Mid-Term Report: Finsearch

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1 About Monte Carlo Model

A certain event can have infinitely many possible outcomes. Monte Carlo simulation is used to estimate the possible outcomes, through generation of random numbers. It basically generates a series of random variables, having properties similar to the factors the problem is trying to simulate.

2 Steps in Monte Carlo Simulation

- Determine the variables of the mathematical problem that have uncertainty.
- Assume a suitable random distribution for each variable (for example Normal distribution)
- Generate iterations of the result with different values of the variables.
- Repeat such iterations a large number of times to get a distribution of the final result.

3 Preliminary example of the idea of Monte Carlo Method

A very simple example is that of rolling two die. There are 36 possible outcomes, out of which 6 are such that the numbers on the faces add upto 7. So, the probability that the sum on faces is 7 should be $\frac{1}{6}$. Suppose we roll the die 30 times, then it will be highly unlikely for getting the sum 7 five times. But as we increase the number of rolls, the probability will become more and more closer to $\frac{1}{6}$. So, each roll represents an iteration of the model, and as the number of iterations increase, the actual result becomes less approximate.

4 Some applications of Monte Carlo Simulation

1. Finding numerical solutions of differential equations
2. Fluid Dynamics: To simulate fluid flows in different conditions
3. Material Science: To estimate structural and physical properties of materials
4. Risk analysis in markets
5. Option Pricing

5 Why Monte Carlo Model for option pricing?

Monte Carlo Model is used for option pricing due to various reasons.

- As seen earlier in the Black-Scholes model, any options' price depends on the underlying assets' price, the strike rate of the option, volatility, interest rates and time to expiration.
- The Monte Carlo simulation is a good model while working with multiple variables. It generated random scenarios for each variable, evaluating their combined effect on the options value.
- Also, as seen in the Black-Scholes model, the dependence of the option price on the above variable is highly non-linear. Monte Carlo model can handle these non-linearities.
- Monte Carlo model also allows the use of risk-neutral probabilities, which are derived from the market's expectation of future events.

Considering the above points, the Monte Carlo simulation model is a fairly good model for option pricing.

6 Pricing an option using Monte Carlo Simulation

Let us consider the Black-Scholes-Merton formula:
$dS_t = rS_t dt + \sigma S_t dZ_t$ where,
$S(t)$ = stock price at time t
r = Risk free rate of interest
σ = Volatility
$Z(t)$ = Brownian motion

The above equation is a Stochastic differential equation, which can be solved by using Euler's Discretization Scheme.

Euler's Discretization Scheme:

Euler's Discretization Method is basically a method to approximate solutions of differential equations which directly cannot be solved. It can b demonstrated on a trivial example as follows:
Consider the differential equation $\frac{dy}{dx} = y, y(0) = 1$.
Now, if from x=0, we take steps of $\Delta x = 0.5$, and find out corresponding values of y using the instantaneous values of $\frac{\Delta y}{\Delta x}$, we get the following table:

x	y	$\frac{\Delta y}{\Delta x} = y$
0	1	1
1	2	2
2	4	4
3	8	8

Using the points acquired above, we get an approximate graph of the solution $y = e^x$. If we increase the number of points and decrease the value of Δx, then a very accurate graph will be obtained, hence giving us the solution of the given differential equation.

So, using a similar approach to the Black-Scholes differential equation, we get the solution to be:
$S_t = S_{t-\Delta t} \cdot exp((r - \frac{\sigma^2}{2})\Delta t + \sigma\sqrt{\Delta t} z_t)$, which in the logarithmic form is useful.
$logS_t = logS_{t-\Delta t} + ((r - \frac{\sigma^2}{2})\Delta t + \sigma\sqrt{\Delta t} z_t)$

7 Code for Monte Carlo Model

https://github.com/DharaSachinShah/Monte-Carlo-Model.git

There are two codes, one which demonstrates the Monte Carlo simulation in general, and other one which shows how an option is priced using the method.

8 Binomial Model for Option Pricing

This model is used for option pricing and is mathematically very simple and uses binomial tree method to predict the option values.

This model assumes a risk-free interest rate.

Following is a demonstration of the model on a hypothetical stock: Current price of the stock is 100 Rs. Its future price in a tenure of 2 years can either go up by 20% or down by 10%. Further let us assume that we are getting a risk-free interest of 8%. Say that the strike price chosen is 110 Rs.

Next, let us take 2 scenarios:

1. After the 1st year, the price went up by 20%. So the resultant price of the stock becomes 120 Rs.

2. After the 1st year, the price went down by 10%. So the resultant price of the stock becomes 90 Rs.

Now further, within 1), there can be 2 possibilities:

1. After the end of the 2nd year, the price again went up by 20%. So the resultant price of the stock becomes 144 Rs.

2. After the end of the 2nd year, the price went down by 10%. So the resultant price of the stock becomes 108 Rs.

Similarly, within 2), there can be 2 possibilities:

1. After the end of the 2nd year, the price went up by 20%. So the resultant price of the stock becomes 108 Rs.

2. After the end of the 2nd year, the price went down further by 10%. So the resultant price of the stock becomes 81 Rs.

Hence, for only one case, we would be able to call the option with a profit of $(144 - 110) = 34$ Rs.

So let us consider that scenario where we are earning a profit of 34 Rs. Here, after the 2nd year ends, we are getting a return of 8% on 120 Rs.

Consider the probability of the stock price to increase by 20% after the 2nd year to be P.

Hence, the following equation follows:

$$120 \times 1.08 = 120 \times 1.2 \times P + 120 \times 0.9 \times (1 - P)$$

Solving this, we get $P = 0.6$.

Now, the value of the option after the end of 2 years will be $34 \times 0.6 = 20.4$ Rs.

Similarly, the value of the option after the end of the 1st year will be $20.4 \times 0.6 = 12.24$ Rs.

Now, the value of the option at the beginning of the 1st year will be $12.24/1.08 = 11.33$ Rs (After removing the risk-free interest).

So, we finally conclude that the value of the option at the beginning is 11.33 Rs.

9 Qualitative difference between Binomial Model and Black Scholes Model

Binomial Model	**Black Scholes Model**
It is a statistical model based on tree.	It is the solution of a differential equation.
Discrete time model	Continuous time model
Both American and European option prices can be determined	Only European option prices can be determined
Can consider dividend paying stocks and taxes	Assumes no dividend or taxes.