

Problem 1 — Scheduling TV Commercials to Maximize Impact (Greedy Job Sequencing)

Description: Schedule commercials (ads) into time slots to maximize total revenue. Each ad has: id, deadline (latest slot), profit. Each slot holds one ad.

Algorithm (Greedy): Sort ads by descending profit and place each ad in the latest available slot on or before its deadline.

```
import matplotlib.pyplot as plt
import numpy as np

# Define an Ad class for better readability
class Ad:
    def __init__(self, ad_id, deadline, profit):
        self.id = ad_id
        self.deadline = deadline
        self.profit = profit

def schedule_ads(ads):
    # Step 1: Sort ads by profit in descending order
    ads.sort(key=lambda x: x.profit, reverse=True)

    # Step 2: Find maximum deadline (slots available)
    max_deadline = max(ad.deadline for ad in ads)
    slots = [-1] * (max_deadline + 1) # -1 means slot empty

    total_profit = 0
    scheduled_ads = []

    # Step 3: Greedy allocation
    for ad in ads:
        # Find a slot for this ad (latest possible before deadline)
        for slot in range(ad.deadline, 0, -1):
            if slots[slot] == -1: # slot available
                slots[slot] = ad.id
                total_profit += ad.profit
                scheduled_ads.append(ad)
                break

    return scheduled_ads, total_profit

# Example usage
ads = [
    Ad("A1", 2, 100),
    Ad("A2", 1, 19),
    Ad("A3", 2, 27),
    Ad("A4", 1, 25),
    Ad("A5", 3, 15),
]

scheduled_ads, total_revenue = schedule_ads(ads)

print("Scheduled Ads:", [ad.id for ad in scheduled_ads])
print("Total Revenue:", total_revenue)
```

Output: Scheduled Ads: ['A1', 'A3', 'A5']
Total Revenue: 142

->Analysis

Time Complexity:

Sorting $\rightarrow O(n \log n)$

Scheduling $\rightarrow O(n * d)$ (n = number of ads, d = max deadline)

Optimized with Disjoint Set Union (DSU) $\rightarrow O(n \log n)$

Space Complexity:

Slots array $\rightarrow O(d)$

Ads storage $\rightarrow O(n)$

-> Visualization

We can simulate varying numbers of ads and plot number of ads vs. total revenue.

```
def experiment(num_ads_list):
    revenues = []

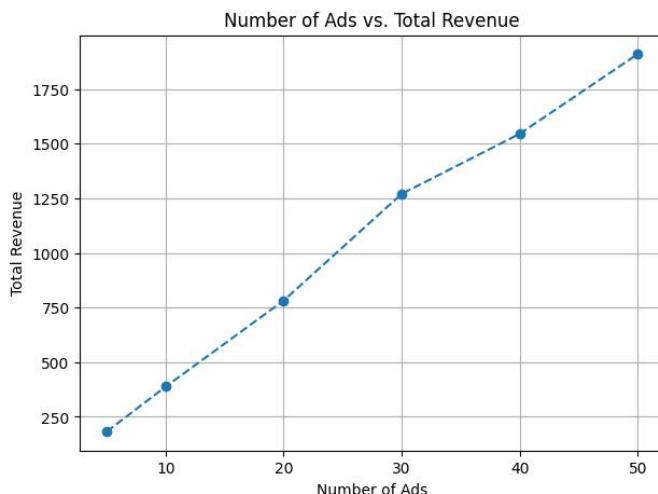
    for n in num_ads_list:
        # Random ads: deadlines [1..n], profits [10..100]
        ads = [Ad(f"A{i}", np.random.randint(1, n//2 + 2), np.random.randint(10, 101)) for i in range(n)]
        _, revenue = schedule_ads(ads)
        revenues.append(revenue)

    return revenues

# Experiment
num_ads_list = [5, 10, 20, 30, 40, 50]
revenues = experiment(num_ads_list)

# Plot
plt.figure(figsize=(7,5))
plt.plot(num_ads_list, revenues, marker='o', linestyle='--')
plt.title("Number of Ads vs. Total Revenue")
plt.xlabel("Number of Ads")
plt.ylabel("Total Revenue")
plt.grid(True)
plt.show()
```

Output:



Problem 2 — Maximizing Profit with Limited Budget (0/1 Knapsack)

Description: Select a subset of items with costs and profits so that total cost \leq budget and profit is maximized. We use dynamic programming to build a DP table $dp[i][w]$.

Python implementation (knapsack):

```
import numpy as np
import matplotlib.pyplot as plt

def knapsack(costs, profits, budget):
    n = len(costs)
    dp = np.zeros((n+1, budget+1), dtype=int)

    # Build DP table
    for i in range(1, n+1):
        for w in range(1, budget+1):
            if costs[i-1] <= w:
                dp[i][w] = max(dp[i-1][w], dp[i-1][w-costs[i-1]] + profits[i-1])
            else:
                dp[i][w] = dp[i-1][w]

    # Reconstruct chosen items
    chosen_items = []
    w = budget
    for i in range(n, 0, -1):
        if dp[i][w] != dp[i-1][w]:
            chosen_items.append(i-1) # index of chosen item
            w -= costs[i-1]

    chosen_items.reverse()
    return dp[n][budget], chosen_items

# Example usage
costs = [2, 3, 4, 5]
profits = [3, 4, 5, 8]
budget = 5

max_profit, chosen = knapsack(costs, profits, budget)
print("Maximum Profit:", max_profit)
print("Chosen Items (indexes):", chosen)
```

Output: Maximum Profit: 8
Chosen Items (indexes): [3]

->Analysis

Time Complexity:

Filling DP table $\rightarrow O(n * \text{budget})$

->Space Complexity:

$O(n * \text{budget})$ (can be optimized to $O(\text{budget})$ using 1D array)

We can vary the budget and see how profit increases.

```

def experiment(costs, profits, max_budget):
    budgets = list(range(1, max_budget+1))
    profits_list = []

    for b in budgets:
        max_profit, _ = knapsack(costs, profits, b)
        profits_list.append(max_profit)

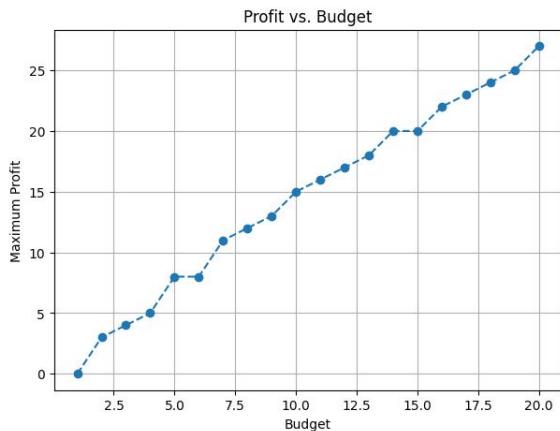
    return budgets, profits_list

# Example experiment
costs = [2, 3, 4, 5, 9, 6]
profits = [3, 4, 5, 8, 10, 7]
budgets, profits_list = experiment(costs, profits, 20)

# Plot
plt.figure(figsize=(7,5))
plt.plot(budgets, profits_list, marker='o', linestyle='--')
plt.title("Profit vs. Budget")
plt.xlabel("Budget")
plt.ylabel("Maximum Profit")
plt.grid(True)
plt.show()

```

Output:



Real-World Considerations

Project Dependencies: In real investment planning, some projects depend on others; this requires constraint-aware knapsack.

Fractional Choices: Sometimes projects can be partially funded (this becomes Fractional Knapsack → solved with Greedy).

Dynamic Budgets: In real life, budgets may change mid-way, requiring re-optimization.

Problem 3 — Solving Sudoku Puzzle (Backtracking)

Description: Use recursive backtracking to fill a 9×9 Sudoku. Check row, column, and 3×3 subgrid validity before placing digits.

Python implementation (solve_sudoku):

```
import numpy as np

# Check if placing num is valid
def is_valid(grid, row, col, num):
    # Check row
    if num in grid[row]:
        return False
    # Check column
    if num in grid[:, col]:
        return False
    # Check 3x3 subgrid
    start_row, start_col = 3 * (row // 3), 3 * (col // 3)
    if num in grid[start_row:start_row+3, start_col:start_col+3]:
        return False
    return True

# Backtracking solver
def solve_sudoku(grid):
    for row in range(9):
        for col in range(9):
            if grid[row][col] == 0: # Empty cell found
                for num in range(1, 10):
                    if is_valid(grid, row, col, num):
                        grid[row][col] = num
                        if solve_sudoku(grid):
                            return True
                        grid[row][col] = 0 # Backtrack
                return False
    return True

# Example Sudoku (0 means empty cell)
sudoku_grid = np.array([
    [5, 3, 0, 0, 7, 0, 0, 0, 0],
    [6, 0, 0, 1, 9, 5, 0, 0, 0],
    [0, 9, 8, 0, 0, 0, 0, 6, 0],
    [8, 0, 0, 0, 6, 0, 0, 0, 3],
    [4, 0, 0, 8, 0, 3, 0, 0, 1],
    [7, 0, 0, 0, 2, 0, 0, 0, 6],
    [0, 0, 0, 0, 0, 0, 2, 8, 0],
    [0, 0, 0, 4, 1, 9, 0, 0, 5],
    [0, 0, 0, 0, 8, 0, 0, 7, 9]
])

print("Original Sudoku:")
print(sudoku_grid)

if solve_sudoku(sudoku_grid):
    print("\nSolved Sudoku:")
    print(sudoku_grid)
else:
    print("\nNo solution exists.")
```

Output:

```
Original Sudoku:
[[5 3 0 0 7 0 0 0 0]
 [6 0 0 1 9 5 0 0 0]
 [0 9 8 0 0 0 0 6 0]
 [8 0 0 0 6 0 0 0 3]
 [4 0 0 8 0 3 0 0 1]
 [7 0 0 0 2 0 0 0 6]
 [0 6 0 0 0 0 2 8 0]
 [0 0 0 4 1 9 0 0 5]
 [0 0 0 0 8 0 0 7 9]]
```

```
Solved Sudoku:
[[5 3 4 6 7 8 9 1 2]
 [6 7 2 1 9 5 3 4 8]
 [1 9 8 3 4 2 5 6 7]
 [8 5 9 7 6 1 4 2 3]
 [4 2 6 8 5 3 7 9 1]
 [7 1 3 9 2 4 8 5 6]
 [9 6 1 5 3 7 2 8 4]
 [2 8 7 4 1 9 6 3 5]
 [3 4 5 2 8 6 1 7 9]]
```

->Analysis

Time Complexity:

Worst-case = $O(9^m)$ $O(9^m)$, where m = number of empty cells.

Practically much faster due to constraint checks pruning invalid paths.

Space Complexity: $O(m)$ $O(m)$ recursion stack for backtracking.

->Visualization (Optional)

We could measure time vs. number of empty cells:

```
import time

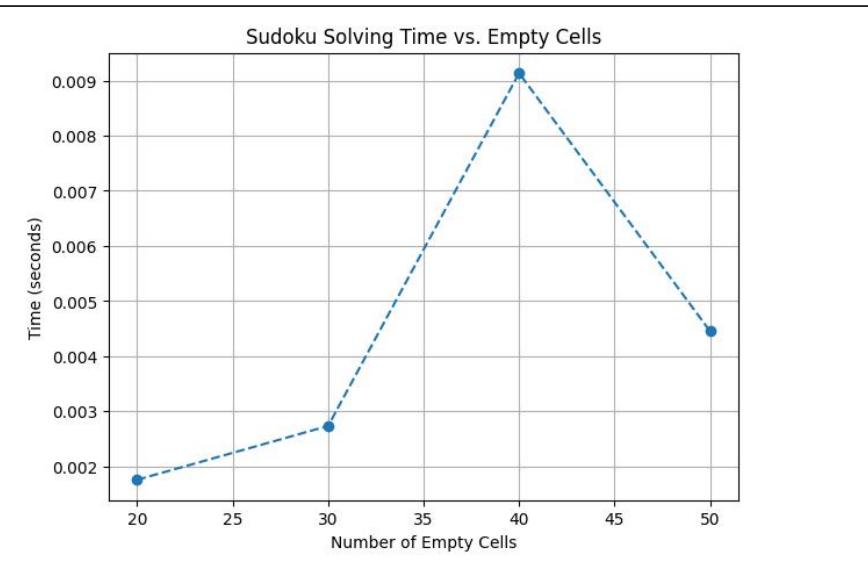
def measure_runtime(empty_cells_list):
    times = []
    for empties in empty_cells_list:
        grid = np.copy(sudoku_grid)
        # Randomly clear some cells
        zero_positions = [(i,j) for i in range(9) for j in range(9)]
        np.random.shuffle(zero_positions)
        for (i,j) in zero_positions[:empties]:
            grid[i][j] = 0

        start = time.time()
        solve_sudoku(grid)
        end = time.time()
        times.append(end - start)
    return times

empty_cells_list = [20, 30, 40, 50]
times = measure_runtime(empty_cells_list)

plt.figure(figsize=(7,5))
plt.plot(empty_cells_list, times, marker='o', linestyle='--')
plt.title("Sudoku Solving Time vs. Empty Cells")
plt.xlabel("Number of Empty Cells")
plt.ylabel("Time (seconds)")
plt.grid(True)
plt.show()
```

Output:



Problem 4 — Brute-force (Safe Simulation & Analysis)

Description: This section provides a safe simulation to estimate how long a brute-force attack would take given charset size and attempts/sec. This script only computes counts and time estimates; it does NOT attempt unauthorized access.

Safe simulation code (time estimates and plots):

```
# Safe simulation: compute combinations and time estimates for brute-force  
  
# Run only on your local machine; this script never accesses networks or other systems.
```

```

# Safe simulation: compute combinations and time estimates for brute-force
# Run only on your Local machine; this script never accesses networks or other systems.

import math
import matplotlib.pyplot as plt
import numpy as np

def combos(charset_size, length):
    """Number of possible strings of given length from charset"""
    return charset_size ** length

def time_seconds(num_attempts, attempts_per_second):
    """Estimate seconds required to try num_attempts at given rate"""
    return num_attempts / attempts_per_second

# Parameters you can change for experiments (all safe)
charsets = {
    'digits (10)': 10,
    'lowercase (26)': 26,
    'alphanum (36)': 36,
    'printable~ (70)': 70 # approximate Larger charset
}

# Example: plot time (in years) vs password length for a chosen charset and attempt rate
chosen_charset_label = 'alphanum (36)'
C = charsets[chosen_charset_label]

attempts_per_second = 1e6 # e.g., 1 million attempts/sec (simulated)
max_length = 10

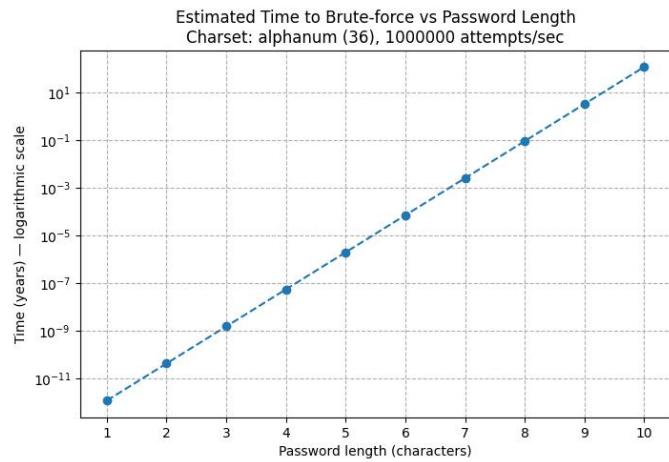
lengths = list(range(1, max_length + 1))
times_years = []
for L in lengths:
    n = combos(C, L)
    secs = time_seconds(n, attempts_per_second)
    years = secs / (60*60*24*365)
    times_years.append(years)

# Plot
plt.figure(figsize=(8,5))
plt.semilogy(lengths, times_years, marker='o', linestyle='--') # Log scale Y
plt.title(f"Estimated Time to Brute-force vs Password Length\nCharset: {chosen_charset_label}, {attempts_per_second:.0f} at {attempts_per_second:.0f} attempts/sec")
plt.xlabel("Password length (characters)")
plt.ylabel("Time (years) - logarithmic scale")
plt.grid(True, which="both", ls="--")
plt.xticks(lengths)
plt.show()

# Additional table-Like printout for reference
print("Length | Combinations | Time (seconds) | Time (years)")
for L, yrs in zip(lengths, times_years):
    n = combos(C, L)
    secs = time_seconds(n, attempts_per_second)
    print(f"[{L:6}] | {n:>20} | {secs:>17,.2f} | {yrs:>12.2e}")

```

Output:



Length	Combinations	Time (seconds)	Time (years)
1	36	0.00	1.14e-12
2	1,296	0.00	4.11e-11
3	46,656	0.05	1.48e-09
4	1,679,616	1.68	5.33e-08
5	60,466,176	60.47	1.92e-06
6	2,176,782,336	2,176.78	6.98e-05
7	78,364,164,096	78,364.16	2.48e-03
8	2,821,109,907,456	2,821,109.91	8.95e-02
9	101,559,956,668,416	101,559,956.67	3.22e+00
10	3,656,158,440,062,976	3,656,158,440.06	1.16e+02