OPTION PRICING MODELS AND THEIR ACCURACY

Introduction

Options are financial Instruments(assets that can be traded, or they can also be seen as packages of capital that may be traded) .

There are 2 broad types of options Call & Puts option, Call option allows the holder to buy an asset at a stated price in a particular time period whereas a Put Option allows the holder to sell that asset at a stated price in a particular time period.

Few Terminologies

- Intrinsic Value (IV) = Max[0, strike price - spot price] for call option.
- Any option that has a non zero intrinsic value is classified as ITM (In the money) Option.
- Otherwise it is classified as OTM (Out of the money) Option.
- If the strike price is almost equal to the spot price(underlying) then the option is considered as At the money option(ATM).

MONTE CARLO SIMULATION

Monte Carlo Simulation (Multiple probability simulation)

- Model was named after a gambling hotspot in Monaco.The model used to predict the probability of a variety of outcomes in a process that cannot be easily predicted due to involvement of different random variable.It is a technique used understand the impact of risk and uncertainty.It has a wide range of applications in multiple fields like investments, business, physics, and engineering.It involves in assigning multiple values to an uncertain variable to achieve multiple results and then averaging the results to obtain an estimate.They are used to estimate the probability of cost overruns in large projects and the likelihood that an asset price will move in a certain way.It takes the variable that has uncertainty and assigns it a random value. The model is then run and a result is provided. This process is repeated again and again while assigning many different values to the variable in question. Once the simulation is complete, the results are averaged to arrive at an estimate.

To understand Monte Carlo simulation with greater clarity we need to understand the meaning of two components:-

Drift
Drift depicts the constant directional movement of an asset's price.

Random Input
It represents a market's volatility.

By studying and analyzing the price data , we can determine the drift, standard deviation, variance, and average price movement of a security.

Monte Carlo Sim is a 4 step process -

Step 1
Tto predict the price trajectory using the current days price and the previous days price. This generates a series of periodic daily returns. We use the natural logarithm function for this :

$$\text{Periodic Daily Return} = ln\left(\frac{\text{Day's Price}}{\text{Previous Day's Price}}\right)$$

Step 2
The process is to find average daily return , standard deviation and variance using AVERAGE , STDEV.P & VAR.P function in MS Excel. Drift is given by :

$$\text{Drift} = \text{Average Daily Return} - \frac{\text{Variance}}{2}$$

Step 3
It involves in finding the second component i.e. the Random Input which is given by:

$$\text{Random Value} = \sigma \times \text{NORMSINV(RAND())}$$

where σ is the Standard deviation which we can find using the STDEV.P function in MS Excel and NORMSINV(RAND()) are excel functions as well.

- Then we can predict the next day's price using the values we found above :

$$\text{Next Day's Price} = \text{Today's Price} \times e^{(\text{Drift}+\text{Random Value})}$$

Step 4
to repeat this step for multiple values in order to obtain the **Price Trajectory.**

When we analyze and plot the data gathered we find that it forms a normal distribution i.e. a **bell shaped** graph which indicates that the most likely return is in the middle of the graph but there is an equal chance of obtaining a lower or higher value.
We need to understand that this model doesn't guarantee the most expected outcome to occur or that the extremes wont be achieved.
This is because Monte Carlo Simulation assumes a perfectly efficient market which happens very rarely.
Overall this model helps us evaluate the risks, possible losses or profits in an investment we wish to take up.

Because of which it is used widely in :

Pricing Stocking Options
The potential price movements of the underlying asset are tracked given every possible variable. The results are averaged and then discounted to the asset's current price. This is intended to indicate the probable payoff of the options.

Portfolio Valuation
Measuring comparative risks in a number of alternative portfolios.

Fixed Income Investments
The random variable here is the short rate. we use it to calculate the probable impact of movements in the short rate on fixed rate investments.

OPTION GREEKS

Greeks are variables or sensitivities which are used to assess the risk characteristics. Each Greek tells about the change in a specific factor related to option pricing with respect to the change in an underlying variable. A few important option greeks are described below:-

- **Delta**

Delta is the rate of change of option price with respect to the change in the underlying asset price. Delta is also known as the Hedge Ratio, it also gives us information about the moneyness of the option. Delta gives exposure to directional risk.

- **Gamma**

Gamma is the derivative of price sensitivity. It is used to determine how sensitive the delta is compared to the asset price. The gamma of a put and a call is always identical and can be positive or negative gamma giving exposure to price fluctuations.

- **Theta**

In simple terms, theta is a measure of how time affects the option premium. It is also called time sensitivity or an options time decay. Theta is usually always negative for bought options.

- **Vega**

Vega is the measure of the option's sensitivity to the asset's implied volatility. When vega is positive, it generally suggests that increasing volatility is helping the position. When vega is negative, it generally suggests that increasing volatility is hurting the position.

- **Rho**

Rho measures the sensitivity of the option price Interest rates relative to changes in the risk-free interest rate.

BLACK-SCHOLES MODEL

Introduction:

- The Black Scholes model is one of the commonly used models to determine the price of an **European option**.
- The model is a **differential equation** derived from **Ito's lemma** that tries to quantify the relationship between the various Greeks of an option.

Idealised Assumptions made by the Black Scholes Model:

- Stock prices follow the Brownian movement.
- Stock returns are normally distributed.
- The volatility of a stock is constant over time.
- There is no possibility of Arbitrage in the market.
- The Markets are Frictionless or lack of transaction costs.
- The compounding occurs over infinitesimally small periods of time.
- The risk-free return rate is constant over the time of the option.
- Options can only be exercised at the expiration date and not earlier.
- Black Scholes also assumes a single stock universe

The Black Scholes Formula in its Differential Equation format

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

The Black Scholes Formula

$$c = S_0\, N(d_1) - K\, e^{-rT} N(d_2)$$

$$p = K\, e^{-rT}\, N(-d_2) - S_0\, N(-d_1)$$

$$\text{where} \quad d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

The Formula takes five inputs.
r - The Risk-Free interest rate.
S - The Current Stock price
K - The Strike price of the option
T - The Time to Expiry (Years)
σ - The Volatility of the stock

N(d) refers to the cumulative distribution function, which gives the probability of a variable being lesser than or equal to a specific value.

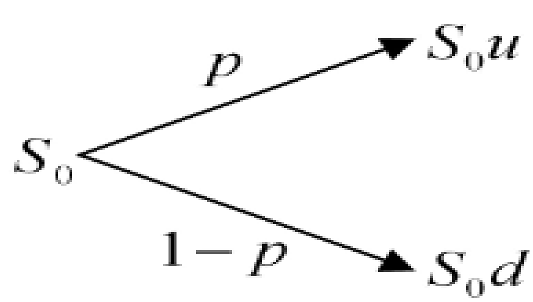
The Aim Behind the Pricing:
The Premium on the Option should be such that the option seller should be able to invest the premium in either the stock itself or a risk free investment (as the model assumes a single stock universe) till the time of expiry and be able to completely hedge against the losses incurred by the option contract (in an idealised Enviroment).
This should ideally imply that the option seller is completely risk free, however due to the market not being ideal this is not the case.

BINOMIAL OPTION PRICING MODEL

Introduction
The Binomial Option pricing model is used to evaluate financial options developed by Cox, Ross, and Rubinstein in 1979. It allows us to model stock prices in discrete time, assuming that the stock price will change to one of two possible values at each step.

Assumptions for Calculation
- The risk-free rate does not change
- At any given point in time, the price can only move one of two ways: either up or down.
- The option can be exercised only after specified discrete time periods,
- The market is efficient, and there are no arbitrage opportunities.
- There are no returns on the underlying stock
- There are no transaction costs, taxes, or dividend payments during the option's life.

Formulation
Let us assume the **initial stock price** is S_0, **strike price** K and two positive numbers d and u such that $0 < d < 1 < u$. The value uS_0 and dS_0 denote the upward and downward movement of stock price respectively.
The **probability** of the price going up be p and going down be q. These events are mutually exclusive as assumed in the model, therefore, $p = 1 - q$.
Let r be the **risk-free interest rate** per time period with the assumption $d < 1 + r < u$ as rate of return of the market is generally less than return on the stock.



Now, The **option payoff** in the **up move** state is:-

$$max\{0, uS_0 - K\}$$

The **option payoff** in the **down move** state is:-

$$max\{0, dS_0 - K\}$$

Hence the net **option payoff** (V_1) after unit time period is:-

$$V_1 = p * max\{0, uS_0 - K\} + q * max\{0, dS_0 - K\}$$

The last step is to discount back to find **current payoff**(V_0) which gives:-

$$V_0 = \frac{V_1}{1+r} = \frac{p*max\{0,uS_0-K\}+q*max\{0,dS_0-K\}}{1+r}$$

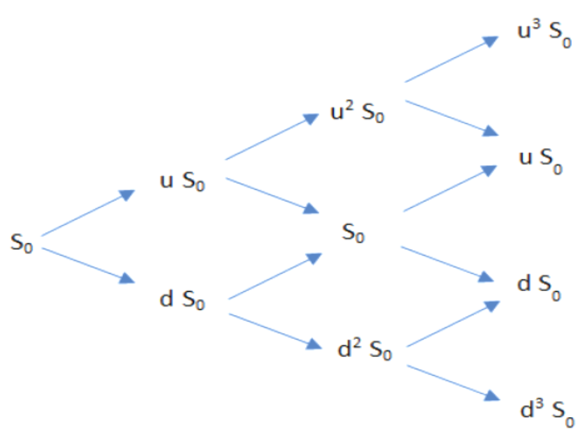
For the **up move probability** we use the formula given below:-

$$p = \frac{(1+tr)-d}{u-d}$$

t is period mutiplier

- The above formula is a discrete-time version of the **"delta-hedging"** formula for derivative securities, according to which the number of shares of an underlying asset a hedge should hold is the derivative (in the sense of calculus) of the value of the derivative security with respect to the price of the underlying asset.
- Formula can be derived by solving for delta in the portfolio value equations for the payoff in both the up move state and the down move state.

Multistep Binomial Model



- The multi-step binomial model is a simple extension of the principles given in the two-step binomial model. We simply step forward in time, increasing or decreasing the stock price by a factor u or d each time.
- The formula derived above works perfectly fine for the multistep model. We can find the individual payoffs at any node and retrace the path back to initial value to calculate the current payoff of the option.
- The inaccuracy caused by considering discrete time periods can be minimized by increasing the branches of the Binomial tree.
- This model also provides simplicity for finding payoffs for the American Options which can be exercised anytime before the expiry of the option

Limitations of the Model

- It assumes a discrete-time framework, which may not accurately capture the continuous nature of market movements.
- The model's accuracy heavily depends on the number of steps in the binomial tree. Increasing the number of steps can enhance accuracy but also increases computational complexity.
- Constant Volatility is assumed which is not true in case of real markets.

BLACK-SCHOLES VS BINOMIAL MODEL

Here we will see the differences between the Black-Scholes method and the Monte Carlo method for pricing of options.

BLACK SCHOLES	BINOMIAL MODEL
1. Assumes stock price to be continuous and changes continuously over time.	1. Assumes bi-directional movement of stock price that is either up or down between a time interval.
2. Complex but single step that is , it involves complex mathematical equations , partial differential equations, but option's price is calculated in single calculation.	2. .The binomial model is relatively straightforward to implement and understand. It involves constructing a binomial tree and calculating option values at each node using simple probability calculations.
3.The Black-Scholes model is primarily designed for European options, which can only be exercised.	3. The binomial model is more flexible than the Black-Scholes model in terms of handling various option types and features.
4. The Black-Scholes model provides a closed-form solution for European options, which allows for efficient and quick calculation of option prices.	4. The number of nodes in the binomial tree grows exponentially with the number of time steps, making it more time-consuming to calculate option prices.
5.The Black-Scholes model assumes that the underlying stock does not pay dividends during the life of the option.	5.In the binomial option pricing model, dividends can be incorporated into the calculation by adjusting the stock price at each node of the binomial tree. This allows for a more precise valuation of options.
6. Continuos time framework.	6. Discrete time framework.

Workings of the Code of Monte Carlo simulation of Nifty

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import datetime as dt
from pandas_datareader import data as pdr
np.random.seed(1)

'''imports past year data of nifty from yahoo finance using pandas'''

def gd(stocks, start, end):
    stockData = pdr.get_data_yahoo(stocks, start, end)
    stockData = stockData['Close']
    returns = stockData.pct_change()
    meanReturns = returns.mean()
    std = returns.std()
    return meanReturns, std

‘’’Gets the mean and standard deviation of the past year data.’’’

stocks = ['NSEI.AX']
endDate = dt.datetime.now()
startDate = endDate - dt.timedelta(days=365)
meanReturns, StandardD = gd(stocks, startDate, endDate)

mc_sims = 400
T = 100
portfolio_sims = np.full(shape=(T, mc_sims), fill_value=0.0)

'''Uses the mean and variance Data of nifty to randomly predict the outcome of nifty for
# the next 100 Days using Bell Curve Distribution that has the same mean and
variance'''

for m in range(0, mc_sims):
    Portfolio = 10000
    dr =np.random.normal(loc=meanReturns, scale=StandardD, size=T)
    for z in range(0,T)
        portfolio_sims[z , m] = Portfolio
```

```
    Portfolio=Portfolio*(1+dr[z])

'''Plots a graph of the different possible outcome of NIfty within the next 100 Days'''

plt.plot(portfolio_sims)
plt.ylabel('NIFTY VALUE')
plt.xlabel('Days')
plt.title('MonteCarlo simulation of NIFTY')
plt.show()
```

Limitations

Since this works on the previous data supposing the past year had a stock Crash this model will predict a further downfall instead of a recovery which is what usually happens.

Also note that this code just uses the Monte carlo simulation to predict the movement of nifty and not outcome of options

References

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