Answer to Question 3

i. False

Explanation: The Leftmost outermost is a normalizing term, meaning, it can find the normal form if it exists, in cases were leftmost innermost may fail to find it. Consider the following example.

Given input: (λy.λz.z (λx.(x x) λx.(x x)))

- If we try to normalize it using leftmost outermost, we get
  Step 1: λz.z

  This happens because since y does not appear in the function's body, the argument of the function (λx.(x x) λx.(x x)) is copied into y and then discarded.

- On the other hand, if we try to normalize the above example with leftmost innermost (Lila), we get the innermost redex, (λx.(x x) λx.(x x)) to output itself, hence it is a non-terminating case.

- So we see that Lola can be terminating on some term where Lila is non-terminating.

ii. True

Explanation: We know leftmost outermost is a normalizing reduction technique and it will always find the normal form if there exists a normal form. Hence if Lola cannot find the normal form and if it yields a non-terminating form, we can conclude that Lila will also yield a non-terminating form.

iii. False

Explanation: Lila is call by value. We know call by value diverges more as stated in [1]. So Lila will not always find a normal form in lesser steps than Lola. So the assumption is false. If we consider the following computation:
(λx.λy.y (λx.(x x) (λx.x y)))

By doing Lola, we get,
Given Input: (λx.λy.y (λx.(x x) (λx.x y)))

Step 1: λy1.y1 [x <- (λx.(x x) (λx.x y)]

So Lola takes only 1 step for this computation. We have verified the output using lambda calculus calculator.

By doing Lila, we get,

Given Input: (λx.λy.y (λx.(x x) (λx.x y)))

Step 1: (λx.λy.y (λx.(x x) y))

Step 2: (λx.λy.y (y y))
Step 3: λy1.y1 [x <- (y y)]

So Lila takes 2 steps more for this computation.

iv. False

Explanation: This is not always true. Consider the following redex:
(λx.(x x) (λx.x y))

- If we normalize is using Lola, then it yields:

➔ (λx.(x x) (λx.x y))
➔ ((λx.x y) (λx.x y))
➔ (y (λx.x y))
➔ (y y)

- On the other hand, if we use Lila, then it yields:

➔ (λx.(x x) (λx.x y))
➔ (λx.(x x) y)
➔ (y y)

Hence we see that Lola takes 2 steps more than Lila for the chosen example.