

PYQ 2007, 2009

If d and e are points on sides AB and AC respectively of a $\triangle abc$ such that $DE \parallel BC$ and $BD = CE$. Prove triangle abc is isosceles.

NCERT EXEMPLAR

Question

Legs (sides other than the hypotenuse) of a right triangle are of lengths 16 cm and 8 cm. Find the length of the side of the largest square that can be inscribed in the triangle.

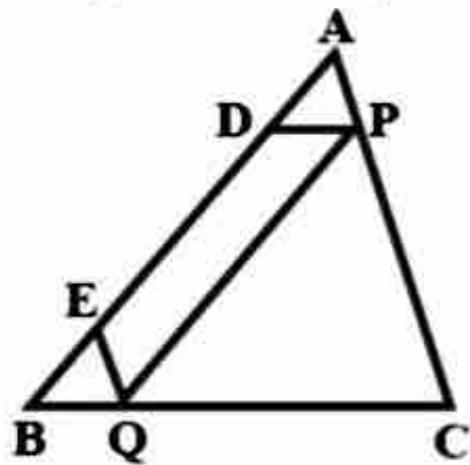
PYQ 2009

Two triangles, $\triangle ABC$ and $\triangle DBC$, lie on the same side of the base BC. From a point P on BC, $PQ \parallel AB$ and $PR \parallel BD$ are drawn. They intersect AC at Q and DC at R. Prove that $QR \parallel AD$.

PYQ 2010, 2013, 2014

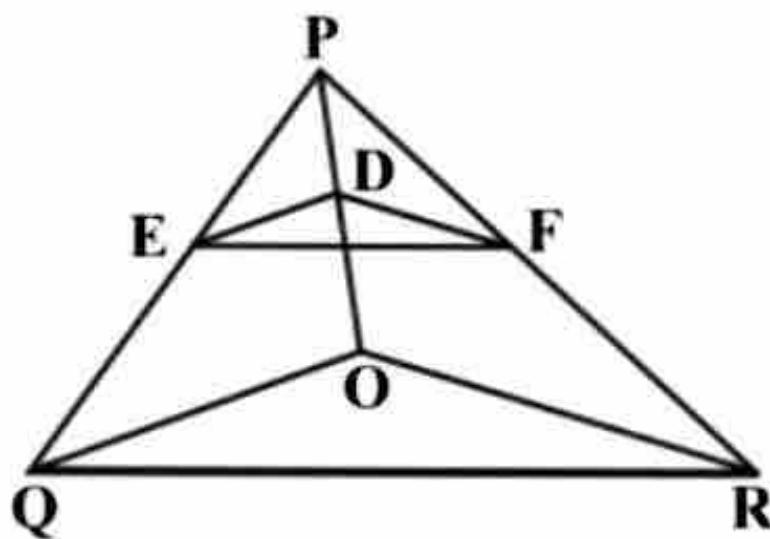
Question

Let ABC be a triangle and D and E be two points on side AB such that $AD = BE$. If $DP \parallel BC$ and $EQ \parallel AC$, then prove that $PQ \parallel AB$.



NCERT, PYQ 2008, 2014

In the following figure, $DE \parallel OQ$ and $DF \parallel OR$, show that $EF \parallel QR$.



NCERT, PYQ 2005

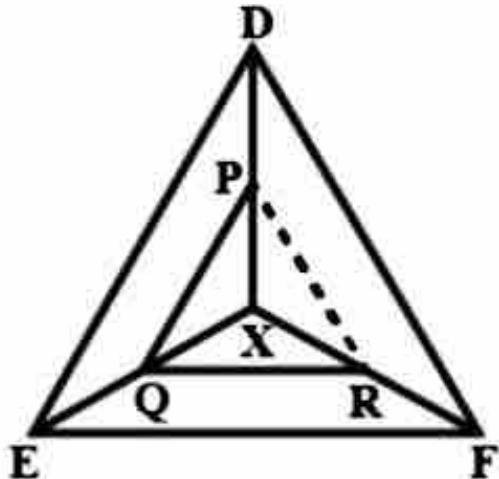
Question

The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

NCERT, PYQ 2002

Question

Any point X inside $\triangle DEF$ is joined to its vertices. From a point P on DX, PQ is drawn parallel to DE meetingXE at Q and QR is drawn parallel to EF meeting XF at R. Prove that PR \parallel DF.



NCERT, PYQ 2010

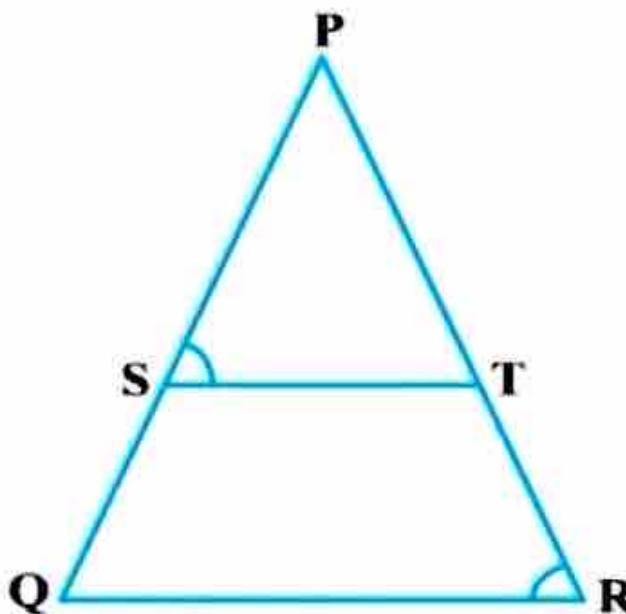
Question

In A, B and C are points on OP, OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$.

NCERT, PYQ 2020 NCERT EXEMPLAR

Question

$\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that $\triangle PQR$ is an isosceles triangle.



**NCERT, NCERT EXEMPLAR
PYQ 2002, 2003, 2005, 2006
2007, 2008, 2009, 2010
2019**

Question

State and prove Basic Proportionality theorem.

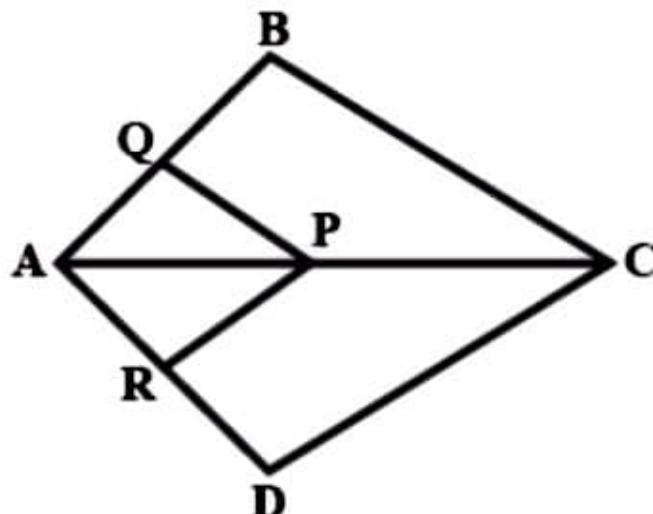
PYQ 2006, 2010

Question

In the given figure, If $PQ \parallel BC$ and $PR \parallel CD$. Prove that

$$(i) \frac{AR}{AD} = \frac{AQ}{AB}$$

$$(ii) \frac{QB}{AQ} = \frac{DR}{AR}$$



NCERT PYQ 2009, 2012, 2017

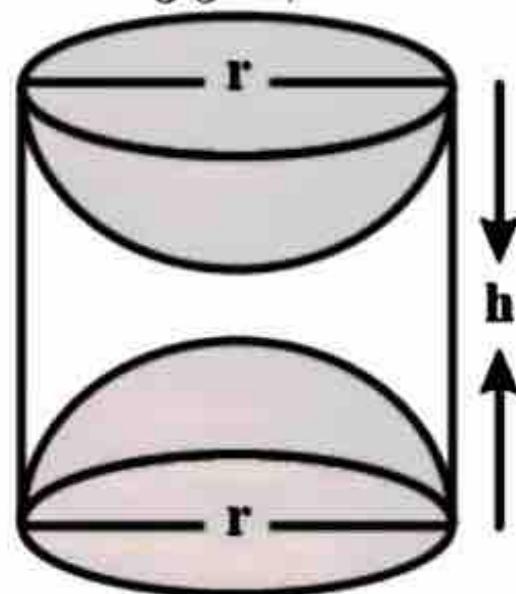
Question

From a solid circular cylinder with height 10cm and radius of the base 6cm, a right circular cone of the same height and same base is removed. Find the volume of the remaining solid. Also, find the whole surface area.

NCERT, PYQ 2014, 2018

Question

A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in figures. If the height of the cylinder is 10cm and its base is of radius 3.5cm, find the total surface area of the article.



NCERT, PYQ 2017

Question

A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold Pens. The dimensions of the cuboid are 15cm by 10cm by 3.5cm . The radius of each of the depression is 0.5cm and the depth is 1.4cm .Find the volume of the wood in the entire stand,correct to 2 decimal places.

NCERT PYQ 2010, 2014

Question

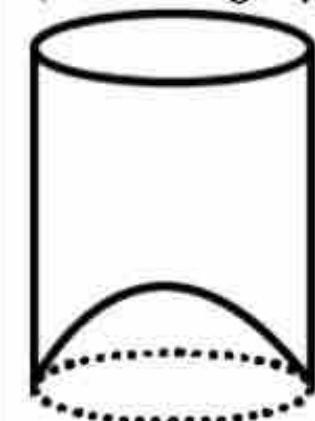
A hemispherical depression is cut out from one face of the cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

NCERT, PYQ 2009

Question

A juice seller was serving his customers using glass. The inner diameter of the cylindrical glass was 5 cm , but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of the glass was 10 cm, find what the apparent capacity of the glass was and what the actual capacity was

(Use $\pi = 3.14$)



NCERT, PYQ 2009, 2016, 2019

In Figure 3, a decorative block is shown which is made of two solids, a cube, and a hemisphere. The base of the block is a cube with an edge 6 cm and the hemisphere fixed on the top has a diameter of 4·2 cm. Find

1. the total surface area of the block.
2. the volume of the block formed. (Take $\pi = \frac{22}{7}$)

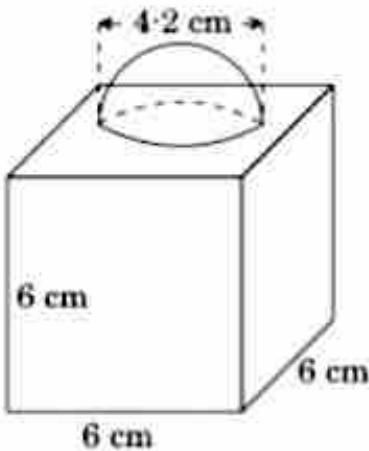


Figure 3

PYQ 2018 NCERT EXEMPLAR

Question

A heap of rice is in the form of a cone with a diameter of 9m and a height of 3.5m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?

NCERT, PYQ 2019

Water in a canal 1.5 m wide and 6 m deep is flowing with a speed of 10 km / hr. How much area will it irrigate in 30 minutes if 8 cm of standing water is desired?

NCERT EXEMPLAR PYQ 2014, 2017

A solid metallic sphere of radius 10.5 cm is melted and recast into a number of smaller cones, each of radius 3.5 cm and height 3 cm. Find the number of cones so formed.

PYQ 2014, 2019

Water in a canal, 30 dm wide and 12 dm deep, is flowing with a velocity of 20 km per hour. How much area will it irrigate in 30 min, if 9 cm of standing water is desired ?

NCERT EXEMPLAR PYQ 2012, 2014

Question

A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, then find the radius and slant height of the heap.

NCERT EXEMPLAR PYQ 2017

Question

Water is flowing at the rate of 5 km per hour through a pipe of diameter 14 cm into a rectangular tank, which is 50 m long and 44 m wide. Find the time in which the level of water in the tank will rise by 7 cm

NCERT, PYQ 2015

A 20 m deep well with diameter 7 cm is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

NCERT EXEMPLAR PYQ 2010

The rain water from a roof of $22\text{m} \times 20\text{m}$ drains into a cylindrical vessel having diameter of base 2m and height 3.5m . If the vessel is just full, find the rainfall in cm.

NCERT, NCERT EXEMPLAR

Question 1

2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

PYQ 2016, 2019

Question

A cylindrical container of radius 6 cm and height 15 cm is fullled with ice-cream. The whole ice-cream has to be distributed to 10 children in equal cones with hemispherical tops. If the height of the conical portion is four times the radius of its base, find the radius of the ice-cream cone

PYQ 2010, 2016

Question

The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from Q(2, -5) and R(-3, 6), then find the coordinates of P.

PYQ 2015

Question

Show that the points (a, a) , $(-a, -a)$ and $(-\sqrt{3}a, \sqrt{3}a)$ are the vertices of an equilateral triangle. Also, find its area.

PYQ 2006, 2013

Question

Prove that the points $(3, 0)$, $(6, 4)$ and $(-1, 3)$ are the vertices of a right angled isosceles triangle.

PYQ 2014

Question

If the point $P(2, 2)$ is equidistant from the points $A(-2, k)$ and $B(-2k, -3)$, find

k . Also, find the length of AP .

PYQ 2016 NCERT EXEMPLAR

Question

The line segment joining the points A(3, 2) and B(5, 1) is divided at the point P in the ratio 1 : 2 and it lies on the line $3x - 18y + k = 0$. Find the value of k.

PYQ 2006, 2018

Question

If $A(-2, 1)$, $B(a, 0)$, $C(4, b)$ and $D(1, 2)$ are the vertices of a parallelogram ABCD, find the values of a and b . Hence find the lengths of its sides.

PYQ 2010

Question

Point P divides the line segment joining the point A(2, 1) and B(5,-8) such that $\frac{AP}{AB} = \frac{1}{3}$. If P lies on the line $2x-y+k=0$, find the value of k.

PYQ 2019, NCERT

Question

Find the coordinate of a point A, where AB is the diameter of a circle whose centre is $(2, -3)$ and B is $(1, 4)$

PYQ 2004, 2018

Question

Find the ratio in which $P(4, m)$ divides the line segment joining the points $A(2, 3)$ and $B(6, -3)$. Hence find m .

PYQ 2006, 10, 16, 19

Question

Find the ratio in which the y-axis divides the line segment joining the points $(5, -6)$ and $(-1, -4)$. Also find the point of intersection.

PYQ 2008, 2009 NCERT

Question

If A and B are two points having coordinates $(-2, -2)$ and $(2, -4)$ respectively, find the coordinates of P such that $AP = \frac{3}{7}AB$.

NCERT, PYQ 2004

Question

If the midpoints of sides of a triangle are $(3, 4)$, $(4, 6)$ and $(5, 7)$, find the vertices of the triangle.

NCERT PYQ 2005, 2013, 2019

Question

Two pillars of equal heights stand on either side of a road which is 150 m wide. At a point on the road between the pillars, the angles of elevation of the tops of the pillars are 60° and 30° . Find the height of each pillar and the position of the point on the road.

PYQ 2006

Question

From a window 15 m high above the ground in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 30° and 45° , respectively. Show that the height of the opposite house is 23.66 m.
[take $\sqrt{3} = 1.732$]

PYQ 2004, 2005, 2010

Question

A man is standing on the deck of a ship which is 10m above the water level. He observes the angle of elevation of the top of hill as 60° and the angle of the base of hill as 30° . Find the height of the hill from the base.

PYQ 2008, 2009, 2016

Question

An aeroplane when flying at a height of 4,000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60 and 45 respectively. Find the vertical distance between the aeroplanes at that instant (take $\sqrt{3} = 1.73$)

PYQ 2008, 2014

Question

The angle of elevation of a jet plane from a point A on the ground is 60° . After a flight of 15 second, the angle of elevation changes to 30° . If the jet plane is flying at a constant height of $1500\sqrt{2}$ m. Find the speed of the jet plane.

NCERT PYQ 2008, 2009, 2017

Question

A straight highway leads to the foot of a tower. A man standing on the top of the tower spots a van at an angle of depression of 30° . The van is approaching the tower with a uniform speed. After 6 minutes, the angle of depression of the van is found to be 60° . How many more minutes will be needed for the van to reach the foot of the tower?

PYQ 2004, 2018

Question

As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45° . If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. [Use $\sqrt{3} = 1.732$]

NCERT PYQ 2008, 2009, 2017

Question

A straight highway leads to the foot of a tower. A man standing on the top of the tower spots a van at an angle of depression of 30° . The van is approaching the tower with a uniform speed. After 6 minutes, the angle of depression of the van is found to be 60° . How many more minutes will be needed for the van to reach the foot of the tower?

PYQ 2010, 2017

Question

The angle of elevation of a cloud from a point 60 m above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60° . Find the height of the cloud.

NCERT EXEMPLAR PYQ 2019

Question

The shadow of a tower standing on a level ground is found to be 40 m longer when Sun's altitude is 30° than when it was 60° . Find the height of the tower.

PYQ 2015, 2016, 2019, 2020

Question

A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height 5 meters. At a point on the plane the angle of elevation of the bottom and the top of the flag staff are respectively 30° and 60° . Find the height of tower.

NCERT PYQ 2008, 2014

Question

A statue 1.6 m tall stands on the top of pedestal. From a point on the elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45° . Find the height of the pedestal.

NCERT EXEMPLAR

Question

A window of a house is h metre above the ground. From the window, the angles of elevation and depression of the top and bottom of another house situated on the opposite side of the lane are found to be α and β respectively. Prove that the height of the house is $h(1 + \tan \alpha \tan \beta)$ metres.

PYQ 2017

Question

From the top of a $120m$ high tower, a man observes two cars on the opposite sides of the tower and in a straight line with the base of the tower with angles of depression as 60^0 and 45^0 . Find the distance between the cars. (Take $\sqrt{3} = 1.732$)

NCERT EXEMPLAR

Question

A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the B wall through a distance p , so that it slides a distance q down the wall

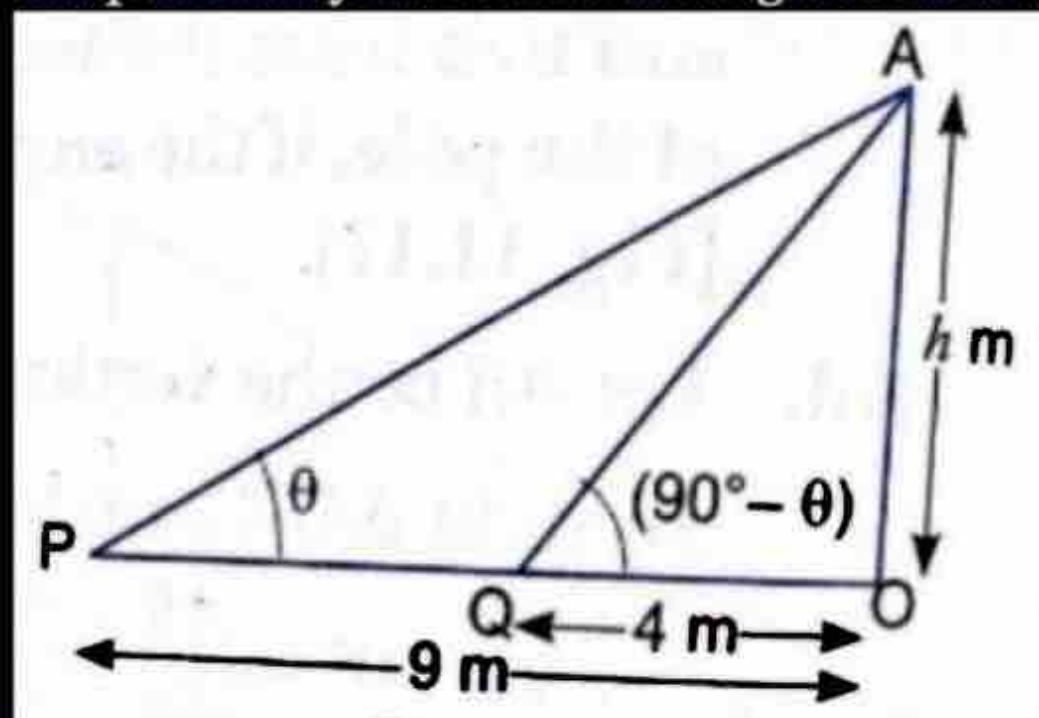
making an angle β with the horizontal. Prove that :

$$\frac{p}{q} = \frac{\cos \beta - \cos \alpha}{\sin \alpha - \sin \beta}$$

NCERT, PYQ 2016

Question

The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.



NCERT, PYQ 2012, 2015, 2017

Question

The angle of elevation of the top of the building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

PYQ 2006

for what value of n the nth term of the following 2
A.P.s are same? i) 1,7,13,19... ii) 69,68,67

PYQ 2017

Question

If the m^{th} term of an AP is $1/n$ and the n^{th} term of it is $1/m$.

Show that: $(mn)^{\text{th}}$ term of this AP is 1.

PYQ 2008, 2017

If the p^{th} term of an A.P be $\frac{1}{q}$ and q^{th} term be $\frac{1}{p}$,
then sum of its pq^{th} term will be

PYQ 2008, 2019

If m times the m^{th} term of an A.P. is equal to n times n^{th} term, show that the $(m + n)^{\text{th}}$ term of the A.P. is zero.

NCERT, PYQ 2013

How many three digit natural numbers are divisible by 7 ?

NCERT, PYQ 2018 20

The sum of four consecutive numbers in AP is 32 and the ratio of the product of the first and the last to the product of two middle terms is 7 : 15 . Find the numbers.

PYQ 2016

Divide 56 into 4 parts which are in AP such that the ratio of products of extremes to the products of means is 5 : 6.

PYQ 2009

The sum of first 6 terms of an arithmetic progression is 42 the ratio of its 10th term to the 30th term is 1 is to 3 calculate the 1st and 13th term of the AP

PYQ 2010 2014

The first and the last terms of an A.P. are 7 and 49 respectively. If sum of all its terms is 420, find its common difference.

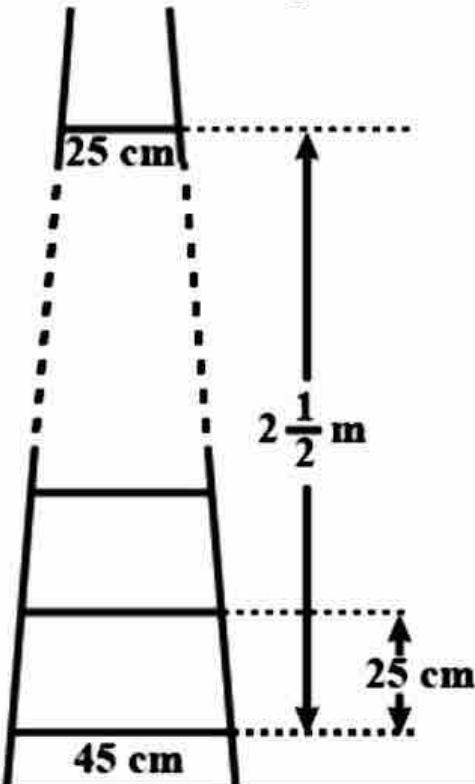
PYQ 2017 NCERT EXEMPLAR

Find the sum of n terms of the series

$$\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \dots \dots \dots$$

NCERT

A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs



PYQ 2013 2016

NCERT

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

PYQ 2015 NCERT EXEMPLAR

If S_n denotes the sum of first n terms of an A.P
prove that $S_{12} = 3(S_8 - S_4)$

PYQ 2016, 2017

The ratio of the sums of m and n terms of an A.P is $m^2 : n^2$. Show that the ratio of m th and n th term is $(2m - 1) : (2n - 1)$.

PYQ 2017

In an A.P, the sum of m terms of an AP is n and sum of n terms of AP is m , then prove that sum of $(m+n)$ terms of AP is $-(m+n)$

NCERT EXEMPLAR

An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.

PYQ 2015, 2017

If m th term of an AP is $1/n$ and n th term is

$1/m$ then find the sum of its first mn terms.

PYQ 2015, 2017

If m th term of an AP is $1/n$ and n th term is

$1/m$ then find the sum of its first mn terms.

PYQ 2014, NCERT

In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

PYQ 2015, NCERT

A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find :

- (i) the production in the 1st year
- (ii) the production in the 10th year
- (iii) the total production in first 7 years

NCERT EXEMPLAR

Question

The area of a circular playground is 22176 m^2 . Find the cost of fencing this ground at the rate of Rs. 50 per metre.

PYQ 2017

Question

The difference between the radii of the smaller circle and the bigger circle is 7cm and the difference between the areas of the two circles is 1078 sq cm. What is the radius of the smaller circle in cm?

NCERT EXEMPLAR

Question

Find the number of revolutions made by a circular wheel of area 1.54 m^2 in rolling a distance of 176 m .

NCERT PYQ 2019

Question

A car has two wipers which do not overlap. Each wiper has a blade of length 25cm sweeping through an angle of 115° . Find the total area cleaned at each sweep of the blades

PYQ 2016

Question

An elastic belt is placed around the rim of a pulley of radius 5 cm . From one point C on the belt, elastic belt is directly away from the centre O of the pulley until it is at P , 10 cm from the point O . Find the length of the belt that is still in contact with the pulley. Also find the shaded area.

PYQ 2017

A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

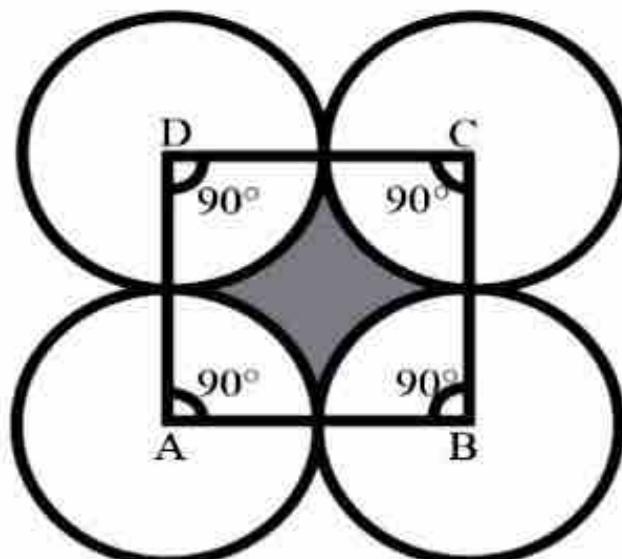
NCERT EXEMPLAR

Question

On a square cardboard sheet of area 784 cm^2 , four congruent circular plates of maximum size are placed such that each circular plate touches the other two plates and each side of the square sheet is tangent to two circular plates. Find the area of the square sheet not covered by the circular plates.

NCERT, NCERT EXEMPLAR

Question

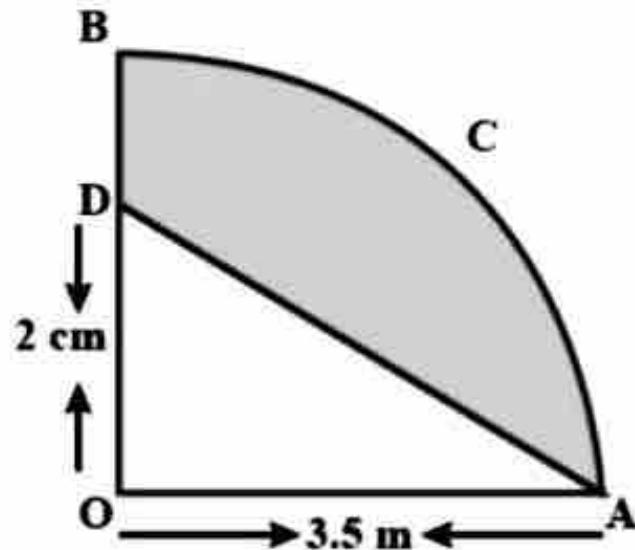


Four equal circles are described about the four corners of a square so that each touches two of the others, as shown in the figure. Find the area of the shaded region, if each side of the square measures 14 cm.

NCERT, PYQ 2014, 2017

Question

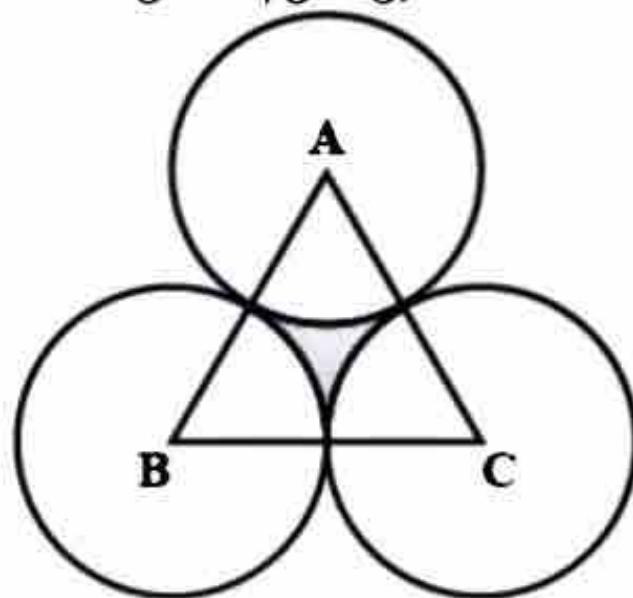
In figure, AOBCA represents a quadrant of a circle of radius 3.5cm with centre O. Calculate the area of the shaded portion (Take $\pi = \frac{22}{7}$)



NCERT

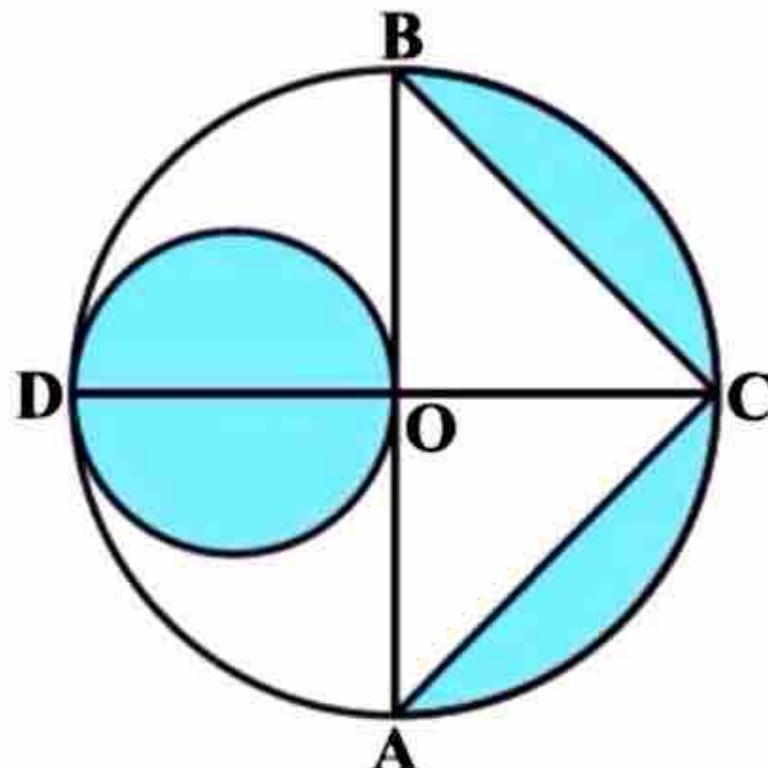
Question

In given figure the area of an equilateral triangle ABC is 17320.5cm^2 . With each vertex of the triangle as a center, a circle is drawn with a radius equal to half the length of the side of the triangle. Find the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$)



NCERT, PYQ 2010, 2013

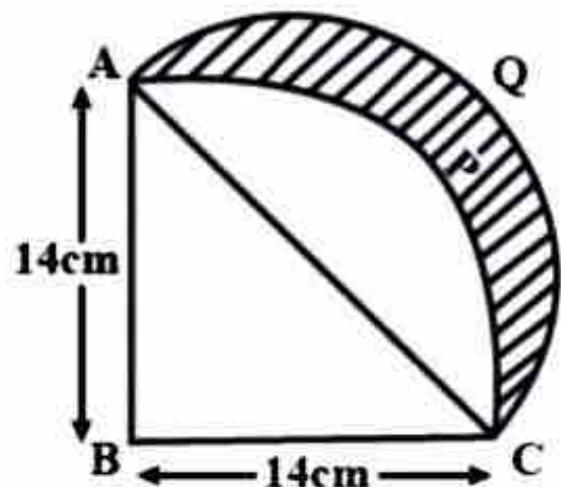
In Fig. 12.27, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.



NCERT, PYQ 2008, 2014

Question

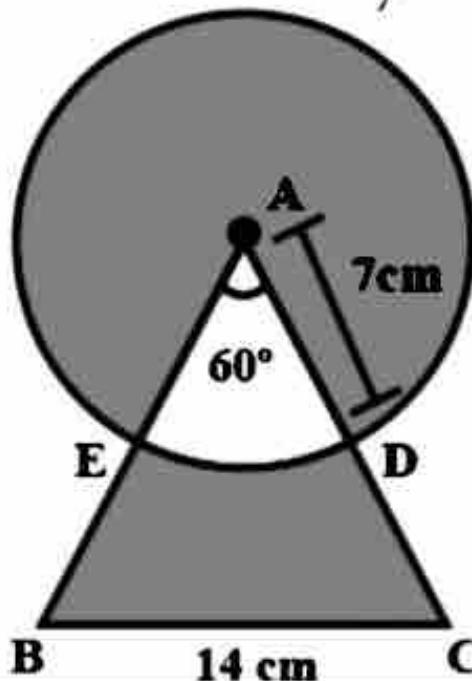
ABCP is a quadrant of a circle of radius 14 cm. With AC as diameter, a semicircle is drawn. Find the area of the shaded portion.



PYQ 2015, 2016

Question

Find the area of a shaded region in the given figure, where a circular arc of radius 7 cm has been drawn with vertex A of an equilateral triangle ABC of side 14 cm as centre. (Use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.73$).



NCERT EXEMPLAR

Question

Find the number of revolutions made by a circular wheel of area 1.54 m^2 in rolling a distance of 176 m.

If $x = \frac{2}{3}$ and $x = -3$ are the roots of the quadratic equation $ax^2 + 7x + b = 0$ then find the values of a and b.

If $x = \frac{2}{3}, x = -3$ are roots, then the eqn will be

$$(x - \frac{2}{3})(x - (-3)) = 0$$

$$\Rightarrow (x - \frac{2}{3})\{(x+3)\} = 0$$

$$\Rightarrow x^2 - \frac{2}{3}x + 3x - 2 = 0$$

$$\Rightarrow 3x^2 - 2x + 9x - 6 = 0$$

$\Rightarrow 3x^2 + 7x - 6 = 0$ is the required eqn.

Compare the eqn with $ax^2 + 7x + b = 0$

We have $a = 3$, and $b = -6$

Hence, a = 3, and b = -6

PYQ 2016,2019

Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Let the time taken by the smaller pipe to fill the tank be x hr.

Time taken by the larger pipe = $(x - 10)$ hr

Part of tank filled by smaller pipe in 1 hour =

$$\frac{1}{x}$$

Part of tank filled by larger pipe in 1 hour =

$$\frac{1}{x-10}$$

It is given that the tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together. Therefore,

$$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$$

$$\Rightarrow \frac{x-10+x}{x(x-10)} = \frac{8}{75}$$

$$\Rightarrow \frac{2x-10}{x(x-10)} = \frac{8}{75}$$

$$\Rightarrow 75(2x-10) = 8x^2 - 80x$$

$$\Rightarrow 150x - 750 = 8x^2 - 80x$$

$$\Rightarrow 8x^2 - 230x + 750 = 0$$

$$\Rightarrow 8x^2 - 200x - 30x + 750 = 0$$

$$\Rightarrow 8x(x-25) - 30(x-25) = 0$$

$$\Rightarrow (x-25)(8x-30) = 0$$

$$\Rightarrow x = 25, \frac{30}{8}$$

$$\Rightarrow x = 25, \frac{30}{8}$$

Time taken by the smaller pipe cannot be $\frac{30}{8} = 3.75$ hours. As in this case, the time taken by the larger pipe will be negative, which is logically not possible.

Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and $25 - 10 = 15$ hours respectively.

Solve the following quadratic equations by factorization:

$$\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}, x \neq 0$$

We have been given

$$\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$$

$$x^2 - x + 2x^2 - 4x = 6(x^2 - x - 2x + 2)$$

$$3x^2 - 13x + 12 = 0$$

Therefore,

$$3x^2 - 9x - 4x + 12 = 0$$

$$3x(x-3) - 4(x-3) = 0$$

$$(3x-4)(x-3) = 0$$

Therefore,

$$3x - 4 = 0$$

$$3x = 4$$

$$x = \frac{4}{3}$$

or,

$$x - 3 = 0$$

$$x = 3$$

Hence, $x = \frac{4}{3}$ or $x = 3$.

**PYQ
2013**

Solve by factorization method:

$$\left(\frac{4}{x}\right) - 3 = \frac{5}{(2x+3)}, \quad x \neq 0, \quad \frac{-3}{2}$$

Given expression is,

$$\left(\frac{4}{x}\right) - 3 = \frac{5}{(2x+3)}, \quad x \neq 0, \quad \frac{-3}{2}$$

On simplifying the given expression, we get;

$$\Rightarrow \frac{4-3x}{x} = \frac{5}{2x+3}$$

$$\Rightarrow (4-3x)(2x+3) = 5x$$

$$\Rightarrow 8x - 12 - 6x^2 - 9x = 5x$$

$$\Rightarrow 6x^2 + 9x - 8x + 12 + 5x = 0$$

$$\Rightarrow \quad \quad \quad 6x^2 - 6x + 12 = 0$$

$$\Rightarrow x^2 + x - 2 = 0 \quad [\text{Divide the equation by 6}]$$

By following factorization method,

$$x^2 + 2x - x - 2 = 0$$

$$\Rightarrow x(x+2) - 1(x+2) = 0$$

$$\Rightarrow \quad \quad (x+2)(x-1) = 0$$

$$\Rightarrow \quad \quad \quad x+2 = 0 \text{ or } x-1 = 0$$

$$\Rightarrow \quad \quad \quad x = -2, 1$$

Hence, the solution of the given expression

is $x = -2, x = 1$

Solve the following quadratic equation for x.

$$4x^2 - 4a^2x + (a^4 - b^4) = 0$$

We have

$$4x^2 - 4a^2x + (a^4 - b^4) = 0$$

$$\Rightarrow (4x^2 - 4a^2x + a^4) - b^4 = 0$$

$$\Rightarrow [(2x)^2 - 2(2x)(a) + (a^2)^2] - b^4 = 0$$

$$\Rightarrow (2x - a^2)^2 - (b^2)^2 = 0$$

$$[\because (a - b)^2 = a^2 + b^2 - 2ab]$$

$$\Rightarrow (2x - a^2 + b^2)(2x - a^2 - b^2) = 0$$

$$[\because a^2 - b^2 = (a - b)(a + b)]$$

$$\Rightarrow 2x = a^2 - b^2 \text{ or } 2x = a^2 + b^2$$

$$\Rightarrow x = \frac{a^2 - b^2}{2} \text{ or } \frac{a^2 + b^2}{2}$$

Solve each of the following quadratic equations:

$$a^2b^2x^2 + b^2x - a^2x - 1 = 0$$

$$a^2b^2x^2 + b^2x - a^2x - 1 = 0$$

$$b^2x(a^2x + 1) - 1(a^2x + 1) = 0$$

$$(b^2x - 1)(a^2x + 1) = 0$$

$$\text{if } b^2x - 1 = 0$$

$$b^2x = 1$$

$$x = \frac{1}{b^2}$$

$$\text{if } a^2x + 1 = 0$$

$$a^2x = -1$$

$$x = \frac{-1}{a^2}$$

PYQ 2005

Solve each of the following quadratic equations:

$$\frac{a}{(x-b)} + \frac{b}{(x-a)} = 2, x \neq b, a$$

$$\frac{a}{(x-b)} + \frac{b}{(x-a)} = 2$$

$$\frac{a(x-a) + b(x-b)}{(x-a)(x-b)} = 2$$

$$\Rightarrow ax - a^2 + bx - b^2 = 2x^2 - 2ax - 2bx + 2ab$$

$$\Rightarrow 2x^2 - 2ax - ax - 2bx - bx + a^2 + b^2 + 2ab = 0$$

$$\Rightarrow 2x^2 - 3x(a+b) + (a+b)^2 = 0$$

$$\Rightarrow 2x^2 - 2x(a+b) - x(a+b) + (a+b)^2 = 0$$

$$\Rightarrow 2x(x-(a+b)) - (a+b)(x-(a+b)) = 0$$

$$\Rightarrow (2x-(a+b))(x-(a+b)) = 0$$

$$\Rightarrow x = \frac{a+b}{2} \text{ or } x = a+b$$

PYQ 2016

18. $\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$, then find x

$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$$

$$\frac{a(x-b) + b(x-a)}{(x-a)(x-b)} = \frac{2c}{x-c}$$

$$\frac{ax-ab+bx-ab}{x^2-(a+b)x+ab} = \frac{2c}{x-c}$$

$$(x - c)(ax + bx - 2ab) = 2c \{x^2 - (a + b)x + ab\}$$

$$(x - c)\{(a + b)x - 2ab\} = 2cx^2 - 2cx(a + b) + 2ab$$

$$x^2(a + b) - 2abx - cx(a + b) + 2abc = 2cx^2 - 2cx(a + b)$$

$$x^2(a + b) - 2cx^2 - 2abx - cx(a + b) + 2cx(a + b) = 0$$

$$x^2(a + b - 2c) - 2abx + cx(a + b) = 0$$

$$x^2(a + b - 2c) - x(2ab + ac + bc) = 0$$

$$x = 0$$

or

$$x(a + b - 2c) - (2ab + ac + bc) = 0$$

$$x = \frac{(2ab+ac+bc)}{(a+b-2c)}$$

PYQ 2015

Solve each of the following quadratic equations:

$$\frac{a}{(x-b)} + \frac{b}{(x-a)} = 2, x \neq b, a$$

$$\frac{a}{(x-b)} + \frac{b}{(x-a)} = 2$$

$$\frac{a(x-a) + b(x-b)}{(x-a)(x-b)} = 2$$

$$\Rightarrow ax - a^2 + bx - b^2 = 2x^2 - 2ax - 2bx + 2ab$$

$$\Rightarrow 2x^2 - 2ax - ax - 2bx - bx + a^2 + b^2 + 2ab = 0$$

$$\Rightarrow 2x^2 - 3x(a + b) + (a + b)^2 = 0$$

$$\Rightarrow 2x^2 - 2x(a + b) - x(a + b) + (a + b)^2 = 0$$

$$\Rightarrow 2x(x - (a + b)) - (a + b)(x - (a + b)) = 0$$

$$\Rightarrow (2x - (a + b))(x - (a + b)) = 0$$

$$\Rightarrow x = \frac{a+b}{2} \text{ or } x = a + b$$

PYQ 2016

Solve the following quadratic equation using quadratic formula .

$$9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$$

The correct option is A

The roots are $\frac{2a+b}{3}$ and $\frac{a+2b}{3}$

We have,

$$9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$$

Comparing this equation with

$Ax^2 + Bx + C = 0$, we have

$$A = 9, B = -9(a+b) \text{ and } C = 2a^2 + 5ab + 2b^2$$

$$\therefore D = B^2 - 4AC$$

$$\Rightarrow D = 81(a+b)^2 - 36(2a^2 + 5ab + 2b^2)$$

$$\Rightarrow D = 81(a^2 + b^2 + 2ab) - (72a^2 + 180ab + 72b^2)$$

$$\Rightarrow D = 9a^2 + 9b^2 - 18ab$$

$$\Rightarrow D = 9a^2 + 9b^2 - 18ab$$

$$\Rightarrow D = 9(a^2 + b^2 - 2ab)$$

$$\Rightarrow D = 9(a-b)^2 \geq 0$$

$$\Rightarrow D \geq 0$$

So, the roots of the given equation are real and are given by

$$\alpha = \frac{-B + \sqrt{D}}{2A} = \frac{9(a+b) + 3(a-b)}{18} = \frac{12a + 6b}{18} = \frac{2a+b}{3}$$

$$\text{and, } \beta = \frac{-B - \sqrt{D}}{2A} = \frac{9(a+b) - 3(a-b)}{18} = \frac{6a + 12b}{18} = \frac{a+2b}{3}$$

PYQ 2004, 09, 16

Solve the following quadratic equation by factorization.

$$3x^2 - 2\sqrt{6}x + 2 = 0$$

$$3x^2 - 2\sqrt{6}x + 2 = 0$$

$$\Rightarrow 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$$

$$\Rightarrow \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$$

$$\Rightarrow (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$$

$$\Rightarrow x = \sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$$

Find the roots of quadratic equation

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$

Given,

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$

$$\text{or, } \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$$

$$\text{or, } x\sqrt{2}(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0$$

$$\text{or, } (x\sqrt{2} + 5)(x + \sqrt{2}) = 0$$

$$\Rightarrow x = -\frac{5}{\sqrt{2}}, -\sqrt{2}.$$

These are the required roots.

find the value of k for which the following equation has equal roots $(k-12)x^2+2(k-12)x+2=0$

$$\text{Sol. } (k - 12)x^2 + 2(k - 12)x + 2 = 0$$

Comparing with $ax^2 + bx + c = 0$ we have $a = k - 12$, $b = 2(k - 12)$, $c = 2$

We know that,

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= [2(k - 12)]^2 - 4(k - 12)(2) \\ &= (2k - 24)^2 - 8(k - 12) \\ &= 4k^2 - 96k + 576 - 8k + 96 \\ &= 4k^2 - 104k + 672\end{aligned}$$

\therefore The roots of given equation are real and equal.

$\therefore \Delta$ must be zero.

$$\begin{aligned}4k^2 - 104k + 672 &= 0 \\ 4(k^2 - 26k + 168) &= 0 \\ k^2 - 14k - 12k + 168 &= 0 \\ k(k - 14) - 12(k - 14) &= 0 \\ (k - 14)(k - 12) &= 0 \\ k - 14 = 0 \text{ or } k - 12 &= 0\end{aligned}$$

$$\therefore k = 14 \text{ or } k = 12$$

For what value of k , are the roots of the quadratic equation $(k + 4)x^2 + (k + 1)x + 1 = 0$ equal?

Given: Roots are equal \Rightarrow Discriminant = 0

$$\Rightarrow \Delta = b^2 - 4ac = 0$$

$$\Rightarrow (k + 1)^2 - 4(k + 4) \times 1 = 0$$

$$\Rightarrow k^2 + 2k + 1 - 4k - 16 = 0$$

$$\Rightarrow k^2 - 2k + 1 = 16$$

$$\Rightarrow (k - 1)^2 = 4^2$$

$$\Rightarrow k - 1 = \pm 4$$

$$\therefore k = 1 \pm 4$$

$$\therefore k = 1 + 4, 1 - 4 \text{ or } k = 5, -3$$

PYQ 2000, 13

If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k .

-5 is a root of quadratic equation $2x^2 + px - 15 = 0$

$$\text{so, } 2(-5)^2 + p(-5) - 15 = 0$$

$$\Rightarrow 50 - 5p - 15 = 0$$

$$\Rightarrow 35 - 5p = 0$$

$$p = 7$$

now, put $p = 7$ in second quadratic equation,

$$p(x^2 + x) + k = 0$$

$$\Rightarrow 7(x^2 + x) + k = 0$$

$$\Rightarrow 7x^2 + 7x + k = 0$$

above equation has equal roots

$$\text{so, } D = b^2 - 4ac = 0$$

$$\Rightarrow 7^2 - 4 \times 7 \times k = 0$$

$$\Rightarrow 49 - 28k = 0$$

$$\Rightarrow k = 49/28 = 1.75$$

hence, the value of $k = 1.75$

**CBSE 2002
2009, 14**

The sum of two numbers is 15. If the sum of their reciprocal is $\frac{3}{10}$ Find smallest number.

Let first number = x

then the other number = $15 - x$

According to the question

$$\frac{1}{x} + \frac{1}{15-x} = \frac{3}{10}$$

$$10(15-x+x) = 3(x(15-x))$$

$$150 = 45x - 3x^2$$

$$3x^2 - 45x + 150 = 0$$

$$x^2 - 15x + 50 = 0$$

$$x^2 - 10x - 5x + 50 = 0$$

$$(x - 10)(x - 5) = 0$$

$$x = 10, x = 5$$

PYQ 2000, 2005

A two digit number is 4 times the sum of its digits and twice the product of its digits. The number is ——

Let the digit in the ones place be x and tens place be y

Hence the two digit number = $10y + x$

Given that the two digit number = 4 times sum of its digits

According to the question,

$$\Rightarrow 10y + x = 4(x + y)$$

$$\Rightarrow 10y + x = 4x + 4y$$

$$\Rightarrow 3x - 6y = 0$$

$$\Rightarrow x = 2y \quad (1)$$

It is also given that the two digit number = 2 times product of its digits

$$\Rightarrow 10y + x = 2xy$$

Divide by xy both the sides, we get

$$\Rightarrow \frac{10}{x} + \frac{1}{y} = 2$$

Put $x = 2y$ from (1), we get

$$\Rightarrow \frac{10}{2y} + \frac{1}{y} = 2$$

$$\Rightarrow \frac{5}{y} + \frac{1}{y} = 2$$

$$\Rightarrow \frac{6}{y} = 2$$

$$\therefore y = 3$$

$$\Rightarrow x = 2y = 2(3) = 6$$

$$\Rightarrow \frac{6}{y} = 2$$

$$\therefore y = 3$$

$$\Rightarrow x = 2y = 2(3) = 6$$

\Rightarrow The two digit number

$$= (10y + x) = 10(3) + 6 = 36$$

PYQ 2016

A motorboat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Speed of boat in still water = 18km/hr

Let speed of the stream = s

Speed of boat upstream = Speed of boat in still water - speed of stream = $18 - s$

Speed of boat down stream = Speed of boat in still water + speed of stream = $18 + s$

Time taken for upstream = Time taken to cover downstream + 1

$$\Rightarrow \frac{\text{Distance}_{\text{upstream}}}{\text{Speed}_{\text{upstream}}} = \frac{\text{Distance}_{\text{downstream}} + 1}{\text{Speed}_{\text{downstream}}}$$

$$\Rightarrow \frac{24}{18 - s} = \frac{24}{18 + s} + 1$$

$$\Rightarrow 24(18 + s) = 24(18 - s) + (18 - s)(18 + s)$$

$$\Rightarrow s^2 + 48s - 324 = 0$$

$$\Rightarrow s^2 + 54s - 6s - 324 = 0$$

$$\Rightarrow (s + 54)(s - 6) = 0$$

$$\Rightarrow s = 6, -54$$

$$\Rightarrow (s + 54)(s - 6) = 0$$

$$\Rightarrow s = 6, -54$$

$$\Rightarrow s \neq -54$$

Thus, $s = 6$ km/hr, Speed of steam cannot be negative.

**PYQ 2014
2018**

A train travels at a certain average speed for a distance 63 km and then travels a distance of 72 km at an average speed of 6 km/hr more than the original speed. If it takes 3 hours to complete total journey, what is its original average speed?

Let the original average speed of train be x .

Time taken by train to cover 63 km with

$$\text{original speed} = \frac{63}{x}$$

Time taken by train to cover 72 km with

$$\text{increased speed} = \frac{72}{x+6}$$

It is given that ;

$$\frac{63}{x} + \frac{72}{x+6} = 3$$

$$\frac{63(x+6) + 72x}{x(x+6)} = 3$$

$$\frac{63x + 378 + 72x}{x^2 + 6x} = 3$$

$$378 + 135x = 3x^2 + 18x$$

$$3x^2 - 117x - 378 = 0$$

$$x^2 - 39x - 126 = 0$$

$$378 + 135x = 3x^2 + 18x$$

$$3x^2 - 117x - 378 = 0$$

$$x^2 - 39x - 126 = 0$$

$$x(x-42) + 3(x-42) = 0$$

$$(x+3)(x-42) = 0$$

$$x = 42, -3$$

Speed cannot be negative, so the original average speed is 42km/hr.

**NCERT EXEMPLAR
PYQ 2018**

Sum of the area of two squares is 640 m^2 . If the difference of their perimeters is 64 m , find the sides of the two squares.

Let the sides of the two squares be of a and b .

According to the problem

$$a^2 + b^2 = 640 \dots\dots\dots(1)$$

$$\text{Again } 4(a - b) = 64$$

$$\text{or, } a - b = 16 \dots\dots\dots(2)$$

Using (2) in (1) we get,

$$a^2 + (-16 + a)^2 = 640$$

$$\text{or, } 2a^2 - 32a + 256 = 640$$

$$\text{or, } 2a^2 - 32a - 384 = 0$$

$$\text{or, } a^2 - 16a - 192 = 0$$

$$\text{or, } (a - 24)(a + 8) = 0$$

Since a is the side of a square then

$$a = 24 \Rightarrow b = 8$$

So the sides of the squares be 24 m and 8 m .

**NCERT, PYQ
2003 2018**

Champa went to a 'sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, " The number of skirts is two less than twice the number of pants purchased. Also the number of skirts is four less than tbur times the number of pants purchased". Find how many pants and skirts Champa bought?

Let the number of pants = x

Let the number of skirts = y

According to the equation

$$\text{Number of skirts} = 2(\text{Number of pants}) - 2$$

Also, Number of skirts = 4(Number of pants) - 4

$$y = 4x - 4 \quad \text{---(ii)}$$

Substituting the value of $y = 4x - 4$ in cq^y

(ii), we get

$$y = 4(1) - 4 = 0$$

$\therefore x = 1, y = 0$ is the solution of the pair of linear equations.

Hence, the number of pants she purchased is 1 and the number of skirts, she purchased is zero i.e. she didn't buy any skirt.

NCERT

Solve

$$\frac{5}{(x-1)} + \frac{1}{(y-2)} = 2$$

$$\frac{6}{(x-1)} - \frac{3}{(y-2)} = 1 [4 MARKS]$$

Given equations are,

$$\frac{5}{(x-1)} + \frac{1}{(y-2)} = 2 \dots \dots \text{(i)}$$

$$\frac{6}{(x-1)} - \frac{3}{(y-2)} = 1 \dots \dots \text{(ii)}$$

Let, $\frac{1}{(x-1)} = p$ & $\frac{1}{(y-2)} = q$

Then equations (i) and (ii) reduces to,

$$5p + q = 2 \dots \dots \text{(iii)}$$

$$6p - 3q = 1 \dots \dots \text{(iv)}$$

Equation (iii) $\times 3$ gives

$$15p + 3q = 6 \dots \dots \text{(v)}$$

Adding (iv) and (v) we get,

$$6p - 3q + 15p + 3q = 1 + 6$$

$$21p = 7 \therefore p = \frac{1}{3}$$

Substituting value of p in (iv)

$$6p - 3q = 1$$

$$6(\frac{1}{3}) - 3q = 1$$

$$q = \frac{1}{3}$$

$$\text{Now, } p = \frac{1}{3} = \frac{1}{(x-1)} = \frac{1}{x}$$

$$\therefore x - 1 = 3 \therefore x = 4$$

$$\text{and } q = \frac{1}{3} = \frac{1}{(y-2)} = \frac{1}{3}$$

$$\Rightarrow x - 1 = 3 \therefore x = 4$$

$$\text{and } q = \frac{1}{3} \Rightarrow \frac{1}{(y-2)} = \frac{1}{3}$$

$$\Rightarrow y - 2 = 3 \therefore y = 5$$

$$\therefore x = 4, y = 5$$

PYQ 2009, NCERT

Solve $\frac{7x-2y}{xy} = 5$; $\frac{8x+7y}{xy} = 15$

$$\begin{aligned}\frac{7x-2y}{xy} &= 5; \frac{8x+7y}{xy} = 15 \\ \frac{7x}{xy} - \frac{2y}{xy} &= 5 \quad \& \quad \frac{8x}{xy} + \frac{7y}{xy} = 15 \\ \therefore \frac{x}{y} - \frac{2}{x} &= 5 \quad \& \quad \frac{8}{y} + \frac{7}{x} = 15\end{aligned}$$

$$\text{Let } \frac{1}{y} = y \text{ & } \frac{1}{x} = x$$

$$\therefore 7y - 2x = 5 \dots(1)$$

$$8y + 7x = 15 \dots(2)$$

$$(1) \times 7 \text{ and } (2) \times 2$$

we get,

$$49y - 14x = 35$$

$$\underline{+16y + 14x = 30}$$

$$65y = 65$$

$$y = 1$$

$$\text{From (1), } 7y - 2x = 5 \Rightarrow x = \frac{7y-5}{2}$$

$$\therefore x = \frac{7(1)-5}{2} = \frac{2}{2}$$

$$x = 1$$

$$y = \frac{1}{y} \Rightarrow y = \frac{1}{y} = \frac{1}{1} = 1$$

$$x = \frac{1}{x} \Rightarrow x = \frac{1}{x} = \frac{1}{1} = 1$$

$$\therefore x = 1, y = 1.$$

NCERT

Question 14

If $(x + 1)$ is a factor of $f(x) = 2x^3 + ax^2 + 2bx + 1$, then find the value of a and b is given that $2a - 3b = 4$.

Given that, $(x + 1)$ is a factor $f(x) = 2x^3 + ax^2 + 2bx + 1$ then $f(-1) = 0$
[if $(x + a)$ is a factor of $f(x) = ax^2 + bx + c$, then $f(-a) = 0$]
 $\rightarrow 2(-1)^3 + a(-1)^2 + 2b(-1) + 1 = 0$
 $\rightarrow -2 + a - 2b + 1 = 0$
 $\rightarrow a - 2b - 1 = 0$
 $\rightarrow 2a - 3b = 4$
 $\rightarrow 3b = 2a - 4$
 $\rightarrow b = \left(\frac{2a-4}{3}\right)$

Now, put the value of b in Eq. (i), we get
 $a - 2\left(\frac{2a-4}{3}\right) - 1 = 0$
 $\rightarrow 3a - 2(2a - 4) - 3 = 0$
 $\rightarrow 3a - 4a + 8 - 3 = 0$
 $\rightarrow -a + 5 = 0$
 $\rightarrow a = 5$

Now, put the value of a in Eq. (i), we get
 $a - 2\frac{2a-4}{3} - 1 = 0$
 $\rightarrow 3a - 2(2a - 4) - 3 = 0$
 $\rightarrow -a + 5 = 0$
 $\rightarrow a = 5$

Now, put the value of a in Eq (i), we get
 $5 - 2b - 1 = 0$
 $\rightarrow 2b = 4$
 $\rightarrow b = 2$
Hence, the required values of a and b are 5 and 2, respectively

NCERT EXEMPLAR

Question 14

If $(x + 1)$ is a factor of $f(x) = 2x^3 + ax^2 + 2bx + 1$, then find the value of a and b is given that $2a - 3b = 4$.

Given that, $(x + 1)$ is a factor $f(x) = 2x^3 + ax^2 + 2bx + 1$ then $f(-1) = 0$
[if $(x + a)$ is a factor of $f(x) = ax^2 + bx + c$, then $f(-a) = 0$]
 $\Rightarrow 2(-1)^3 + a(-1)^2 + 2b(-1) + 1 = 0$
 $\Rightarrow -2 + a - 2b + 1 = 0$
 $\Rightarrow a - 2b - 1 = 0$
 $\Rightarrow 2a - 3b = 4$
 $\Rightarrow 3b = 2a - 4$
 $\Rightarrow b = \left(\frac{2a-4}{3}\right)$

Now, put the value of b in Eq. (i), we get:

$$\begin{aligned} & a - 2\left(\frac{2a-4}{3}\right) - 1 = 0 \\ & \Rightarrow 3a - 2(2a - 4) - 3 = 0 \\ & \Rightarrow 3a - 4a + 8 - 3 = 0 \\ & \Rightarrow -a + 5 = 0 \\ & \Rightarrow a = 5 \end{aligned}$$

Now, put the value of a in Eq. (i), we get

$$\begin{aligned} & a - 2\left(\frac{2a-4}{3}\right) - 1 = 0 \\ & \Rightarrow 3a - 2(2a - 4) - 3 = 0 \\ & \Rightarrow -a + 5 = 0 \\ & \Rightarrow a = 5 \end{aligned}$$

Now, put the value of a in Eq (i), we get

$$\begin{aligned} & 5 - 2b - 1 = 0 \\ & \Rightarrow 2b = 4 \\ & \Rightarrow b = 2 \end{aligned}$$

Hence, the required values of a and b are 5 and 2, respectively

Find the value of k for which each of the following systems of equations have infinitely many solution:

$$2x + 3y = 2$$

$$(k+2)x + (2k+1)y = 2(k-1)$$

$$2x + 3y - 2 = 0$$

$$(k+2)x + (2k+1)y - 2(k-1) = 0$$

The above equations are of the form

$$a_1 x + b_1 y - c_1 = 0$$

$$a_2 x + b_2 y - c_2 = 0$$

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -2$

$a_2 = (k+2)$, $b_2 = (2k+1)$, $c_2 = -2(k-1)$

So according to the question,

For unique solution, the condition is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{(k+2)} = \frac{3}{(2k+1)} = \frac{-2}{2(k-1)}$$

$$\frac{2}{(k+2)} = \frac{3}{(2k+1)} \text{ and } 3(2k+1) = -2(2(k-1))$$

$$\Rightarrow 2(2k+1) = 3(k+2) \text{ and } 3(k-1) = (2k+1)$$

$$\Rightarrow 4k+2 = 3k+6 \text{ and } 3k-3 = 2k+1$$

$$\Rightarrow k=4 \text{ and } k=4$$

Hence, the given system of equations will have infinitely many solutions, if $k = 4$.

Find the value of k if the system has unique solution, no solution, infinite solution.

$$x+2y=5 \quad 3x+ky=-15$$

, no solution, infinite solution.

Sol:- $x+2y=5 = 0 \quad 3x+ky+15=0$

For unique solution :- $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
 $\Rightarrow \frac{1}{3} \neq \frac{2}{k} \Rightarrow k \neq 6$

So, (Except $k=6$) for all value of k , Eqⁿ has unique solution

For no solution :- $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
 $\Rightarrow \frac{1}{3} = \frac{2}{k} \neq \frac{-5}{15}$
 $\Rightarrow k=6 \quad \text{&} \quad \frac{2}{k} \neq \frac{-5}{15} \quad k \neq -6$.

So, Value of $k=6$ (but not equal to -6).

For infinite solution :- $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
 $\Rightarrow \frac{1}{3} = \frac{2}{k} = \frac{-5}{15}$.
 $\Rightarrow k=6 \quad \text{&} \quad k=-6$.
No common value of k .
So for no value of k , Given Eqⁿ has infinite solution

PYQ 2001, 2019

One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II]

Let A have x and B have y initially,
If B gives A 100, then A becomes twice
as rich as B
 $So x+100 = 2(y-100)$ (1)
If A gives B 10 then B becomes 6 times
as rich as A
 $So y+10 = 6(x-10)$ (2)

From (1) $x+100 = 2y-200$
 $\Rightarrow x-2y = -300$ (3)
From (2) $y+10 = 6x-60$
 $\Rightarrow 6x-y = 70$ (4)
Multiply (3) by -6 and add to (4)
 $-6x+12y=1800$
 $6x-y = 70$

$11y = 1870$
 $y = 170$
substitute in (3) $x-2(170) = -300$
 $x-340 = -300$
 $x = 40$

NCERT

Jamila sold a table and a chair for ₹ 1050 , thereby making a profit of 10% on the table and 25% on the chair. If she had taken a profit of 25% on the table and 10% on the chair she would have got ₹ 1065. Find the cost price of each .

Let the CP of the table be Rs x and that of the chair be Rs y.

$$\text{Case I: Profit on table} = \frac{10}{100}x$$

$$\text{We know } SP - CP = \text{Profit}$$

$$\Rightarrow SP = \text{Profit} + CP$$

$$\Rightarrow SP = \frac{10}{100}x + x = \frac{110}{100}x = \frac{11}{10}x$$

$$\text{Profit on chair} = \frac{25}{100}y$$

$$SP = \frac{25}{100}y + y = \frac{125}{100}y$$

$$\text{Total SP} = \frac{110}{100}x + \frac{125}{100}y = 1050$$

$$110x + 125y = 105000 \quad \dots(i)$$

$$\text{Case II: Profit on table} = \frac{25}{100}x$$

$$\Rightarrow SP = \text{Profit} + CP$$

$$\Rightarrow SP = \frac{25}{100}x + x = \frac{125}{100}x$$

$$\text{Profit on chair} = \frac{10}{100}y$$

$$SP = \frac{10}{100}y + y = \frac{110}{100}y$$

$$\text{Total SP} = \frac{125}{100}x + \frac{110}{100}y = 1065$$

$$125x + 110y = 106500 \quad \dots(ii)$$

From (i) and (ii), we have

$$235(x + y) = 211500$$

$$\Rightarrow x + y = 900 \quad \dots(iii)$$

Subtracting (i) from (ii), we have

$$15(x - y) = 1500$$

$$\Rightarrow x - y = 100 \quad \dots(iv)$$

Solving (iii) and (iv), we get

$$x = 500 \text{ and } y = 400$$

Thus, CP of table = Rs 500 and CP of chair = Rs 400.

NCERT EXEMPLAR

The sum of two digit number and the number formed by interchanging the digits is 132. If 12 is added to the number, the new number becomes 5 times the sum of the digits. Find the number.

Let the digits at units and tens place in the given number be x and y respectively.

Then, number = $10y + x \dots \text{(i)}$

Number formed by interchanging the digits = $10x + y$

According to the given condition, we have $(10y + x) + (10x + y) = 132$

and $(10y + x) + 12 = 5(x + y)$

$\Rightarrow 11x + 11y = 132$ and $4x - 5y = 12$

$\Rightarrow x + y - 12 = 0 \dots \text{(ii)}$ and $4x - 5y - 12 = 0 \dots \text{(iii)}$

Solving these two equations:

Multiplying eq. (i) $\times 4$, we get

$4x + 4y = 48 \dots \text{(iv)}$

On subtracting (iii) from (iv), we get

$9y = 36$

$\Rightarrow y = 4$

On substituting $y = 4$ in eq. (ii), we get

$x + 4 - 12 = 0$

$\Rightarrow x = 8$

$\Rightarrow x = 8$ and $y = 4$

On substituting the values of x and y in equation (1), we have

$10y + x = 10 \times 4 + 8 = 48$

\therefore The number is 48.

PYQ 2002

A two-digit number is 4 times the sum of its digits and twice the product of its digits. Find the number.

Let the digit in the ones place be x and tens place be y

Hence the two digit number = $10y + x$

Given that the two digit number = 4 times sum of its digits

$$\Rightarrow 10y + x = 4(x + y)$$

$$\Rightarrow 10y + x = 4x + 4y$$

$$\Rightarrow 3x - 6y = 0$$

$$\Rightarrow 3x = 6y$$

$$\therefore x = 2y \rightarrow (1)$$

It is also given that the two digit number = 2 times product of its digits

$$\Rightarrow 10y + x = 2xy$$

Divide by xy both the sides, we get

$$10/x + 1/y = 2$$

put $x = 2y$ from 1, we get

$$6/y = 3$$

so $y = 2$ and hence $x = 6$

Hence $x = 6$

The two digit number is $(10y + x) =$

$$10(3) + 6 = 36$$

PYQ 2005

Question 18

Two numbers are in the ratio 5:6. If 8 is subtracted from each of the numbers, the ratio becomes 4:5, then find the numbers.

Let the two numbers be x and y

Then, by first condition, ratio of these two numbers = 5 : 6

$$x:y = 5:6$$

$$\frac{x}{y} = \frac{5}{6} \Rightarrow y = \frac{6x}{5}$$

And by second condition, then, 8 is subtracted from each of the numbers, then ratio becomes 4:5

$$\frac{x-8}{y-8} = \frac{4}{5}$$

$$\Rightarrow 5x - 40 = 4y - 32$$

$$\Rightarrow 5x - 4y = 8$$

Now, put the value of y in Eq. (ii), we get

$$5x - 4\left(\frac{6x}{5}\right) = 8$$

$$\Rightarrow 25x - 24x = 40$$

$$\Rightarrow x = 40$$

Put the value of x in Eq (i), we get

$$y = \frac{6}{5} \times 40$$

$$= 6 \times 8 = 48$$

The denominator of a fraction is 4 more than twice its numerator. Denominator becomes 12 times the numerator, if both the numerator and the denominator are reduced by 6. Find the fraction.

Let the fraction be $\frac{x}{y}$.

Denominator of a fraction is 4 more than twice its numerator.

So,

$$y = 4 + 2x$$

$$\Rightarrow 2x - y = -4 \dots\dots (I)$$

Also, denominator becomes 12 times the numerator, if both the numerator and the denominator are reduced by 6.

So,

$$y - 6 = 12(x - 6)$$

$$\Rightarrow y - 6 = 12x - 72$$

$$\Rightarrow 12x - y = 72 - 6 = 66$$

$$\Rightarrow 12x - y = 66 \dots\dots (II)$$

Subtracting (I) from (II)

$$12x - 2x - y - (-y) = 66 - (-4)$$

$$\Rightarrow 10x = 70$$

$$\Rightarrow x = \frac{70}{10} = 7$$

$$\Rightarrow x = 7$$

Putting the value of $x = 7$ in (I)

$$2(7) - y = -4$$

$$\Rightarrow 14 - y = -4$$

$$\Rightarrow y = 14 + 4 = 18$$

Thus, the fraction obtained is $\frac{7}{18}$.

PYQ 2008

the sum of the numerator the denominator
of a fraction is 4 more than twice the
numerator if 3 is added to each of the
numerator and denominator their ratio
becomes 2:3. find the fraction

Let the numerator be x and denominator be y .

$$x + y = 2x + 4 \Rightarrow y = x + 4$$

$$\frac{x+3}{y+x} = \frac{2}{3}$$

$$\Rightarrow \frac{x+3}{x+4+3} = \frac{2}{3}$$

$$\Rightarrow 3x+9 = 2x+14$$

$$\Rightarrow x=5$$

$$y=9$$

Fraction is $\frac{5}{9}$.

PYQ 2001, 2010

The sum of the numerator and denominator of a fraction is 3 less than twice the denominator. If the numerator and denominator are decreased by 1, the numerator becomes half the denominator. Determine the fraction.

Let the numerator and denominator of the fraction be x and y respectively.

Then the fraction is $\frac{x}{y}$.

The sum of the numerator and denominator of the fraction is 3 less than twice the denominator. Thus, we have:

$$x + y = 2y - 3$$

$$\Rightarrow x + y - 2y + 3 = 0$$

$$\Rightarrow x - y + 3 = 0$$

If the numerator and denominator are decreased by 1, the numerator becomes half the denominator. Thus, we have

$$x - 1 = \frac{1}{2}(y - 1)$$

$$\Rightarrow \frac{x - 1}{y - 1} = \frac{1}{2}$$

$$\Rightarrow \frac{x - 1}{y - 1} = \frac{1}{2}$$

$$\Rightarrow 2(x - 1) = y - 1$$

$$\Rightarrow 2x - 2 = y - 1$$

$$\Rightarrow 2x - y - 1 = 0$$

So, we have two equations

$$x - y + 3 = 0$$

$$2x - y - 1 = 0$$

Here x and y are unknowns. We have to solve the above equations for x and y .

By using cross-multiplication, we have

$$\frac{x}{(-1)(-1) - (-1)3} = \frac{y}{(1)(-1) - 2 \times 3} = \frac{1}{(1)(-1) - 2 \times (-1)}$$

$$\Rightarrow \frac{x}{1+1-1+6} = \frac{1}{-1+6-1+2}$$

$$\Rightarrow \frac{x}{4} = \frac{1}{4}$$

$$\Rightarrow \frac{x}{4} = \frac{1}{4}$$

$$\Rightarrow x = 4, y = 7$$

PYQ 2001, 2010

Father's age is 3 times the sum of Ages of his two children . after 5 years his age will be twice the sum of Ages of two children find the age of father

let the age of two children be x and y .
father's present age is $3(x+y)$.

$$\begin{aligned}\text{age of children after 5 years} &= x+5+y+5 \\ &= x+y+10\end{aligned}$$

$$\text{age of the father after 5 years} =$$

$$2(x+y+10) \text{ or } 3(x+y)+5$$

$$2x+2y+20 = 3x+3y+5$$

$$2x-3x + 2y-3y = -20+5$$

$$-(x+y) = -15$$

$$x+y = 15$$

therefore, present age of father is

$$3(x+y) = 3(15)$$

$$=45.$$

PYQ 2003, 2019

A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car, it takes him 4 hours. but if he travels 130 km by train and the rest by car, it takes 18 minutes more. Find the speed of the train and that of the car.

Let the speed of the train be x km/hr and that of the car be y km/hr. We have following cases:

Case I When he travels 250 km by train and the rest by car.

In this case, we have

$$\text{Time taken by the man to travel } 250 \text{ km by train} = \frac{250}{x} \text{ hrs}$$

Time taken by the man to travel

$$(370 - 250) = 120 \text{ km by car} = \frac{120}{y} \text{ hrs}$$

\therefore Total time taken by the man to cover

$$370 \text{ km} = \frac{250}{x} + \frac{120}{y}$$

It is given that the total time taken is 4 hours

$$\therefore \frac{250}{x} + \frac{120}{y} = 4 \\ \Rightarrow \frac{125}{x} + \frac{60}{y} = 2 \quad (\text{i})$$

$$130u + 240v = \frac{43}{10} \quad (\text{iv})$$

Multiplying equation (iii) by 4 the given system of equations becomes

$$500u + 240v = 8 \quad (\text{v})$$

$$130u + 240v = \frac{43}{10} \quad (\text{vi})$$

Subtracting equation (vi) from equation (v), we get

$$370u = 8 - \frac{43}{10} \Rightarrow 370u = \frac{37}{10} \Rightarrow u = \frac{1}{100}$$

Putting $u = \frac{1}{100}$ in equation (v), we get

$$5 + 240v = 8 \Rightarrow 240v = 3 \Rightarrow v = \frac{1}{80}$$

Now, $u = \frac{1}{100}$ and $v = \frac{1}{80}$

$$\Rightarrow \frac{1}{x} = \frac{1}{100} \text{ and } \frac{1}{y} = \frac{1}{80}$$

$$\Rightarrow x = 100 \text{ and } y = 80$$

Hence, Speed of the train = 100 km/hr

Speed of the car = 80 km/hr.

Case II When he travels 130 km by train and the rest by car:

In this case, we have

$$\text{Time taken by the man to travel } 130 \text{ km by train} = \frac{130}{x} \text{ hrs}$$

Time taken by the man to travel

$$(370 - 130) = 240 \text{ km by car} = \frac{240}{y} \text{ hrs.}$$

In this case, total time of the journey is 4 hrs 18 minutes.

$$\begin{aligned} & \therefore \frac{130}{x} + \frac{240}{y} = 4 \text{ hrs 18 minutes} \\ & \Rightarrow \frac{130}{x} + \frac{240}{y} = 4 \frac{18}{60} \\ & \Rightarrow \frac{130}{x} + \frac{240}{y} = \frac{43}{10} \quad (\text{ii}) \end{aligned}$$

Thus, we obtain the following system of equations:

$$\begin{aligned} & \frac{125}{x} + \frac{60}{y} = 2 \\ & \frac{130}{x} + \frac{240}{y} = \frac{43}{10} \end{aligned}$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the given system reduces to

$$125u + 60v = 2 \quad (\text{iii})$$

PYQ 2001

Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

Step 1: Given data

Let the speed of the first car at A be x kmph and the speed of the car at B be y kmph

When the cars travel in the same direction Relative Speed is $S=x-y$

Distance $D=100$ km

Time, $t=5$ hours

Step 2: Formula used

$$\text{Since, Speed}(S) = \frac{\text{Distance}(D)}{\text{Time}(T)}$$

Therefore, Distance $(D) = S \times T$

Step 3: Calculating the speed of the car

Substitute the given values in $(D) = S \times T$

we get

$$\Rightarrow 100 = (x - y) \times 5$$

$$\Rightarrow 100 = (x - y) \times 5$$

$$\therefore x - y = 20 \dots\dots\dots (i)$$

When cars travel in opposite directions, the relative Speed is $x+y$
Distance=100 km

Time (T)=1 hours

$$\text{Distance } D = S \times T$$

$$\Rightarrow 100 = (x + y)$$

$$x + y = 100 \dots\dots\dots (ii)$$

Adding equations (i) & (ii), we get

$$x - y + x + y = 20 + 100$$

$$\Rightarrow 2x = 120$$

$$\therefore x = 60 \text{ kmh}^{-1}$$

From equation(i), $y = 60 - 20$

$$\therefore y = 40 \text{ kmh}^{-1}$$

Hence, the speed of the car at A = 60 kmh^{-1} and

Speed of the car at B = 40 kmh^{-1}

PYQ 2009, NCERT

Two years ago, a father was five times as old as his son. Two years later, his age will be 8 more than three times the age of the son. Find the present ages of father and son.

Let father age be x , his son age be y .

Before 2 years,

$$\Rightarrow (x - 2) = 5(y - 2) \dots\dots\dots(1)$$

After 2 years

$$\Rightarrow (x + 2) = 8 + 3(y + 2) \dots\dots\dots(2)$$

$$\text{From (1)} \ x - 5y + 8 = 0$$

From (2)

$$\Rightarrow x + 2 - 3y - 6 - 8 = 0$$

$$\Rightarrow x - 3y - 12 = 0$$

Solving them

$$\Rightarrow -2y + 20 = 0$$

$$\Rightarrow y = 10$$

$$\text{When } y = 10, x = 42$$

PYQ 2004

A boat goes 30 km upstream and 44 downstream in 10 hours. The same boat goes 40 km upstream and 55 km downstream in 13 hours. On this information some student guessed the speed of the boat in still water as 8.5 km/h and speed of the boat in still water as 8.5 km/h and speed of the stream as 3.8 km/h. Do you agree with their guess?

Let the speed of boat = x km/hr.
 Let the speed of stream = y km/hr.
 Net Speed of boat in upstream = $(x - y)$ km/hr.
 Net Speed of boat in downstream = $(x + y)$ km/hr.
 Time taken to cover 30 km upstream
 $= \frac{30}{x-y}$ hrs.
 Time taken to cover 40 km downstream
 $= \frac{44}{x+y}$ hrs.
 According to question,
 Total time taken = 10 hrs.
 $\frac{30}{x-y} + \frac{44}{x+y} = 10 \dots (i)$
 Now, Time taken to cover 55 km downstream
 $= \frac{55}{x+y}$ hrs.
 Time taken to cover 40 km upstream
 $= \frac{40}{x-y}$ hrs.
 Total time taken = 13 hrs.

Solving eq. (i) and eq. (ii).
 Let $\frac{1}{x-y} = u$, $\frac{1}{x+y} = v$
 $30u + 44v = 10 \dots (iii)$
 $40u + 55v = 13 \dots (iv)$
 Multiplying eq. (iii) by 4 and eq. (iv) by 3, and subtracting we get
 $120u + 176v = 40$
 $120u + 165v = 39$
 $- \quad - \quad -$
 $11v = 1$
 $v = \frac{1}{11}$
 Putting the value of v in eq. (iii)
 $30u + 44v = 10 \rightarrow 30u + 44 \times \frac{1}{11} = 10$
 $\rightarrow 30u + 4 = 10$
 $\rightarrow 30u = 6$
 $\rightarrow u = \frac{6}{30}$
 or $u = \frac{1}{5}$
 Now,
 $v = \frac{1}{11}$
 $\rightarrow \frac{1}{x+y} = \frac{1}{11}$
 $\rightarrow x + y = 11 \dots (v)$

And $u = \frac{1}{5}$
 $\rightarrow \frac{1}{x-y} = \frac{1}{5}$
 $\rightarrow x - y = 5 \dots (vi)$
 On solving eq. (v) and (vi)
 $x + y = 11$
 $x - y = 5$

$2x = 16$
 or $x = 8$
 Put the value of x in eq. (v)
 $8 + y = 11$
 $y = 11 - 8$
 $y = 3$

The speed of boat in still water = 8 km/hr.
 The speed of stream = 3 km/hr.
 We learn that the speed of boat is slow in upstream and fast in downstream

PYQ 2019

A train covered a certain distance at a uniform speed. If the train would have been 10km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10km/h ; it would have taken 3 hours more than the scheduled time. The distance covered by the train is

Let the speed of the train be x km/h and the time has taken be y h.

The total distance is then xy km.

Case1:

Speed increases by 10 km/h and the time taken reduces by 2 hours.

The distance traveled remains xy km.

$$(x + 10)(y - 2) = xy$$

$$\text{Therefore, } xy - 2x + 10y - 20 = xy$$

$$\Rightarrow -2x + 10y - 20 = 0$$

$$\Rightarrow -2x + 10y = 20 \quad \text{--- (1)}$$

Case 2:

Speed decreases by 10 km/h, then the time taken increases by 3 hours.

However, the distance remains, xy km.

$$(x - 10)(y + 3) = xy$$

$$\text{Therefore, } xy + 3x - 10y - 30 = xy$$

$$\Rightarrow 3x - 10y - 30 = 0$$

$$\Rightarrow 3x - 10y = 30 \quad \text{--- (2)}$$

$$\Rightarrow x = 50 \text{ km/h}$$

Using this in (2), we get

Using this in (2), we get

$$150 - 10y = 30$$

$$\Rightarrow y = 12$$

Therefore, the distance is $50 \times 12 = 600$ km.

PYQ 2020

A boat goes 30 km upstream and 44 downstream in 10 hours. The same boat goes 40 km upstream and 55 km downstream in 13 hours. On this information some student guessed the speed of the boat in still water as 8.5 km/h and speed of the boat in still water as 8.5 km/h and speed of the stream as 3.8 km/h. Do you agree with their guess?

Let the speed of boat = x km/hr.
 Let the speed of stream = y km/hr.
 Net Speed of boat in upstream = $(x - y)$ km/hr.
 Net Speed of boat in downstream = $(x + y)$ km/hr.
 Time taken to cover 30 km upstream
 $= \frac{30}{x-y}$ hrs.
 Time taken to cover 40 km downstream
 $= \frac{40}{x+y}$ hrs.
 According to question,
 Total time taken = 10 hrs.
 $\frac{30}{x-y} + \frac{40}{x+y} = 10 \dots (i)$
 Now, Time taken to cover 55 km downstream
 $= \frac{55}{x+y}$ hrs.
 Time taken to cover 40 km upstream
 $= \frac{40}{x-y}$ hrs.
 Total time taken = 13 hrs.

Solving eq. (i) and eq. (ii).

Let $\frac{1}{x-y} = u$, $\frac{1}{x+y} = v$

$$30u + 44v = 10 \dots (iii)$$

$$40u + 55v = 13 \dots (iv)$$

Multiplying eq. (iii) by 4 and eq. (iv) by 3, and subtracting we get

$$120u + 176v = 40$$

$$120u + 165v = 39$$

$$\begin{array}{rcl} 11v &=& 1 \\ v &=& \frac{1}{11} \end{array}$$

Putting the value of v in eq. (iii)

$$30u + 44v = 10 \Rightarrow 30u + 44 \times \frac{1}{11} = 10$$

$$\Rightarrow 30u + 4 = 10$$

$$\Rightarrow 30u = 6$$

$$\Rightarrow u = \frac{6}{30}$$

$$\text{or } u = \frac{1}{5}$$

Now,

$$v = \frac{1}{11}$$

$$\Rightarrow \frac{1}{x+y} = \frac{1}{11}$$

$$\Rightarrow x + y = 11 \dots (v)$$

And $u = \frac{1}{5}$

$$\Rightarrow \frac{1}{x-y} = \frac{1}{5}$$

$$\Rightarrow x - y = 5 \dots (vi)$$

On solving eq. (v) and (vi)

$$x + y = 11$$

$$x - y = 5$$

$$2x = 16$$

$$\text{or } x = 8$$

Put the value of x in eq. (v)

$$8 + y = 11$$

$$y = 11 - 8$$

$$y = 3$$

The speed of boat in still water = 8 km/hr.

The speed of stream = 3 km/hr.

We learn that the speed of boat is slow in upstream and fast in downstream.

PYQ 2019 NCERT

A shopkeeper buys a number of books for ₹ 80. If he had bought 4 more books for the same amount, each book would have cost ₹ 1 less. He bought ____ books.

Let the number of books bought be x .
Then,

Cost of x books = ₹ 80 \Rightarrow Cost of one book = ₹ $\frac{80}{x}$

If the number of books bought is $x + 4$, then

Cost of one book = ₹ $\frac{80}{x+4}$

It is given that the cost of one book is reduced by one rupee.

$$\therefore \frac{80}{x} - \frac{80}{x+4} = 1$$

$$\Rightarrow 80 \left(\frac{1}{x} - \frac{1}{x+4} \right) = 1$$

$$\Rightarrow 80 \left\{ \frac{x+4-x}{x(x+4)} \right\} = 1$$

$$\Rightarrow \frac{320}{x^2+4x} = 1$$

$$\Rightarrow x^2 + 4x = 320$$

$$\Rightarrow x^2 + 4x - 320 = 0$$

$$\Rightarrow x^2 + 20x - 16x - 320 = 0$$

$$\Rightarrow x(x+20) - 16(x+20) = 0$$

$$\Rightarrow (x+20)(x-16) = 0$$

$$\Rightarrow x = -20 \text{ or, } x = 16 \Rightarrow x = 16 [\because x \text{ cannot be negative}]$$

Hence, the number of books is 16.

A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days, she has to pay Rs. 1000 as hostel charges whereas a student B, who takes food for 12 days, pays Rs. 1180 as hostel charges. Find the fixed charges and the cost of food per day.

Let the fixed charge be Rs. x and the variable charge be Rs. y per day.

It is given that when a student A takes food for 20 days she has to pay Rs. 1000 as hostel charges, therefore,

$$\begin{aligned}x - 20y &= 1000 \\ \Rightarrow x &= 1000 - 20y \quad \dots\dots(1)\end{aligned}$$

It is also given that when a student B takes food for 12 days she has to pay Rs. 1180 as hostel charges, therefore, we have:

$$x - 26y = 1180$$

Let us now substitute the value of x from equation 1 as follows:

$$\begin{aligned}x - 26y &= 1180 \\ \Rightarrow 1000 - 20y + 26y &= 1180 \\ \Rightarrow -20y + 26y &= 1180 - 1000 \\ \Rightarrow 6y &= 180\end{aligned}$$

$$\begin{aligned}\Rightarrow y &= \frac{180}{6} \\ \Rightarrow y &= 30\end{aligned}$$

Now, substitute the value of y in equation 1:

$$x = 1000 - (20 \times 30) = 1000 - 600 = 400$$

Therefore, $x = 400$ and $y = 30$.

Hence, the fixed charges is Rs. 400 and the variable charges is Rs. 30 per day.

PYQ 2000, NCERT

A shopkeeper buys a number of books for ₹ 80. If he had bought 4 more books for the same amount, each book would have cost ₹ 1 less. He bought ____ books.

Let the number of books bought be x .
Then,

Cost of x books = ₹ 80 \Rightarrow Cost of one book = ₹ $\frac{80}{x}$

If the number of books bought is $x + 4$, then

Cost of one book = ₹ $\frac{80}{x+4}$

It is given that the cost of one book is reduced by one rupee.

$$\therefore \frac{80}{x} - \frac{80}{x+4} = 1$$

$$\Rightarrow 80 \left(\frac{1}{x} - \frac{1}{x+4} \right) = 1$$

$$= 80 \left(\frac{x+4-x}{x(x+4)} \right) = 1$$

$$\Rightarrow \frac{320}{x^2+4x} = 1$$

$$\Rightarrow x^2 + 4x = 320$$

$$\Rightarrow x^2 + 4x - 320 = 0$$

$$\Rightarrow x^2 + 20x - 16x - 320 = 0$$

$$\Rightarrow x(x+20) - 16(x+20) = 0$$

$$\Rightarrow (x+20)(x-16) = 0$$

$\Rightarrow x = -20$ or, $x = 16 \Rightarrow x = 16$ [$\because x$ cannot be negative]

Hence, the number of books is 16.

NCERT

Points A and B are 70 km apart on a highway. A car starts from A and another car starts from B simultaneously. If they travel in the same direction, they meet in 7 hours. But, if they travel towards each other, they meet in 1 hour. Find the speed of each car.

Let the speed of the car starting from A be x km/hr and that from B be y km/hr
If they are travelling in opposite direction ie towards each other then effective speed will be $(x + y)$ km/hr
If they are travelling in same direction then effective speed will be $(x - y)$ km/hr
According to the question,

$$\Rightarrow \frac{70}{x-y} = 7 \dots \text{(i)} \text{ and } \frac{70}{x+y} = 1 \dots \text{(ii)}$$

From (i) and (ii), we get,

$$\Rightarrow x - y = 10 \dots \text{(iii)} \text{ and } x + y = 70 \dots \text{(iv)}$$

Adding (iii) and (iv),

$$\Rightarrow 2x = 80$$

$$\therefore x = 40 \text{ km/hr}$$

$$\text{Hence, } y = 70 - 40 = 30 \text{ km/hr}$$

PYQ 2008

Question

In a $\triangle ABC$, right angled at A, if $\tan C = \sqrt{3}$, find the value of $\sin B \cos C + \cos B \sin C$.

NCERT EXEMPLAR

Question

Prove that $\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta} = \tan \theta + \cot \theta$.

PYQ 2016

Question

Simplify: $(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta)$.

PYQ 2000, 2000C

Prove that :

$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{\sin^2 A - \cos^2 A} = \frac{2}{2\sin^2 A - 1} = \frac{2}{1 - 2\cos^2 A}$$

NCERT PYQ 2000, 2019

Question

$$(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A.$$

NCERT EXEMPLAR CBSE 2000

$$\frac{\sin \theta}{(1 + \cos \theta)} + \frac{(1 + \cos \theta)}{\sin \theta} = 2 \csc \theta$$

PYQ 2008, 2019

Question

Prove that: $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$.

NCERT PYQ 2002, 2019

Sum

Prove that : $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} = \sec A \cdot \csc A + 1.$

NCERT EXEMPLAR PYQ 2002

Sum

If $\sin \theta + \sin^2 \theta = 1$ show that: $\cos^2 \theta + \cos^4 \theta = 1$

PYQ 2009

Question

Prove that

$$\sin^3 \theta + \cos^3 \theta = (1 - \sin \theta \cdot \cos \theta)(\sin \theta + \cos \theta)$$

NCERT EXEMPLAR PYQ 2002

Sum

Prove that $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$.

PYQ 2000, 2019

Question

Prove that: $\frac{\sin A}{(\cot A + \operatorname{cosec} A)} = 2 + \frac{\sin A}{(\cot A - \operatorname{cosec} A)}$

PYQ 2000, 2019

Question

Prove that: $\frac{\sin A}{(\cot A + \operatorname{cosec} A)} = 2 + \frac{\sin A}{(\cot A - \operatorname{cosec} A)}$

NCERT EXEMPLAR PYQ 2012

Question

If $\sin \theta + \cos \theta = \sqrt{2}$, then find the value of $\tan \theta + \cot \theta$.

NCERT PYQ 2000, 2006

Prove that :

$$(\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A =$

Solution

Given:

$$\cos A + \cos^2 A = 1$$

$$\Rightarrow 1 - \cos^2 A = \cos A$$

So,

$$\sin^2 A + \sin^4 A$$

$$= \sin^2 A + \sin^2 A \cos^2 A$$

$$= \sin^2 A + (1 - \cos^2 A)(1 - \cos^2 A)$$

$$= \sin^2 A + \cos A \cos A$$

$$= \sin^2 A + \cos^2 A$$

$$= 1$$

Question

Question 1

If $\operatorname{cosec} \theta + \cot \theta = p$, then prove that

$$\cos \theta = \frac{p^2 - 1}{p^2 + 1}$$

Given $\operatorname{cosec} \theta + \cot \theta = p$

$$\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} = p$$

$$1 + \cot \theta = p$$

using properties of cos and sin we get

$$\Rightarrow \operatorname{cosec}^2 \theta = 2 \cos^2 \frac{\theta}{2} - 1 \quad \& \quad \sin \theta = 2 \cos \frac{\theta}{2} \sin \frac{\theta}{2}$$
$$\frac{1 + \cot^2 \frac{\theta}{2} - 1}{2 \cos \frac{\theta}{2} \sin \frac{\theta}{2}} = p$$

$$\Rightarrow \left[\cot^2 \frac{\theta}{2} = p \right] \quad \text{--- (1)}$$

To prove $\cos \theta = \frac{p^2 - 1}{p^2 + 1}$

$$\text{R.H.S.} = \frac{p^2 - 1}{p^2 + 1} \quad \text{using eq (1)}$$

$$\Rightarrow \frac{\cot^2 \frac{\theta}{2} - 1}{\cot^2 \frac{\theta}{2} + 1} = \frac{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}{\sin^2 \frac{\theta}{2}}$$
$$\frac{\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2}}{\sin^2 \frac{\theta}{2}}$$

$$\frac{p^2 - 1}{p^2 + 1} = \cos \theta \cdot \left[\frac{\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} = 1}{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} = \cos \theta} \right]$$

Hence [LHS = RHS]

Evaluate

$$(\sin A + \cos A)(\tan A + \cot A)$$

The value of $(\sin A + \cos A)(\tan A + \cot A)$ is

$$\begin{aligned}&= (\sin A + \cos A) \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right) \\&= (\sin A + \cos A) \left(\frac{\sin^2 A + \cos^2 A}{\cos A \sin A} \right) \\&= \frac{\sin A + \cos A}{\cos A \sin A} \\&= \frac{\sin A}{\cos A \sin A} + \frac{\cos A}{\cos A \sin A} \\&= \frac{1}{\cos A} + \frac{1}{\sin A} \\&= \sec A + \operatorname{cosec} A\end{aligned}$$

If $\sin \theta + 2 \cos \theta = 1$ prove that
 $2 \sin \theta - \cos \theta = 2.$

Consider the given equation.

$$\sin \theta + 2 \cos \theta = 1 \dots\dots\dots (1)$$

On squaring both sides, we get

$$(\sin \theta + 2 \cos \theta)^2 = 1^2$$

$$\sin^2 \theta + 4 \cos^2 \theta + 4 \sin \theta \cos \theta = 1$$

We know that

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

Therefore,

$$1 - \cos^2 \theta + 4(1 - \sin^2 \theta) + 4 \sin \theta \cos \theta = 1$$

$$- \cos^2 \theta + 4 - 4 \sin^2 \theta + 4 \sin \theta \cos \theta = 0$$

$$-\cos^2 \theta - 4 \sin^2 \theta + 4 \sin \theta \cos \theta = -4$$

$$4 \sin^2 \theta + \cos^2 \theta - 4 \sin \theta \cos \theta = 4$$

$$(2 \sin \theta - \cos \theta)^2 = 4$$

$$2 \sin \theta - \cos \theta = 2$$

Hence, proved.

If $1 + \sin^2 A = 3\sin A \cos A$, then prove that
 $\tan A = 1$ or $1/2$

$$1 + \sin^2 A = 3\sin A \cos A.$$

$$\cos^2 A + \sin^2 A + \sin^2 A = 3\sin A \cos A [1 = \sin^2 + \cos^2]$$

$$\cos^2 A + 2\sin^2 A = 3\sin A \cos A \dots (1)$$

DIVIDE (1) BY $\cos^2 A$ we get,

$$1 + 2\tan^2 A = 3\tan A$$

$$1 + 2\tan^2 A - 3\tan A = 0$$

$$(2\tan A - 1)(\tan A - 1) = 0$$

$$2\tan A - 1 = 0. \tan A - 1 = 0$$

$$2\tan A = 1. \tan A = 1$$

$$\tan A = 1/2. \tan A = 1$$

Q. If $\tan \theta + \sec \theta = x$, Prove that $\sec \theta = \frac{x^2 - 1}{2x}$

Ans. $\tan \theta + \sec \theta = x$

$$\therefore \tan \theta = x - \sec \theta \quad] \text{ squaring}$$

$$\tan^2 \theta = x^2 + \sec^2 \theta - 2x \sec \theta$$

$$\sec^2 \theta - 1 = x^2 - 2x \sec \theta + \sec^2 \theta$$

$$-1 = x^2 - 2x \sec \theta$$

$$x^2 + 1 = 2x \sec \theta$$

$$\therefore \sec \theta = \frac{x^2 + 1}{2x}$$

Question 1 (x)

Find the zeroes of the following polynomials by factorization method and verify the relations between the zeroes and the coefficient of the polynomials

(x) $7y^2 - \frac{11}{3}y - \frac{2}{3}$

x) Let $f(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$
= $21y^2 - 11y - 2$
= $21y^2 - 14y + 3y - 2$ [by splitting the middle term]
= $7y(3y - 2) + 1(3y - 2)$
= $(3y - 2)(7y + 1)$

So, the value of $7y^2 - \frac{11}{3}y - \frac{2}{3}$ is zero when

$$3y - 2 = 0 \text{ or } 7y + 1 = 0$$

i.e., when $y = \frac{2}{3}$ or $y = -\frac{1}{7}$

so, the zeroes $7y^2 - \frac{11}{3}y - \frac{2}{3}$ are $\frac{2}{3}$ and $-\frac{1}{7}$

$$\therefore \text{sum of zeroes} = \frac{2}{3} - \frac{1}{7} = \frac{14-3}{21} = \frac{11}{21} = -\left(\frac{-11}{3 \times 7}\right)$$

$$= (-1) \left(\frac{\text{coefficient of } y}{\text{coefficient of } y^2} \right)$$

$$\text{And product of zeroes} = \left(\frac{2}{3}\right)\left(-\frac{1}{7}\right) = \left(-\frac{2}{21}\right)$$

$$= \frac{\text{Constant term}}{\text{Coefficient of } y^2}$$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

If $(x+a)$ is a factor of $2x^2+2ax+5x+10$ find a

Ans:

$x+a$ is a factor of $2x^2+2ax+5x+10$

therefore $f(-a)=0$;

substituting values,

$$2(-a)^2 + 2(a)(-a) + 5(-a) + 10 = 0$$

$$2a^2 - 2a^2 - 5a + 10 = 0$$

$$-5a + 10 = 0$$

$$\text{therefore } a = 2$$

CBSE 2008

Find the quadratic polynomial whose zeros are reciprocals of the zeros of the polynomials $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$.

Let p, q be zeros of $ax^2 + bx + c$

$$\therefore p + q = -\frac{b}{a} \text{ & } pq = \frac{c}{a}$$

Let P & Q be zeros of required polynomial

It is given that $P = 1/p$, $Q = 1/q$

Then,

$$P + Q = \frac{1}{p} + \frac{1}{q} = \frac{q + p}{pq} = \frac{-b/a}{c/a} = -\frac{b}{c}$$

$$PQ = \frac{1}{p} \cdot \frac{1}{q} = \frac{1}{pq} = \frac{a}{c}$$

\therefore Required polynomial =

$$x^2 + \frac{b}{c}x + \frac{a}{c} \Rightarrow cx^2 + bx + a$$

Find the area of the triangle made by the line $2x + 3y = 12$ with the co-ordinate axes.

The given equation of the line $2x + 3y = 12$ can be written as $\frac{x}{6} + \frac{y}{4} = 1$.

From the equation it is clear that the line made intercepts of length 6 and 4 from the x and y axes respectively.

Then we get a triangle whose base is 6 units and perpendicular is 4 units.

Then area of the triangle is $\frac{1}{2} \times 6 \times 4 = 12$ unit².

PYQ 2002

NCERT EXEMPLAR PYQ 2006

Question

The mean of the following frequency table is 50. But the frequencies f_1 and f_2 in class 20-40 and 60-80 are missing. Find the missing frequencies.

Class:	0-20	20-40	40-60	60-80	80-100	Total
Frequency:	17	f_1	32	f_2	19	120

NCERT EXEMPLAR PYQ 2018

Question

The mean of the following distribution is 18.
Find the frequency f of the class 19 - 21.

Class	11 - 13	13 - 15	15 - 17	17 - 19	1
Frequency	3	6	9	13	

19 - 21	21 - 23	23 - 25
f	5	4

NCERT, PYQ 2019

Question

The table below- shows the daily expenditure on food of 25 households in a locality.

100-150	150-200	200-250	250-300	300-350
4	5	12	2	2

Find the mean daily expenses on food by a suitable method