Option Pricing: Finsearch

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1 Black Scholes Model

Black-Scholes formula is used to calculate the price of an option, given certain parameters. This model is only used to price European options, that is the option can only be exercised on their expiration date.
Following are the assumptions we make while using BSM:

1. No dividends are paid out during the life of the option.
2. No transaction costs in buying the option.
3. The risk free interest rate and volatility of the underlying asset are known and are constant.
4. Return of the asset are normally distributed. This assumption is made as a large number of random variables follow the normal distribution, and the Central Limit Theorem provides a theoretical basis for this assumption.

The parameters used in the Black Scholes Model are as follows:

- C = Price of option
- S = Current stock price
- K = Strike price
- r = Risk-free interest rate
- t = Time to expiration
- N = Normal distribution
- σ = Standard deviation of log returns (volatility)

The formula is as follows:
For call option:
$C = S \cdot N(d_1) - Ke^{-rt} \cdot N(d_2)$

For put option:
$C = Ke^{-rt} \cdot N(-d_2) - S \cdot N(d_1)$, where

$$d_1 = \frac{ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$
$$d_2 = \frac{ln(\frac{S}{K}) + (r - \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

Following are the reasons of BSM to follow lognormal distribution:

- Assumes that the price of asset follows geometric Brownian motion, where the logarithmic results are normally distributed.
- Fat tails: Extreme events might occur in more frequently than predicted by a normal distribution. Lognormal distribution accounts for fat tails in financial markets

2 Asian and American Options

Asian Options

- An Asian option is an option type where the payoff depends on the average price of the underlying asset over a certain period of time as opposed to standard options, where the payoff depends on the price of the underlying asset at a specific point in time.
- Asian options have payoff profiles based on the average price of the security over the life of the option. Average price calls and puts pay off the difference between the average stock price and the strike price
- The average price will be much less than the actual price.
- All other things being equal, Asian options are cheaper than regular options.

American Options

- An American option is a derivative contract that can be redeemed or exercised at any moment throughout the Option's life or at the Option's maturity date. This means that the option holder or investors can exercise the Option at any moment before or after the expiration date. American options are typically valued more than their European counterparts, which can only be exercised at maturity.
- Types: Call option: An investor can exercise the American call option at any time up until the maturity date. This means that the investor must choose the most advantageous moment to exercise the Option.
- Put option: A put option in the United States is the polar opposite of a call option. Holders of any option, however, have the choice to exercise it before or on the expiration date. An investor who acquires a put option does not have to wait until the Option expires to exercise it, as is the case with a European option.
- It has more flexibility, and they are usually worth more and command a higher premium.
- They are usually traded on exchanges.