)	O(n)	Very slow for large n
Fibonacci (DP)	O(n)	O(n)	Efficient via memoization
Merge Sort	O(n log n)	O(n)	Stable and efficient
Quick Sort	O(n log n)	O(log n)	Fast but not stable
Insertion Sort	O(n^2)	O(1)	Efficient for small inputs
Bubble Sort	O(n^2)	O(1)	Simple but slow
Selection Sort	O(n^2)	O(1)	Deterministic, in-place
Binary Search	O(log n)	O(1)	Requires sorted array