Tutorial - 1

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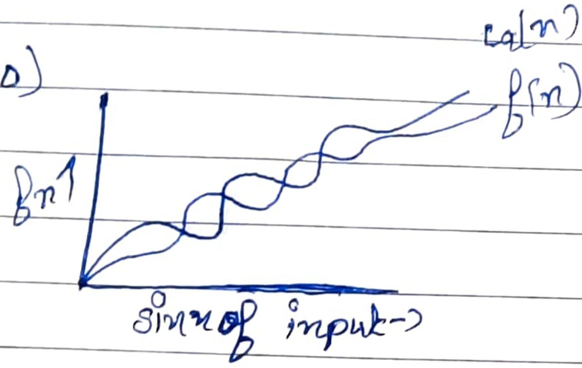
Q1) Asymptotic Notations :-
They help us to find the complexity when input is very large.

i) Big O (O)



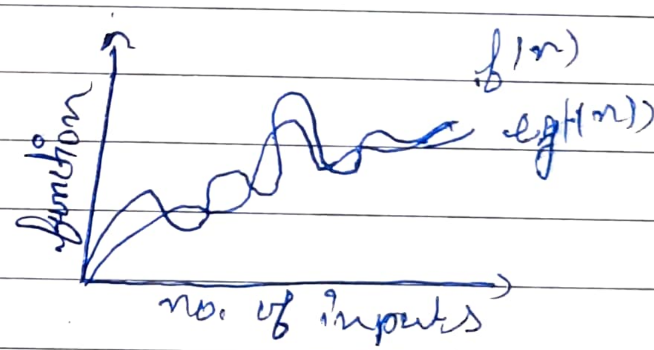
$f(n) = O(g(n))$

iff $f(n) \le c \cdot g(n)$
$\forall n \ge n_0$

for some ~~weight~~ constant
$c > 0$

-> g(n) is tight upper bound of f(n)

ii) Big Omega (Ω)



$f(n) = \Omega(g(n))$
g(n) is tight lower bound of f(n)
$f(n) = \Omega(g(n))$
$\forall$ $n \ge n$ for some constant $c > 0$

Q-10. for functions, n^k & c^n, what is the asymptotic relation between these functions?
assume that $k >= 1$, & $c > 1$ are constant.
Find out the value of c & n_0 for which relation holds

as given n^k & c^n

relation b/w n^k & c^n is

$$n^k = O(c^n).$$

~~[illegible]~~ as $n^k \leq a c^n$
$\forall n \geq n_0$ & some constant ~~[illegible]~~ $a > 0$

for $n_0 = 1$
$c = 2$

$\Rightarrow 1^k \leq a 2^1$

$\Rightarrow$ $\underline{n_0 = 1 \text{ \& } c = 2}$

[illegible]

Ques 9 Time complexity of -

```
void function (int n)
{ for (i=1 to n)                         // O(n)
    { for (j=1 ; j<=n ; j=j+i)
          print ("*")                    // O(n)
    }
}
```

for $i=1$:) $j = 1, 2, 3, 4 \dots n$. $= n$.
for $i=2$:) $j = 1, 3, 5, \dots n$ $= n/2$
for $i=3$:) $j = 1, 4, 7, \dots n$ $= n/3$

$\vdots$

for $i=n$:) $j = 1 \dots$

$$\Rightarrow \sum_{j=n} n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \dots + 1$$

$$\Rightarrow \sum_{j=n} n\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots \frac{1}{n}\right]$$

$$\sum_{j=n} n[\log n]$$

$$\Rightarrow T(n) = [n \log n]$$

$$\underline{\underline{T(n) = O(n \log n)}}$$

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8) Time Complexity of

```
function (int n)
{ int (n==1)
    return;                          // O(1)
  for (i=1 to n)                     // i = 1,2,3,4 --- n => O(n)
  { for (j=1 to n)                   // j = 1,2,3,4, --- n => O(n^2)
    { print ("*");
    }
  }
  function (n-3);                    T(n/3)
}
```

$\Rightarrow \quad T(n) = T(n/3) + n^2$

$\Rightarrow \quad a = 1, \quad b = 3, \quad f(n) = n^2$

$c = \log_3 1 = 0$

$\Rightarrow \quad n^0 = 1 \quad > (f(n) = n^2)$.

$\Rightarrow \quad T(n) = \Theta(n^2)$

Dafeena

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for $k = k*2$

$k = 1, 2, 4, 8, \ldots n.$

$\Rightarrow$ GP $\Rightarrow$ $a = 1$, $r = 2$.

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{1(2^k - 1)}{1}$

$n \Rightarrow 2^k$.

$\Rightarrow \log n = k$.

i	j	k
1	$\log n$	$\log n * \log n$
2	$\log n$	$\log n * \log n$
⋮	⋮	⋮
n	$\log n$	$\log n * \log n$

$\Rightarrow O(n * \log n * \log n)$

$\Rightarrow O(n \log^2 n)$

Ques-6 Time complexity of -

```
void fn(int n)
{ int i, count=0;
    for(i=1 ; i*i <=n ; ++i)
        count++                    // O(1)
}
```

as $i^2 <= n$

$\Rightarrow i <= \sqrt{n}$

$i = 1, 2, 3, 4 \ldots, \sqrt{n}$

$\sum_{i=1} 1+2+3+4+ \ldots + \sqrt{n}$

$\Rightarrow T(n) = \frac{\sqrt{n} \times (\sqrt{n}+1)}{2}$

$\Rightarrow T(n) = \frac{n \times \sqrt{n}}{2}$

$\Rightarrow T(n) = O(n)$

Ques-7 Time complexity of :-

```
void fn(int n)
{ int i, j, k, count=0;
    for(i=n/2 ; i<=n ; ++i)
        for(j=1 ; j<=n ; j=j*2)
            for(k=1 ; k<=n ; k=k*2)
                count++ ;
}
```

Dafona

Qs-5 What should be time complexity of

```
int i=1, s=1;
while (s<=n)
{ i++; s=s+i;
   printf("#");
}
```

i = 1 2 3 4 5 6 - - - - -

$s = 1 + 3 + 6 + 10 + 15 + 21 - - - - n$ — ①

~~Also~~ s<=n

s becomes

Sum of $s = 1+3+6+10+ - - - + T_n$ — ①

also $s = 1+3+6+10 + - - T_{n-1} + T_n$ — ②

from ① - ②

$0 = 1+2+3+4+ - - - - n - T_n$

$\Rightarrow T_k = 1+2+3+4+ - - - k$

$\Rightarrow T_k = \frac{1}{2} k(k+1)$

$\Rightarrow$ for k iterations.

$1+2+3+ - - - - + k <= n$

$\Rightarrow \frac{k(k+1)}{2} <= n$

$\Rightarrow \frac{k^2+k}{2} <= n$

$\Rightarrow O(k^2) <= n$

$\Rightarrow k = O(\sqrt{n})$

$\Rightarrow T(n) = O(\sqrt{n})$

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$\Rightarrow T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} - \dots$

$\Rightarrow GP = 2^{k-1} + 2^{k-2} + 2^{k-3} + \dots$

$a = 2^{k-1}$

$r = 1/2$

$\Rightarrow S_n = \dfrac{a(1-r^n)}{1-r}$

$= \dfrac{2^{k-1}(1-(1/2)^n)}{1-\frac{1}{2}}$

$= 2^k(1-(1/2)^k)$

$= 2^k - 1$

Let $n-k = 0$

$\Rightarrow n = k$

$\Rightarrow T(n) = 2^n T(n-n) - (2^n - 1)$

$\Rightarrow T(n) = 2^n \cdot 1 - (2^n - 1)$

$\Rightarrow T(n) = 2^n - (2^n - 1)$

$\Rightarrow T(n) = O(1)$

Saxena

putting $n = n-2$ in ①

$T(n) = 3(T(n-3))$ — ④

$\Rightarrow T(n) = 27(T(n-3))$

$\Rightarrow T(n) = 3^k(T(n-k))$

putting $n-k=0$

$\Rightarrow n = k$

$\Rightarrow T(n) = 3^n[T(n-n)]$

$\Rightarrow T(n) = 3^n T(0)$

$\Rightarrow T(n) = 3^n \times 1$ $\quad [T(0) = 1]$

$\Rightarrow T(n) = O(3^n)$

4) $T(n) = \{2T(n-1) - 1$ if $n > 0$, otherwise $1\}$

$T(n) = 2T(n-1) - 1$ — ①

Let $n = n-1$

$\Rightarrow T(n-1) = 2T(n-2) - 1$ — ②

$\Rightarrow$ from ① & ②

$\Rightarrow T(n) = 2[2T(n-2) - 1] - 1$

$\Rightarrow T(n) = 4T(n-2) - 2 - 1$ — ③

let $n = n-2$

$\Rightarrow T(n-2) = 2T(n-3) - 1$ — ④

from ③ & ④

$\Rightarrow T(n) = 4[2T(n-3) - 1] - 2 - 1$

$\Rightarrow T(n) = 8T(n-3) - 4 - 2 - 1$

Diksha

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Ques-2 What should be time complexity of
for(i=1 to n) {i = i * 2}

for(i=1 to n) // $i = 1, 2, 4, 8 \dots n$
{ i=i*2} // O(1)

$\Rightarrow \sum_{i=1} 1+2+4+8+\dots n$

GP kth value $\Rightarrow T_k = ar^{k-1}$

$\Rightarrow 1 \times 2^{k-1}$

$\Rightarrow n = \frac{2^k}{2}$

$\Rightarrow 2n = 2^k$

$\Rightarrow \log 2n = k \log 2$

$\Rightarrow \log_2 + \log n = k \log 2$

$\Rightarrow \log n + 1 = k$

$\Rightarrow O(k) = O(1 + \log n)$

$= O(\log n)$

Q-3 $T(n) = \{3T(n-1)$ if $n>0$, otherwise $1\}$

$T(n) = 3T(n-1)$ - ①

put $n = n-1$

$T(n-1) = 3\{T(n-2)$ - ②

from 1 & 2

$\Rightarrow T(n) = 3(3T(n-2))$

$= 9T(n-2)$ - ③

Saksena

4) Small o(O)

$f(n) = o(g(n))$

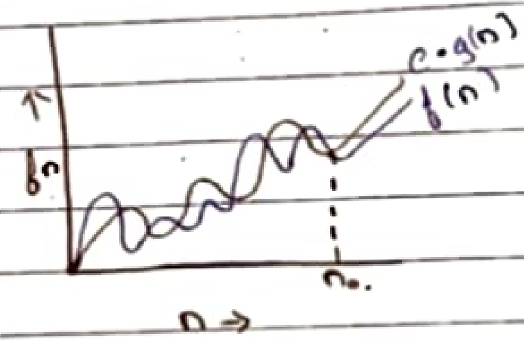
$g(n)$ is upper bound of fn $f(n)$

$f(n) = o(g(n))$

when $f(n) < c \cdot g(n)$

$\forall\ n > n_0$

& $\forall\ c > 0$



5) small omega (ω)

$f(n) = \omega(g(n))$

$g(n)$ is lower bound of fn $f(n)$

$f(n) = \omega(g(n))$

when $f(n) > c \cdot g(n)$

$\forall\ n > n_0$

& $\forall\ c > 0$