SIT718 Real World Analytics

Assessment Task 2: Problem solving task 1

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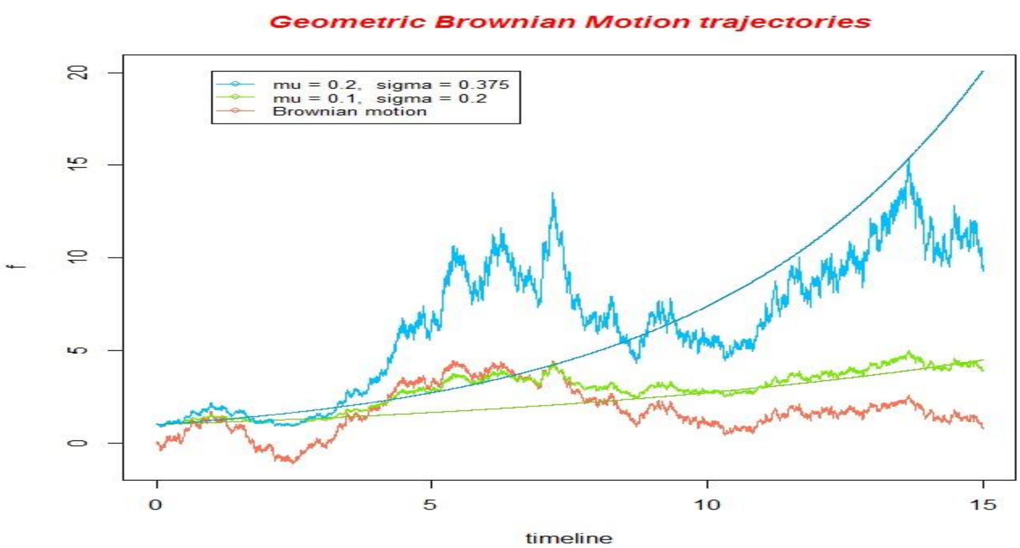
<u>Ans 1:</u>In geometric Brownian motion σ_t and μ_t these values come under stochastic differential equation where μ_t is the percentage drift or mean(average of the values taken) with respect to time t and σ_t is the percentage volatility or variance(squared deviation of a random variable from its mean) with respect to time t.

Yes, these values of σ and μ can be considered as constants with time t.

- The assumptions made in this model are that the values of S_0(The Stochastic process at time 0 or at the initial start) and W_0(Brownian motion at the start or at the initial 0) should be equal to 0 and the standard Brownian motion $W_t \sim N(0,t)$. and we evaluate it till $S_t \sim GBM(\mu, \sigma)$.
- "The assumption for GBM is that that values of closing stock prices are distributed log-normally with a mean of the component that's certain and a standard deviation of the component that is not fixed or certain, as shown below:

  $$\ln\frac{S_T}{S_0} \sim \phi\left[\left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\varepsilon\sqrt{T}\right]$$

  Where, S0 is the stock price now and ST is the price at time T" (Reddy & Clinton, 2016).
- We assumed that the number of days to be 65 for the three months as daily price data is used, we assume that the number of trading days in a year are 250 trading days per year excluding all public holidays and including working days and for the duration of three months we have 65 trading days per three months, the time interval used was 1/65.
- An example figure is shown below:



(delta9hedge, 2018)

Ans 2: I have collected the closing prices of Australia and New Zealand Banking Group ANZ.AX y for each trading day for 3 consecutive months, from 1 August 2018 to 31 October 2018 inclusive.

The closing prices are listed in this table below:

Date	Closing Stc
01-08-2018	29.07
02-08-2018	28.91
03-08-2018	28.64
06-08-2018	28.92
07-08-2018	29.09
08-08-2018	28.82
09-08-2018	29.17
10-08-2018	29.31
13-08-2018	29.17
14-08-2018	29.64
15-08-2018	30.15
16-08-2018	30.06
17-08-2018	30.28
20-08-2018	30.05
21-08-2018	29.51
22-08-2018	29.22
23-08-2018	28.72
24-08-2018	28.5
27-08-2018	28.52
28-08-2018	28.78
29-08-2018	29.43
30-08-2018	29.39
31-08-2018	29.5
03-09-2018	29.21
04-09-2018	28.8
05-09-2018	28.59
06-09-2018	28.53
07-09-2018	28.4

Date	Close Price
10-09-2018	28.26
11-09-2018	28.58
12-09-2018	28.37
13-09-2018	28.06
14-09-2018	28.15
17-09-2018	28.4
18-09-2018	28.39
19-09-2018	28.66
20-09-2018	28.42
21-09-2018	28.46
24-09-2018	28.58
25-09-2018	28.22
26-09-2018	27.96
27-09-2018	27.79
28-09-2018	28.18
01-10-2018	27.73
02-10-2018	27.49
03-10-2018	27.44
04-10-2018	27.61
05-10-2018	27.72
07-10-2018	26.99
08-10-2018	26.83
09-10-2018	26.88
10-10-2018	26.01
11-10-2018	25.91
14-10-2018	25.43
15-10-2018	25.55
16-10-2018	25.81

Date	Close Price
17-10-2018	25.88
18-10-2018	26.04
21-10-2018	25.79
22-10-2018	25.29
23-10-2018	25.45
24-10-2018	24.8
25-10-2018	24.91
28-10-2018	25.2
29-10-2018	25.66
30-10-2018	25.93

Ans 3:The condition that share prices have to satisfy in order to be represented by a geometric Brownian motion is:

We can say that a random process such as stock prices, $\{X_t : t>0\}$, is a Brownian motion with parameters(μ,σ) if

1. For $0 < t_1 < t_2 < . . . < t_{n-1} < t_n$

   $(X_{t2} - X_{t1})$, $(X_{t3} - X_{t2})$, . . ., $(X_{tn} - X_{tn-1})$

   Are mutually independent. {independent increments}
2. For $s > 0$, $X_{t+s} - X_t \sim N(\mu s,\sigma^2 s)$, and
3. X_t is a continuous function of t (Haug & Iyengar, 2018).

- ✓ As is evident from the data that the differences between the values based on their increasing time period(i.e. no of days) is not related on one another for its values or is mutually independent.
  {for example taking values for dates 01-08-2018, 02-08-2018, 03-08-2018 and 06-08-2018 the differences are as -0.16(2nd -1st), -0.270001(3rd – 2nd) and 0.280001(4th - 3rd) (Haug & Iyengar, 2018).
- ✓ Moreover for every value of stock price beyond the initial stock price the difference between the values lie between the drift and volatility(variance) so it follows the second property.
- ✓ Now from the graph below taking some values from observations:

	1	2	3	4	5	6
N(0,1)	0.600484	-0.676	-0.058615	-0.74006	0.753485	-0.319669
Log return	0.045067	-0.05073	-0.004399	-0.05554	0.05655	-0.023991
Price	30.41006	28.90572	28.778837	27.22398	28.80784	28.124927
Actual Price	29.07	28.91	29.09	28.82	29.17	29.309999

  As we do not see any breaks in the line of the graph we can concur that it is continuous over different values of t (Haug & Iyengar, 2018).
- ✓ We could also observe the graph of estimated values to the given values of share prices which would prove as an evidence to our statistical test comparing estimated simulated prices and the actual stock prices for each singular stocks, portfolios prepared based on their volatility and also based on the expected returns of each industry. The graph is shown below:

	1	2	3	4	5	6
N(0,1)	0.161351	-0.11594	0.5613788	-1.08737	-1.2575	-1.803403
Log return	0.012109	-0.0087	0.0421319	-0.08161	-0.09438	-0.135347
Simulatd Estimated Price	29.42416	29.16924	30.424447	28.04019	25.5149	22.285041
Actual Price	29.07	28.91	29.09	28.82	29.17	29.309999

As you could see there aren't any overlapping points for the both simulated values and actual values meaning that the if the difference between the values are taken into account then it shows independency among values also while calculating the differences between the two values that is the next stock price and the previous stock price they all fall under the range specified in the condition 2.And, at last we observe from the graph that there are no breaks in the graphs through which we can insinuate that it follows the continuous property condition of GBM.

Ans 4:The σ and μ for the three months historic data is:

- μ mean or the percentage drift is 27.92742.
- σ standard deviation or the percentage volatility is 1.46453652.

For the calculation of mean and variance over all the values listed in the stock price table we used the following formulas:-

- ✓ Mean(μ) :- $\bar{x} = (\Sigma x_i) / n$; xi "all of the x-values"
  n means "the number of items in the sample"
- ✓ Standard Deviation(σ) :-

$$\sigma^2 = \frac{\sum (X-\mu)^2}{N}$$

; where μ is the mean and N is the number of scores.

Ans 5: "ANZ announced a Statutory Profit after tax for the Full Year ended 30 September 2018 of $6.40 billion, flat on the comparable period and a Cash Profit1 on a continuing basis of $6.49 billion, down 5%. ANZ's Common Equity Tier 1 Capital Ratio was 11.4% up 87 basis points (bps). Return on Equity decreased 67 bps to 11.0% with Cash Earnings per Share down 4% to 223.4 cents (continuing)" (Ries & Higginbottom, 2018).

The Final Dividend is 80 cents per share, fully franked, bringing the Full Year Dividend to 160 cents (Ries & Higginbottom, 2018).
All in all for having an continual profit base upon bps it maintained a constant μ and for when there was an increase in cash profits it had a positive variance/volatility and vice versa for a decreased return on equity it had a negative volatility.

<u>Ans 6:</u> So for this we will need to use the property of GBM over a distributed data using the formula:-

$$\ln\left(\frac{S_{t+\Delta t}}{S_t}\right) = \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}$$

Where, St is the stock price at time t, Δt is the time interval for prediction, μ is the expected annual rate of return, σ is the expected annual volatility, and ε is a randomly drawn number from a normal distribution with a mean of zero and a standard deviation of one, representing random volatility.

- The time interval used is 77 days, as for predicting daily prices over the period from 1 August 2018 to 16 November 2018.
- "We are using the capital asset pricing model (CAPM) to calculate the expected annual return for each stock using the following formula:

$$\mu = r_f + \beta_m(r_m - r_f)$$

;Where, rf is the risk-free rate of return, βm is the beta of stock against the market, and rm is the expected return of the market portfolio. CAPM is used because it is simple to calculate and the input variables are easily accessible. We calculated the daily standard deviation from the daily returns and used the formula: σ = s/ √τ(to calculate the annualised volatility of each stock where s represents the daily standard deviation and τ is the intervals measured in days" (Reddy & Clinton, 2016).

→ So, for 16th of November,2018 we get <u>27.38</u> as the closing estimated stock price of Australia and New Zealand Banking Group ANZ.AX which is close to the actual closing stock price i.e. 25.36.

<u>Ans 7:</u> "For a particular type of stochastic process such as a Markov process(following geometric Brownian motion)only the present value of a variable is relevant for predicting the values/data in future and past historical data of the variable and the way that the present has emerged from the past are irrelevant" (Marathe, 2006).{ A Wiener process is a type of Markov stochastic process in which the mean change in the value of the variable is zero with the variance of change equal to one per unit time.} for some of "the data sets, the assumption of GBM process distribution may not be appropriate (example, cell-phone revenue data and Internet host data). Hence in any given model, caution should be taken before assuming that the particular data set follows the GBM process" (Marathe & Ryan, 2005).
Moreover, as far as analysis goes ANZ is Actions taken since 2016 to simplify the business and reduce cost, position ANZ well to meet the immediate challenges facing the industry. This means that short-term revenue growth and higher margins in Australia were sacrificed, particularly in the investor and interest-only segments, it was the right thing to do for shareholders as there was a steady increase in expenses for the duration of the whole year, primarily accounting due to increase in

compensation given to customer and remediation of the costs prices in Australia as a whole. The company also excluded large notable items, expenses were brought down to 1.5% in the year, positioning them to better manage the headwinds impacting the sector.

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