Digital Modulation

- Continuous-wave(CW) modulation (recap):
  - A parameter of a sinusoidal carrier wave is varied continuously in accordance with the message signal.
    * Amplitude
    * Frequency
    * Phase
- Digital Modulation:
  - Pulse Modulation: Analog pulse modulation: A periodic pulse train isused as a carrier. The following parameters of the pulse are modified in accordance with the message signal. Signal is transmitted at discrete intervals of time.
    * Pulse amplitude
    * Pulse width
    * Pulse duration

- Pulse Modulation: Digital pulse modulation: Message signal represented in a form that is discrete in both amplitude and time.
  - The signal is transmitted as a sequence of coded pulses
  - No continuous wave in this form of transmission

Analog Pulse Modulation

- Pulse Amplitude Modulation(PAM)
  - Amplitudes of regularly spaced pulses varied in proportion to the corresponding sampled values of a continuous message signal.
  - Pulses can be of a rectangular form or some other appropriate shape.
  - Pulse-amplitude modulation is similar to natural sampling, where the message signal is multiplied by a periodic train of rectangular pulses.
  - In natural sampling the top of each modulated rectangular pulse varies with the message signal, whereas in PAM it is maintained flat. The PAM signal is shown in Figure 1.

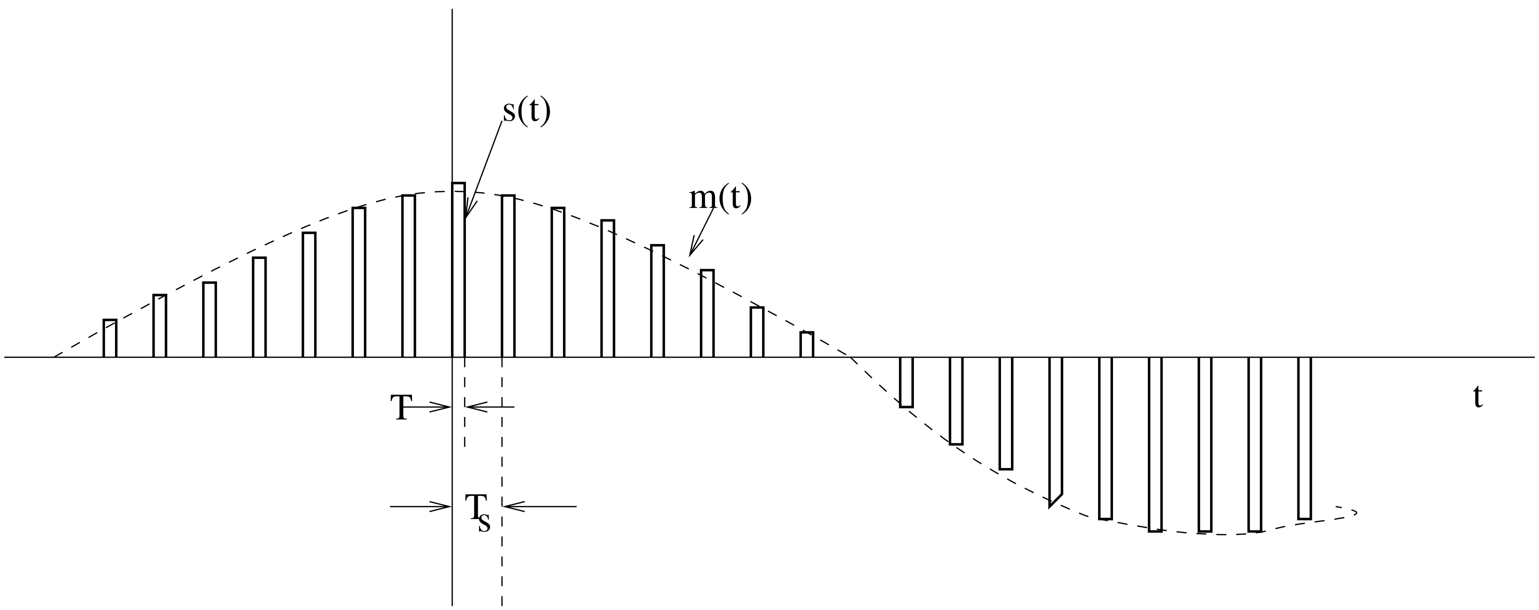


Figure 1: PAM signal

- Mathematical Analysis of PAM signals
  - Let $s(t)$ denote the sequence of the flat-top pulses generated in the manner described Figure 1. We may express the PAM signal as

$$s(t) = \sum_{n=-\infty}^{+\infty} m(nT_s)h(t - nT_s)$$

– $h(T_s)$ is a standard rectangular pulse of unit amplitude and duration T, defined as follows

$$h(t) = \begin{cases} = 1, & 0 \leq t \leq T \\ = \frac{1}{2}, & t = 0, t = T \\ = 0, \text{otherwise} \end{cases}$$

– The instantaneously sampled version of $m(t)$ is given by

$$m_\delta(t) = \sum_{n=-\infty}^{+\infty} m(nT_s)\delta(t - nT_s)$$

– Therefore, we get

$$
\begin{aligned}
m_\delta(t) * h(t) &= \int_{-\infty}^{+\infty} m_\delta(\tau) h(t-\tau)\, d\tau \\
&= \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} m(nT_s)\delta(\tau - nT_s) h(t-\tau)\, d\tau \\
&= \sum_{n=-\infty}^{+\infty} m(nT_s) \int_{-\infty}^{+\infty} \delta(\tau - nT_s) h(t-\tau)\, d\tau
\end{aligned}
$$

using the shifting property of the delta function, we obtain

$$
s(t) = m_\delta(t) * h(t) = \sum_{n=-\infty}^{+\infty} m(nT_s) h(t - nT_s)
$$

$$
\begin{aligned}
S(\omega) &= M_\delta(\omega) * H(\omega) \\
&= \frac{\omega_s}{2\pi} \sum_{k=-\infty}^{\infty} M(\omega - k\omega_s) H(\omega)
\end{aligned}
$$

– Since, we use flat top samples, $H(\omega) = Tsinc\left(\omega \frac{T}{2}\right) e^{-j\omega \frac{T}{2}}$. This results in distortion and a delay of $\frac{T}{2}$. To correct this the magnitude of the equalizer is chosen as $\dfrac{1}{Tsinc\left(\omega \frac{T}{2}\right)}$.

– The message signal $m(t)$ can be recovered from the PAM signal $s(t)$ as shown in Figure 2.

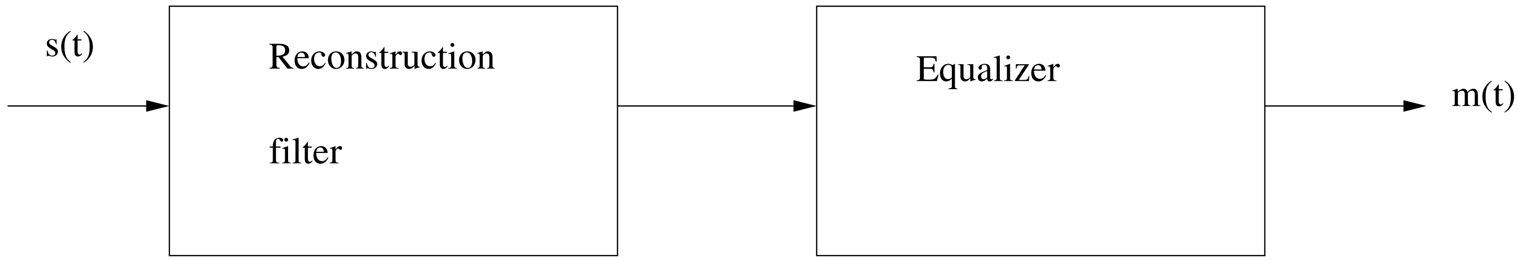


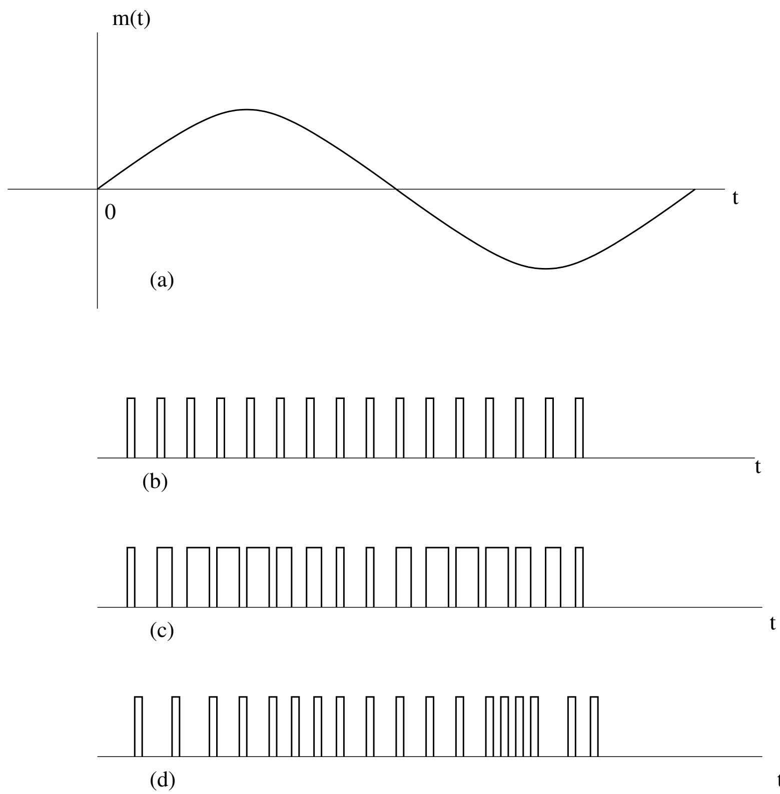
Figure 2: recovering message signal from PAM signal

- Other forms of Pulse Modulation

  1. *Pulse-duration modulation*(PDM), also referred to as *Pulse-width modulation*, where samples of the message signal are used to vary the duration of the individual pulses in the carrier.
  2. *Pulse-position modulation*(PPM), where the position of a pulse relative to its unmodulated time of occurence is varied in accordance with the message signal. It is similar to FM.

The other two types of modulation schemes are shown in

Figure 3.



(a) modulating wave
(b) Pulse carrier
(c) PDM wave
(d) PPM wave

Figure 3: other pulse modulation schemes

Pulse Digital Modulation

Pulse Code Modulation

- Discretisation of both time and amplitude.
- Discretisation of amplitude is called *quantisation.*
  - Quantisation involves conversion of an analog signal amplitude to discrete amplitude.
  - Quantisation process is memoryless and instantaneous.
  - A quantiser can be of uniform or non uniform.
    - Uniform quantiser: the representation levels are uniformly spaced
    - Nonuniform quantiser: the representation levels are non-uniformly spaced.

Uniform Quantisers

- Quantiser type: The quantiser characteristic can be of midtread or midrise quantizer. These two types are shown in Figure 4.

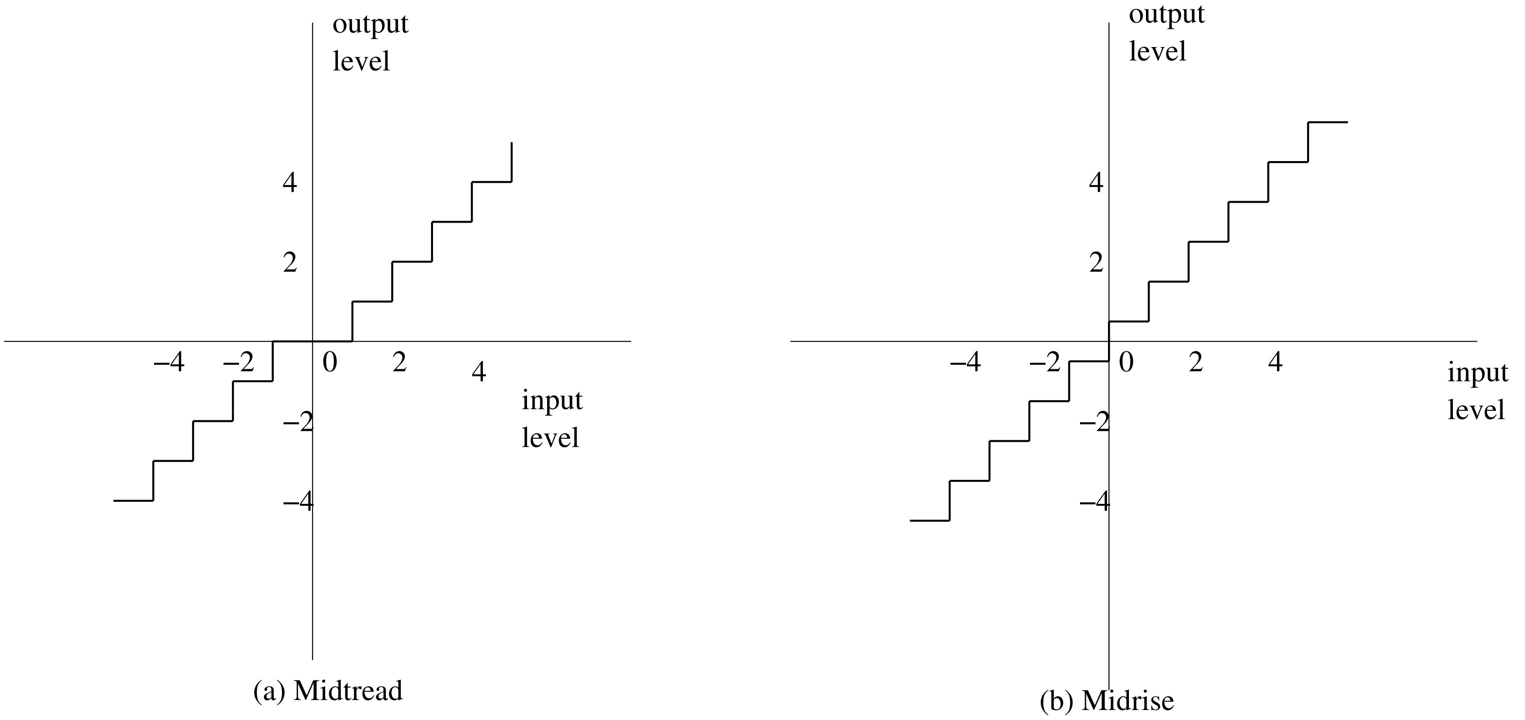


Figure 4: Different types of uniform quantisers

- Quantization Noise: Quantisation introduces an error defined as the difference between the input signal m and the output signal v. The error is called *Quantisation Noise.*
- We may therefore write

$$Q = M - V$$

- Analysis of error in uniform quantisation:
  - The input M has zero mean, and the quantiser assumed to be symmetric $\implies$ Quantiser output V and therefore the quantization error Q also will have a zero mean.
  - Consider that input m has continuous amplitude in the range $(-m_{max}, m_{max})$. Assuming a uniform quantiser of midrise type, we get the step size of the quantiser given by

$$\Delta = \frac{2m_{max}}{L}$$

where, L is the total number of representation levels.

- The quantization error Q will have its sample values bounded by $\frac{-\Delta}{2} \leq q \leq \frac{\Delta}{2}$.
- For small Δ, assume that the quantisation error Q is a uniformly distributed random variable. Thus,
- The probability density function of quantisation error Q can be given like this

$$f_Q(q) = \begin{cases} \frac{1}{\Delta}, & \frac{-\Delta}{2} \leq q \leq \frac{\Delta}{2} \\ 0, & \text{otherwise} \end{cases}$$

- Since we assume mean of Q is zero, the variance is same as the mean square value.

$$
\begin{aligned}
\sigma_Q^2 &= E[Q^2] \\
&= \int_{-\frac{\Delta}{2}}^{+\frac{\Delta}{2}} q^2 f_Q(q)\, dq \\
&= \frac{1}{\Delta} \int_{-\frac{\Delta}{2}}^{+\frac{\Delta}{2}} q^2\, dq \\
&= \frac{\Delta^2}{12}
\end{aligned}
$$

- Typically, the L-ary number k, denotes the kth representation level of the quantiser,
- It is transmitted to the receiver in binary form.
- Let R denote the number of bits per sample used in the construction of the binary code. We may then write

$$L = 2^R$$

$$R = log_2(L)$$

Hence, we get

$$\Delta = \frac{2m_{max}}{2^R}$$

also,

$$\sigma_Q^2 = \frac{1}{3} m_{max}^2 2^{-2R}$$

– Let P denote the average power of the message signal $m(t)$. We may then express the output signal to noise ratio of a uniform quantiser as

$$(SNR)_O = \frac{P}{\sigma_Q^2}$$

$$= \frac{3P}{m_{max}^2} . 2^{2R}$$

- Example: Sinusoidal Modulating Signal

- Let the amplitude be A_m.
- Power of the signal, $P = \dfrac{A_m^2}{2}$.
- Range of the singal : $-A_m, A_m$.
- Hence, $\sigma_Q^2 = \frac{1}{3}A_m^2 2^{-2R}$.
- $\implies (SNR)_O = \dfrac{\dfrac{A_m^2}{2}}{\frac{1}{3}A_m^2 2^{-2R}}$
- Thus, $(SNR)_O = \frac{3}{2}2^{2R}$ or
- $10\log_{10}(SNR)_O = 1.8 + 6RdB$.

* The major issue in using a uniform quantiser is that no effort is made to reduce quantisation power for a presumed set of levels.

Non Uniform Quantisers

The main advantages of using non-uniform quantizer are:
colorblack

1. Protection of weak passages over loud passages.
2. Enable uniform precision over the entire range of the voice signal.
3. Fewer steps are required in comparison with uniform quantizer.

A nonuniform quantiser is equivalent to passing the baseband signal through a compressor and then applying the compressed signal to a uniform quantizer. A particular form of compression law that is used in practice is μ - law, which is defined as follows

$$|v| = \frac{\log(1+\mu|m|)}{\log(1+\mu)}$$

where, m and v are the normalized input and output voltages and

μ is a positive constant.

Another compression law that is used in practice is the so called A - law defined by

$$|v| = \begin{cases} \frac{A|m|}{1+\log A}, & 0 \leq |m| \leq \frac{1}{A} \\ \frac{1+\log(A|m|)}{1+\log A}, & \frac{1}{A} \leq |m| \leq 1 \end{cases}$$

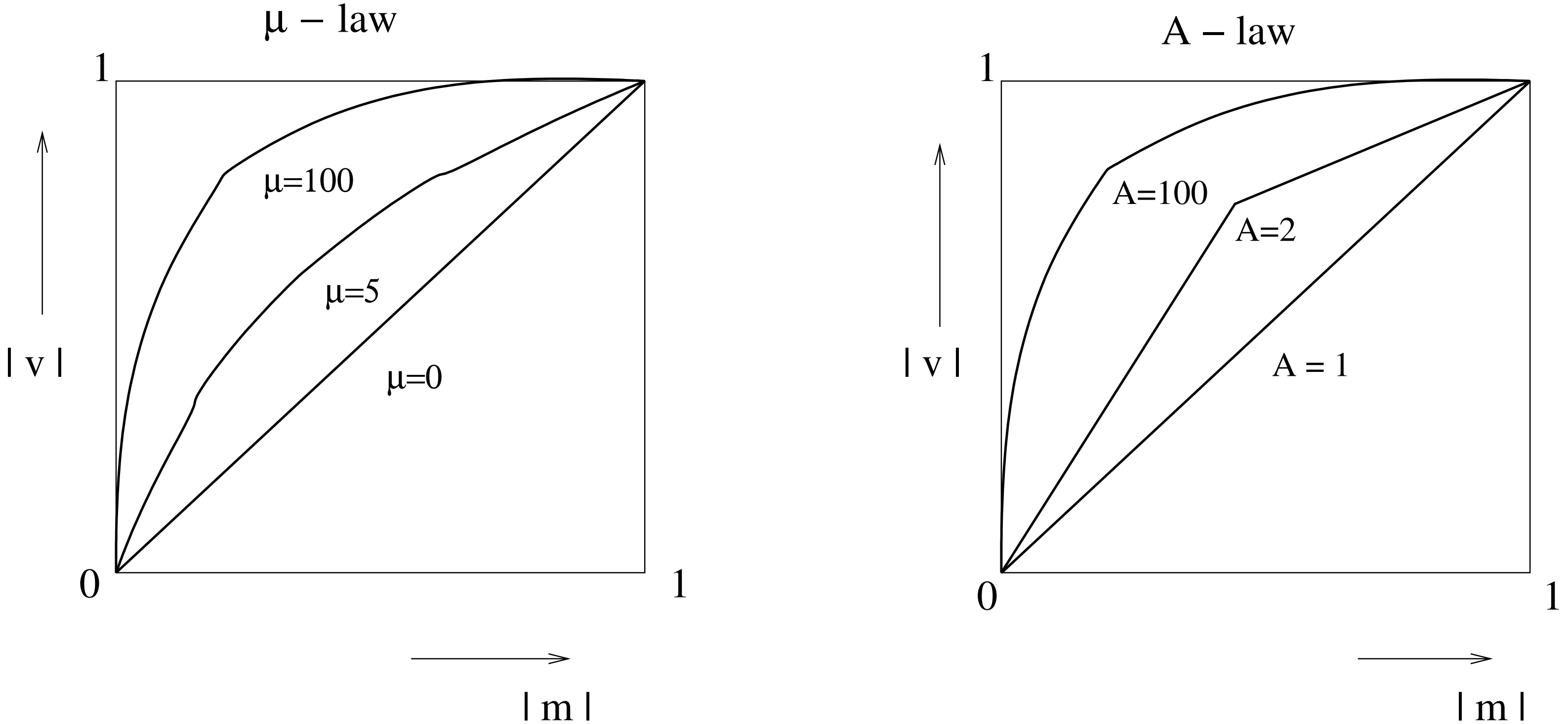


Figure 5: Compressions in non-uniform quantization

Differential Pulse code Modulation(DPCM)

- In DPCM, we transmit not the present sample $m[k]$, but $d[k]$ which is the difference between $m[k]$ and its predicted value $\hat{m}[k]$.
- At the receiver, we generate $\hat{m}[k]$ from the past sample values to which the received $d[k]$ is added to generate $m[k]$. Figure 6 shows the DPCM transmitter.

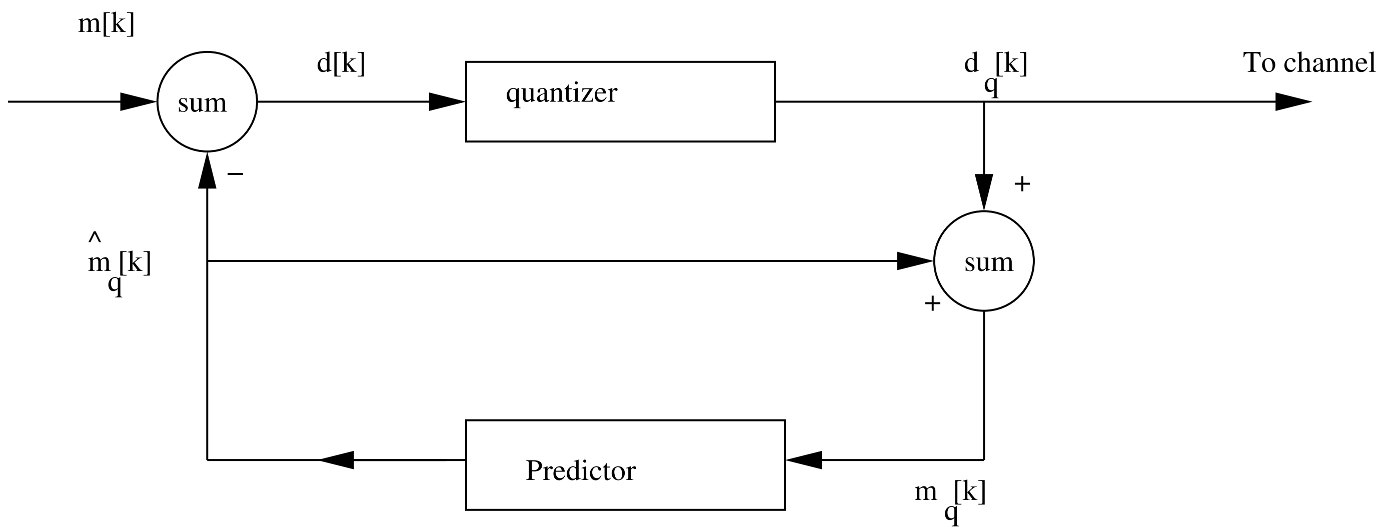


Figure 6: DPCM Transmitter

- Analysis of DPCM: If we take the quantised version, the predicted value as $\hat{m}_q[k]$ of $m_q[k]$. The difference

$$d[k] = m[k] - \hat{m}_q[k]$$

which is quantized to yield

$$d_q[k] = d[k] + q[k]$$

The predictor input $m_q[k]$ is

$$
\begin{aligned}
\hat{m}_q[k] = \hat{m}_q[k] + d_q[k] & \\
& = m[k] - d[k] + d_q[k] \\
& = m[k] + q[k]
\end{aligned}
$$

- The quantized signal $d_q[k]$ is now transmitted over the channel. At the receiver, $\hat{m}[k]$ is predicted from the previous samples and $d[k]$ is added to it to get $m[k]$. A DPCM receiver is shown in Figure 7

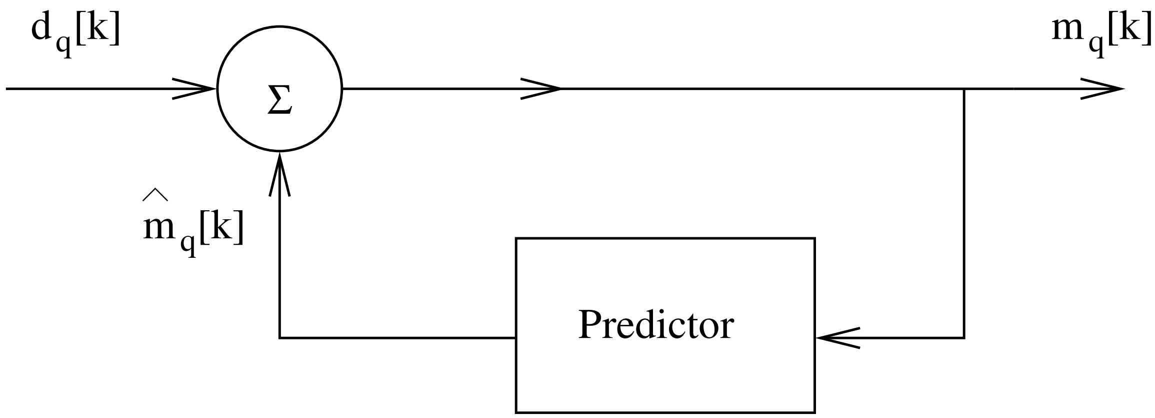


Figure 7: DPCM Receiver

Delta Modulation

- Delta Modulation uses a first order predictor.
- It is a one bit DPCM.
- DM quantizer uses only two levels($L = 2$)
- The signal is oversampled(atleast 4 times the Nyquist rate) to ensure better correlation among the adjacent samples.

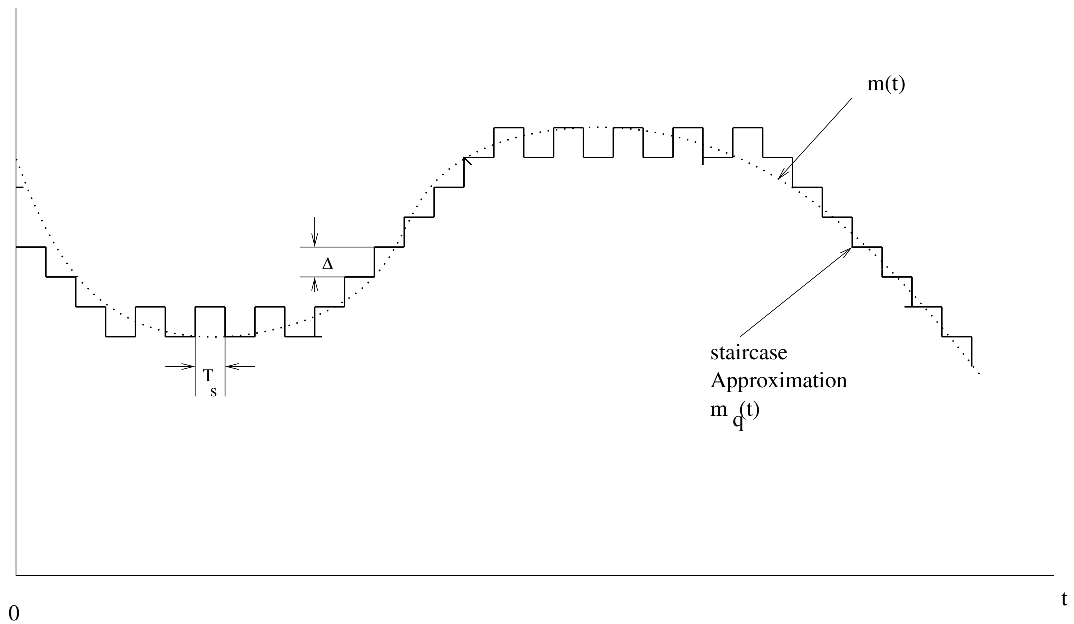


Figure 8: Delta Modulation

- DM provides a staircase approximation to the oversampled version of the message signal.
- The difference between the input and the approximation is quantised into only two levels, namely, $\pm\Delta$, corresponding to

positive and negative differences.

- For this approximation to work, the signal should *not change rapidly.*
- Analysis of Delta Modulation:
  - Let $m(t)$ denote the input signal, and $m_q(t)$ denote its staircase approximation. For convenience of presentation, we adopt the following notation that is commonly used in the digital signal processing literature:

$$m[n] = m(nT_s), n = 0, \pm 1, \pm 2, \cdots$$

    T_s is the sampling period and $m(nT_s)$ is a sample of the signal $m(t)$ taken at time $t = nT_s$, and likewise for the samples of other continuous-time signals.
  - The basic principles of delta modulation is the following set

of discrete-time relations:

$$e[n] = m[n] - m_q[n-1]$$
$$e_q = \Delta sgn(e[n])$$
$$m_q[n] = m_q[n-1] + e_q[n]$$

- $e[n]$ is an error signal representing the difference between the present sample $m[n]$ of the input signal and the latest approximation $m_q[n-1]$ to it.
- $e_q[n]$ is the quantized version of $e[n]$,
- $sgn(.)$ is the signum function.
- The quantiser output $m_q[n]$ is coded to produce the DM signal.

- The transmitter and receiver of the DM is shown in Figure

9.

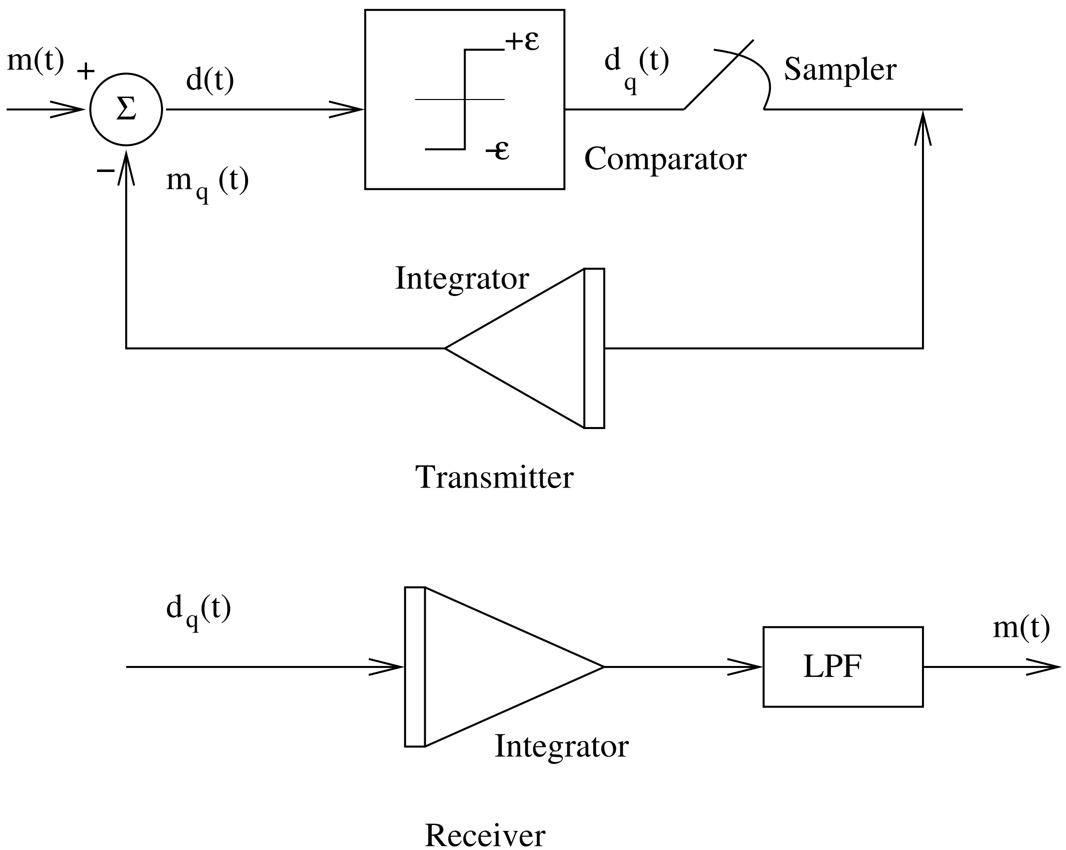


Figure 9: Transmitter and Receiver for Delta Modulation