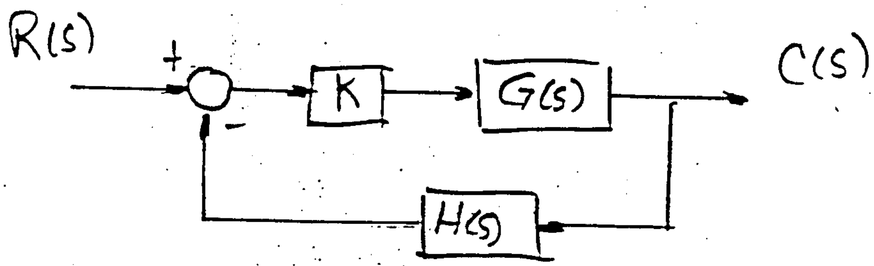
EEL 3657

THE NYQUIST STABILITY CRITERION AND RELATIVE STABILITY

CONSIDER THE CONTROL SYSTEM SHOWN..



OBJECTIVES:

(A) TO DETERMINE WHETHER THE CLOSED LOOP SYSTEM IS STABLE, MARGINALLY STABLE, OR UNSTABLE. FOR THIS WE APPLY THE NYQUIST STABILITY CRITERION.

(B) TO DETERMINE RELATIVE STABILITY, IE, IF THE SYSTEM IS STABLE, HOW STABLE IS IT? FOR THIS WE CONSIDER GAIN MARGIN AND PHASE MARGIN.

THE CLOSED LOOP TRANSFER FUNCTION IS

$$\frac{C(s)}{R(s)} = \frac{KG(s)}{1+KG(s)H(s)}$$

THE OPEN LOOP TRANSFER FUNCTION KG(s)H(s) IS ASSUMED TO BE KNOWN EITHER

(A) IN TERMS OF BODE PLOTS OF MAGNITUDE AND PHASE, IE ITS FREQUENCY RESPONSE, OR

(B) AS A RATIONAL POLYNOMIAL..

$$KG(s)H(s) = \frac{Q_1(s)}{Q_2(s)}.$$

DEFINE F(s):

$$\begin{aligned} F(s) &= 1 + KG(s)H(s) \\ &= 1 + \frac{Q_1(s)}{Q_2(s)} \\ &= \frac{Q_1(s) + Q_2(s)}{Q_2(s)} \\ &= \frac{Q(s)}{Q_2(s)} \end{aligned}$$

THIS IS ALSO A RATIONAL POLYNOMIAL. NOTE THAT:

(A) THE POLES OF THE OPEN LOOP TRANSFER FUNCTION KG(s)H(s) ARE THE SAME AS THOSE OF F(s).

(B) NOTE ALSO THAT BY SETTING F(s) TO ZERO WE OBTAIN THE CHARACTERISTIC EQUATION OF THE CLOSED LOOP SYSTEM; AND RECALL THAT IN ORDER THAT THE CLOSED LOOP SYSTEM BE STABLE, THE CHARACTERISTIC EQUATION

$$F(s) = 1 + KG(s)H(s) = 0$$

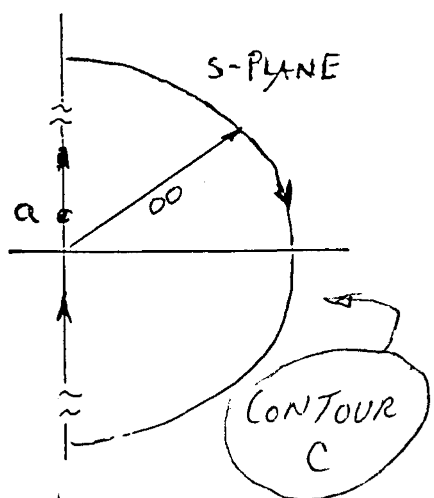
MUST HAVE ZEROS ONLY IN THE LEFT HALF PLANE (LHP), IE NO $j\omega$ AXIS AND NO RIGHT HALF PLANE ZEROS; EQUIVALENTLY, THE CLOSED LOOP TRANSFER FUNCTION MUST HAVE POLES ONLY IN THE LHP, IE, NO $j\omega$ AXIS AND NO RIGHT HALF PLANE POLES.

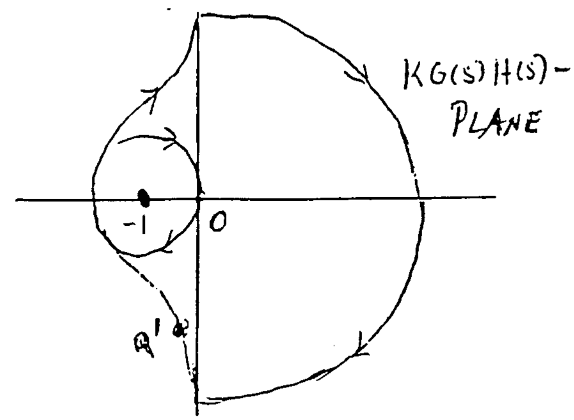
WE WILL USE A THEOREM KNOWN AS <u>CAUCHY'S PRINCIPLE OF ARGUMENT</u>. IN ORDER TO APPLY THIS THEOREM, LET C BE A CLOSED, CLOCKWISE CONTOUR THAT COMPLETELY ENCLOSES THE RHP, AND WHICH EXCLUDES FINITE POLES OF F(s) THAT LIE ON THE $j\omega$ AXIS, WHERE FINITE POLES OF F(s) ARE OBTAINED FROM $Q_2(s)=0$. THE CONTOUR C IS CALLED THE <u>NYQUIST PATH</u>.

FOR EACH POINT ALONG THE NYQUIST PATH C, WE CAN DETERMINE THE CORRESPONDING POINT IN THE COMPLEX F(s) PLANE [THE 1+KG(S)H(S) PLANE].

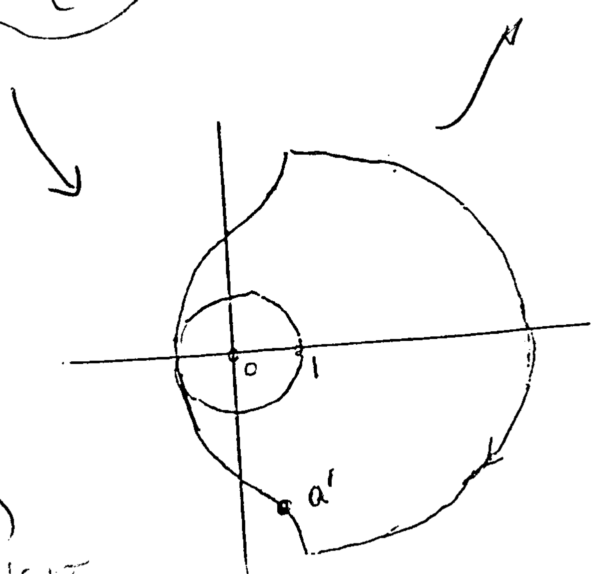
<u>NYQUIST DIAGRAM</u>:
LIKEWISE, FOR EACH POINT ALONG THE NYQUIST PATH C, WE CAN DETERMINE THE CORRESPONDING POINT IN THE COMPLEX KG(s)G(s) PLANE. <u>THIS PLOT FORMS THE NYQUIST DIAGRAM</u>.

SINCE C IS A <u>CLOSED</u> CONTOUR, THE PLOTS OF 1+KG(s)H(s) and KG(s)H(s) WILL LIKEWISE FORM <u>CLOSED</u> CONTOURS.





CONTOUR C ENCLOSES THE ZEROS OF 1+KG(s)H(s) IN THE RIGHT HALF PLANE



F(s)-PLANE, OR 1+KG(s)H(s) PLANE

Z = # OF ZEROS OF 1+KG(s)H(s) THAT ARE IN RIGHT HALF PLANE.

NOTE THAT IF THAT PART OF THE CONTOUR C THAT IS ALONG THE IMAGINARY AXIS PASSES THROUGH ANY FINITE ZEROS OF F(s), THEN

$$F(j\omega)=0,$$

IE,

$$1+KG(j\omega)H(j\omega)=0$$

THIS CONDITION, IE, CHARACTERISTIC EQUATION ZEROS ON THE IMAGINARY AXIS, OR EQUIVALENTLY, CLOSED LOOP TRANSFER FUNCTION POLES ON THE IMAGINARY AXIS, IS A NECESSARY CONDITION FOR MARGINAL STABILITY OF THE CLOSED LOOP CONTROL SYSTEM. THUS FOR MARGINAL STABILITY, WE HAVE THE FOLLOWING NECESSARY, BUT NOT SUFFICIENT, CONDITION..

$$KG(j\omega)H(j\omega) \;=\; -1 \;=\; -1+j0 \;=\; 1\angle\pm180^{o}$$

DEFINE..

N = NUMBER OF CLOCKWISE ENCIRCLEMENTS MADE OF THE ORIGIN IN THE 1+KG(s)H(s) PLANE.

THIS DEFINITION OF N IS EQUIVALENT TO THE FOLLOWING:

N = NUMBER OF CLOCKWISE ENCIRCLEMENTS MADE OF THE POINT $-1+j0$ IN THE KG(s)H(s) PLANE.

NOTE: IF A CLOSED CONTOUR MAKES, SAY, 2 COUNTERCLOCKWISE ENCIRCLEMENTS OF A POINT, THEN, EQUIVALENTLY, IT MAKES -2 CLOCKWISE ENCIRCLEMENTS OF THAT POINT.

Z = THE NUMBER OF FINITE ZEROS OF F(s)=1+KG(s)H(s) THAT ARE IN THE RHP

= THE NUMBER OF FINITE POLES OF THE CLOSED LOOP TRANSFER FUNCTION THAT ARE IN THE RHP

P = THE NUMBER OF FINITE POLES OF F(s)=1 + KG(s)H(s) IN THE RHP

= THE NUMBER OF FINITE POLES OF KG(s)H(s) IN THE RHP; [KG(s)H(s) IN FACTORED FORM IS ASSUMED TO BE KNOWN, THUS P IS READILY OBTAINABLE; IF KG(s)H(s) IS STABLE, P=0].

BY CAUCHY'S PRINCIPLE OF ARGUMENT,

$$Z=N+P$$

WHERE

(A) N CAN BE FOUND BY MAPPING THE NYQUIST PATH C INTO THE PLANE OF THE OPEN LOOP TRANSFER FUNCTION, IE, BY FINDING THE NYQUIST DIAGRAM

(B) P IS KNOWN SINCE WE ARE ASSUMING THAT THE OPEN LOOP TRANSFER FUNCTION KG(s)H(s) IS KNOWN IN FACTORED FORM

(A) AND, AS LONG AS THE NYQUIST DIAGRAM DOES NOT PASS THROUGH THE POINT $-1+j0$, WE MUST HAVE Z=0 IN ORDER THAT CLOSED LOOP CONTROL SYSTEM BE STABLE, IE, WE MUST HAVE

$$N=-P, \quad OR$$
$$0=N+P$$

FOR CLOSED LOOP SYSTEM STABILITY

THUS, IN ORDER TO ANSWER THE QUESTION "IS THE CLOSED LOOP SYSTEM STABLE?" WE DO THE FOLLOWING:

(1) DETERMINE P

(2) PLOT THE NYQUIST DIAGRAM

(A) SEE NEXT PAGE

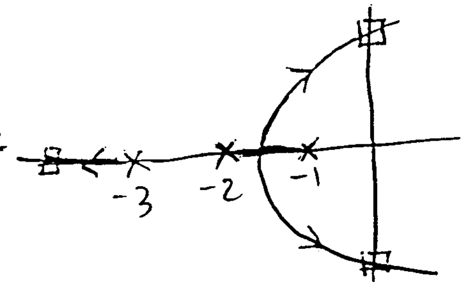
(A) IF NYQUIST DIAGRAM PASSES <u>THROUGH</u> THE POINT $-1+j0$, THE SYSTEM WILL BE EITHER

(1) MARGINALLY STABLE

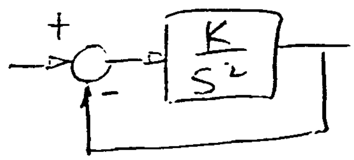
FOR EXAMPLE, ROOT LOCUS MAY BE

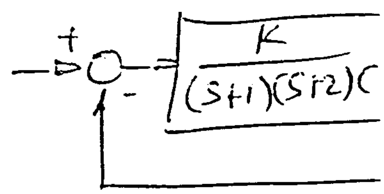
JW AXIS POLES ALONE

OR



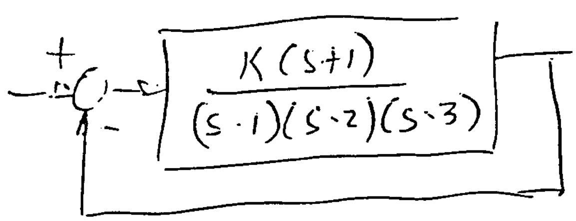
JW AXIS POLES TOGETHER WITH LEFT HALF PLANE POLE

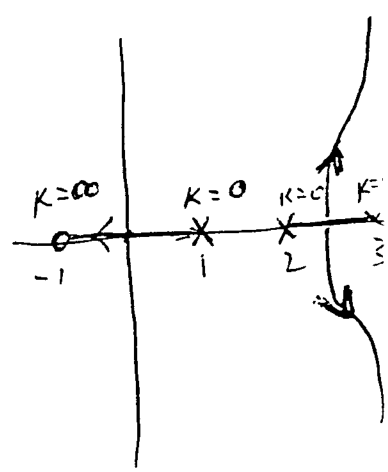




OR (2) <u>UNSTABLE</u> (CONTAINING JW AXIS POLES TOGETHER WITH AT LEAST ONE RIGHT HALF PLANE POLE).

FOR EXAMPLE —





(B) SEE NEXT PAGE

(B) IF THE NYQUIST DIAGRAM DOES NOT PASS THROUGH THE POINT $-1+j0$, DETERMINE N.

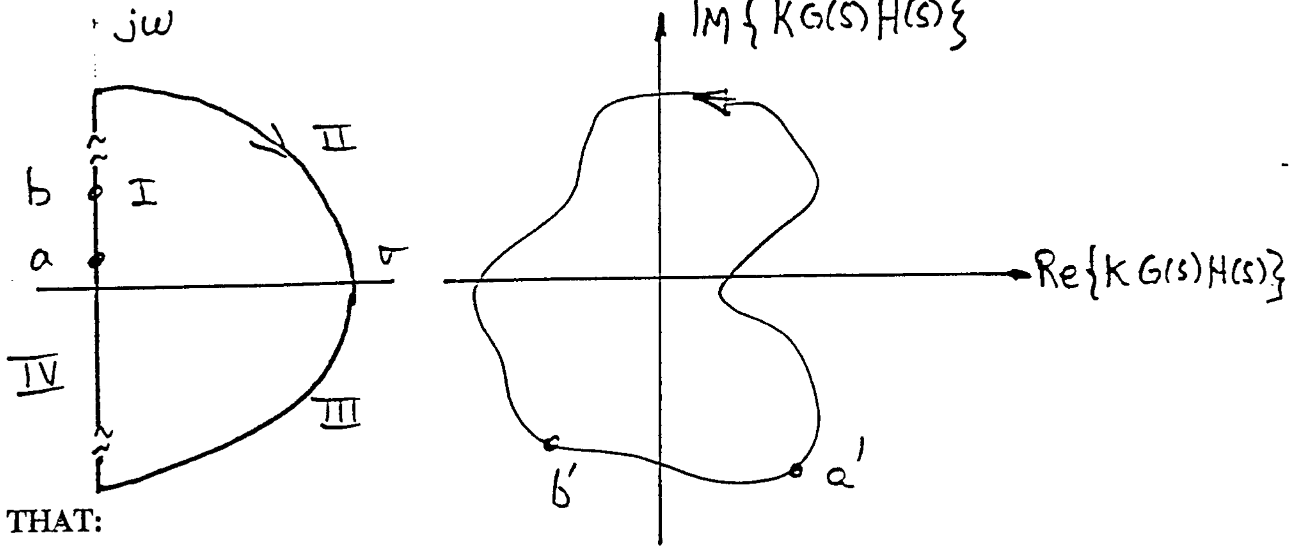
(3) SEE WHETHER THE CONDITION: $N+P=0$ IS TRUE.

(A) IF TRUE, THE SYSTEM IS STABLE, IE, ALL OF THE POLES OF THE CLOSED LOOP TRANSFER FUNCTION WILL BE IN THE LEFT HALF PLANE (LHP).

(B) IF NOT TRUE, THE SYSTEM IS NOT STABLE, IE, THERE WILL BE AT LEAST ONE POLE OF THE CLOSED LOOP SYSTEM TRANSFER FUNCTION THAT IS IN THE RIGHT HALF PLANE (RHP).

PLOTTING THE NYQUIST DIAGRAM

THE NYQUIST DIAGRAM IS OBTAINED BY MAPPING POINTS ALONG THE NYQUIST PATH, C, INTO THE PLANE OF THE OPEN LOOP TRANSFER FUNCTION KG(s)H(s), IE, FOR EACH POINT ON THE s-PLANE NYQUIST PATH, WE OBTAIN THE CORRESPONDING POINT ON THE KG(s)H(s) PLANE.



NOTE THAT:

(1) WE CAN START ON THE $j\omega$ AXIS, AND LET ω INCREASE FROM ZERO TO INFINITY, AND FIND $KG(j\omega)H(j\omega)$. THIS IS PRECISELY THE FREQUENCY RESPONSE OF THE OPEN LOOP TRANSFER FUNCTION.

(2) WE CAN CONTINUE ALONG THE INFINITE RADIUS SEMICIRCLE, WHERE $s=\infty$; TYPICALLY $|KG(s)H(s)|=0$ ALONG THIS PORTION OF THE PATH, IE, SEGMENTS II AND III TYPICALLY MAP INTO THE ORIGIN OF THE PLANE OF THE OPEN LOOP TRANSFER FUNCTION.

(3) THE NYQUIST DIAGRAM IS SYMMETRICAL WITH RESPECT TO THE REAL AXIS: IF WE LET

$$KG(j\omega)H(j\omega)=\alpha(\omega)+j\beta(\omega),$$

IT FOLLOWS THAT

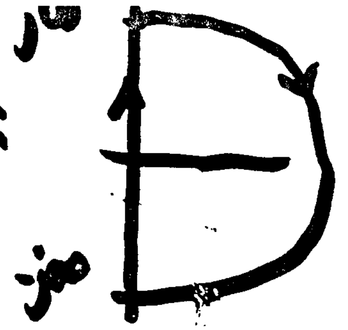
$$KG(-j\omega)H(-j\omega)=\alpha(\omega)-j\beta(\omega).$$

REFERRING TO NYQUIST PATH, THIS MEANS THAT THE NYQUIST DIAGRAM FOR SEGMENTS III AND IV WILL BE THE MIRROR IMAGE ABOUT THE REAL AXIS OF THE NYQUIST DIAGRAM FOR SEGMENTS I AND II.

THUS, EVALUATING KG(s)H(s) FOR $s=j\omega$, WHERE $0\le\omega\le\infty$, IS USUALLY SUFFICIENT FOR DETERMINING THE COMPLETE NYQUIST DIAGRAM. IN OTHER WORDS, IF WE KNOW THE FREQUENCY RESPONSE OF THE OPEN LOOP TRANSFER FUNCTION, WE CAN USUALLY FIND THE NYQUIST DIAGRAM.

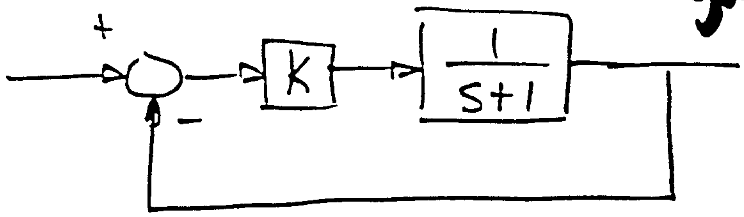
FOR A STABLE OPEN LOOP TRANSFER FUNCTION (IE, P = 0), WE CAN DETERMINE THE FREQUENCY RESPONSE EXPERIMENTALLY.

C:



EXAMPLES:

(1) GIVEN THE CONTROL SYSTEM..

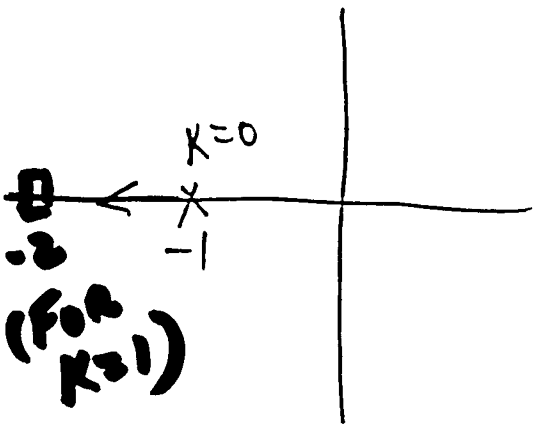


THE OPEN LOOP TRANSFER FUNCTION IS: $\frac{K}{s+1}$.

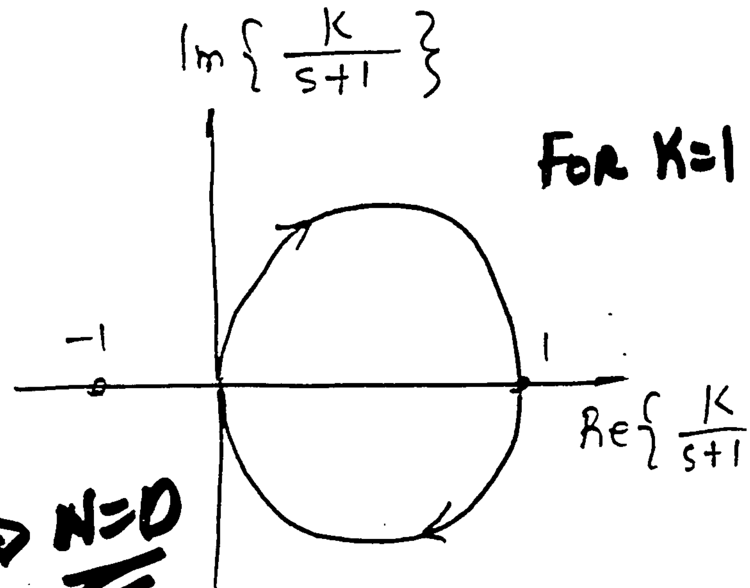
P=0

THE ROOT LOCUS AND NYQUIST PLOTS ARE...

Root Locus:



For K=1



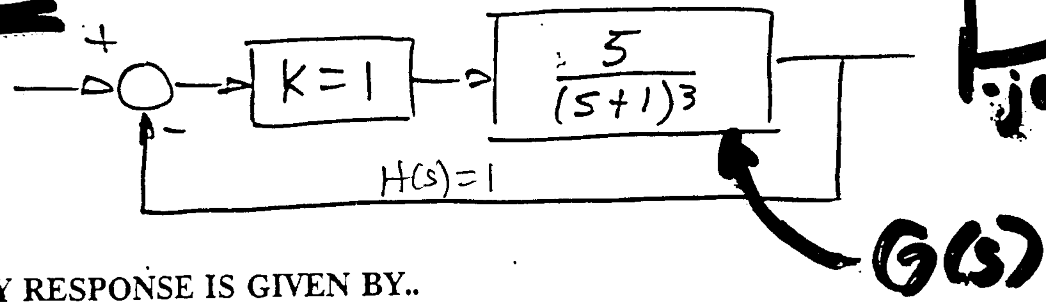
⇒ N=0

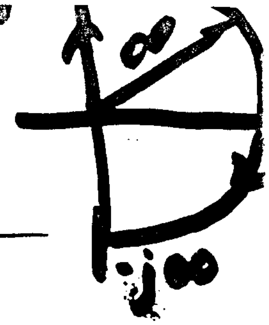
NYQUIST DIAGRAM

N+P=0

THIS SYSTEM IS CLEARLY STABLE FOR ANY $K>0$.

(2) **EXAMPLE 8.10:** GIVEN THE CONTROL SYSTEM..

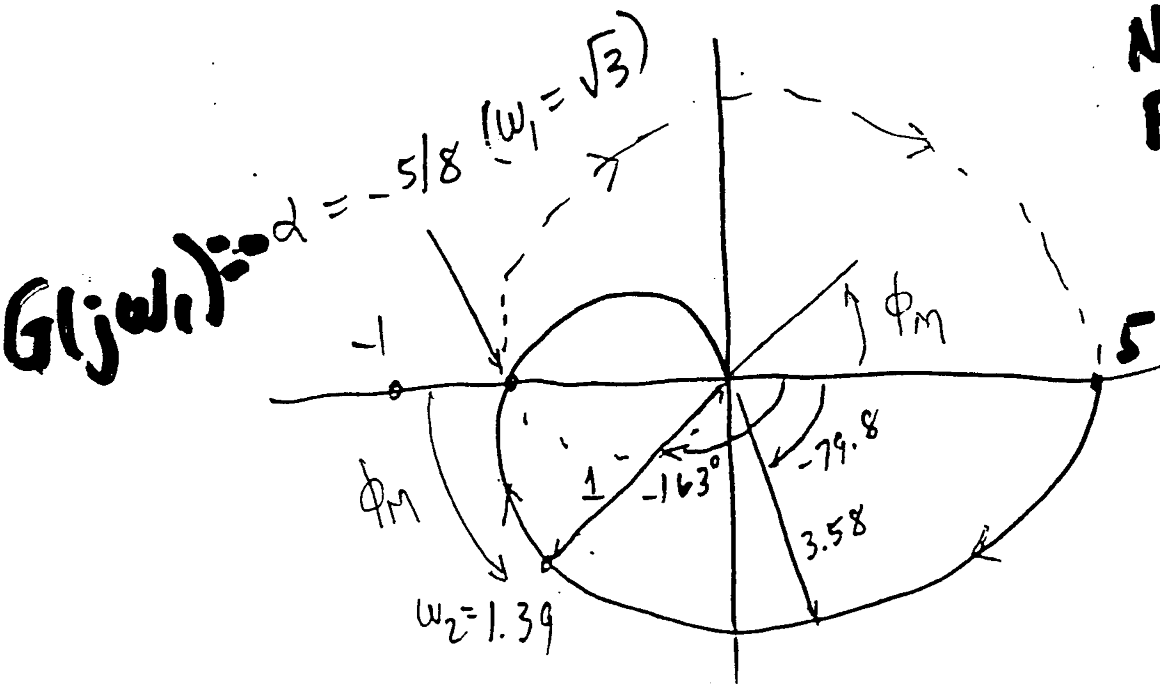




THE FREQUENCY RESPONSE IS GIVEN BY..

ω	$\lvert KG(j\omega H(j\omega)\rvert$	$\angle KG(j\omega)H(j\omega)$, degrees
0	5	0
0.5	3.58	-79.8
1	1.77	-135
$\omega_2=1.39$	1 (0dB GAIN CROSSOVER)	-163 ($\Rightarrow \phi_M=17°$)
1.5	0.85	-169
$\omega_1=\sqrt{3}$	$\frac{5}{8}=0.63$ ($\Rightarrow GM=\frac{8}{5}=1.6$ or $4.1dB$)	-180 (-180° PHASE CROSSOVER)
2	0.45	-190
5	0.04	-236

THE CORRESPONDING NYQUIST DIAGRAM IS..

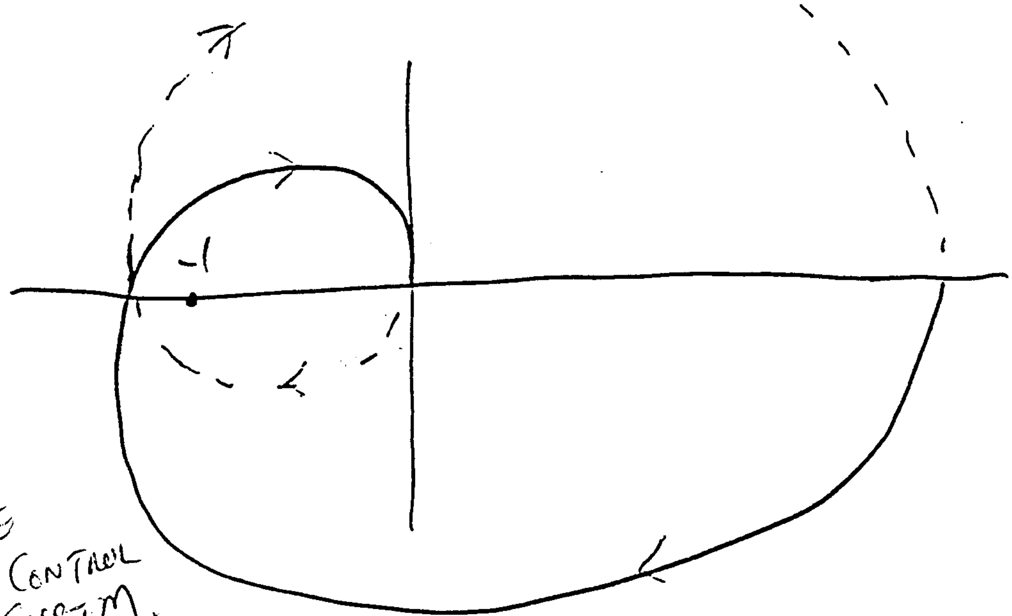


N=0
P=0
$\Rightarrow Z=N+P=0$
∴ STABLE
FOR K=1

For any $K < 8/5$, Closed Loop System is STABLE

NOTICE THAT IF K WERE GREATER THAN 8/5, THE NYQUIST DIAGRAM, AS SHOWN BELOW, WOULD ENCLOSE THE -1 + j0 POINT. THUS N=2; THEREFORE, Z=N+P=2+0=2...UNSTABLE SYSTEM.

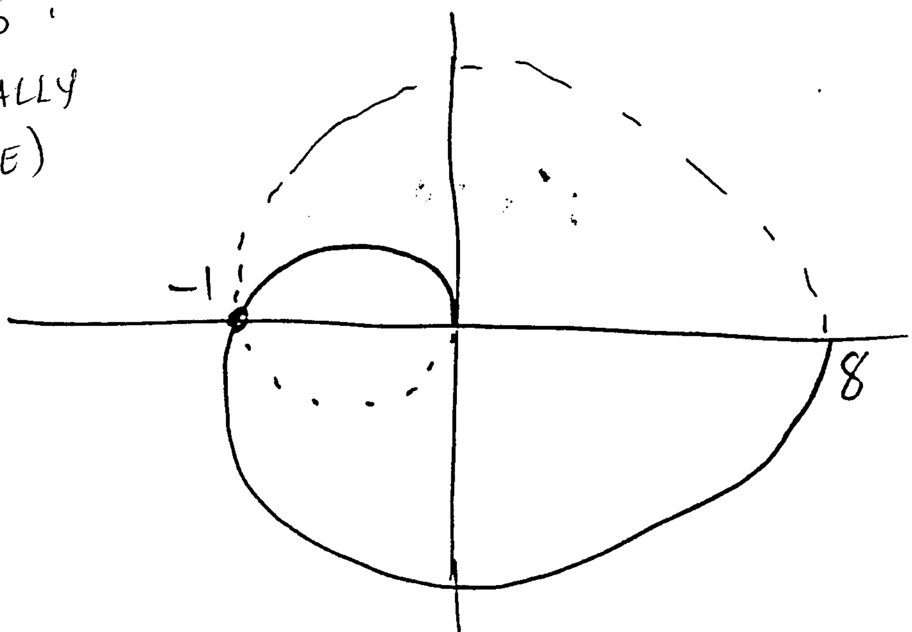
$K > 8/5$:
(UNSTABLE)



$N = 2$
$P = 0$
$Z = N + P = 2$
$\Rightarrow$ UNSTABLE FEEDBACK CONTROL SYSTEM.

AND IF K WERE EQUAL TO 8/5, THE NYQUIST DIAGRAM WOULD PASS THROUGH THE -1 + j0 POINT..

$K = 8/5$:
(MARGINALLY STABLE)



NYQ. DIAG

CONCLUDE THAT IF THIS [redacted] PASSES THROUGH THE -1 +j0 POINT (WITH K=8/5), THE CLOSED LOOP SYSTEM WILL BE MARGINALLY STABLE.

WE CAN ALSO OBTAIN THESE ABOVE RESULTS USING THE ROUTH HURWITZ STABILITY CRITERION AND THE ROOT LOCUS:

$$1 + KG(s)H(s) = 0 \Rightarrow s^3 + 3s^2 + 3s + 1 + 5K = 0$$

s^3	1	3
s^2	3	$1+5K$
s^1	$8-5K$	
s^0	$1+5K$	

$\Rightarrow K < 8/5$

$\Rightarrow K > -1/5$

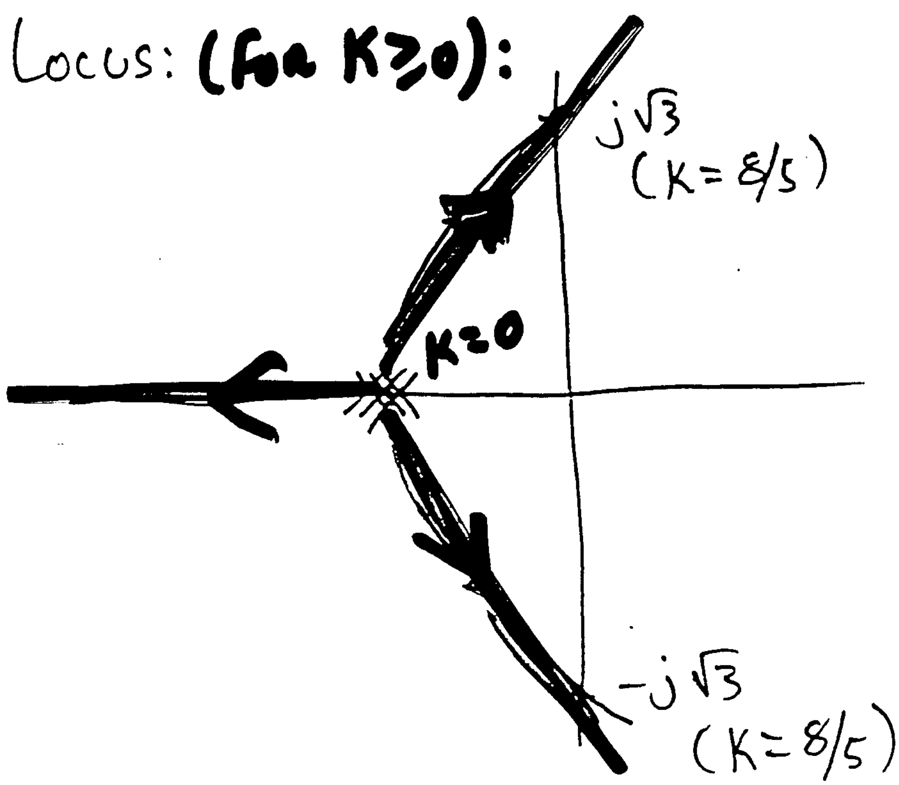
$$\therefore -0.2 < K < 8/5$$

WITH $K = 8/5$, $\varphi_a(s) = 3s^2 + 9 = 0$

$\Rightarrow s = \pm j\sqrt{3}$

$\Rightarrow \omega_1 = \sqrt{3}$

Root Locus: **(for K≥0):**



RELATIVE STABILITY: GAIN MARGIN AND PHASE MARGIN

GAIN MARGIN: $GM=\frac{1}{\alpha}$ = THE FACTOR BY WHICH THE GAIN K MAY BE INCREASED BEFORE THE CLOSED LOOP SYSTEM BECOMES UNSTABLE.

FOR THE PREVIOUS EXAMPLE, $\cancel{GM=\frac{1}{\frac{8}{5}}=\frac{5}{8}}$. $\cancel{GM_{dB}=20\log\frac{5}{8}=}$

PHASE MARGIN: ϕ_M. FOR A STABLE CONTROL SYSTEM, IT IS THE MAGNITUDE OF THE MINIMUM ANGLE BY WHICH THE NYQUIST DIAGRAM MUST BE ROTATED IN ORDER TO INTERSECT THE -1 POINT. IT CAN BE EXPRESSED AS

$$\phi_M=\angle KG(j\omega_2)H(j\omega_2)-180^o$$

WHERE ω_2 = FREQUENCY AT WHICH $KG(j\omega)H(j\omega)=1$, ALSO CALLED THE UNITY MAGNITUDE (OR ZERO dB) CROSSOVER FREQUENCY.

FOR THE PREVIOUS EXAMPLE, $\phi_M=17^O$.

$$GM=\frac{1}{\frac{5}{8}}=\frac{8}{5}$$

$$GM_{dB}=20\log\frac{8}{5}=4.1\,dB$$

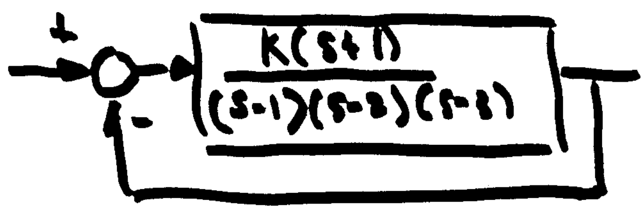
$K G(j\omega) H(j\omega) = -1 = -1 + j0 = 1 \angle \pm 180$

TO SHOW THAT IT IS A NECESSARY BUT NOT A SUFFICIENT CONDITION, FOR MARGINAL STABILITY CONSIDER THE FOLLOWING EXAMPLE...

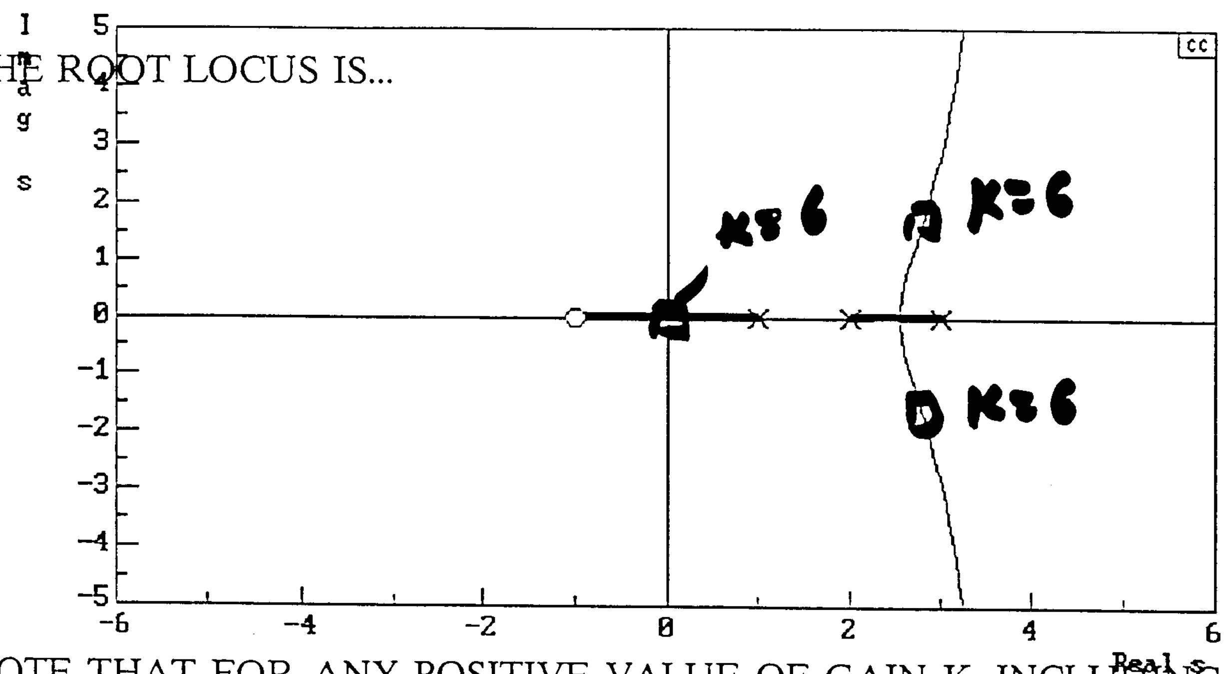
EXAMPLE: LET THE OPEN LOOP TRANSFER FUNCTION BE

$$KG(s)H(s) = \frac{K(s+1)}{(s-1)(s-2)(s-3)}$$

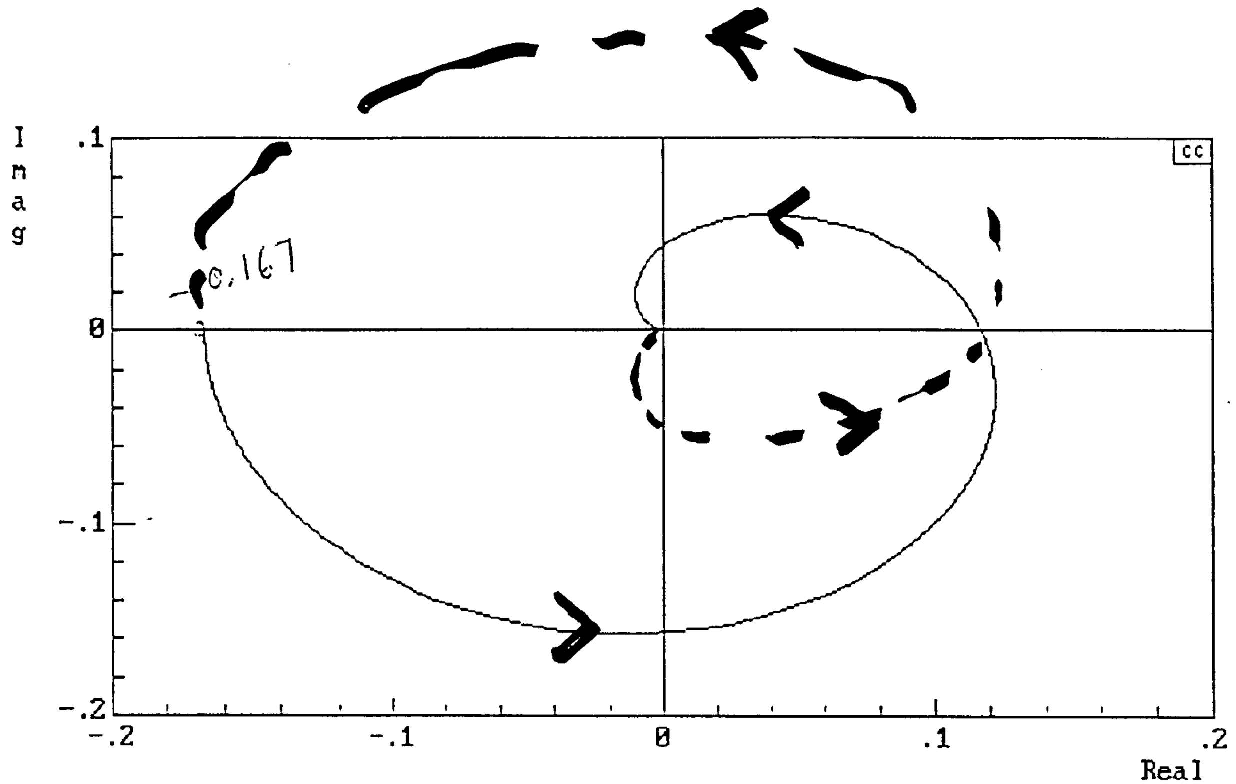
THE CLOSED LOOP SYSTEM BLOCK DIAGRAM IS GIVEN BY

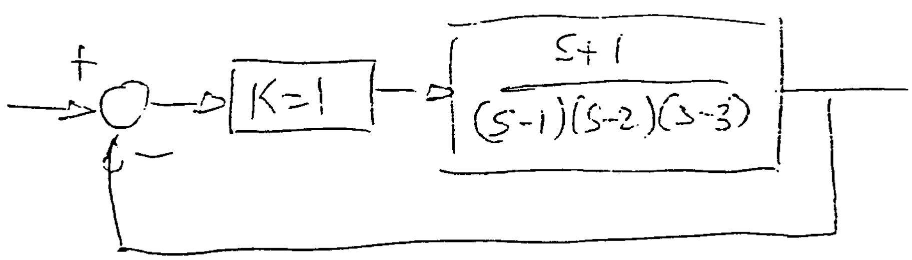


THE ROOT LOCUS IS...



NOTE THAT FOR ANY POSITIVE VALUE OF GAIN K, INCLUDING THAT VALUE OF K FOR WHICH THE ROOT LOCUS CROSSES THE IMAGINARY AXIS, THERE WILL ALWAYS BE CLOSED LOOP SYSTEM POLES THAT ARE IN THE RIGHT HALF PLANE, IE THE CLOSED LOOP CONTROL SYSTEM IS ALWAYS UNSTABLE. THUS THE SYSTEM CANNOT BE MARGINALLY STABLE.

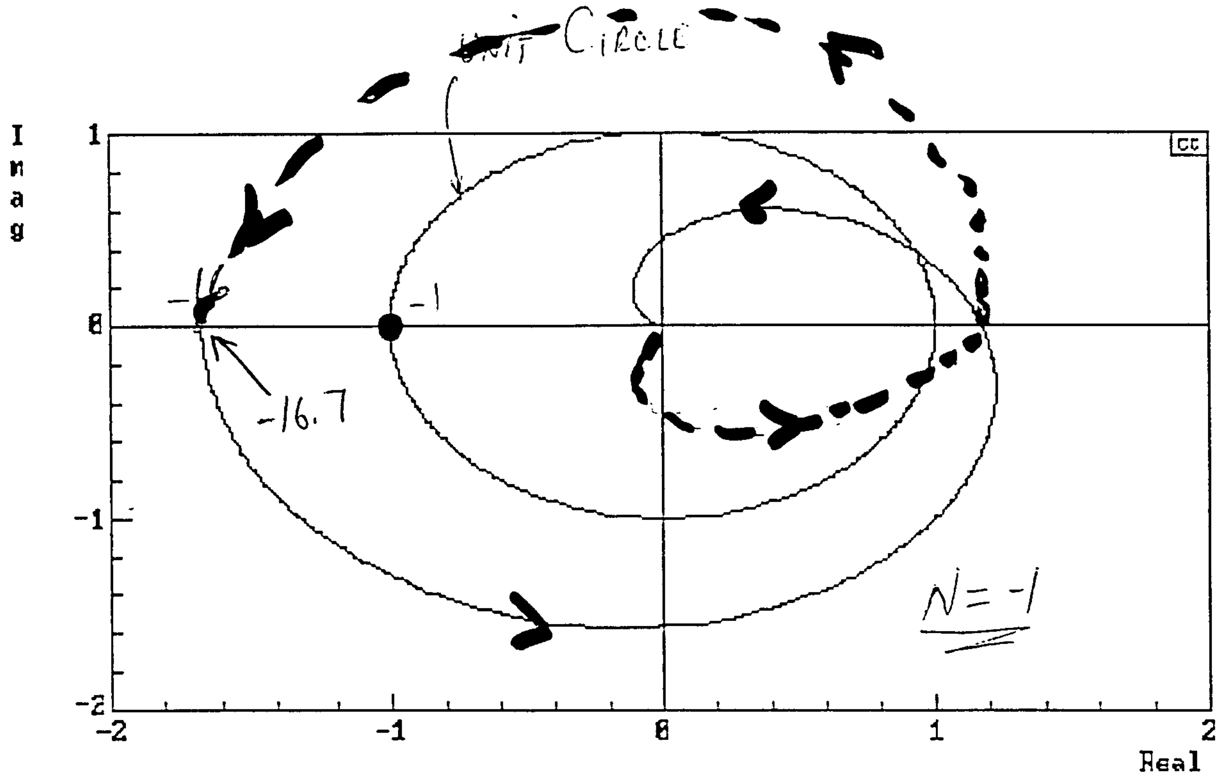


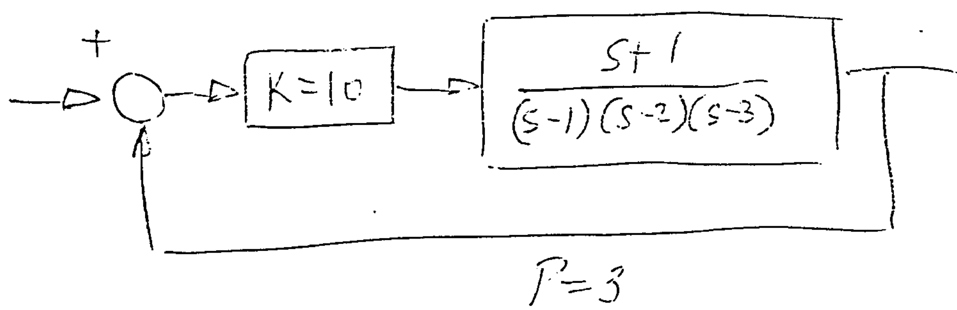


$N+P \stackrel{?}{=} 0$

$0+3=3\neq 0$ ∴ NOT STABLE

FOR $K = 1/0.167 = 6$, NYQUIST DIAGRAM PASSES <u>THROUGH</u> THE POINT -1, YET IT IS STILL <u>UNSTABLE</u>.





$N + P \stackrel{?}{=} 0$

$-1 + 3 = 2 \neq 0$ ∴ Not stable

EEL 3657
FARAYANAKIS

$$\lim_{s\to\infty} G_c(s) = \lim_{s\to\infty} \frac{1 + s/\omega_0}{1 + s/\omega_p} = \frac{\omega_p}{\omega_0}$$

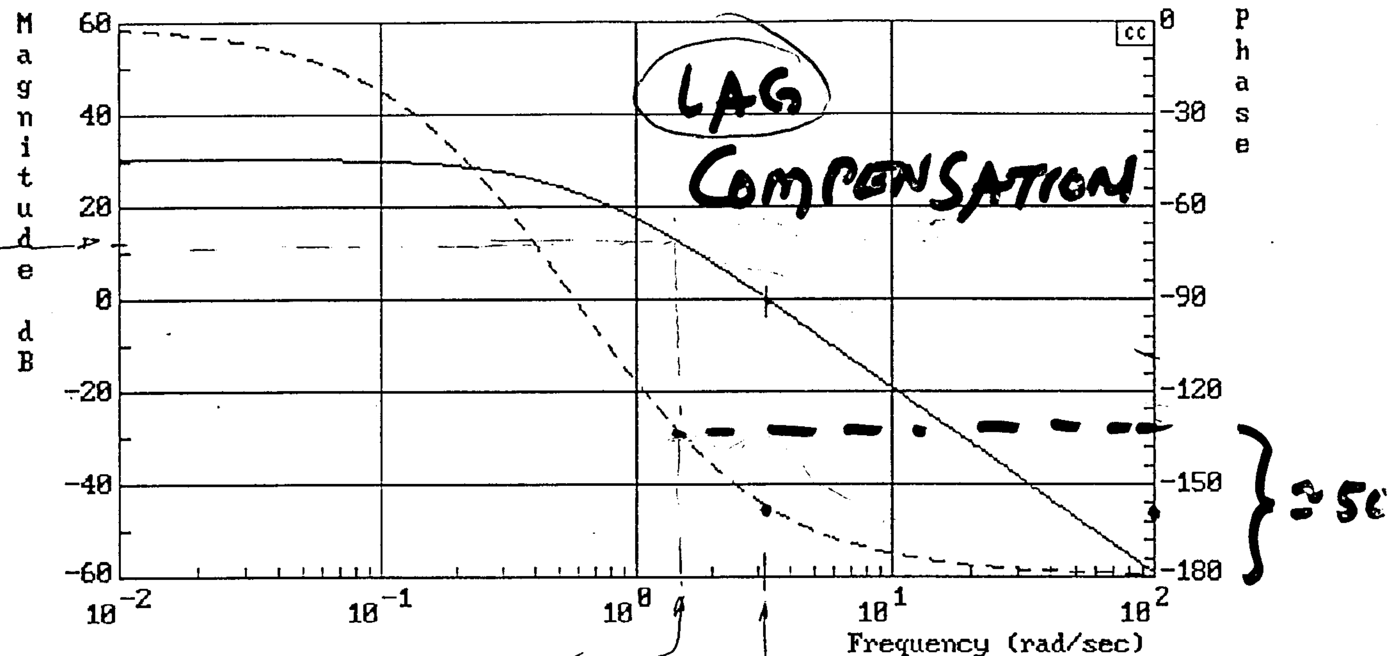
$$\underline{\omega_0 = 0.1\,\omega_d}$$

$$\lim_{s\to\infty} G_c(s) = \frac{1}{|G_p(j\omega_d)|}$$

$$\underline{\omega_p = \frac{0.1\,\omega_d}{|G_p(j\omega_1)|}}$$

12

12 dB



$12\,dB = 20 \log x$

$\Rightarrow x = 4$

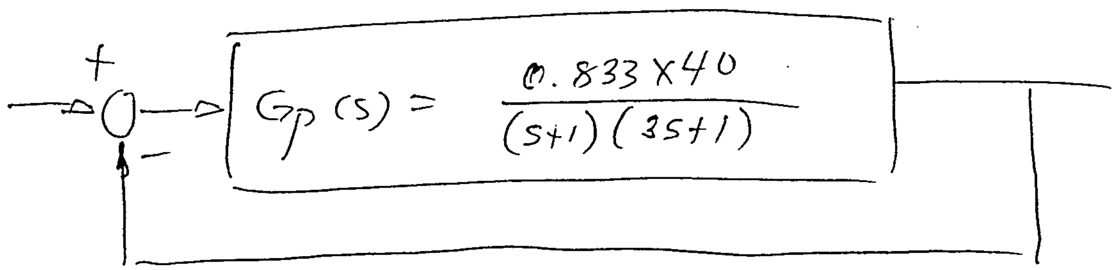
$\omega_d = 1.5$

AT $\omega = 3.3$,

$\angle G_p(s) = -157$

$\Rightarrow \phi_m = 23°$

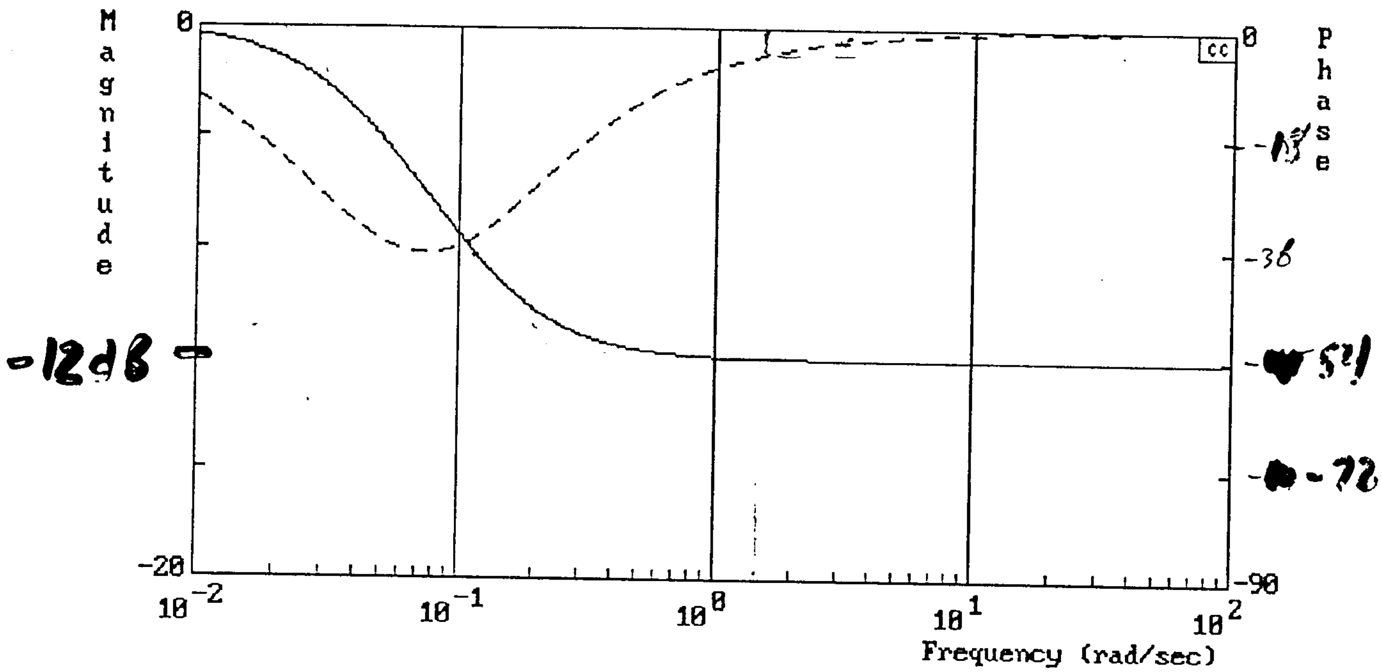
$\phi_{M\,DESIRED} = 45°$



$\omega_0 = 0.1\,\omega_d = 0.1(1.5) = 0.15$

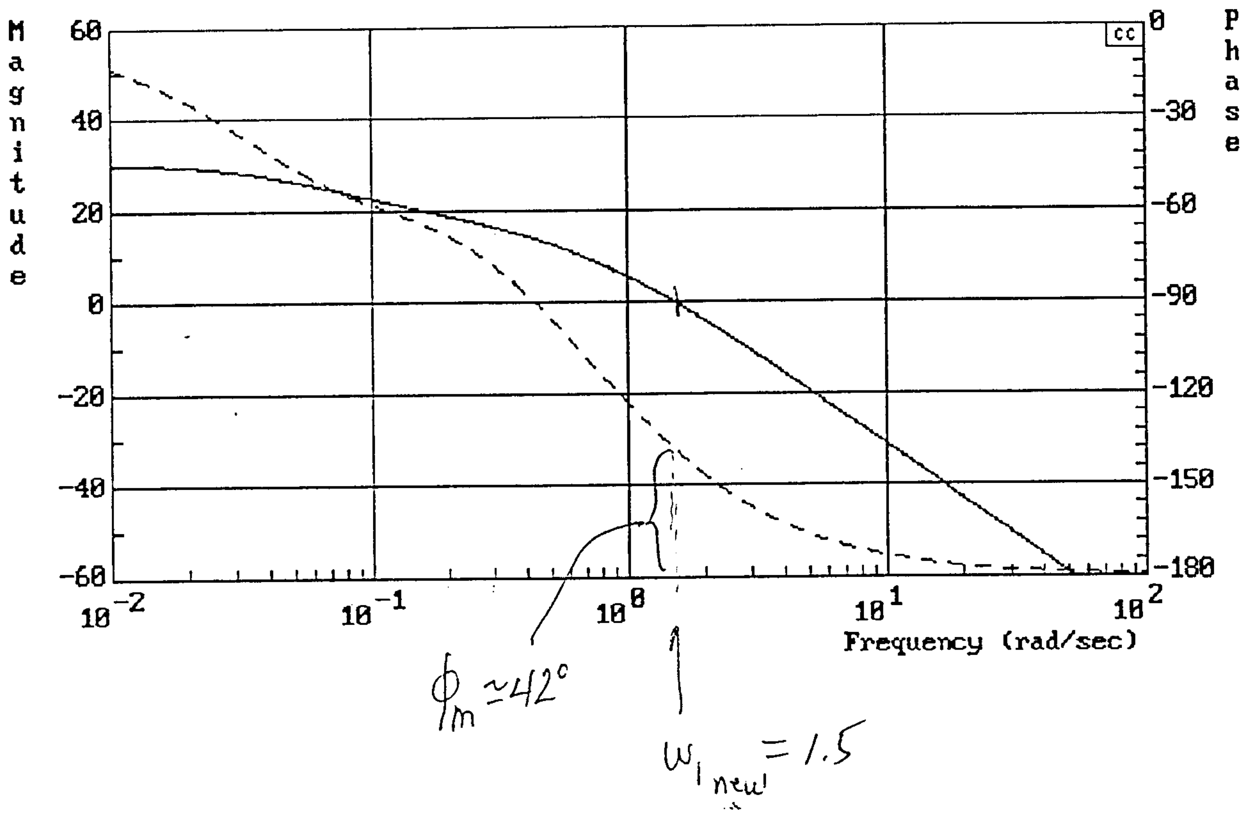
$\omega_p = \frac{0.1\,\omega_d}{|G_p(j\omega_1)|} = \frac{0.15}{4} = 0.0375$

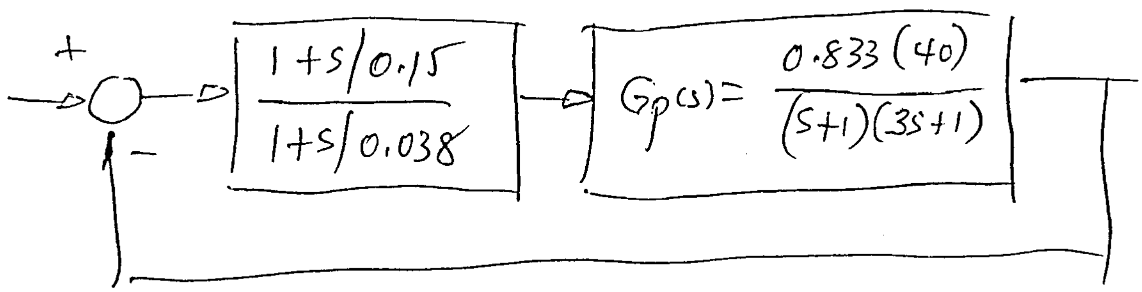
RESERVE

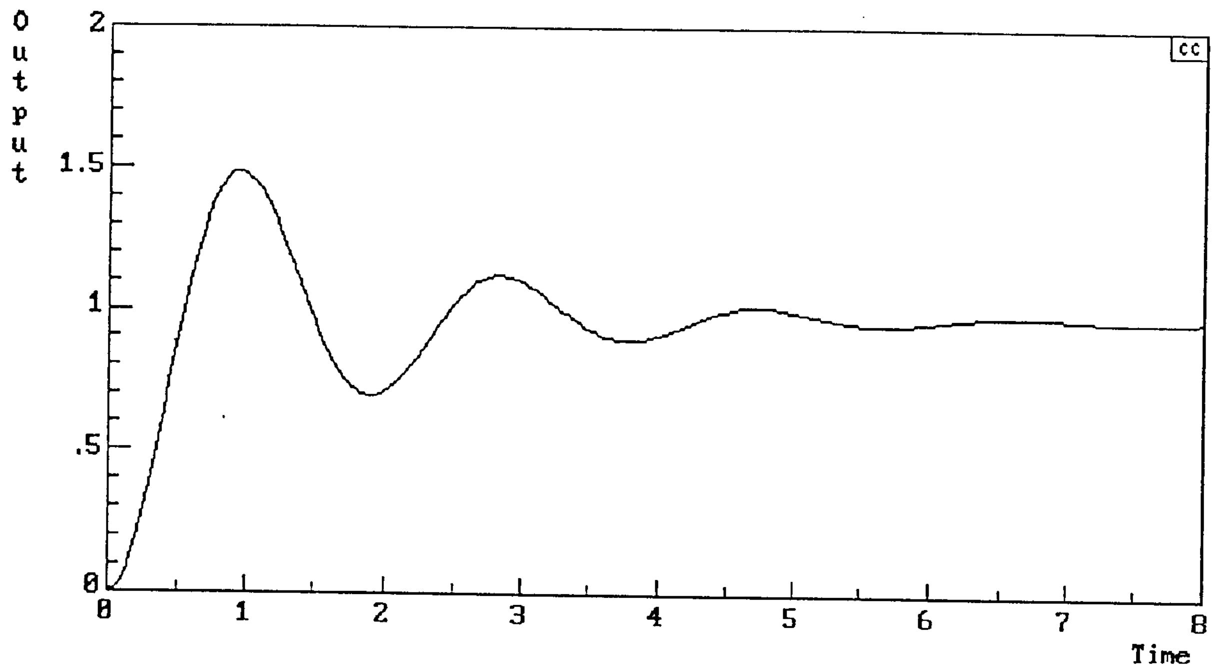


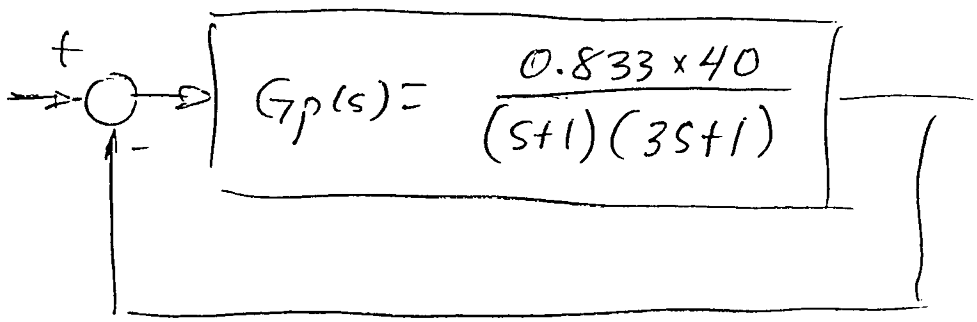
$$G_c(s) = \frac{1 + s/0.15}{1 + s/0.038}$$

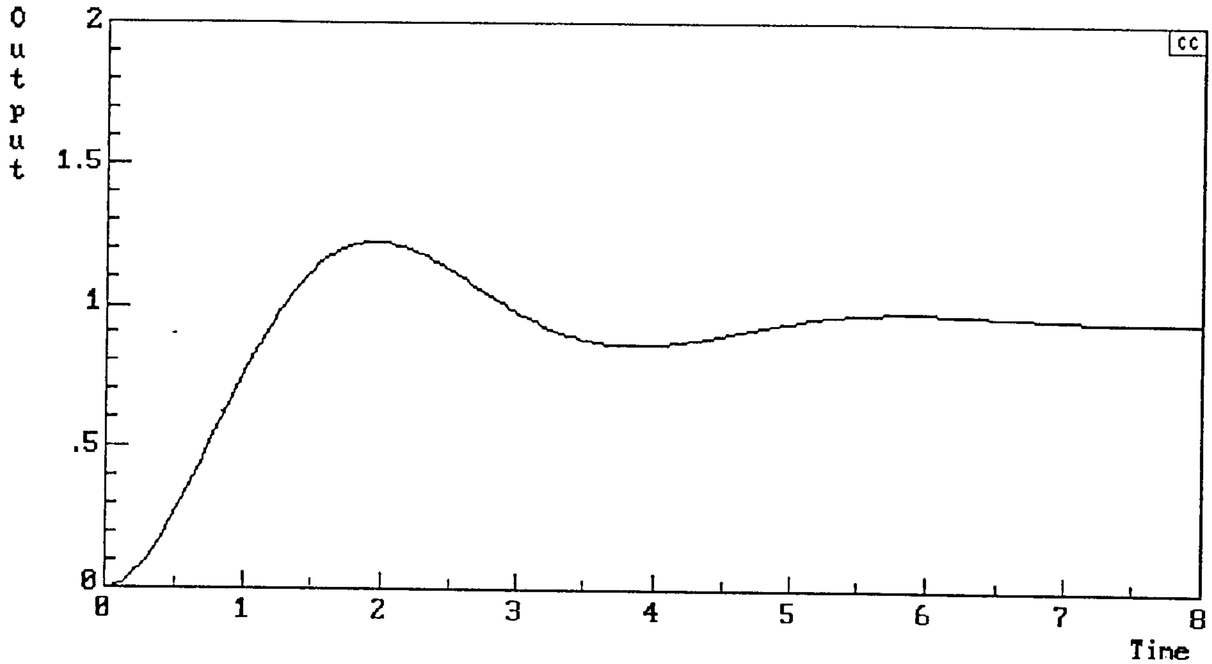
RESERVE

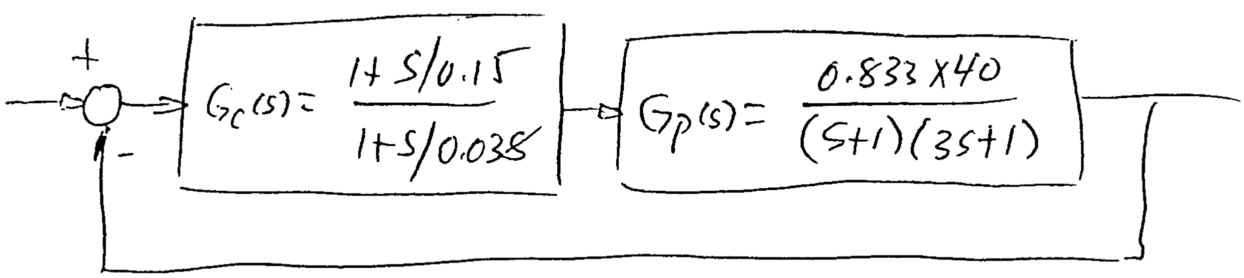

Magnitude
60
40
20
0
-20
-40
-60
Phase
0
-30
-60
-90
-120
-150
-180
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)
cc
$\phi_m \simeq 42°$
$\omega_{1_{new}} = 1.5$

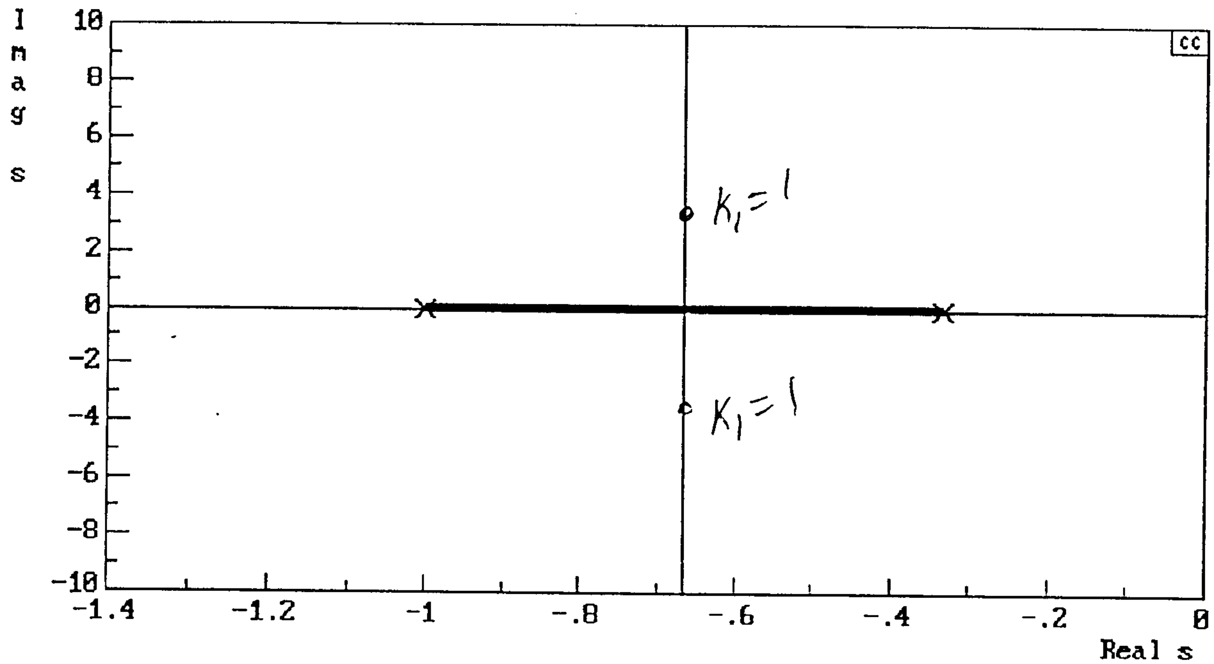

+
−
$\frac{1+s/0.15}{1+s/0.038}$
$G_p(s) = \frac{0.833\,(40)}{(s+1)(3s+1)}$


Output
2
1.5
1
.5
0
0
1
2
3
4
5
6
7
8
Time
cc

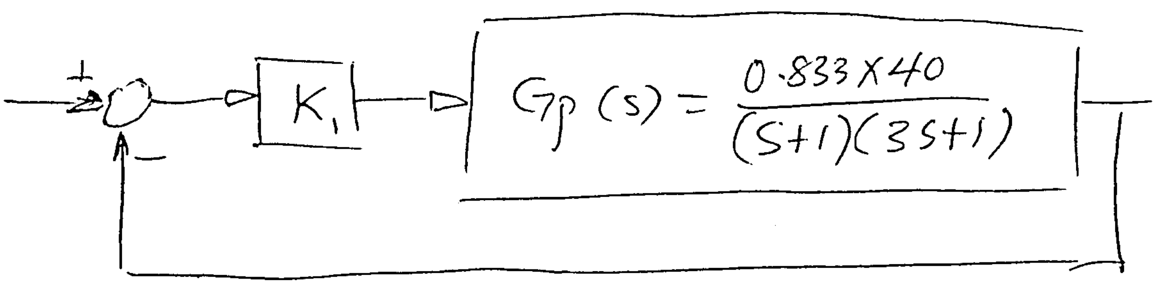

+
-
$G_p(s) = \frac{0.833 \times 40}{(s+1)(3s+1)}$


Output
2
1.5
1
.5
0
0
1
2
3
4
5
6
7
8
Time
cc

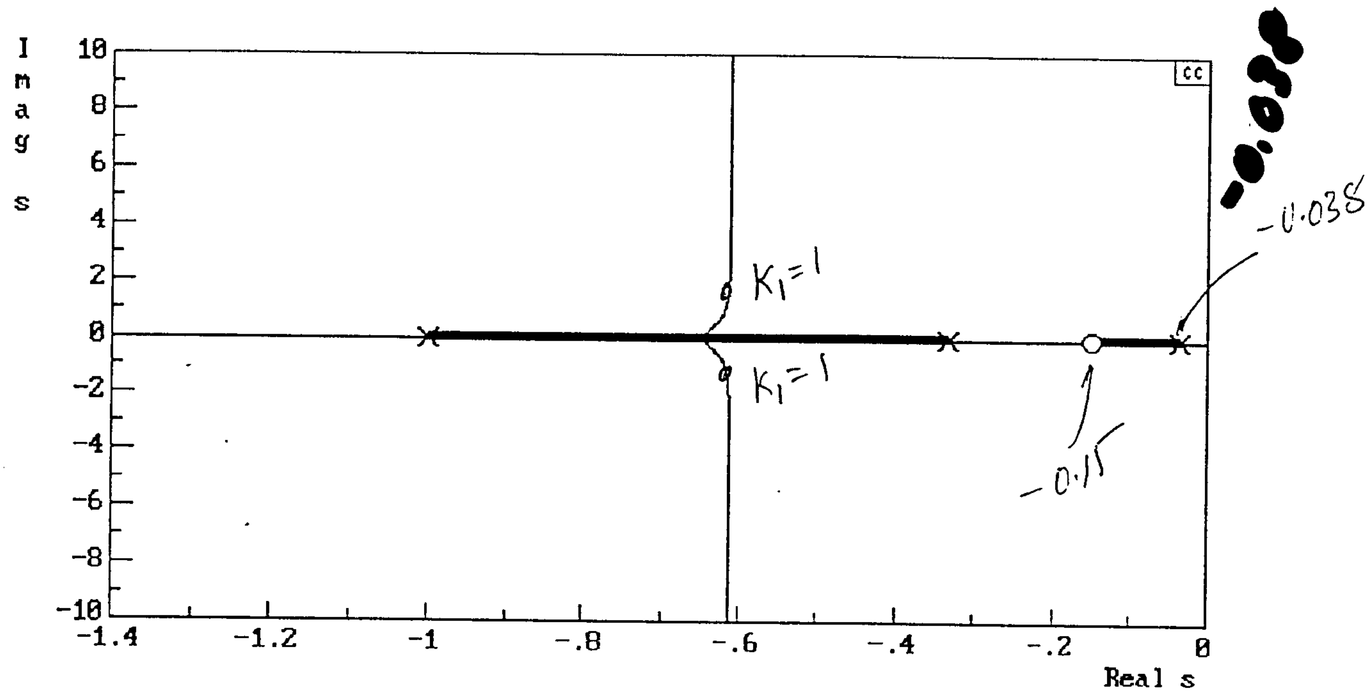

+
-
$G_c(s) = \frac{1+s/0.15}{1+s/0.038}$
$G_p(s) = \frac{0.833 \times 40}{(s+1)(3s+1)}$

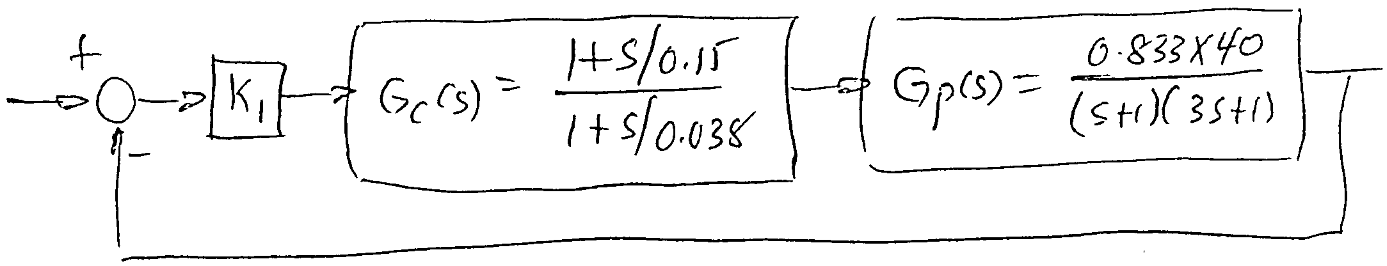


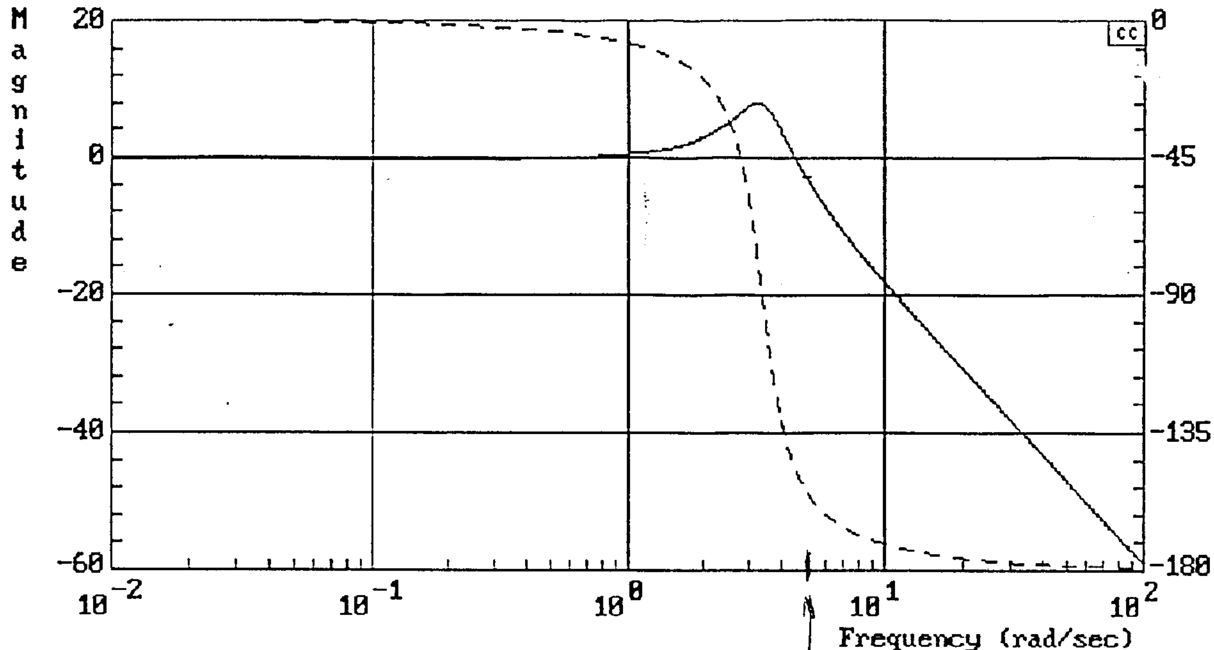
Imag s
10
8
6
4
2
0
-2
-4
-6
-8
-10
-1.4
-1.2
-1
-.8
-.6
-.4
-.2
0
Real s
cc
$K_1 = 1$
$K_1 = 1$



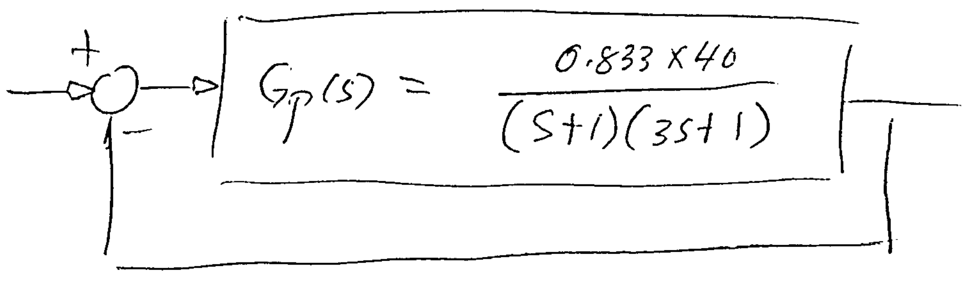
+
−
K_1
$G_p(s) = \frac{0.833 \times 40}{(s+1)(3s+1)}$

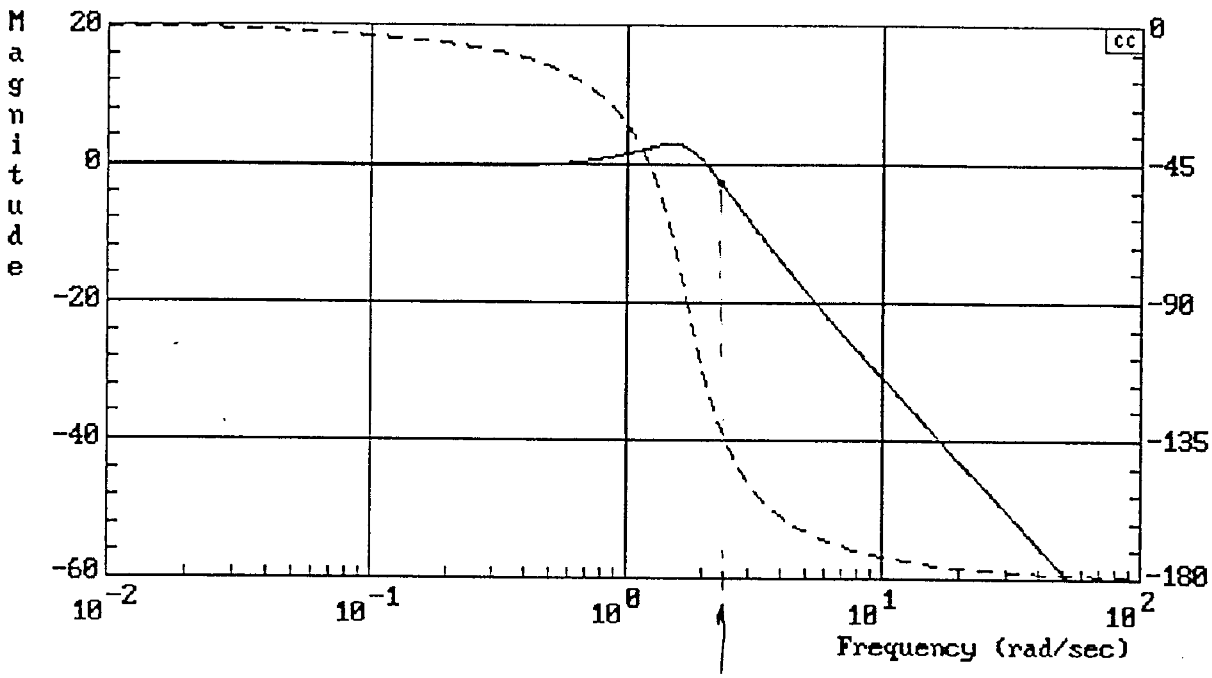

Imag s
10
8
6
4
2
0
-2
-4
-6
-8
-10
-1.4
-1.2
-1
-.8
-.6
-.4
-.2
0
Real s
cc
-0.038
-0.038
$K_1=1$
$K_1=1$
-0.15


+
−
K_1
$G_c(s) = \dfrac{1+s/0.15}{1+s/0.038}$
$G_p(s) = \dfrac{0.833 \times 40}{(s+1)(3s+1)}$

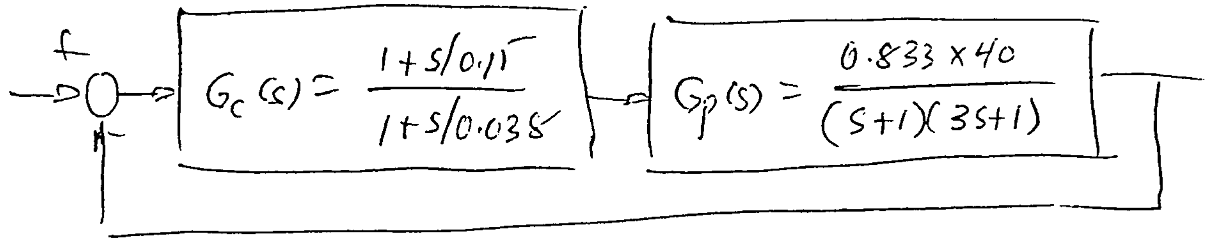

Magnitude
Phase
20
0
-20
-40
-60
0
-45
-90
-135
-180
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)
CC

3dB DOWN @ 5 R/s.


+
−
$G_p(s) = \frac{0.833 \times 40}{(s+1)(3s+1)}$



3dB DOWN @ 2.4 R/S => SMALLER BANDWIDTH
=> SLOWER RESPONSE

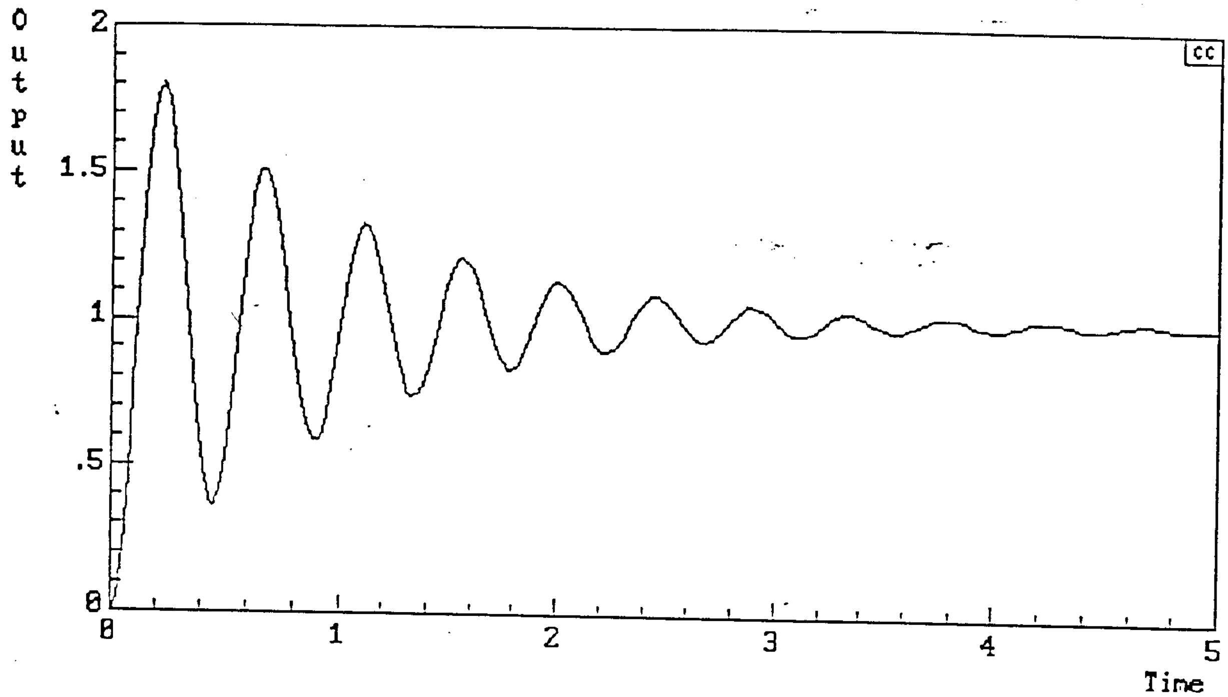


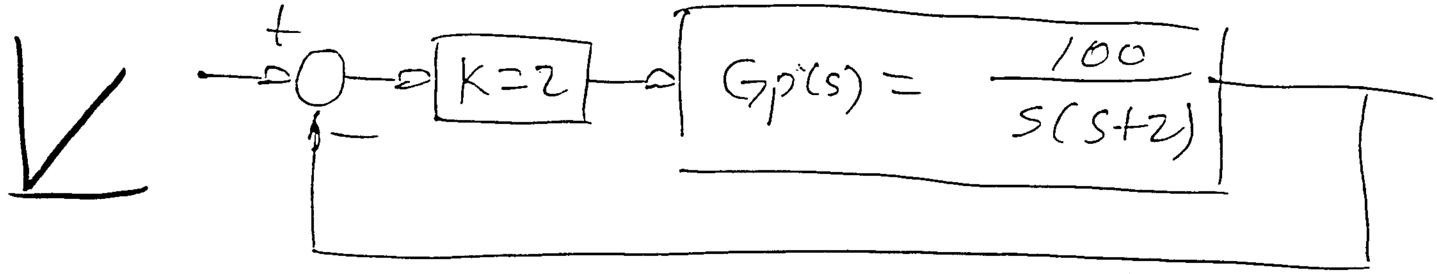
IF $K_V = 100$ THEN USING

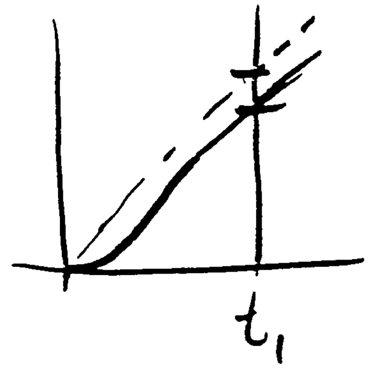
LEAD
MPENSATION

$$K_V = \lim_{s \to 0} s K G_p(s) \Rightarrow 100 = \lim_{s \to 0} s K \frac{100}{s(s+2)}$$

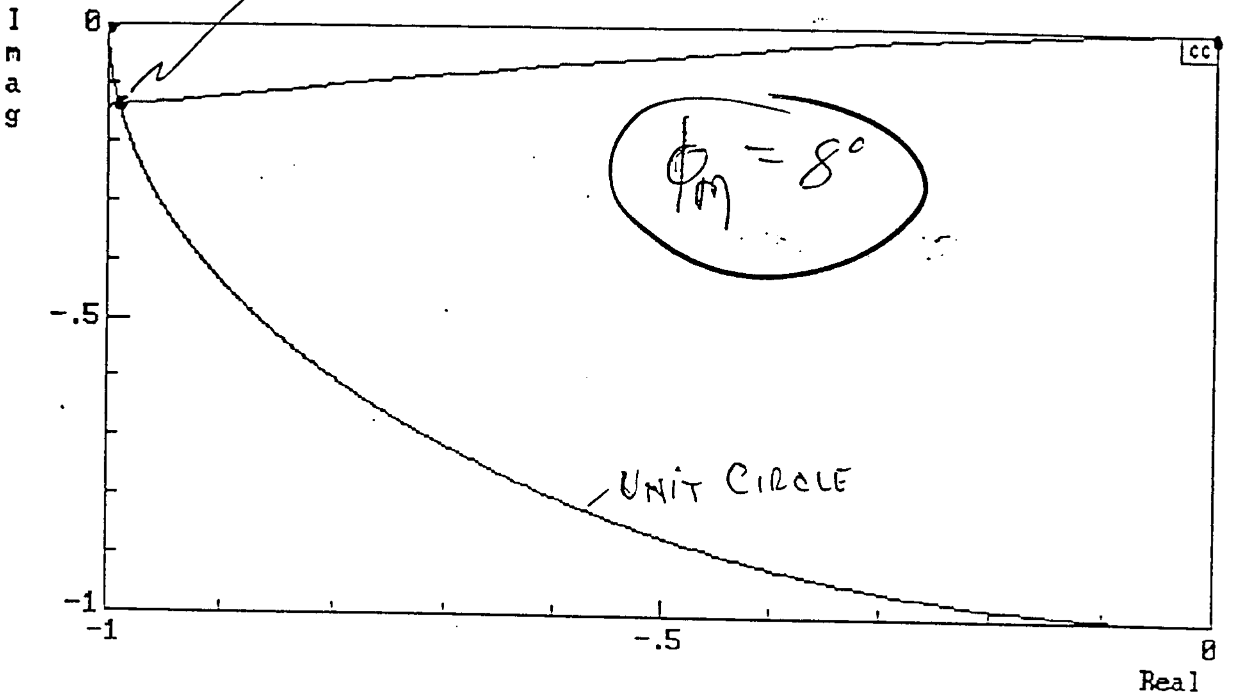
$\Rightarrow$ $\underline{K = 2}$ SATISFIES ACCURACY SPEC

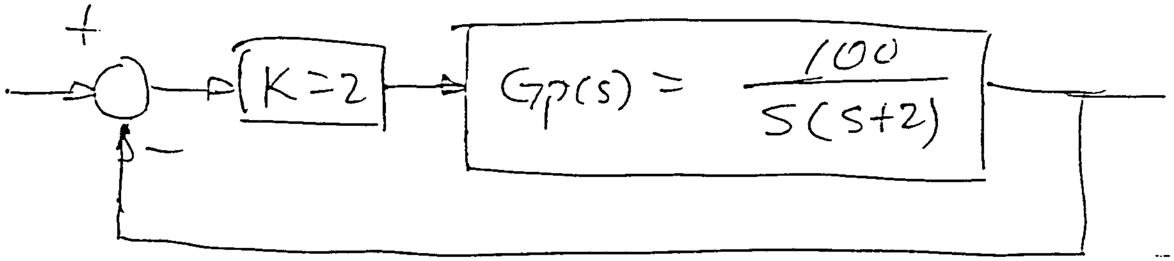




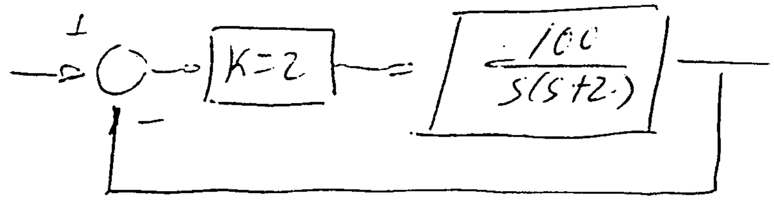


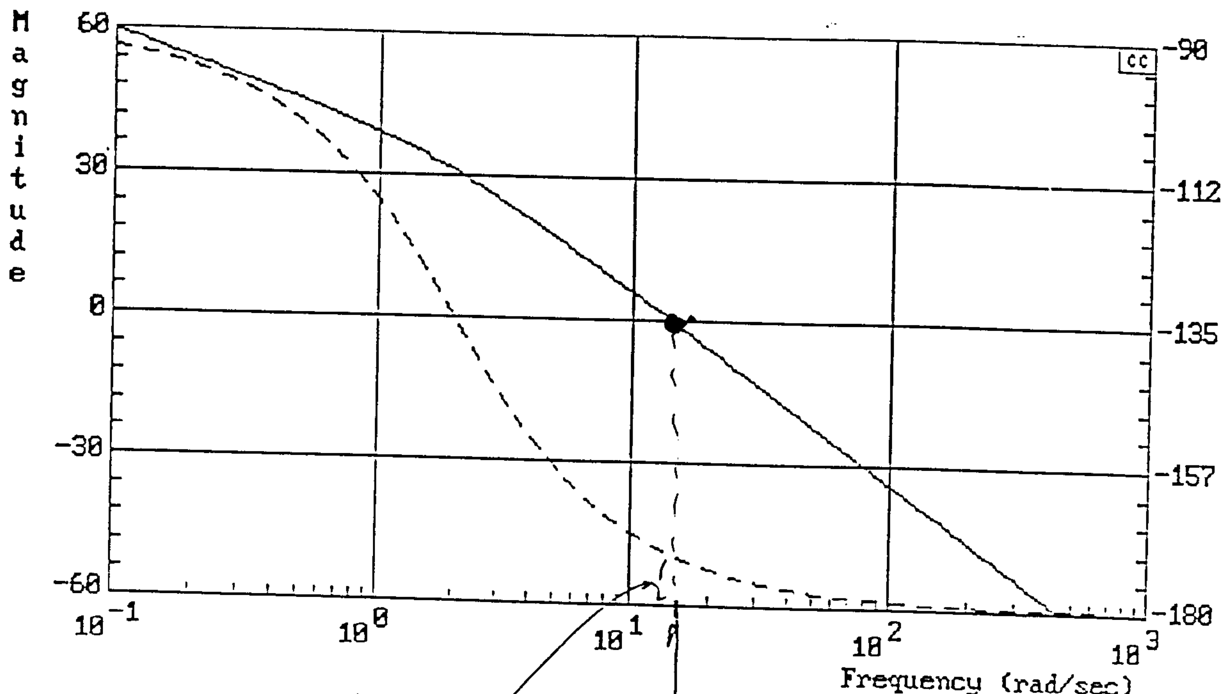
$\omega = 14.1;\ |KG_p(s)| = 1;\ \angle KG_p(s) = -172°$





LEAD COMPENSATION





$\Phi_{M\,UNCOMP} = 8^\circ$ $\quad$ $\omega_1 = 14.1$ R/S

(0 dB GAIN CROSSOVER FREQ)

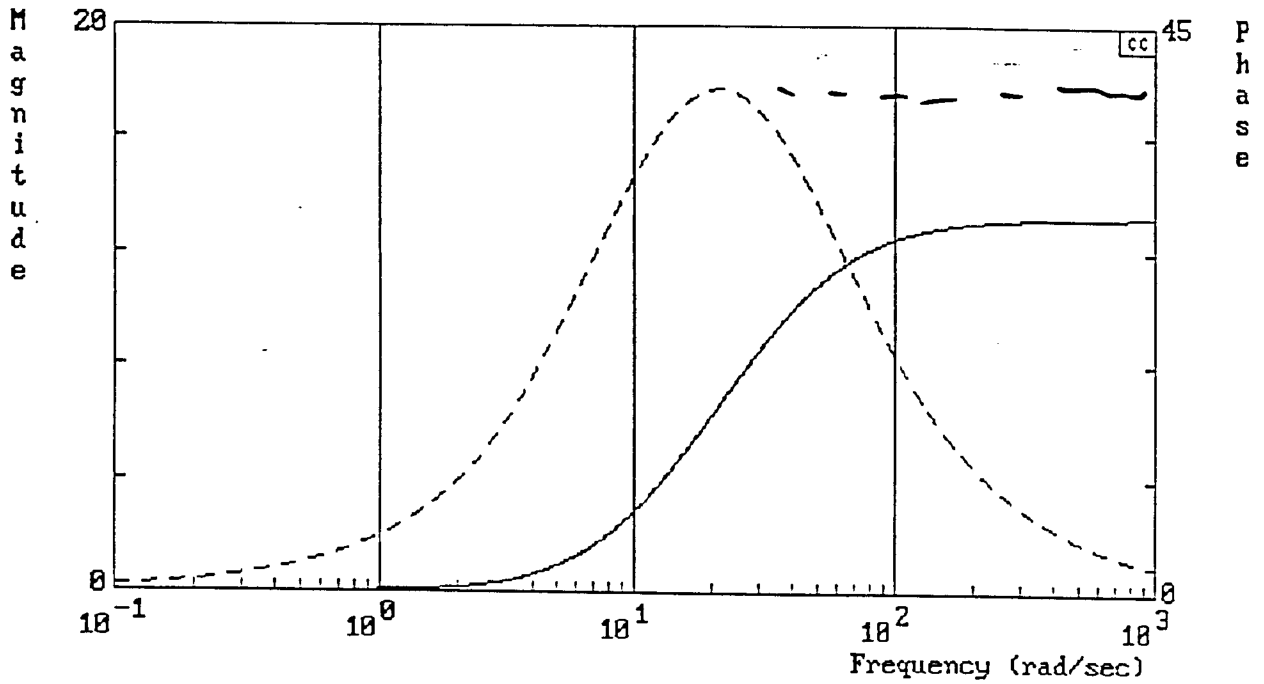
$\Phi_{M\,DESIRED} = 45^\circ$ $\quad$ MAKE $\omega_d = \omega_1 + 0.5\omega_1 = 14.1 + 0.5(14.1) = 21.15$

DESIGN THE LEAD COMP SUCH THAT THE PEAK (ϕ_{MAX}) OF THE PHASE OF THE COMPENSATOR IS USED TO ACHIEVE $\Phi_{M\,DESIRED}$; SINCE PEAK PHASE OCCURS AT A FREQ $= \sqrt{\omega_0 \omega_p}$, WE LET $\omega_d = \sqrt{\omega_0 \omega_p} \Rightarrow 21.15 = \sqrt{\omega_0 \omega_p}$

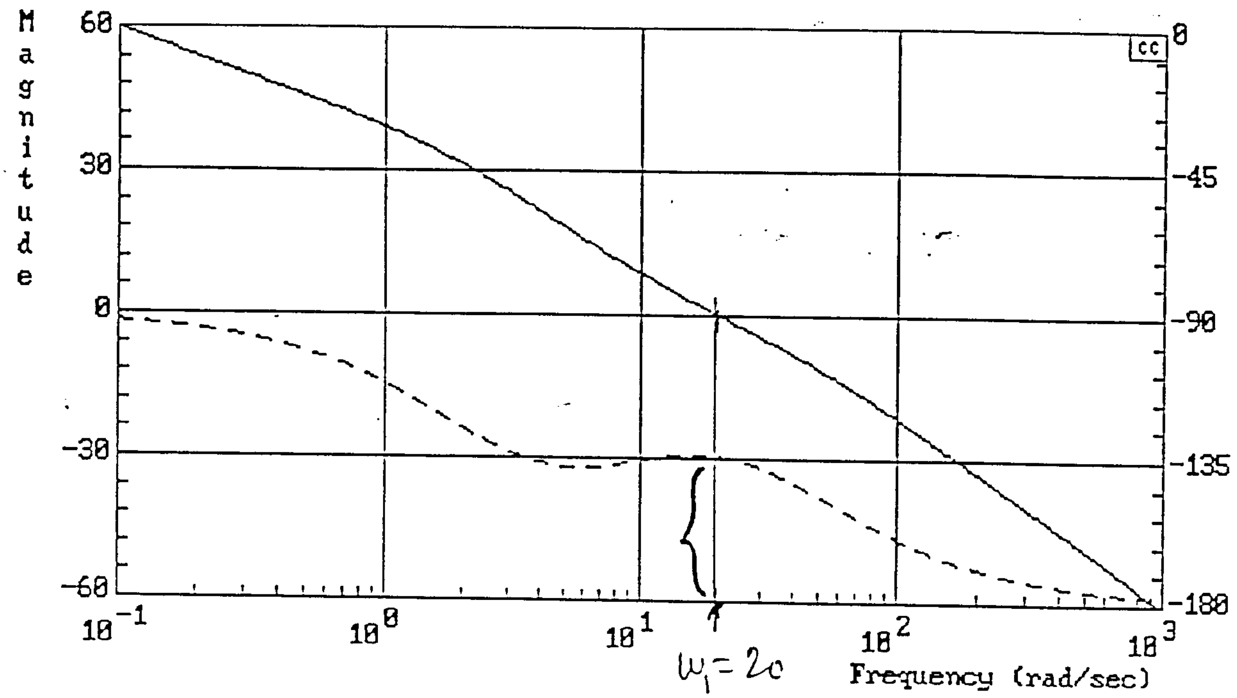
WE ALSO KNOW THAT $\frac{\omega_p}{\omega_0} = \frac{1+\sin\phi_{max}}{1-\sin\phi_{max}}$

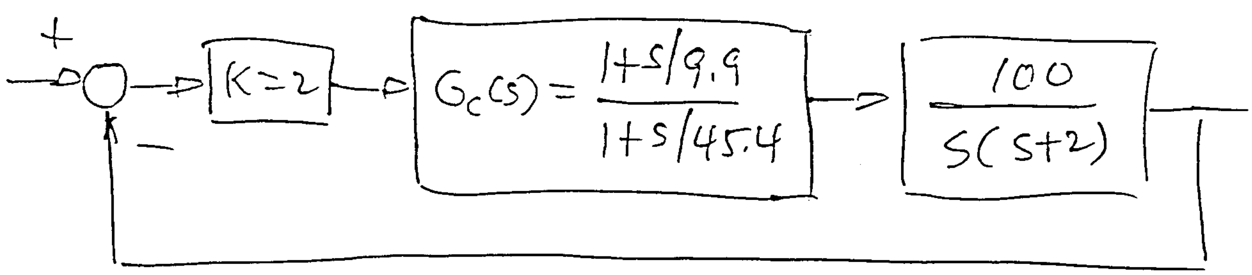
WHERE $\phi_{max} = 45 - 8 + 3^\circ = 40^\circ \Rightarrow \omega_0 = 9.9$ & $\omega_p = 45.4$

RESERVE

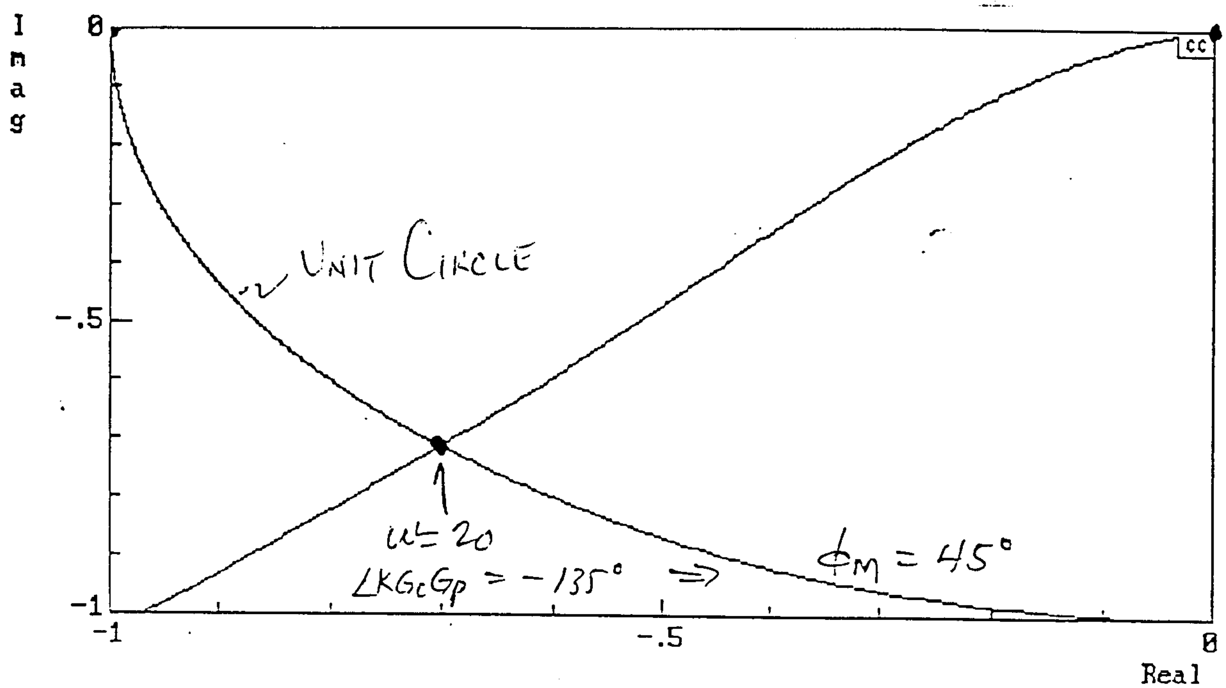


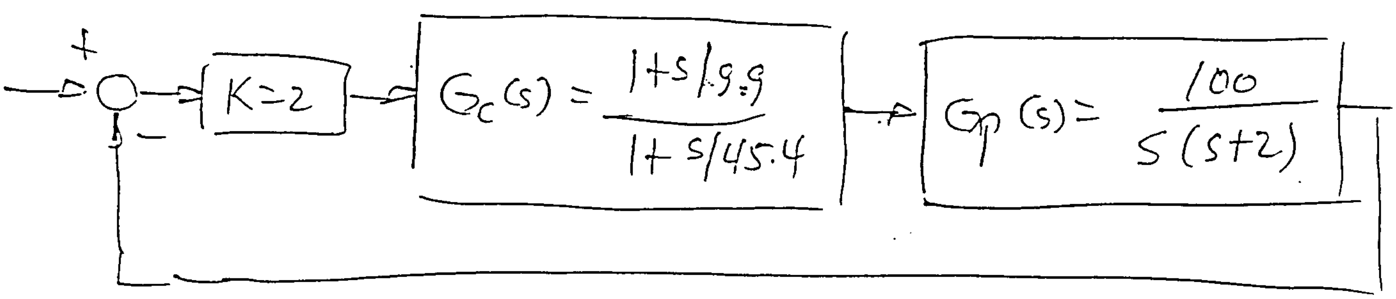
$$G_c(s) = \frac{1 + s/9.9}{1 + s/45.4}$$

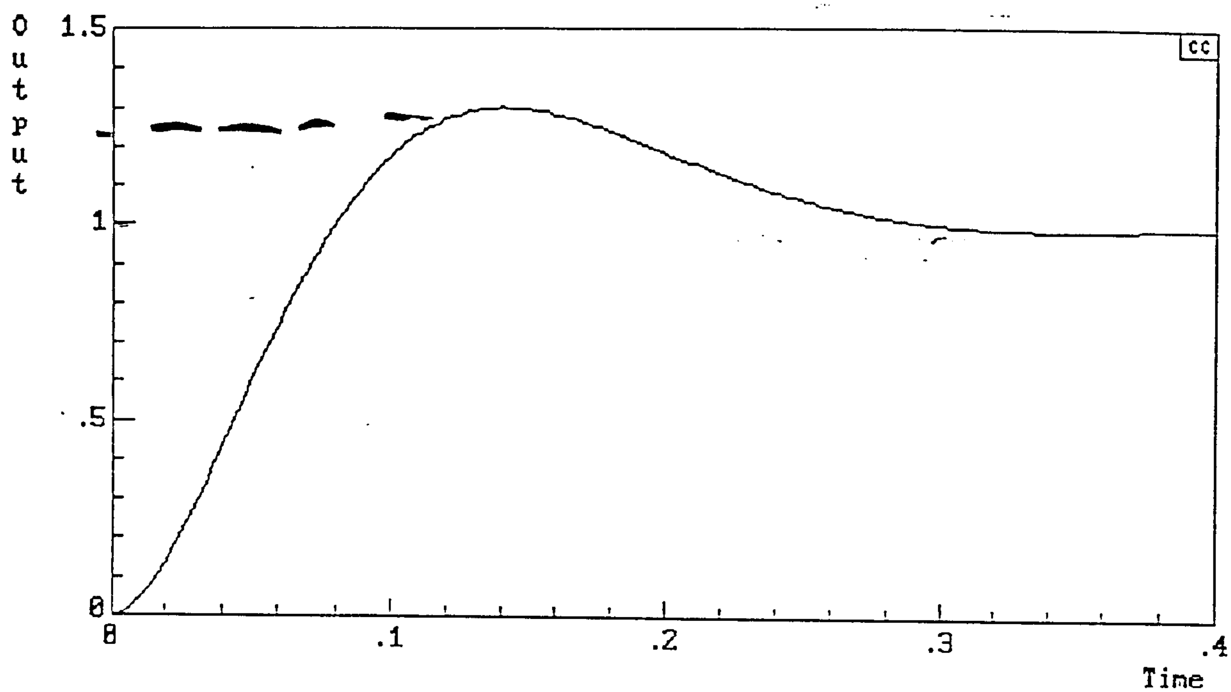


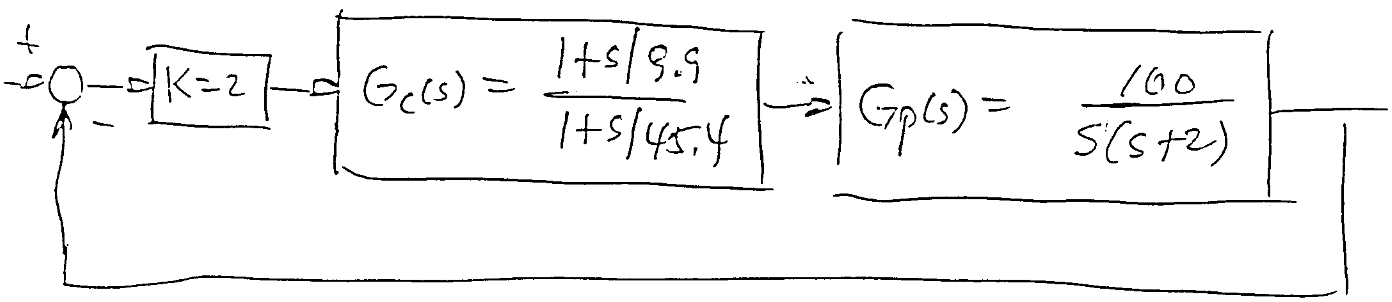


FROM ABOVE BODE PLOTS, $\phi_M = 45°$


Imag
0
-.5
-1
-1
-.5
0
Real
CC
UNIT CIRCLE
ω ≈ 20
∠KGcGp = −135° ⇒
φM = 45°


+
−
K=2
$G_c(s) = \frac{1+s/9.9}{1+s/45.4}$
$G_p(s) = \frac{100}{s(s+2)}$


Output
1.5
1
.5
0
0
.1
.2
.3
.4
Time
CC


+
−
K=2
Gc(s) = 1+s/9.9 / 1+s/45.4
Gp(s) = 100 / s(s+2)