

Hyundai_Clustering

November 10, 2021

```
[117]: import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

```
[118]: mall_customers = pd.read_csv("Mall_Customer.csv")
```

We are trying to form a cluster of customers based on Annual Income and Spending Score

```
[119]: mall_customers.head()
```

```
[119]:
```

	CustomerID	Gender	Age	Annual Income (k\$)	Spending Score (1-100)
0	1	Male	19	15	39
1	2	Male	21	15	81
2	3	Female	20	16	6
3	4	Female	23	16	77
4	5	Female	31	17	40

```
[120]: # missing values
mall_customers.isnull().sum()
```

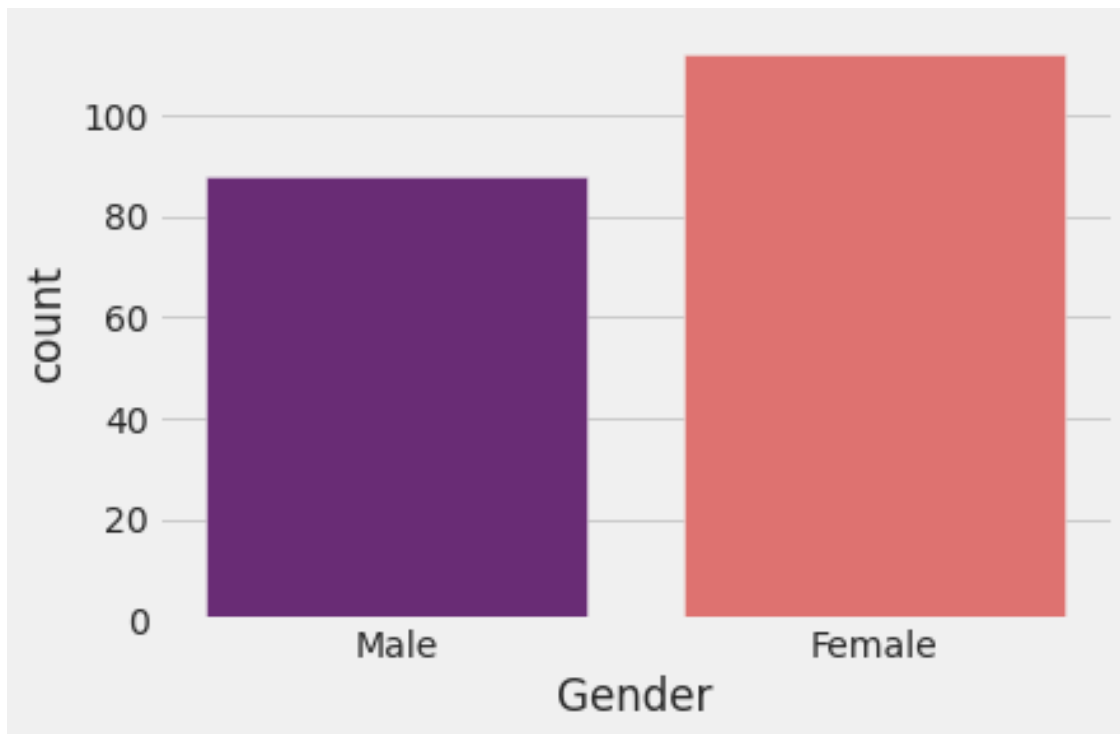
```
[120]: CustomerID          0
Gender                  0
Age                    0
Annual Income (k$)     0
Spending Score (1-100) 0
dtype: int64
```

```
[121]: mall_customers['Gender'].value_counts()
```

```
[121]: Female    112
Male       88
Name: Gender, dtype: int64
```

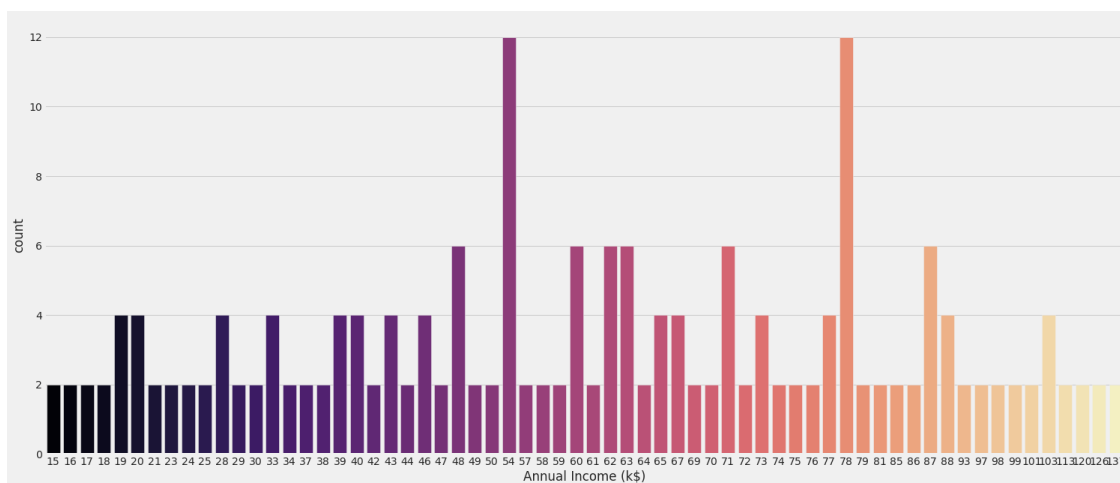
```
[122]: sns.countplot(x = 'Gender', data = mall_customers, palette='magma')
```

```
[122]: <matplotlib.axes._subplots.AxesSubplot at 0x7f0a0e2ceb50>
```



```
[123]: plt.figure(figsize=(23,10))
sns.countplot(x = 'Annual Income (k$)', data = mall_customers, palette='magma')
```

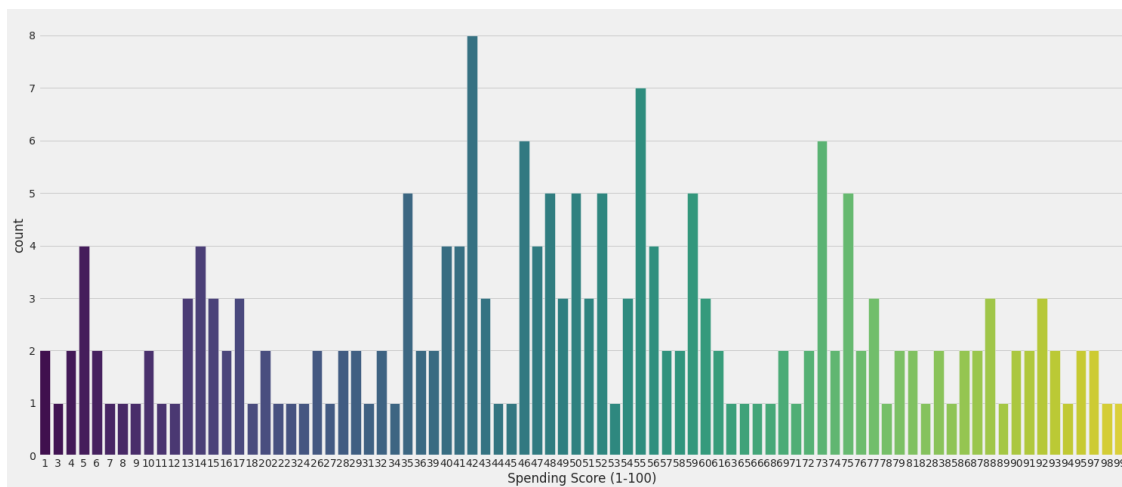
```
[123]: <matplotlib.axes._subplots.AxesSubplot at 0x7f0a0eb03fd0>
```



Top two Annual incomes are 54,78

```
[124]: plt.figure(figsize=(23,10))
sns.countplot(x = 'Spending Score (1-100)', data = mall_customers,
             palette='viridis')
```

```
[124]: <matplotlib.axes._subplots.AxesSubplot at 0x7f0a0e32f710>
```



Top two spending scores are 42, 55

Features

```
[125]: X = mall_customers.iloc[:, [3,4]]
```

```
[126]: X
```

```
[126]:
```

	Annual Income (k\$)	Spending Score (1-100)
0	15	39
1	15	81
2	16	6
3	16	77
4	17	40
..
195	120	79
196	126	28
197	126	74
198	137	18
199	137	83

```
[200 rows x 2 columns]
```

```
[127]: from sklearn.cluster import KMeans
```

Elbow Method

To find optimum number of clusters, k

- In the Elbow method, we are actually varying the number of clusters (K) from 1 – 15.
- For each value of K, we are calculating WCSS (Within-Cluster Sum of Square).
- WCSS is the sum of squared distance between each point and the centroid in a cluster.
- When we plot the WCSS with the K value, the plot looks like an Elbow.
- As the number of clusters increases, the WCSS value will start to decrease.
- When we analyze the graph we can see that the graph will rapidly change at a point and thus creating an elbow shape.
- From this point, the graph starts to move almost parallel to the X-axis.
- The K value corresponding to this point is the optimal K value or an optimal number of clusters.

```
[128]: # Use elbow method to find optimum number of cluster i.e. value of 'K'
wcss = []
for i in range(1, 16):
    kmeans = KMeans(n_clusters=i, init='k-means++', max_iter=300, n_init=10,
    random_state=0)
    kmeans.fit(X)
    wcss.append(kmeans.inertia_)
```

kmeans++ - uses smarter selection of centroids

n_init - Number of time the k-means algorithm will be run with different centroid seeds. The final results will be the best output of n_init consecutive runs in terms of inertia.

max_iter - Maximum number of iterations of the k-means algorithm for a single run

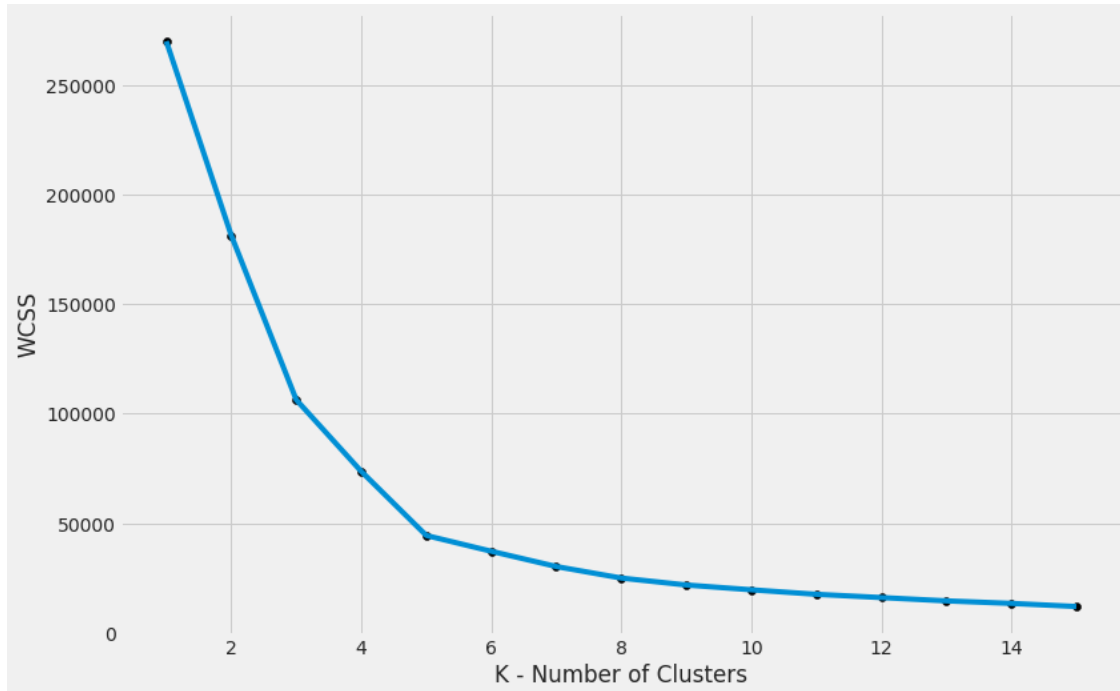
Inertia_ : Sum of squared distances of samples to their closest cluster center

```
[129]: wcss
```

```
[129]: [269981.28,
181363.59595959596,
106348.37306211118,
73679.78903948834,
44448.45544793371,
37239.83554245604,
30273.394312070042,
25018.576334776335,
21850.165282585633,
19664.685196005543,
17602.19046838677,
16115.215606639838,
14600.44364738564,
13450.08023381847,
12038.745689262341]
```

```
[130]: # Plot visualization b/w WCSS and Number of Clusters(K)
plt.figure(figsize=(12,8))
```

```
plt.plot(range(1,16), wcss)
# to view the points clearly we add scatter plot in addition to line
plt.scatter(range(1,16), wcss, c = 'black', marker='o')
plt.xlabel("K - Number of Clusters")
plt.ylabel("WCSS")
plt.show()
```



Optimal Number of Clusters K=5

K-Means with random centroid initialization

```
[131]: kmeans = KMeans(n_clusters = 5, init = 'random', max_iter= 300, n_init=10,
    ↪ random_state=0)
```

```
[132]: kmeans.fit(X)
```

```
[132]: KMeans(algorithm='auto', copy_x=True, init='random', max_iter=300, n_clusters=5,
            n_init=10, n_jobs=None, precompute_distances='auto', random_state=0,
            tol=0.0001, verbose=0)
```

```
[133]: y_clusters = kmeans.predict(X)
       y_clusters
```

```
[133]: array([2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4,
            2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 0,
            2, 4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

```

0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 1, 3, 0, 3, 1, 3, 1, 3,
0, 3, 1, 3, 1, 3, 1, 3, 1, 3, 0, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3,
1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3,
1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3,
1, 3], dtype=int32)

```

Performance Evaluation using Silhouette Score

```

[134]: from sklearn.metrics import silhouette_score
silhouette_avg = silhouette_score(X, y_clusters)
silhouette_avg

```

```

[134]: 0.553931997444648

```

K-Means with k-means++ centroid initialization

```

[135]: kmeans = KMeans(n_clusters = 5, init = 'k-means++', max_iter= 300, n_init=10,
↳random_state=0)

```

```

[136]: kmeans.fit(X)

```

```

[136]: KMeans(algorithm='auto', copy_x=True, init='k-means++', max_iter=300,
n_clusters=5, n_init=10, n_jobs=None, precompute_distances='auto',
random_state=0, tol=0.0001, verbose=0)

```

```

[137]: y_clusters = kmeans.predict(X)
y_clusters

```

```

[137]: array([3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1,
3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 0,
3, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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0, 2, 4, 2, 4, 2, 4, 2, 4, 2, 0, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2,
4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2,
4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2,
4, 2], dtype=int32)

```

Performance Evaluation using Silhouette Score

```

[138]: from sklearn.metrics import silhouette_score
silhouette_avg = silhouette_score(X, y_clusters)
silhouette_avg

```

```

[138]: 0.553931997444648

```

```
[139]: # Centroids
kmeans.cluster_centers_
```

```
[139]: array([[55.2962963 , 49.51851852],
            [25.72727273, 79.36363636],
            [86.53846154, 82.12820513],
            [26.30434783, 20.91304348],
            [88.2       , 17.11428571]])
```

Clustering Visualization

convert x to numpy arrays, y is already numpy array

```
[140]: y_clusters.astype
```

```
[140]: <function ndarray.astype>
```

```
[141]: x_array = np.array(X)
x_array
```

```
[141]: array([[ 15,  39],
            [ 15,  81],
            [ 16,   6],
            [ 16,  77],
            [ 17,  40],
            [ 17,  76],
            [ 18,   6],
            [ 18,  94],
            [ 19,   3],
            [ 19,  72],
            [ 19,  14],
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            [ 20,  15],
            [ 20,  77],
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            [ 25,  73],
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            [ 28,  82],
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```

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```

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[103, 69],
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[120, 16],
[120, 79],
[126, 28],
[126, 74],
[137, 18],
[137, 83]])

```

```
[142]: x_array.astype
```

```
[142]: <function ndarray.astype>
```

```

[143]: # Visualization of Cluster
plt.style.use("ggplot")
plt.figure(figsize=(15,8))
plt.title('Clusters of Customers')
plt.xlabel('Annual Income (k$)')
plt.ylabel('Spending Score (1-100)')

# x_array contain two values, Annual Income and Spending Score
# Index 0 - contains Annual Income and index 1 contains spending score
sns.scatterplot( x = x_array[y_clusters == 0, 0], y = x_array[y_clusters == 0, 1], label = "Cluster 1", s = 50)

```

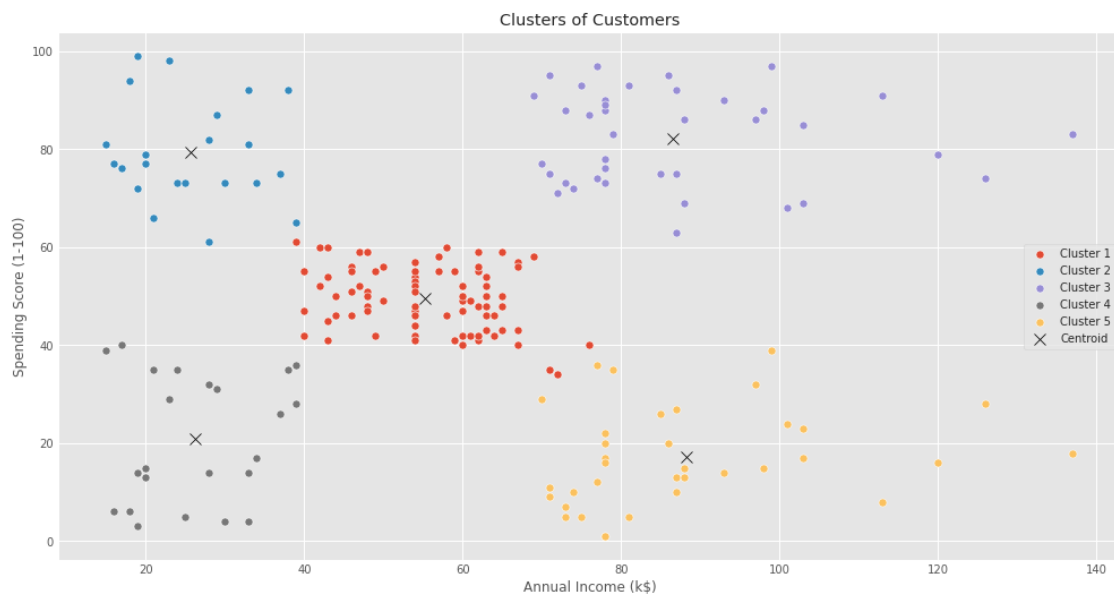
```

sns.scatterplot( x = x_array[y_clusters == 1, 0], y = x_array[y_clusters == 1, 1], label = "Cluster 2", s = 50)
sns.scatterplot( x = x_array[y_clusters == 2, 0], y = x_array[y_clusters == 2, 1], label = "Cluster 3", s = 50)
sns.scatterplot( x = x_array[y_clusters == 3, 0], y = x_array[y_clusters == 3, 1], label = "Cluster 4", s = 50)
sns.scatterplot( x = x_array[y_clusters == 4, 0], y = x_array[y_clusters == 4, 1], label = "Cluster 5", s = 50)
#plot the centroid of the cluster inside the plot
sns.scatterplot(kmeans.cluster_centers_[0],kmeans.cluster_centers_[1],marker='x',color='black',label='Centroid',s=100)
plt.show()

```

/usr/local/lib/python3.7/dist-packages/seaborn/_decorators.py:43: FutureWarning: Pass the following variables as keyword args: x, y. From version 0.12, the only valid positional argument will be `data`, and passing other arguments without an explicit keyword will result in an error or misinterpretation.

FutureWarning



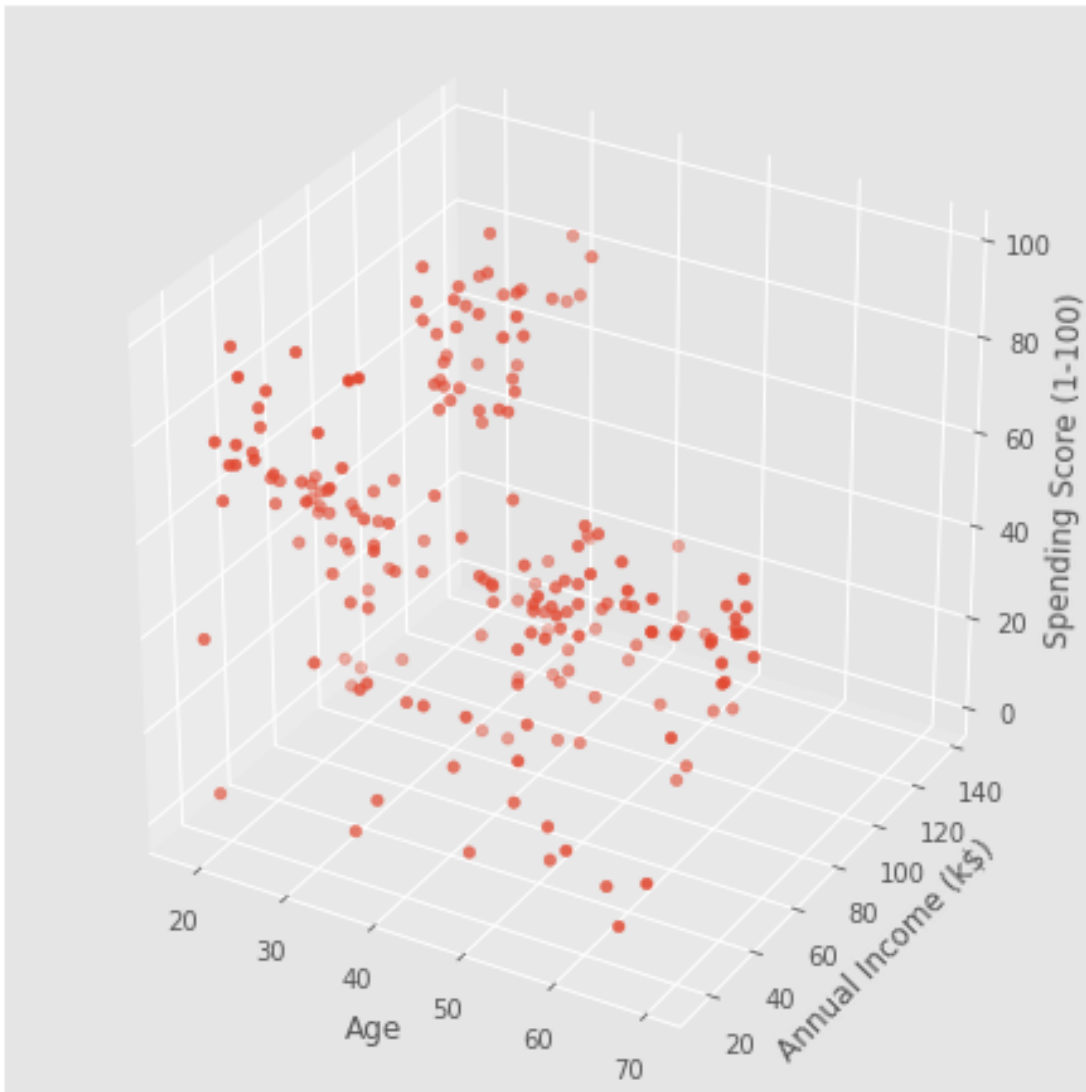
```

[144]: from mpl_toolkits.mplot3d import Axes3D
fig = plt.figure(figsize=(6,6))
ax = Axes3D(fig)
ax.scatter(mall_customers['Age'], mall_customers['Annual Income (k$)'], mall_customers['Spending Score (1-100)'])
#ax.scatter(dataframe['x2'], dataframe['x1'], predicted)
ax.set_xlabel('Age')
ax.set_ylabel('Annual Income (k$)')

```

```
ax.set_zlabel('Spending Score (1-100)')
```

```
[144]: Text(0.5, 0, 'Spending Score (1-100)')
```



```
[145]: x = mall_customers.iloc[:, 2:].values
```

```
# let's check the shape of x  
print(x.shape)
```

```
(200, 3)
```

```
[146]: x
```

```
[146]: array([[ 19,  15,  39],
               [ 21,  15,  81],
               [ 20,  16,   6],
               [ 23,  16,  77],
               [ 31,  17,  40],
               [ 22,  17,  76],
               [ 35,  18,   6],
               [ 23,  18,  94],
               [ 64,  19,   3],
               [ 30,  19,  72],
               [ 67,  19,  14],
               [ 35,  19,  99],
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```

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[58, 88, 15],
[27, 88, 69],
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[35, 93, 90],
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[46, 98, 15],
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[28, 101, 68],

```

[ 41, 103, 17],
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[ 33, 113, 8],
[ 38, 113, 91],
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[ 35, 120, 79],
[ 45, 126, 28],
[ 32, 126, 74],
[ 32, 137, 18],
[ 30, 137, 83]])

```

```

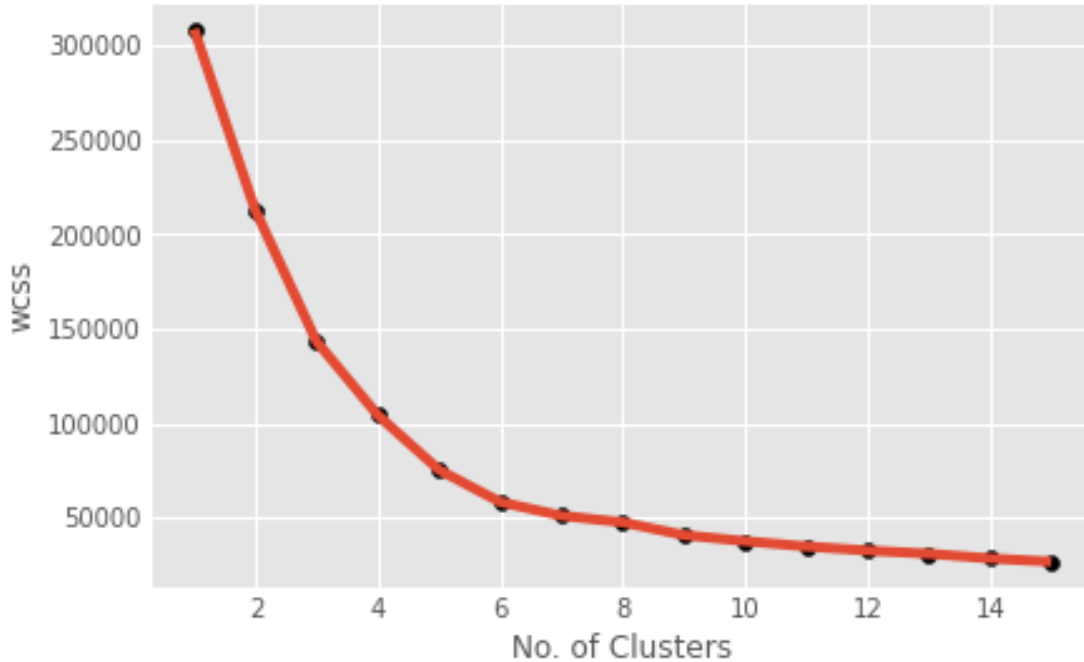
[147]: from sklearn.cluster import KMeans

wcss = []
for i in range(1, 16):
    km = KMeans(n_clusters = i, init = 'k-means++', max_iter = 300, n_init = 10, random_state = 0)
    km.fit(x)
    wcss.append(km.inertia_)

plt.plot(range(1, 16), wcss)
# to view the points clearly we add scatter plot in addition to line
plt.scatter(range(1,16), wcss, c = 'black', marker='o')
plt.title('The Elbow Method', fontsize = 20)
plt.xlabel('No. of Clusters')
plt.ylabel('wcss')
plt.show()

```

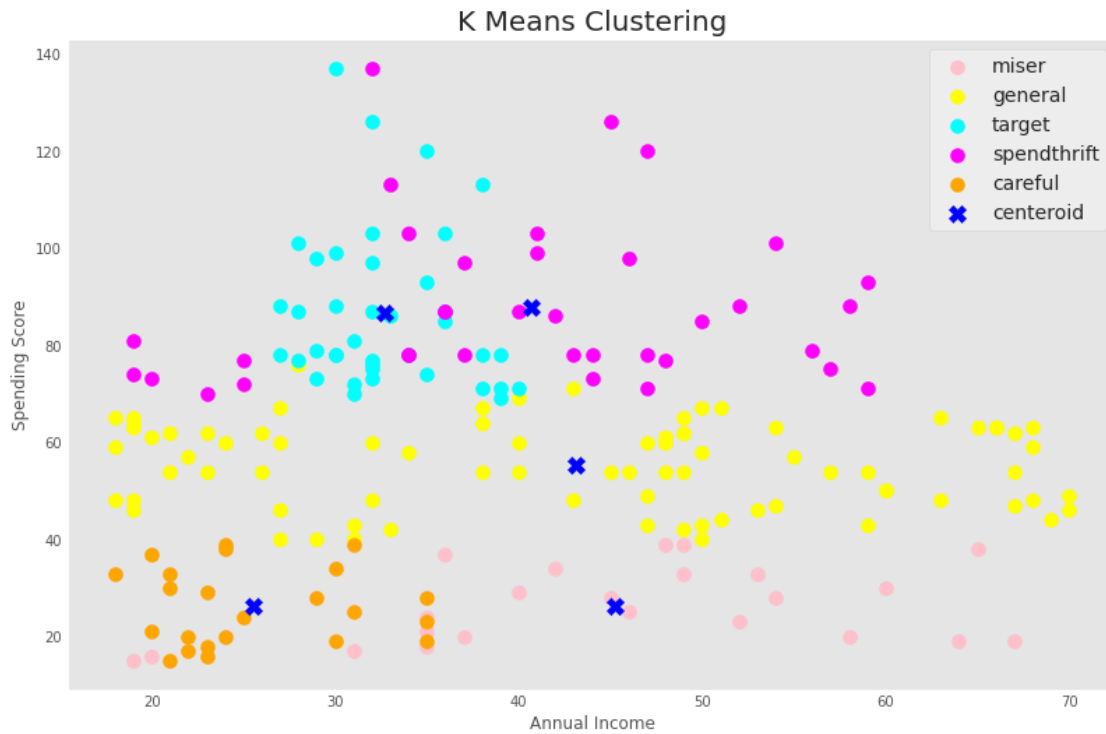
The Elbow Method



```
[148]: km = KMeans(n_clusters = 5, init = 'k-means++', max_iter = 300, n_init = 10,
    random_state = 0)
y_means = km.fit_predict(x)
plt.figure(figsize=(12,8))
plt.scatter(x[y_means == 0, 0], x[y_means == 0, 1], s = 100, c = 'pink', label =
    'miser')
plt.scatter(x[y_means == 1, 0], x[y_means == 1, 1], s = 100, c = 'yellow',
    label = 'general')
plt.scatter(x[y_means == 2, 0], x[y_means == 2, 1], s = 100, c = 'cyan', label =
    'target')
plt.scatter(x[y_means == 3, 0], x[y_means == 3, 1], s = 100, c = 'magenta',
    label = 'spendthrift')
plt.scatter(x[y_means == 4, 0], x[y_means == 4, 1], s = 100, c = 'orange',
    label = 'careful')
plt.scatter(km.cluster_centers[:,0], km.cluster_centers[:, 1], s = 150, c =
    'blue', marker = 'X', label = 'centeroid')

plt.style.use('fivethirtyeight')
plt.title('K Means Clustering', fontsize = 20)
plt.xlabel('Annual Income')
plt.ylabel('Spending Score')
plt.legend()
plt.grid()
```

```
plt.show()
```



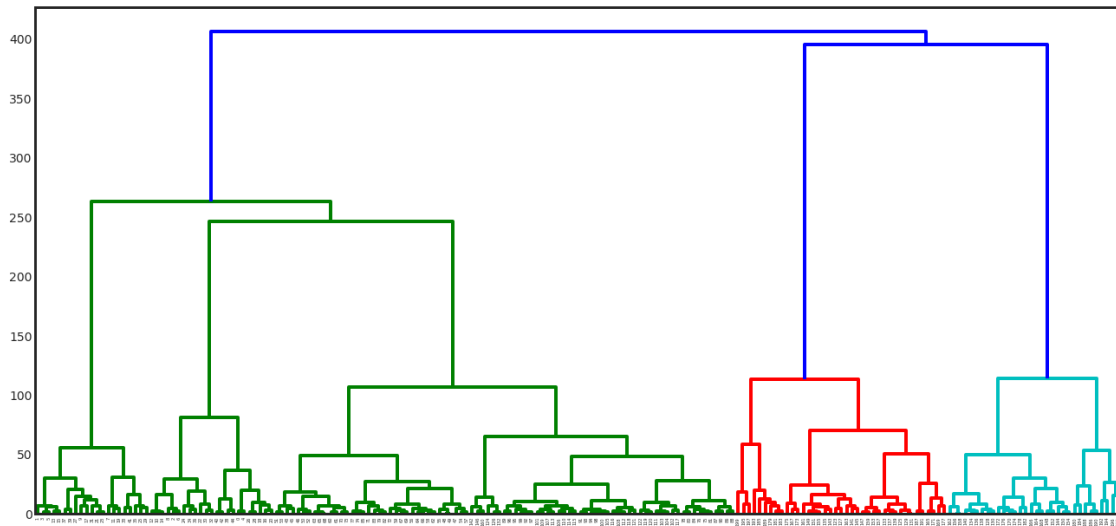
0.1 Hierarchical Clustering

Agglomerative Clustering - Bottom up approach

Dendrogram

- It is a tree like diagram, it illustrates the arrangement of cluster produced by analysis of cluster
- Dendrogram is used by hierarchical clustering to find number of clusters
- Dendrogram is summary of distance matrix between two different clusters

```
[149]: import scipy.cluster.hierarchy as sch
sns.set_style('white')
fig = plt.figure(figsize=(20,10))
dendrogram = sch.dendrogram(sch.linkage(X, method = 'ward'))
plt.show()
```



Number of clusters can be found by number of vertical lines in the top

```
[150]: from sklearn.cluster import AgglomerativeClustering
hc = AgglomerativeClustering(n_clusters = 5, affinity = 'euclidean', linkage = 'ward')
```

```
[151]: y_hc = hc.fit_predict(X)
```

```
[152]: y_hc
```

```
[152]: array([4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3,
            4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 1,
            4, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
            1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
            1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
            1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
            1, 2, 1, 2, 1, 2, 0, 2, 0, 2,
            1, 2, 0, 2, 0, 2, 0, 2, 0, 2, 1, 2, 0, 2, 1, 2, 0, 2, 0, 2, 0, 2,
            0, 2, 0, 2, 0, 2, 1, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2,
            0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2,
            0, 2])
```

[152] :