Unit 1: DIFFRACTION

Q1.1. What do you mean by diffraction? State its types and differentiate between them.

(M.U. May 09, 11; Dec. 2009, 11, 15) (3 m)

As waves come into contact with obstacles (or openings), they bend around the edges of the obstacles if the obstacles' dimensions are equal to the waves' wavelength. Diffraction is the bending of waves around the edges of an obstacle (or opening).

Basically, the diffraction phenomenon has two main types:

(A) Fresnel Diffraction
(B) Fraunhoffer Diffraction

Difference between Fresnel diffraction and Fraunhoffer diffraction

Fresnel Diffraction	Fraunhoffer Diffraction
The source and slits are both at finite distance from the slit.	The source and slits are at infinite distance from the slit.
Both the incident and diffracted wavefronts are cylindrical or spherical.	Both the incident and diffracted wavefronts are plane.
The incident and diffracted rays are divergent.	The incident and diffracted rays are parallel.
Lenses are not required in actual experiment.	Lenses are used in experiment to achieve parallel wavefront.
Path difference between the rays forming the diffraction pattern depends on distance of slit from source as well as the screen and the angle of diffraction. Hence mathematical treatment is complicated.	Path difference between the rays forming the diffraction pattern depends only on the angle of diffraction. Hence mathematical treatment is comparatively easier.

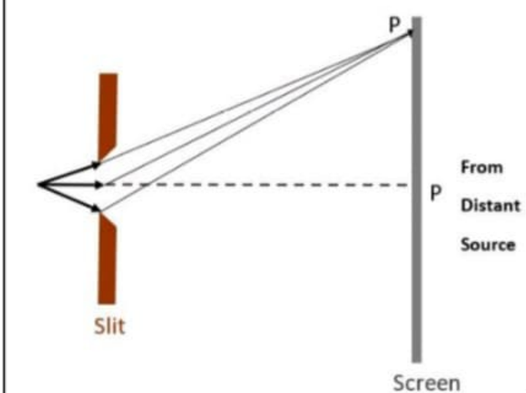


Figure 1.1.1: Fresnel Diffraction

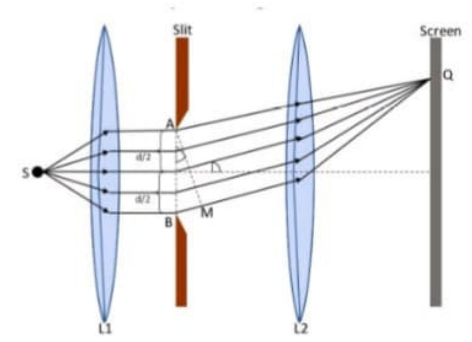


Figure 1.1.2: Fraunhofer Diffraction

Q1.2. Explain Fraunhoffer Diffraction at a single slit, obtain expression for the resultant intensity and derive expressions for maxima and minima for a single slit.

(M.U. May 2007) (5 m)

Let *AB* be a single narrow slit of width ***d*** on which a parallel beam of monochromatic light is incident as a plane wave front as shown in *Figure 1.2.1*.

The incident wave front is diffracted by the slit and is then focused on the screen by the lens L. According to Huygens principle, each point on AB acts as a source of secondary waves.

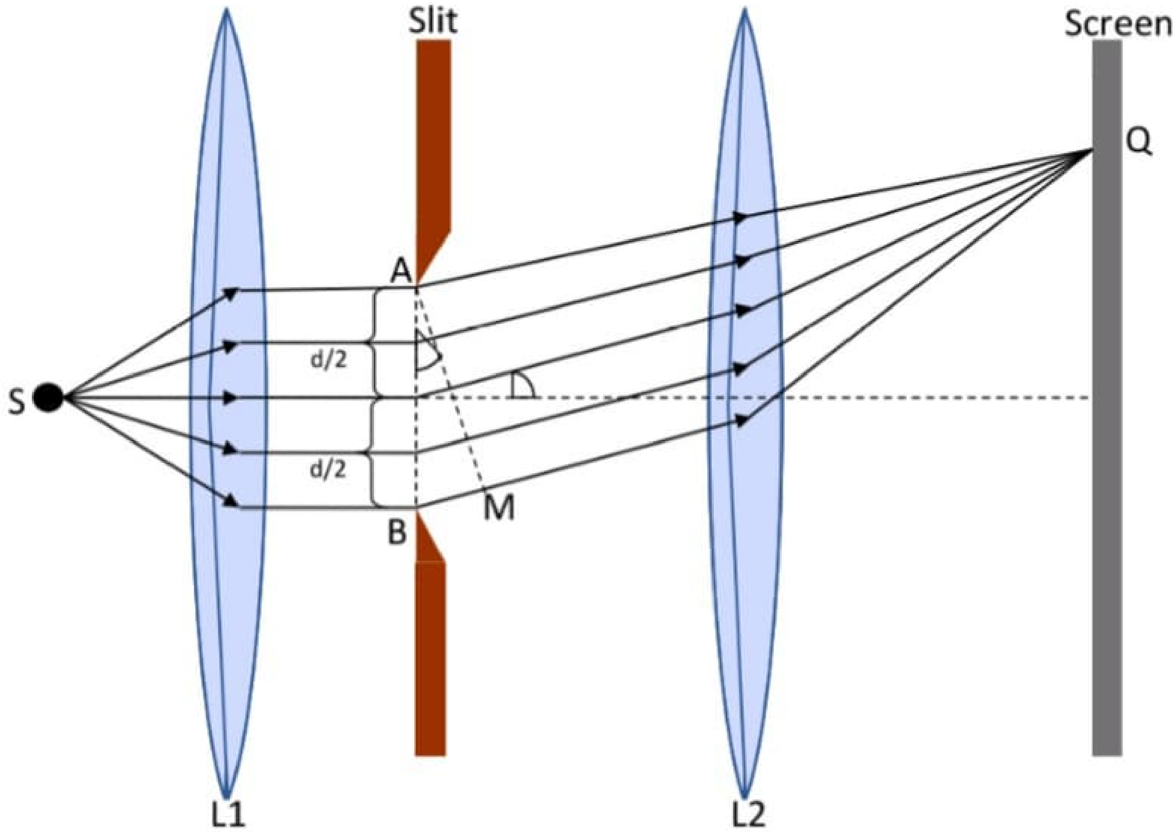


Figure 1.2.1: Fraunhofer Diffraction at a single slit

As all the points on AB are in phase, the point sources will be coherent. Hence the light from one portion of the slit can interfere with light from another portion and the resultant intensity on the screen will depend on the direction θ of the diffracted waves.

When the waves traveling in straight direction without diffraction, they are in the same phase and after covering equal optical path lengths their superposition produces zero order central maxima where the dotted line meets the screen. Lets refer this point as P.

Let us consider another point Q on the screen. The waves that leave the slit at an angle θ reach the point Q. The point Q will be dark or bright depending upon the path difference between the waves arriving at Q from different points on the wave front. The total path difference between the waves that are travelling from point A and point B on the slit is :-

$$\text{path difference} = d\,sin\theta$$

This path difference corresponds to a total phase difference of :-

$$\phi = \frac{2\pi}{\lambda} d \sin\theta \qquad \text{...............(1)}$$

Derivation for Intensity variation :-

In order to determine the intensity distribution for the single slit diffraction pattern, we follow the graphical approach.

Consider the slit is divided into a large number (N) of narrow strips of equal width Δy .

Each strip acts as a source of coherent radiation and the light emanating from it can be represented by a short phasor.

The path difference between the rays diffracted from its upper and lower edges :-

$$\text{path difference for a strip} = dy.sin\theta$$

hence the corresponding phase difference between them is,

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta y \sin\theta \qquad \text{...............(2) such that } \phi = N\Delta\phi$$

(i) At the centre of diffraction pattern θ=0 hence net phase difference is zero, and the phasors in this case are laid end to end as shown in *Figure 1.2.2*

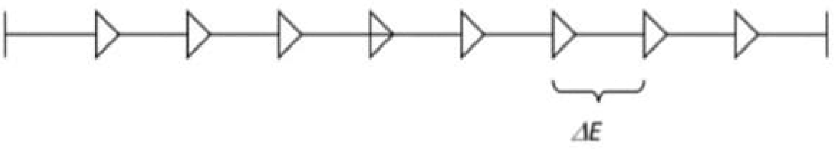


Figure 1.2.2: Resultant amplitude for diffraction angle=0

The amplitude of the resultant has its maximum value E_m given as :-

$$E_m = N\Delta E \quad \text{(slit is divided into N no. of parts)}$$

(ii) At a value of θ other than zero, $\Delta\phi$ has a finite value. The amplitude E_θ of the resultant is the vector sum of phasors, and hence given by the length of chord as shown in *Figure 1.2.3*. It can be seen that E_θ is less than E_m.

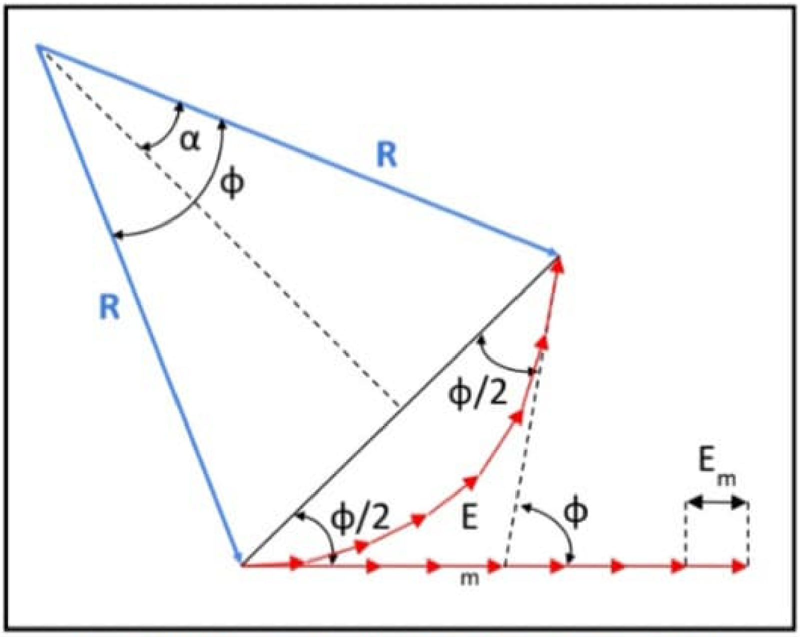


Figure 1.2.3: Resultant amplitude for diffraction angle=θ

The resultant amplitude E_θ equals the length of chord and E_m is the length of an arc as shown in the figure and ϕ is the measure of total phase difference between initial and final phasors and as opposite angles of quadrilateral are supplementary ϕ is also the angle of the sector formed between the two radii R.

Inside this sector the perpendicular drawn from the centre of the circle to the chord bisects the chord, hence

$$\sin\phi/2 = \frac{E_\theta/2}{R}$$

$$\therefore E_\theta = 2R\sin\phi/2$$

$$\text{But } \phi = \frac{E_m}{R} = \frac{arc}{radius}$$

$$E_\theta = 2\left(\frac{E_m}{\phi}\right)\sin\frac{\phi}{2}$$

Using all the above we can write expression for resultant amplitude at pt Q

$$\boxed{E_\theta = E_m\left[\frac{\sin\phi/2}{\phi/2}\right]} \quad \ldots\ldots\ldots\ldots\ldots (3)$$

The intensity I_θ is proportional to amplitude square E_θ^2,

$$I_\theta = I_m\left[\frac{\sin\phi/2}{\phi/2}\right]^2 \qquad \text{.................(4)}$$

Let $\frac{\phi}{2} = \alpha = \frac{\pi d \sin\phi}{\lambda}$ ....(5)

Hence, $$\boxed{I_\theta = I_m\left[\frac{\sin^2\alpha}{\alpha}\right]} \qquad \text{.................(6)}$$

The following graph shows the intensity variation with respect to angle ϕ in a single slit diffraction as shown in *Figure 1.2.4.*

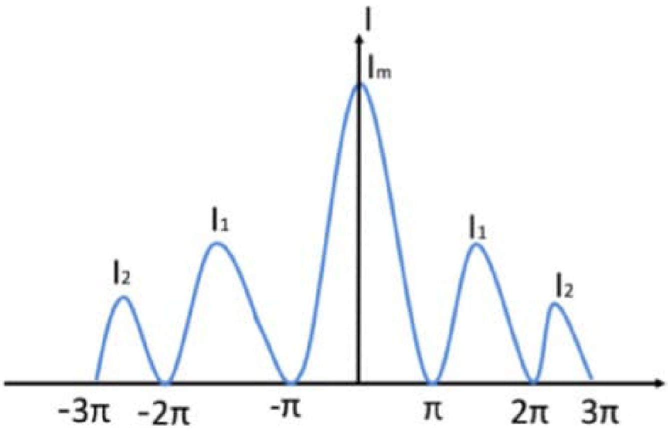


Figure 1.2.4: Intensity variation with respect to angle φ

Conditions for maxima and minima :-

(a) Principle Maximum:

The resultant amplitude in diffraction pattern is given by

$$E_\theta = E_m\left[\frac{\sin\alpha}{\alpha}\right]$$

For E_θ to be maximum, we need $\alpha = 0$ i.e $\alpha = \frac{\pi}{\lambda} d\sin\theta = 0$

$$\therefore \sin\theta = 0 \quad i.e.\ \theta = 0.$$

Thus principal maxima is obtained at $\theta = 0$.

(b) Minimum intensity positions (minima):

The intensity $I_\theta = I_m\left[\frac{\sin^2\alpha}{\alpha^2}\right]$ will be zero where *sin* $\alpha = 0$ but $\alpha \neq 0$

The values of α which satisfy that equation are :-

$\alpha = n\pi$ where $n = \pm 1, \pm 2, \pm 3, \ldots\ldots$

$$\alpha = \frac{\pi}{\lambda} d\sin\theta = n\pi$$

Hence, the condition for minima is

$$d \sin\theta = n\lambda \text{, where } n = \pm 1, \pm 2, \pm 3, \ldots\ldots$$

(Since θ becomes zero which corresponds to the principal maximum. The positions of minima are on either side of principal maximum.)

(c)Secondary maxima

Analysis shows that the secondary maxima lie approximately half way between the minima. i.e.

$$\alpha = \pm\left(n + \frac{1}{2}\right)\pi \qquad n = 1, 2, 3, \ldots\ldots$$

$$d \sin\theta = (2n+1)\frac{\lambda}{2}$$

Substituting this value of α in I_θ.

$$I_\theta = I_m\left[\frac{\sin^2\alpha}{\alpha^2}\right] \text{ we get,}$$

$$\frac{I_\theta}{I_m} = \left[\frac{\sin\left(n+\frac{1}{2}\right)\pi}{\left(n+\frac{1}{2}\right)\pi}\right]^2$$

$$= \frac{1}{\left(n+\frac{1}{2}\right)^2\pi^2} \qquad m = 1, 2, 3, \ldots\ldots$$

$$\frac{I_\theta}{I_m} = 0.045, 0.016, 0.0083, \ldots\ldots$$

Thus, the successive maxima decrease in intensity rapidly.

Q1.3. What is Diffraction Grating? Explain the construction of diffraction grating. Determination of Wavelength of Light using Grating.

(M.U. May 2008, 11, 13, 16, 17; Dec. 2009, 12, 17, Nov. 2018) (5 m)

The diffraction grating is an arrangement consisting of a large number of parallel slits of equal width separated from one another by equal opaque spaces.

A diffraction grating is formed by ruling a plane glass plate with fine lines using a diamond point. Ideally a diffraction grating will have 5000 to 15000 lines per inch.

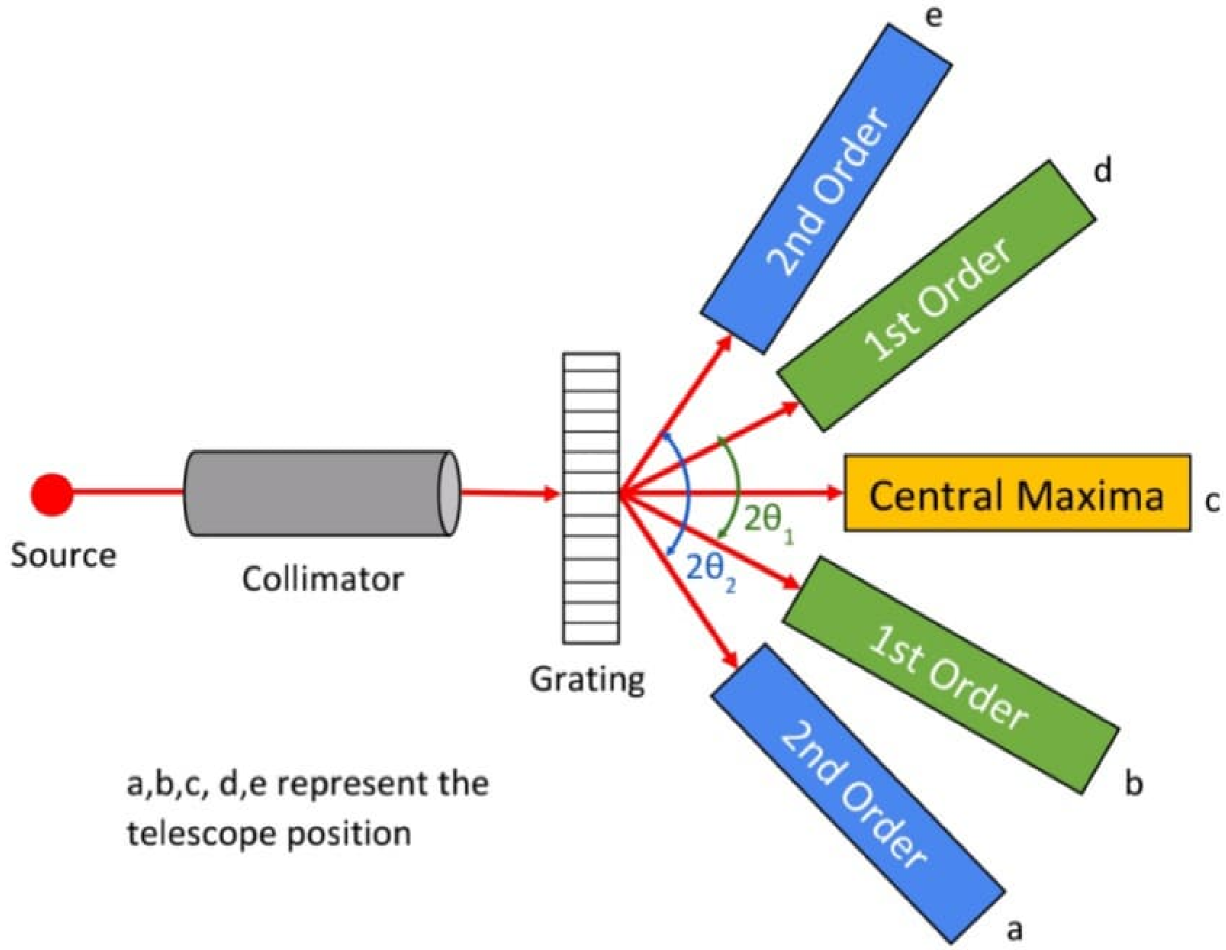


Figure 1.3.1: Determination of wavelength using diffraction grating

The diffraction formula for a principal maximum in a grating's diffraction is given by

$$(a+b)\sin\theta = n\lambda$$

Where; (a+b) = grating element

n = order of spectrum

λ= wavelength of incident light.

A diffraction grating is often used in laboratories to determine the unknown wavelength of light. The grating spectrum of the given source of monochromatic light is obtained by using a spectrometer. The arrangement is as shown in *Figure 1.3.1*

1. First the source for which wavelength is to be determined, the collimator and the telescope are kept in one line subsequently, the spectrometer is adjusted for parallel rays using a prism.
2. Then the prism is replaced by a diffraction grating on a prism table. One has to make sure that the grating is kept perpendicular to the light rays in order to achieve normal incidence.
3. The 0^{th} order spectrum can be seen from the telescope, when It is directly in line with the incident light.
4. As the telescope is moved in clockwise and anticlockwise direction with respect to its 0^{th} order position, higher order spectrums are seen.
5. The angle between the zero-order position and n^{th} order position is called as the diffraction angle for that order and denoted by θ_n.
6. For first and second order spectrums the diffraction angle is measured using the spectrometer.
7. The unknown wavelength is calculated using the grating formula for both the orders:

$$\lambda = \frac{(a+b)\sin\theta_n}{n}$$

Where, (a+b) is the grating element in centimeters,

which is calculated using the formula:

(a+b) = 1/N

Where, N is the number of lines per cm on the diffraction grating, which is mentioned on the grating itself by the manufacturer.

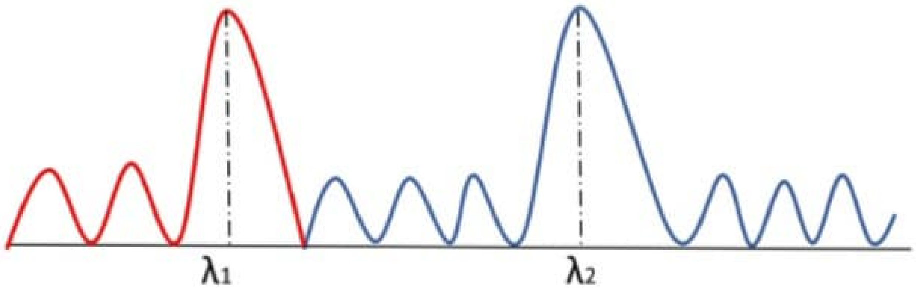
Mean wavelength from both the orders is the unknown wavelength of the source which is now determined.

Q1.4. Explain Rayleigh's Criterion of Resolution.

(M.U. May 2010, 11, 13, 14, 15; Dec. 2016, 17) (3 m)

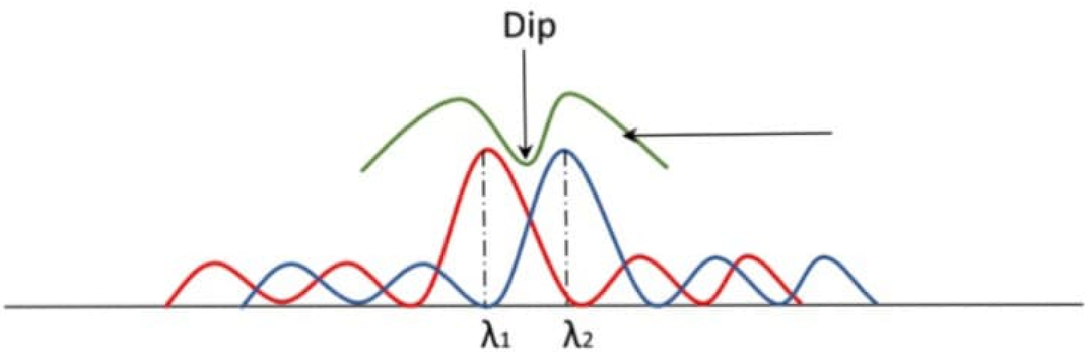
Rayleigh's criterion of resolution states that:

When the central maximum of the diffraction pattern of one source falls over the 1st minimum of the diffraction pattern of another source, which is placed close to the previous source, the two-point sources of light are said to have been just resolved.



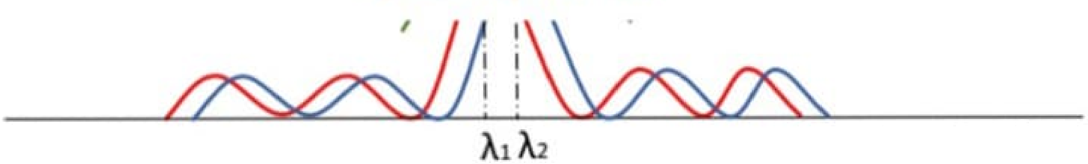
Objects well Resolved

Figure 1.4.1a: Well resolved



Objects Just Resolved

Figure 1.4.1b: Just resolved



Objects not Resolved

Figure 1.4.1c: Not resolved

Q1.5. What is Resolving Power of an optical Instrument? Obtain an expression for resolving power of a diffraction grating.

(M.U. May 2010, 11, 13, 14, 15; Dec. 2016, 17) (3 m)

If the accompanying diffraction patterns are distinguishable from each other, an optical instrument is said to be able to resolve two-point objects. **Resolving power** refers to an instrument's capacity to generate only different diffraction patterns of two close objects.

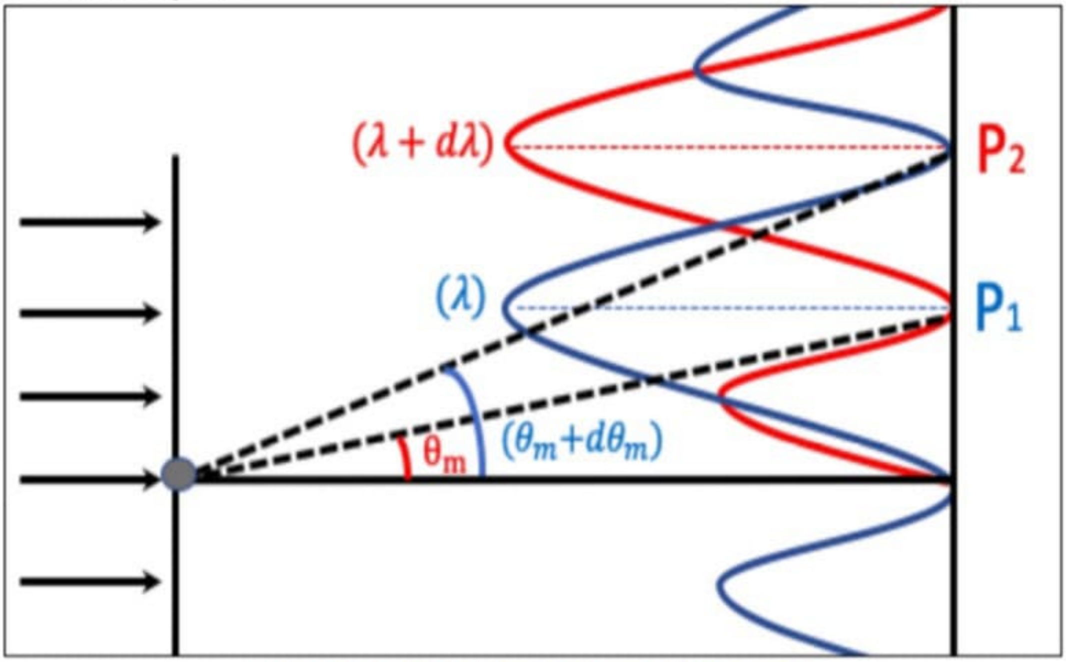


Figure 1.5.1: Resolving Power of an optical Instrument

Resolving Power of a grating:

A grating is capable of resolving the image of slit formed by the two spectral lines of wavelength λ and (λ + dλ) as shown in *Figure 1.5.1*.

The resolving power of a grating is defined as the smallest wavelength $d\lambda$ for which the spectral lines can be first resolved at the wavelength λ, is mathematically given as $R.P = \frac{\lambda}{d\lambda}$ ---------(1a)

Let XY be the grating surface and MN is the field of view of the telescope.

P_1 is the mth primary maximum of spectral line wavelength λ at an angle of diffraction θ_m.

P_2 is the m^{th} primary maximum of a second spectral line of wavelength $\lambda+d\lambda$ at an angle of diffraction $\theta_m+d\theta_m$.

Conditions for maxima P_1 and P_2 are:

$$(a+b)\sin(\theta_m + d\theta_m) = m(\lambda + d\lambda)\text{--------}(1)$$

$$(a+b)\sin(\theta_m) = m\lambda\text{------}(2)$$

Distance between two maxima can be calculated by subtracting (1)-(2) we get:

$$P1\ \mathrm{P2} = (a+b)\sin(\theta_m + d\theta_m) - (a+b)\sin(\theta_m) = m(\lambda + d\lambda) - m\lambda$$

Which results into

$$P1\ \mathrm{P2} = m(d\lambda)\text{-------}(3)$$

According to Rayleigh's criterion $P1\ \mathrm{P2}$ will also be equal to the distance between the principal maxima and first minima of source λ

Principal Maxima condition: $(a+b)\sin(\theta_m) = 0$---------(4)

First Minima condition: $(a+b)\sin(\theta_m) = \frac{\lambda}{N}$----------(5)

Thus, we get $P1\ \mathrm{P2}$ by subtracting (5)-(4):

$$P1\ \mathrm{P2} = \frac{\lambda}{N}\text{-------}(6)$$

Comparing equation (6) and (3):

$$md\lambda = \frac{\lambda}{N}$$

Then by the definition of resolving power of grating in equation (1a) we obtain the expression for resolving power of grating in terms of number of lines N for the grating as:

$$\therefore R.P = \frac{\lambda}{d\lambda} = mN$$

Q1.6. Why is diffraction not evident in daily life?

(M.U. May 2008) (3 m)

In daily life the objects that we come across are very small. Objects of size of the order of wavelength of light i.e. around 0.1 micro meter are needed for diffraction to be observed, hence it is not evident in daily life easily.

Formula List for Diffraction

1. Grating formula

$$(a+b)\sin\theta = n\lambda$$

Where;

- $(a+b)$= Grating element
- θ = Angle of Diffraction
- n = order of Diffraction
- λ = Wavelength of wave getting Diffracted

2. Grating Element

$$(a+b) = \frac{1}{N}$$

Where;

- $(a+b)$= Grating element
- N = Number of lines per unit length of Grating

3. Resolving power of Grating (R.P.)

$$\mathrm{R.P.} = \frac{\lambda}{\mathrm{d}\lambda} = mN$$

Where;

- N = Number of lines per unit length of Grating
- m = order of Diffraction
- λ = Mean Wavelength of sources to be resolved
- d λ = Difference between Wavelength of sources to be resolved

DIFFRACTION PROBLEMS

Q1. A diffraction grating used at normal incidence gives a yellow line (λ =6000A^0)in a certain spectral order superimposed on a blue line (λ =4800A^0) of next higher order. If the angle of diffraction is sin^{-1}(3/4) calculate the grating element.

Given:- λ_1=6000A^0=6x10^{-5} cm; λ_2=4800A^0=4.8 x 10^{-5}cm ; $\theta = \sin^{-1}\left(\frac{3}{4}\right)$

Formula:- $(\mathrm{a}+\mathrm{b})\sin\theta = n\lambda$; n=1,2,3,4 .....

<u>**Solution:-**</u> for given $(\mathrm{a}+\mathrm{b})$ and θ; n α 1/ λ

$$(\mathrm{a}+\mathrm{b})\sin\theta = n\ \lambda_1$$
$$(\mathrm{a}+\mathrm{b})\sin\theta = (n+1)\ \lambda_2$$
$$n\,\lambda_1 = (n+1)\,\lambda_2$$
$$\frac{\lambda_1}{\lambda_2} = 1 + \frac{1}{n}$$

Therefore, $n = 4$

$$(a+b) = \frac{n\lambda_1}{\sin\theta} = \frac{4\times 6\text{x}\ 10^{-5}}{\frac{3}{4}} = 32 \times 10^{-5}\ \text{cm}$$

<u>**Ans:- The grating element is 3.2 x 10^{-4} cm**</u> .

Q2. Monocromatic light of wavelength 6560A falls normally on a grating 2 cm wide. The frist order spectrum is produced at an angle of 16°17' from the normal. Calculate the total number of lines on the grating.

Given:- λ=6560A=6560 x 10^{-8}cm; width = 2cm ; n=1 ; θ=16.28°

Formula:- $(\mathrm{a}+\mathrm{b})\sin\theta = n\lambda$

$$\text{a}+\text{b} = \frac{1}{(N)Number\ of\ lines\ per\ cm}$$

Total number of lines= N x width

Solution:- (a+b)=$\frac{n\lambda}{sin\theta}$ =$\frac{6560}{sin16.28} \times 10^{-8}$ = 2.34 x 10^{-4} cm

Number of lines per cm = $\frac{1}{a+b}$ =4273

Total no. of lines = 4273 x 2 = 8547

Ans:- There will be 8547 lines on the grating.

Q3. A parallel beam of light is incident on a plane transmission grating having 3000 lines/cm. A third order diffraction is observed at 30°. calculate the wavelength of the line.

Given:-a+b = 1/3000 ; n=3 ; θ=30

Formula:- $(\mathrm{a} + \mathrm{b})\sin\theta = n\lambda$, n=1,2,3...

Solution:- λ = $\frac{a+b}{n} \times sin\theta$

= $\frac{1}{3000\times3} \times sin30$

= $\frac{1}{9000\times2}$= 5.555×10^{-5} cm

Ans:- The wavelength of the line is 5.555×10^{-5} cm.

Q4. The visible spectrum ranges from 4000A^0 to 7000A^0 . Find the angular breath of the first order visible spectrum produced by a plane grating having 6000 lines/cm when light is incident normally on the grating.

Given:- l1 = 4000A=4×10^{-5} cm l2= 7000A=7x10^{-5} cm n=1
a+b=1/6000lines per cm

Formula:- $(\mathrm{a} + \mathrm{b})\sin\theta = n\lambda$

Solution:- (a +b)sin θ_1= λ_1

$\theta_1 = \sin^{-1}\frac{\lambda 1}{a+b}$ = $\sin^{-1}(4\times10^{-5}\times 6000)$ = 13.88°

(a +b)sin $\theta_2 = \lambda_2$

$\theta_2 = \sin^{-1}\frac{\lambda 2}{a+b}$ = $\sin^{-1}(7\times10^{-5}\times 6000)$ = 24.83°

$\theta_2 - \theta_1$ =24.83 - 13.88 = 10.95°

Ans :- The Angular separation = 10.95°

Q5. In plane transmission grating the angle of diffraction for the second order principal maxima for the wavelength 5 x 10^{-5} cm is 35°. Calculate the number of lines/cm on the diffraction grating.

Given:- l=5x10^{-5}cm ; θ =35° ; n=2

Formula:- $(a+b)\sin\theta = n\lambda$; $\frac{1}{a+b}$= number of lines/cm

Solution:- a+b=$\frac{n\lambda}{sin\theta}$ = $\frac{2\times5\times10^{-5}}{sin35°}$ = 1.74 x 10^{-4}

Number of lines per cm = $\frac{1}{a+b}$ = $\frac{1}{1.74\times10^{-4}}$ = 5735

Ans:- The number of lines pre cm is 5735.

Q6. A grating has 620 ruling/mm and is 0.5 mm wide. What is the smallest wavelength interval that can be resolved in the third order at λ =481mm?

Given:- N = 620 x 0.5=310 ; λ =481x10^{-9}m ; m=3

Formula:-$\frac{\lambda}{d\lambda} = mN$

Solution:- d λ = $\frac{\lambda}{mN}$ = $\frac{481\times10^{-9}}{3\times310}$ = 0.5172 x 10^{-9} m

$d\lambda$ = 0.5172 A°

Ans:- The smallest wavelength interval is 0.5172 A°

Q7. Calculate the minimum number of lines required on grating that can just resolve the two sodium lines λ 1=5890A and λ 2=5893 A in third order.

Given:- λ_1=5890 x 10^{-8} cm, λ_2=5893 x 10^{-8}cm , m=3

Formula:- Resolving power $=\frac{\lambda}{d\lambda} = mN$

<u>**Solution:-**</u> $\lambda = \frac{\lambda_1+\lambda_2}{2} = \frac{(5890+5896)10^{-8}}{2}$ = 5893 x 10^{-8} cm

d= (5896-5890)x10^{-8} = 6x10^{-8} cm

$$N = \frac{\lambda}{m\, d\lambda} = \frac{5893 \times 10^{-8}}{3 \times 6 \times 10^{-8}} = 327$$

<u>**Ans:- Minimum of 327 lines are required on the grating.**</u>

Q8. Calculate the maximum order of diffraction maxima seen from plane transmission grating with 2500 lines per inch if light of wavelength 6900 A falls normally on it.

Given:- N=$\frac{1}{a+b}$ = 2500lines/inch = 2500x 2.52 x 10^{-2} = 63 lines/m

λ= 6900A=6900x10^{-10} m

Formula:- $(\mathrm{a} + \mathrm{b})\sin\theta = n\lambda$

<u>**Solution:-**</u> for n= n_{max} , sin θ =1

$$n_{max} = \frac{a+b}{\lambda} = 2.3$$

<u>**Ans:- Maximum order of diffraction is 2**</u>

Unit 2a: LASER

Q2a.1. What is the full form of LASER? Distinguish between laser source and ordinary source.

(M.U. May 2012; Nov. 2018) (7m)

Full form of LASER is **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation.

Light Laser Source	**Ordinary source**
Monochromatic in nature	Polychromatic in nature
Coherent waves are in phase.	Waves have no definite phase relation.
Highly focused has low divergence hence very intense.	Highly divergent hence diffuse because intensity decreased due to spreading.
Laser light is directional, it is emitted in one direction.	Emitted in all directions.
eg. Nd-YAG laser, He- Ne	eg. candle, LED, bulb

Q2a.2. Explain main three processes involved in production of LASER with appropriate diagrams.

(M.U. Dec 2006, 12; May 2009) (7m)

Radiation consists of photons and Matter consists of atoms and molecules. Thus, Interaction of radiation with Matter means interaction of photons with atoms and molecules. The three processes that coexist at all temperatures whenever radiation interacts with matter are absorption, emission and stimulated emission.

Absorption:

The transition of atoms from lower energy ground state E_1 to a higher energy excited state E_2 after acceptance of an incident photon is known as stimulated absorption as shown in *Figure 2a.2.1*. The incident photon should be of energy $E = h\nu = E_2 - E_1$.

Absorption can be expressed as $A + h\nu \Rightarrow A^*$

Where, A = atom in ground state and A^* =atom in excited state.

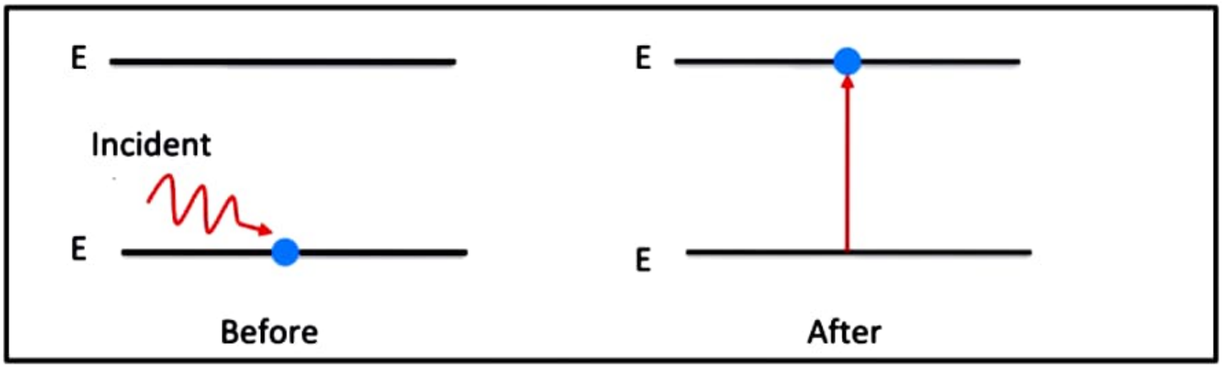


Figure 2a.2.1: Absorption

Spontaneous Emission:
If an excited atom returns to the ground state on its own accord by releasing an energy equivalent to $E = h\nu = E_2 - E_1$, then the process is known as spontaneous emission as shown in *Figure 2a.2.2* .
It can be expressed as $A^* \Rightarrow A + h\nu$

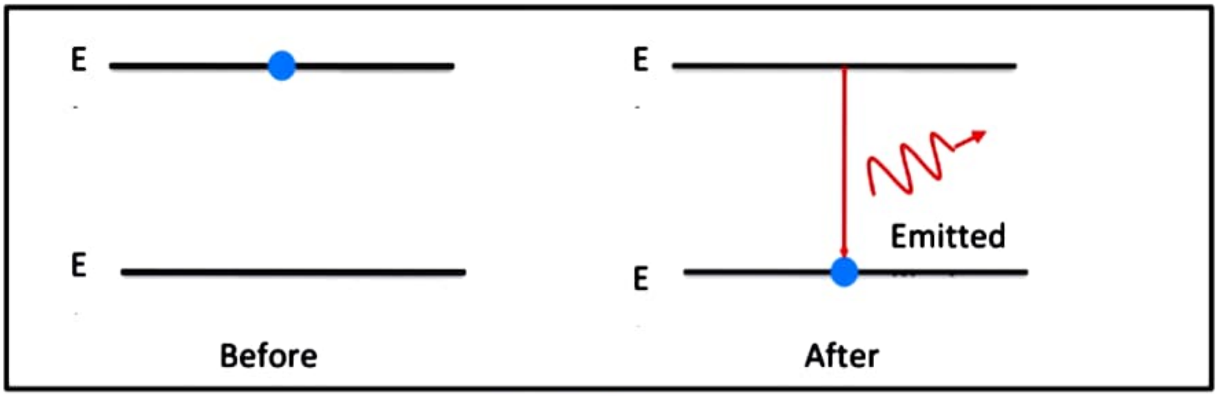


Figure 2a.2.2: Spontaneous

Stimulated Emission:
If an atom in the excited state E_2 returns to the ground state E_1 in presence of an external photon of energy $\mathbf{h\nu = E_2 - E_1}$ giving out another photon of same energy the process is called stimulated emission as shown in *Figure 2a.2.3*.
It can be written as $A^* + h\nu \Rightarrow A + 2h\nu$.

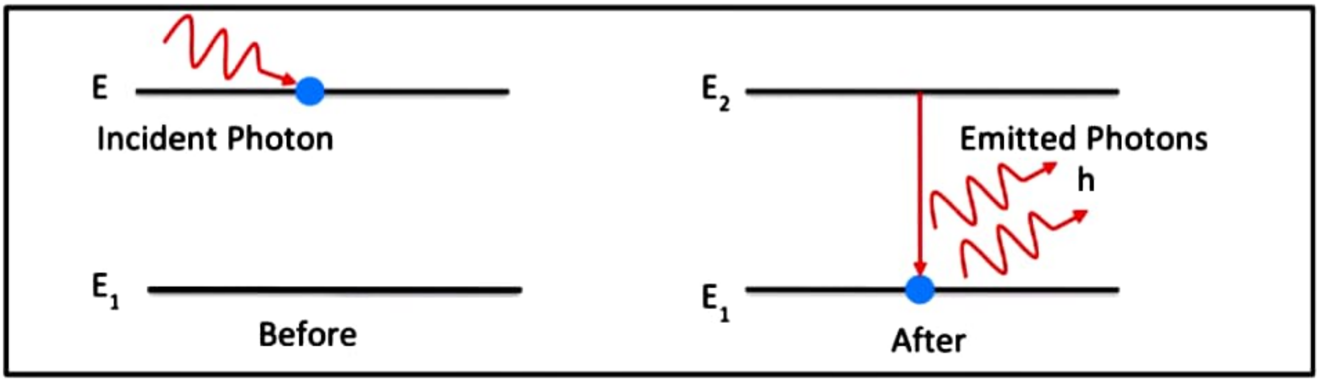


Figure 2a.2.3: Stimulated

Characteristics of stimulated emission:

- The emitted photon and the incident photon are identical in all respects and have the same frequency.
- Both photons travel in the same direction.
- Multiplication of photons takes place in the process.

Q2a.3. Explain the terms population inversion, Active medium, meta stable state, pumping.

(M.U. Dec. 2008,09,10,17) (8m)

Population inversion: Every system in nature tries to achieve minimum potential energy. Thus, naturally majority of atoms in every system lie in the ground state and a very few are present in excited state, this is called the natural population. In order for the stimulated emission to occur we require majority of atoms to be present in the excited state and very few in the ground state i.e., just inverse of the natural population. Once this condition is achieved. The state of population inversion is said to be achieved.

Active medium: The medium in which light gets amplified is called active medium. It may be solid, liquid or gases. Out of all atoms present in medium only few are responsible for stimulated emission and consequent light amplification, they are called active centres.

Meta stable state: The state which has energy in between that of ground state and excited state is called meta stable state. This is a partially stable state. The life time of atoms is in between their respective life time is ground state and excited states.

Pumping: To obtain and maintain a state of population inversion atoms have to be raised continuously to excited state. It requires energy to be supplied to the system. The process of supplying energy to the medium with a view to transfer it into the excited state is pumping.

Techniques for pumping are,

1. **Optical pumping:** Light source used for pumping.
2. **Electric pumping:** Electric field is applied to medium producing ionisation creating excitation.
3. **Chemical pumping:** When chemical reaction is used for excitation.

Q2a.4. Derive expression for Einstein's coefficients.

Consider a two level laser system with ground state $E_1$and excited stateE_2. Let N_1 and N_2 be the numbers of atoms in $E_1$and E_2 respectively. Let Q(x) be the density of photons incident on this system two level laser system. Letthe probability absorption i.e. the probability of N_1atoms to get excited to E_2 state be denoted by'P_{12}'. This probability will depend on number of atoms in the ground state 'N_1' and the density of photons 'Q' and can be written as:

Probability of stimulated absorption: $P_{12} = B_{12}N_1Q$ ---------(1)

where B_{12} is constant.

The probability of emission will have two terms, one corresponding to stimulated and other corresponding to spontaneous emission. Let the total emission probability be denoted by 'P_{21}'.This can be written as :

Probability of emission : $P_{21} = P_{21sp} + P_{21st}$--------------(2)

The probability of spontaneous emission i.e. N_2 atoms to go to ground state without any stimulus is $P_{21sp} = A_{21}N_2$---------(3a)

The probability of stimulated emission i.e. N_2 atoms to go to ground state with stimulus is $P_{21st} = B_{21}N_2$------------(3b)

Using equation (2) and (3) we get,

$P_{21} = N_2(B_{21}Q + A_{21})$--------------(4)

Here B_{21}, Q are constant coefficients.

At Equilibrium the probability of absorption i.e atoms going from E_1 to E_2 will be equal to the probability of emission i.e.atoms coming back from E_2 to E_1

Therefore ,$P_{21} = P_{12}$-------------(5)

Putting Equation (1) and (4) in (5) we get:

$$B_{12}N_1Q = N_2(B_{21}Q + A_{21})$$

After simplifying the above equation for Q we obtain:

$$Q = \frac{A_{21}N_2}{\left(\frac{B_{12}N_1}{B_{21}N_2}-1\right)N_2B_{21}} \text{-------------(6)}$$

Maxwell distribution gives the relation between N_1 and N_2, it is given as:

$$N_1 = N_2e^{\frac{h\upsilon}{kT}} \text{- -----------(7)}$$

Using (7) in equation(6) we have:

$$Q = \frac{A_{21}}{\left(\frac{B_{12}}{B_{21}}e^{\frac{h\upsilon}{kT}}-1\right) B_{21}} \text{-------------(8)}$$

Planck's law gives us the expression for energy density given as:

$$Q = \frac{8\pi h\upsilon^3}{c^3}\left(\frac{1}{e^{\frac{h\upsilon}{kT}}-1}\right) \text{-----------------(9)}$$

Comparing equation (8) and equation (9) we get expressions:

$$B_{21} = B_{12} \text{ and}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\upsilon^3}{c^3}$$

Where B_{12}, $B_{21}$and $A_{21}$are called Einstein's coefficients.

Q2a.5. Describe action of resonant cavity.

(M.U. May 2012) (3 m)

When stimulated emission is generated in medium it has to be sustained in the desired direction and suppressed in remaining direction. This goal can be achieved by resonant cavity. Resonant Cavity consist of two oppositely facing parallel mirrors with active medium placed in between them as shown in *Figure 2a.5.1*.

Figure 2a.5.1: Resonant Cavity

The action of such a cavity can be explained as follows:

1. Ground state: Initially active centres are in ground state as shown in *Figure 2a.5.2*.

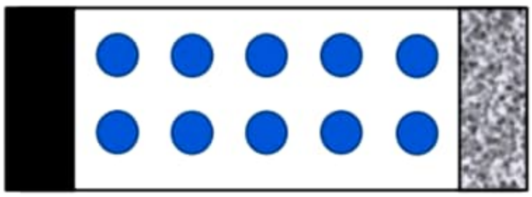


Figure 2a.5.2: Non excited state

2. Optical pumping: By optical pumping the material is taken to a state of population inversion as shown in *Figure 2a.5.3*.

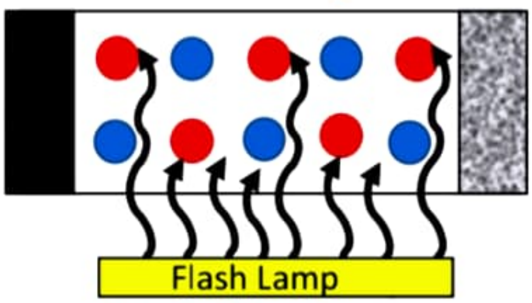


Figure 2a.5.3: optical pumping

3. Spontaneous and stimulated emission: Spontaneous occur in random direction and they thus produce stimulated emission as shown in *Figure 2a.5.4*.

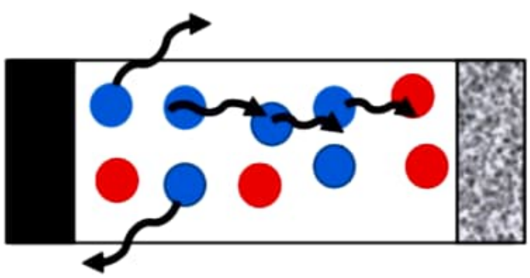


Figure 2a.5.4: Spontaneous and Stimulated emission

4. Optical feedback: Due to reflecting mirrors photons in favourable direction are feedback as shown in *Figure 2a.5.5*.

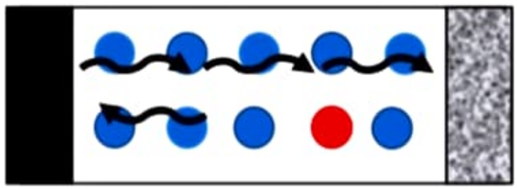


Figure 2a.5.5: Optical Feedback and light oscillation

5. Light amplification: Feedback from mirrors combined with stimulated emissions will lead to light multiplication and then light amplification.

6. Light oscillation : Light beam in the cavity begins to oscillate when the amount of amplified light becomes equal to the total amount of light lost through the sides of the resonator as shown in *Figure 2a.5.5*. Then waves propagating in the cavity take the standing wave pattern. If L is the length of the cavity.

$$2L = m\lambda; \qquad 2L = \frac{mc}{\upsilon}\ ; \upsilon = \frac{mc}{2L} \text{-------------}(1)$$

A resonator may support several standing waves of slightly different wavelength, these are called longitudinal modes. Each mode has distinct frequency (υ) as in equation (1).

Q2a.6. Explain the working of He-Ne laser along with Principle.

(M.U. May 2007, 08, 13, 15, 18; Nov. 2018; Dec. 2007, 14, 15)

Principle: Gas lasers employ electrical pumping of gas mixtures where electron impact excites A gas . Then A gas molecules transfer their energy in collisions to B Gas molecules that are the actual active centres. It can be expressed as

$e_1 + A \Rightarrow e_2 + A^*$

$A^* + B \Rightarrow A + B^*$

Note: A* is metastable state and B* is excited state.

Construction :

He- Ne laser comprises of long and narrow tube that is filled with He and Ne gas in the ratio 10:1 at a pressure of 1 mm of mercury. Tube length is 50cm and 1cm in diameter .Electrodes are provided to produce discharge in the gas and they are connected to a high voltage power supply .Tube ends are sealed at ends using silica windows inclined as brewsters angle. This tube is placed in between two minorset one is fully reflecting other is partially reflecting. This forms the resonant cavity of He-Ne laser as shown in *Figure 2a.6.1*.

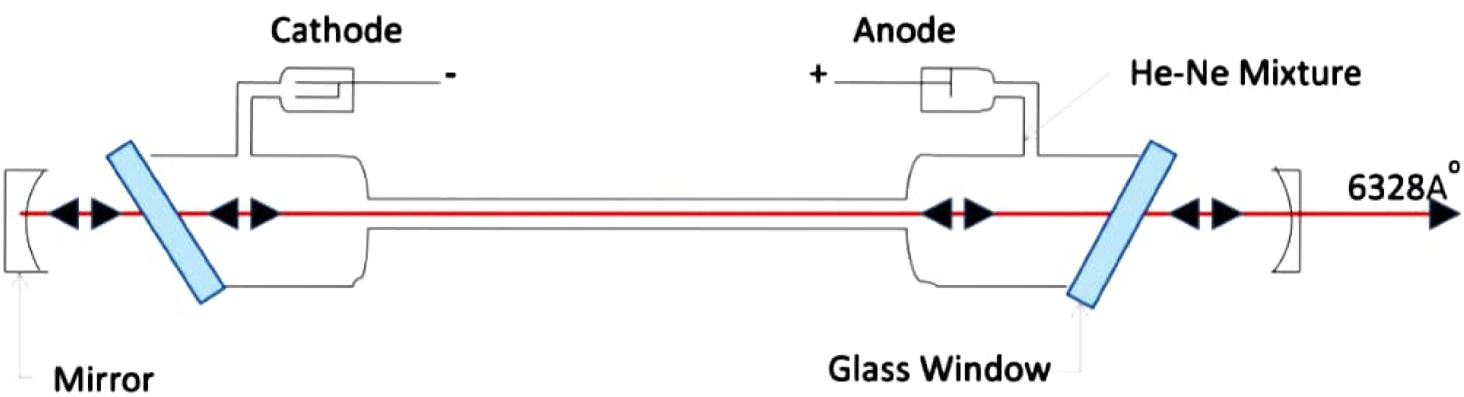


Figure 2a.6.1: Construction of He-Ne Laser

Working :

He-Ne laser employs a four level pumping scheme. When the power (1kv) is switched on , the electric field ion is essome of gas atoms and ions move towards oppositely charged electrode . Electron lighter in weight hence gain higher velocity and He atoms excited more readily because they are lighter in weight .

e_1 + He => e_2 + He*

He atoms get excited to F_2 and F_3 from F_1 where F_2 and F_3 are metastable state where atoms remain for a longer time creating population inversion.

Ne energy states E_6 and E_4 are very close to metastable states of F_2 and F_3 of He atom. Thus when He collides Ne atom, resonant transfer of energy takes place.

He* + Ne => He + Ne*

Population increases rapidly in E_6 and E_4. Thus population inversion takes place in E_6 and E_4 with respect to E_5 and E_3 as shown in *Figure 2a.6.2*

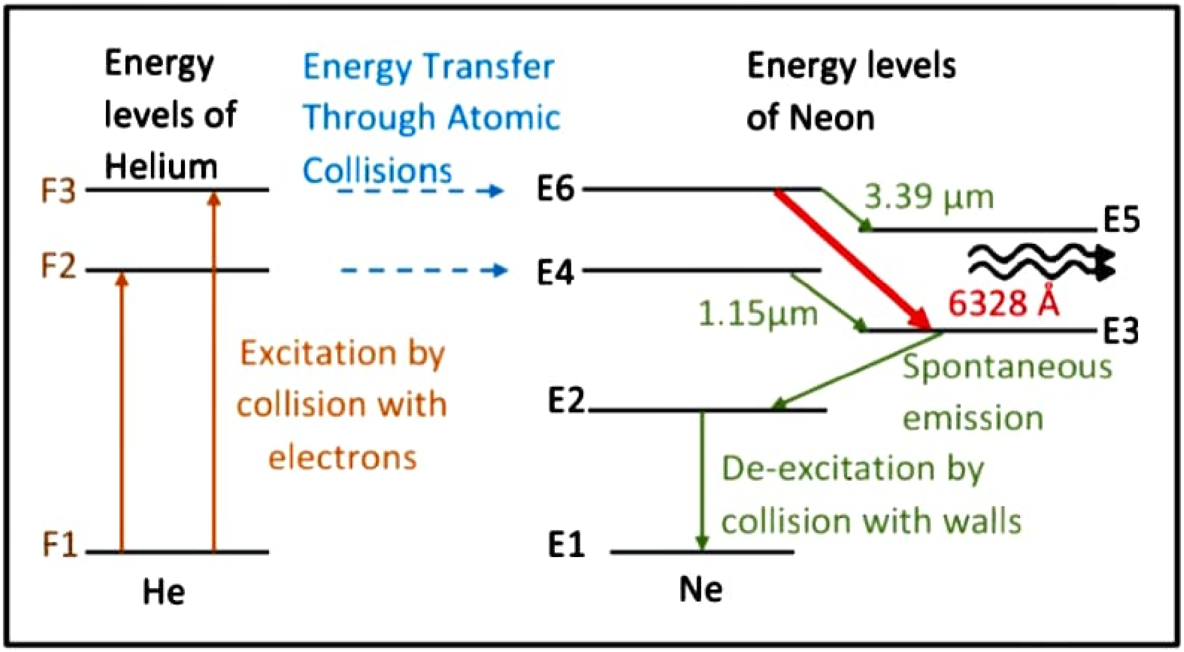


Figure 2a.6.2: Working of He-Ne Laser

three main transitions are

1. E_6-E_5 of 3.39 µm in IR region not visible.
2. E_6-E_3 of 6328 A^0red color visible.
3. E_4-E_3 of 1.15µm in IR region not visible.

Atoms in E_3 collected undergo spontaneous emission to reach E_2 .E_2 is a metastable state in Neon .So, there is a chance of accumulation of atoms in this state.The atoms in E_2 are de-energized by keeping diameter of the discharge tube small . So that number of collisions with the tube walls takes away energy of atoms in E_2 and they rapidly reach ground state to get excited to E_6 and E_4 again. The discharge is maintained continuously thus this cycle of events continues , giving out a continuous laser output.

Q2a.7. Explain Nd-YAG laser, Principle, construction and working.
(M.U. May 2011,12; Dec 08,09,19) (8m)

Principle: Optically pumped Nd-YAG rod inside the resonant cavity undergoes stimulated emission and light thus produced gets amplified in the cavity to produce Nd -YAG laser.

Construction:
Nd-YAG assembly consists of an elliptically cylindrical cavity that has Nd-YAG rod along if one focal axis and Xenon flash lamp at the other as shown in *Figure 2a.7.1b*.

The cavity is silvered internally so that light leaving the lamp at one focus meets the rod at other focus after every reflection. Thus, light is focused on the rod. Two ends of the rod are polished and silvered for resonator formation as shown in *Figure 2a.7.1a*.

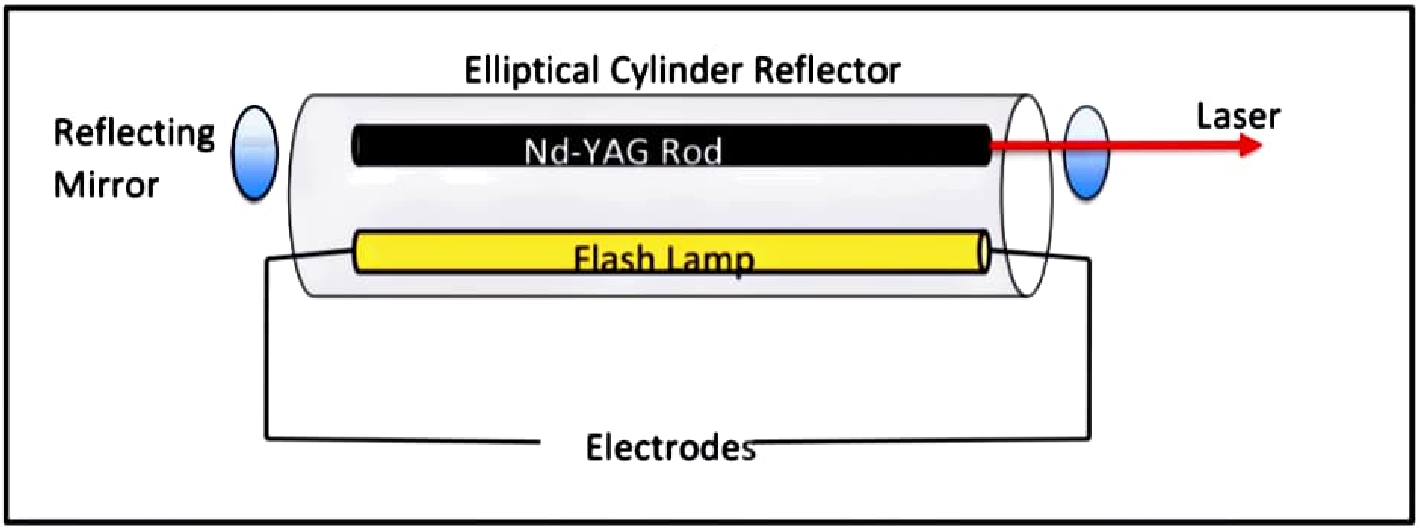


Figure 2a.7.1a: Construction of Nd-YAG Laser

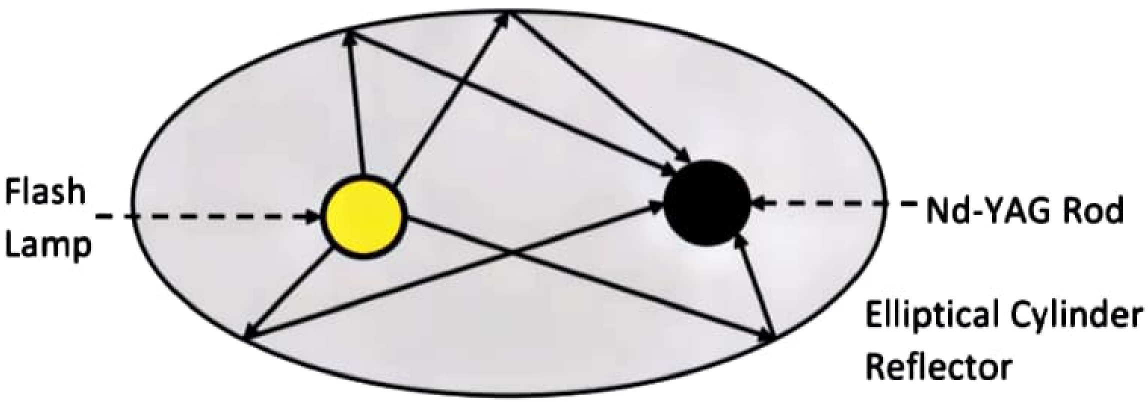


Figure 2a.7.1b: Construction of Nd-YAG Laser

Working:
As shown in *Figure 2a.7.2* the energy level of Nd. E_1 is the ground state and E_3 is the metastable state. Light in the range 5000-8000 A^0 is used to pump the Nd $^{+3}$ ions that act as active centres to higher states.

E_3 is the metastable state that gets rapidly populated due to downward transitions from higher energy levels as none of them is metastable. Thus population inversion takes place between E_3 and E_2. Thus by resonant cavity action with help of pumping a Continuous laser of 1.6 µm in IR region is given out between E_3 and E_2.

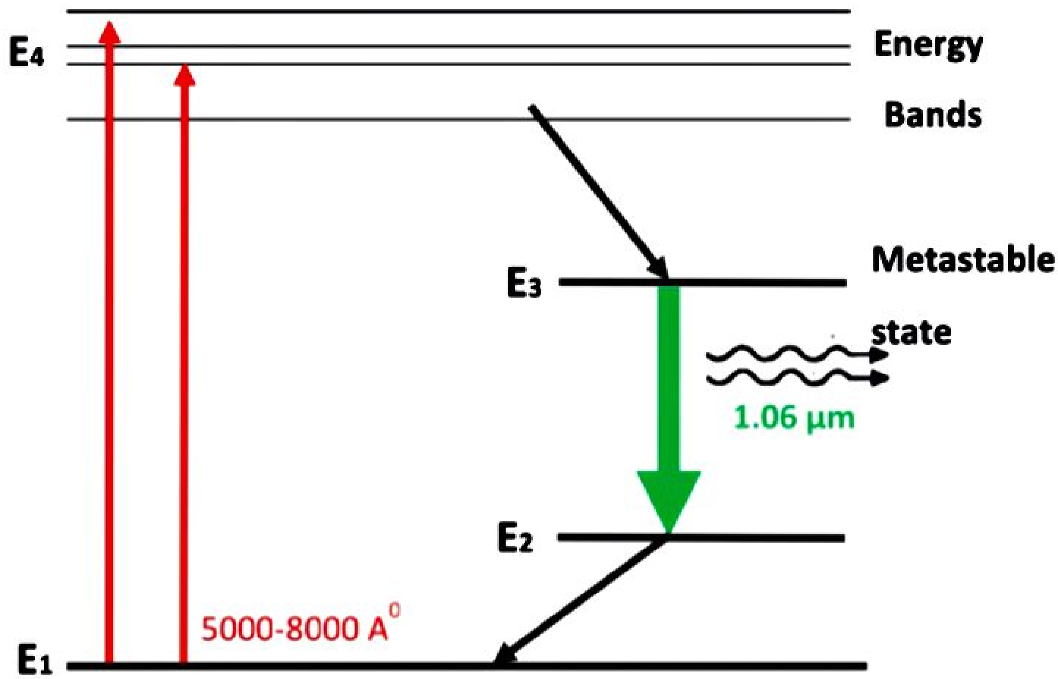


Figure 2a.7.2: Energy Level of Nd-YAG Laser

Q2a.8. Explain semiconductor diode laser on the basis of principle , construction and working . (M.U. May 2010; Dec 2012,16)(7m)

Principle: When a current higher than a Threshold value is passed in a forward bias on junction that is basically doped it emits a laser that is called semiconductor diode laser.

Construction:

A diode is heavily doped for the purpose of being used as a semiconductor diode laser. The semiconductor diode is of the order 1mm in size. The front and the rear ends of the diode are polished perpendicular to the junction in order to create a resonator cavity. The thin junction here acts as the active region as shown in *Figure 2a.8.1*.

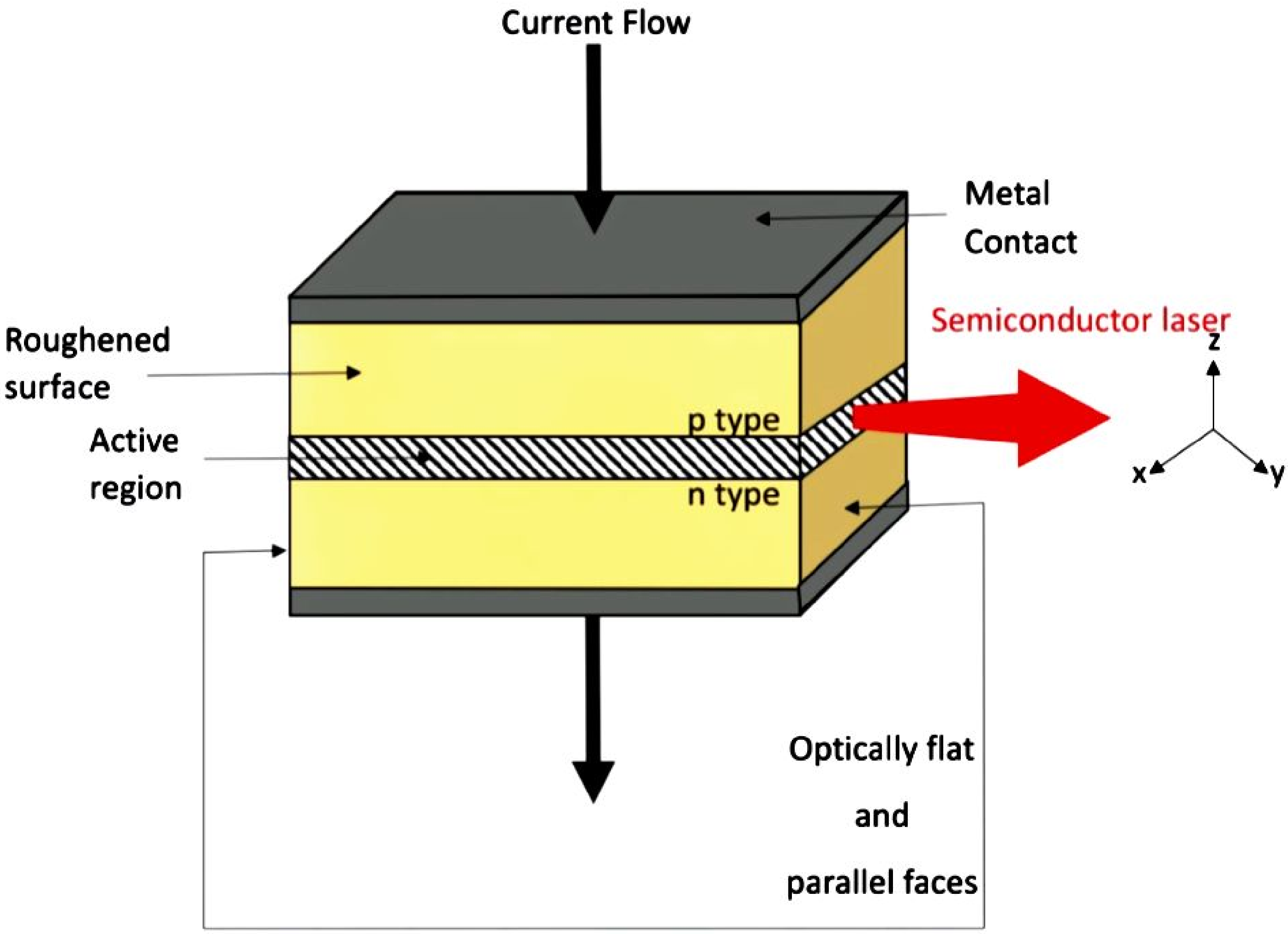


Figure 2a.8.1: Construction of Semiconductor laser LASER

Working:

A heavily doped pn junction is forward biased. This puts fermi level within conduction band. So, on heavily doped p side the acceptor levels enter with E_{fp} into the valence band creating holes in valence band. Zero bias condition is shown at the top of the adjoining *Figure 2a.8.2*.

When a forward bias is applied zero bias changes to the condition shown below, electrons injected into depletion region hence it's holes appear in lower region. Low forward bias current causes spontaneous emissions of photons at the junction due to electron hole pair recombination.

When the forward bias current increases **threshold value** carrier concentration increases to very high value. The narrow region where this is achieved is called active region. Forward bias current plays the role of pumping agent in diode lasers. eg. In GaAs laser light of 9000 A° in IR. GaAsP in visible region red colour 6500 A°. Diode laser are simple, efficient low power compact, less monochromatic and highly temperature sensitive.

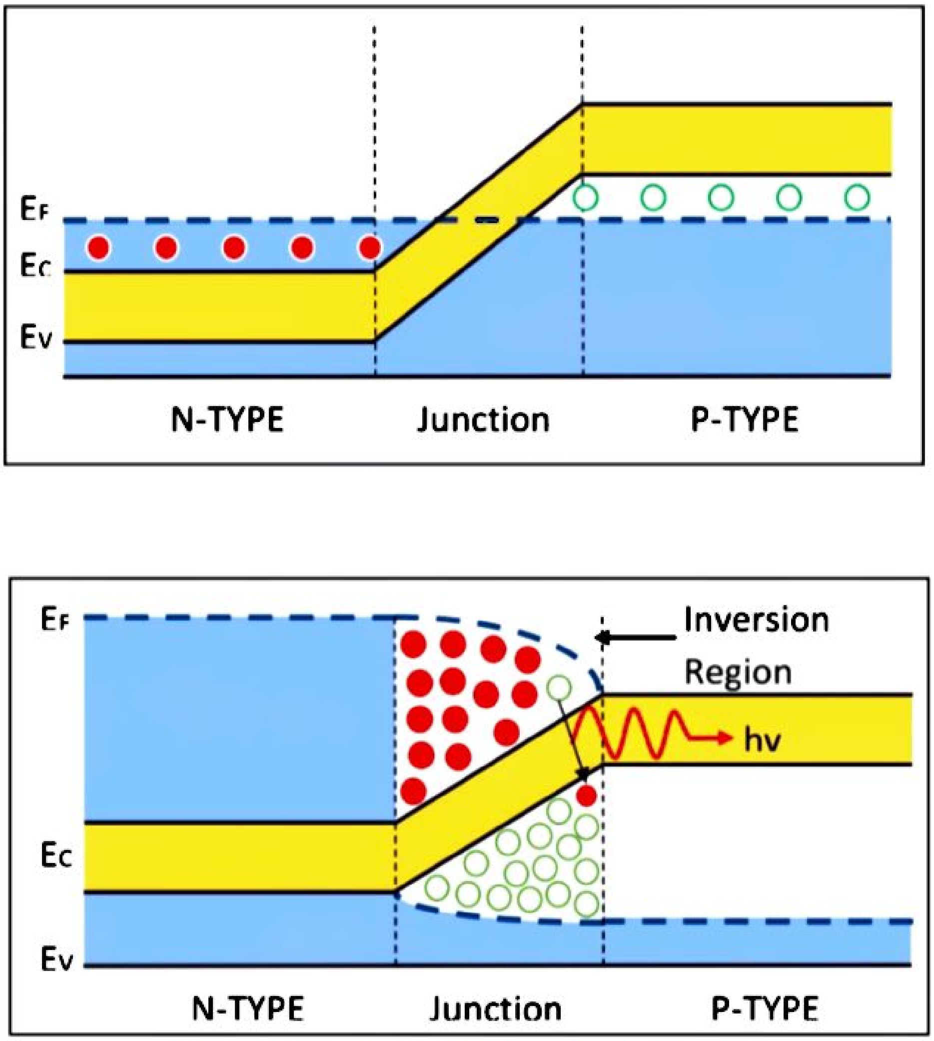


Figure 2a.8.2: Construction of Semiconductor laser LASER

Q2a.9. Write a note on Holography.

(M.U. May 2007; Dec. 2007, 10, 11, 16) (5 m)

A Photograph records a two-dimensional record of a three-dimensional scene where the information of the third dimension is lost. While a hologram is the three-dimensional record of a three-dimensional scene. Thus, the hologram is "holos" which means "complete" and "gram" that means a "record" in greek. The process of creating a 3D image "Hologram" is called holography it has two main stages namely recording and reading they are described as follows:

Recording hologram:
A broad LASER beam is incident on the object to be recorded. The wave scattered from object called the object beam is made to interfere with a coherent unscattered wave (reference beam) to obtain interference pattern that becomes the record of phase information i.e. 3rd dimension.
The hologram thus obtained is similar to the image as in case of photograph. On the contrary, it consists of alternate bright and dark bands (interference pattern) that have phase information to give additional 3rd dimension information as shown in *Figure 2a.9.1*.

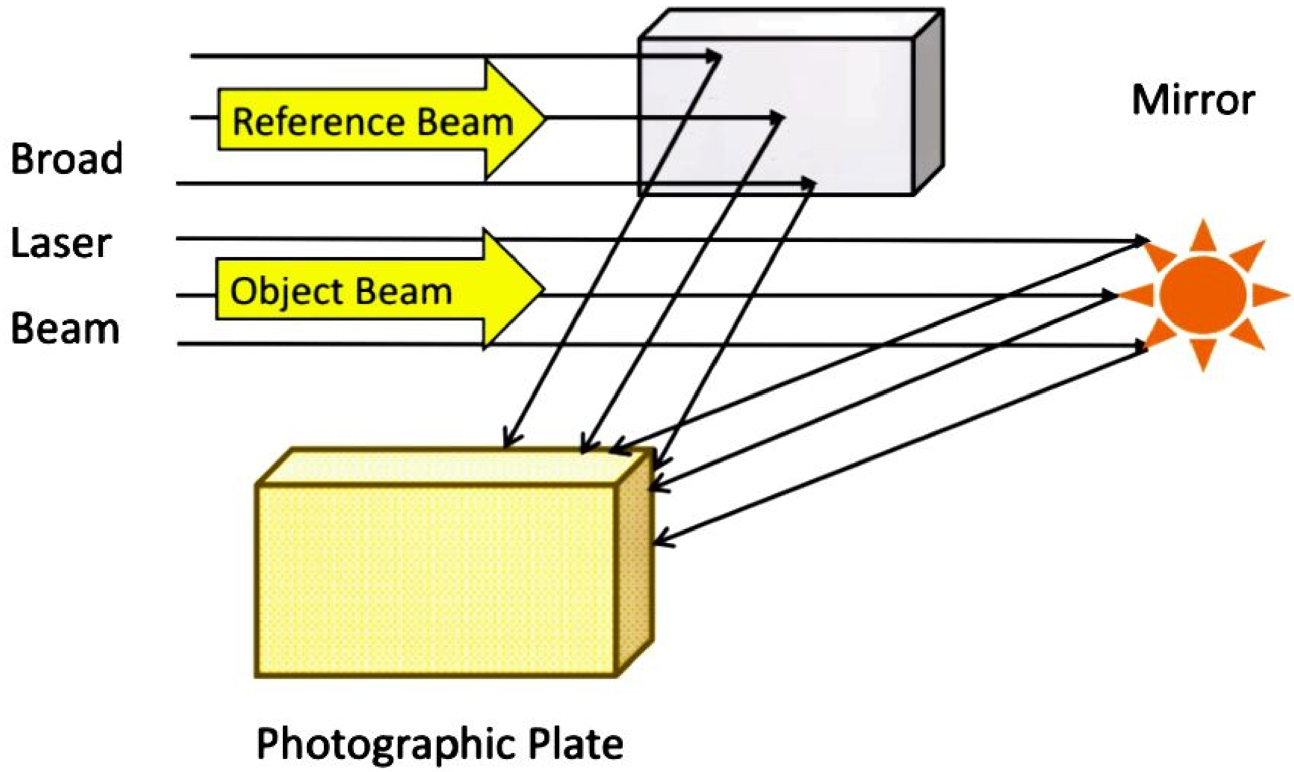


Figure 2a.9.1: Recording a Holography

Reading a hologram:

When hologram is illuminated by the reconstruction two waves are produced. One appears to diverge from the object and provides virtual image of the object and second converges to form real image as shown in *Figure 2a.9.2*.

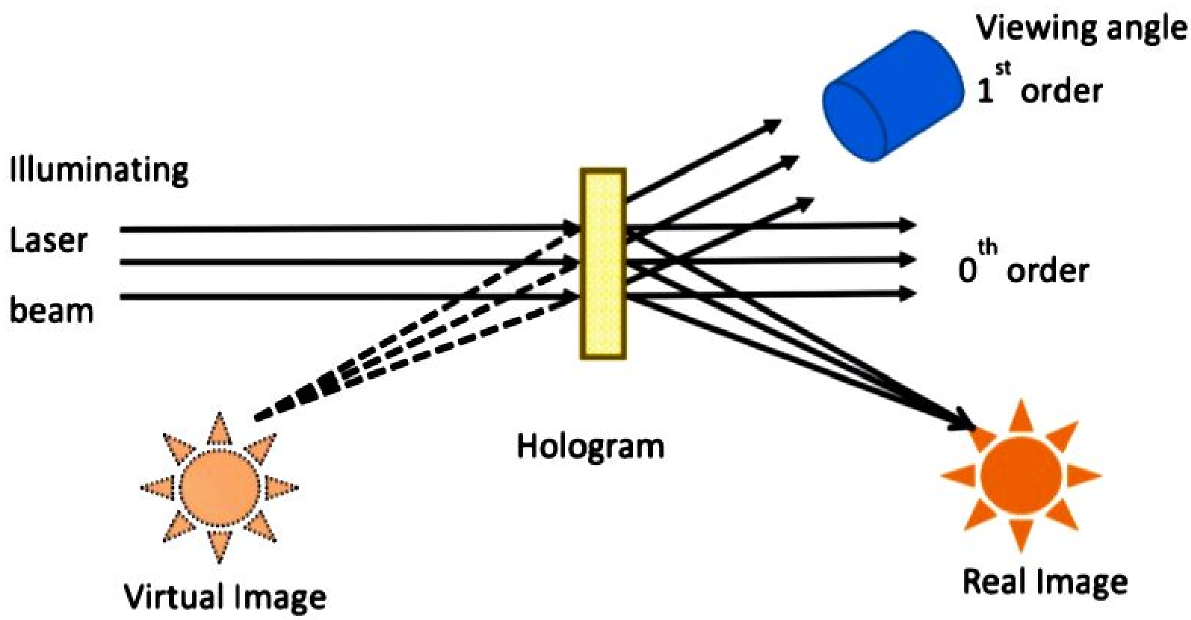


Figure 2a.9.2: Reading a Hologram

Q2a.10.Distinguish between photograph and hologram.
(M.U. May 2007, 19) (3 m)

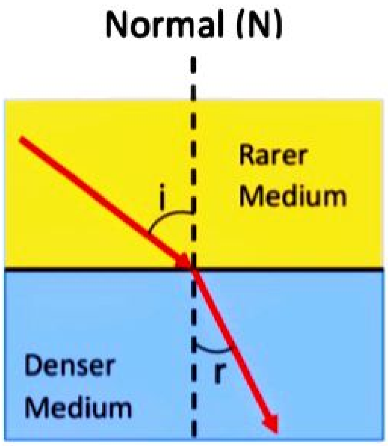
Photograph	**Hologram**
2D representation of the object.	3D representation of the object.
The negative resembles the object.	The hologram is just an interference pattern that does not resemble object.
Ordinary light is used for photography.	Laser light is used for holography.
Photograph reading is not encoded	Hologram reading is encoded with the wavelength of the light used for recording it.

Unit 2b: LASER AND FIBRE OPTICS

Q2b.1. Explain the phenomenon of Total internal reflection.

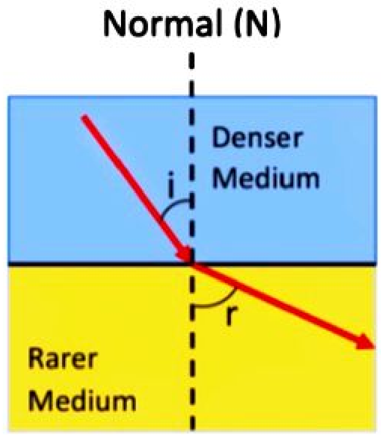
(M.U. May 2009; Dec. 2013, 14, 15) (3 m)

When a ray of light suffers refraction at a boundary while travelling from a rarer medium to a denser medium, it bends towards the normal as shown in *Figure 2b.1.1a*. Similarly, if a ray of light travels from a denser medium to a rarer medium, it bends away from the normal as shown in *Figure 2b.1.1b*



Ray of light bends towards N

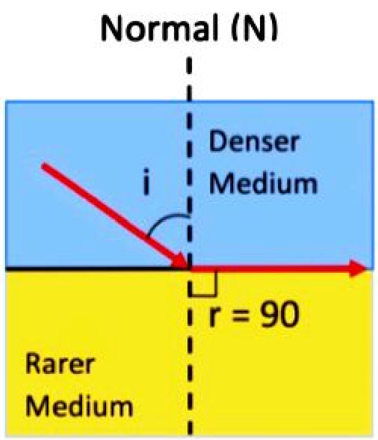
Figure 2b.1.1a: Reflection at the boundary of a denser medium



Ray of light bends away from N

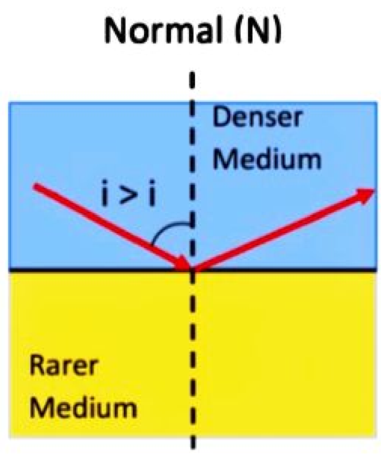
Figure 2b.1.1b: Reflection at the boundary of a rarer medium

In both the cases angle of refraction (r) increases with increase in the angle of incidence (i). When the ray of light is travelling from denser medium towards the rarer medium similar to the case as shown in *Figure 2b.1.1b*, then the angle of incidence for which the angle of refraction



Critical angle of incidence (i_c)

Figure 2b.1.2a: Critical angle of incidence (ic)



Total Internal Reflection

Figure 2b.1.2b: Total Internal Reflection becomes 90° is called the critical angle of incidence (i_c) as shown in *Figure 2b.1.2a*.

When the angle of incidence increases further ($i > i_c$) the refracted ray does not enter the rarer medium and is reflected back to the denser medium .

Thus, when a ray of light is travelling from the denser medium towards the rarer medium is incident at the boundary with an angle greater than the critical angle ($i > i_c$), the ray of light instead of getting refracted to the rarer medium gets reflected back to the denser medium.

This phenomenon where light is reflected back into the denser medium, is called **Total Internal Reflection** as shown in *Figure 2b.1.2b*.

Q2b.2. Write a note on optical fibres.

Optical fibres are long thin hair like cables made of plastic or glass to electric light along their length.

An Optical fibre generally has three co-axial regions as shown in *Figure 2b.2.1*.

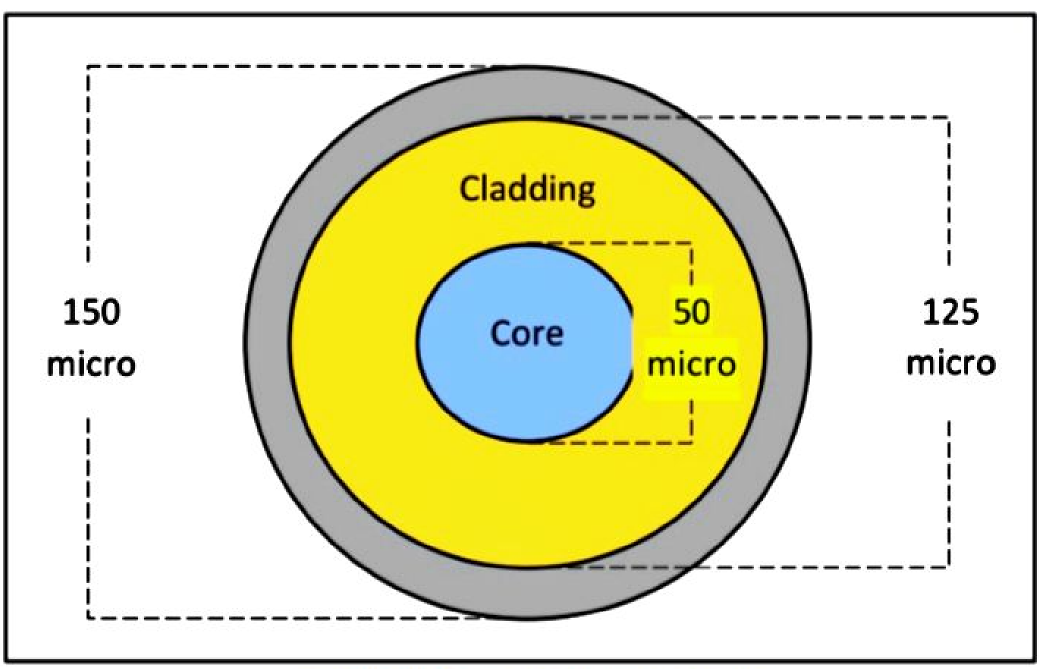


Figure 2b.2.1: Dimensions of Fibre optic cable

1. **Core:** The innermost region of nearly 50μm in diameter which is optically dense as compared to cladding is called core.
2. **Cladding:** The region surrounding the core with 125 μm diameters which is rarer as compared to core is called cladding.
3. **Sheath:** The outermost skin of optical fire to protect it from external damage is called sheath.

Q2b.3. Why is cladding required when light travels through core only?

Cladding is required as:

1. It enhances the mechanical strength of fiber.
2. Protects core from surface contamination.
3. Reduces scattering loss at the core.

Q2b.4. Derive expression for acceptance angle of an optical fibre.

(M.U. Dec. 2002, 05,07,08,11,12,15, 16: May 2013, 15) (5 m)

The maximum angle of incidence for which the light incident on the core propagates successfully through the fibre is called acceptance angle (θ_0).

Consider a step index optical fibre with core of refractive index μ_1 and cladding of refractive index μ_2 placed in air which has refractive index μ_0 as shown in *Figure 2b.4.1*.

For a ray of light that is travelling from air to core the relative refractive index can be written as:

$$\mu_1^0 = \frac{\mu_1}{\mu_0} \text{------------------------} (1a)$$

For a ray of light that is travelling from core to cladding the relative refractive index can be written as :

$$\mu_2^1 = \frac{\mu_2}{\mu_1} \text{------------------------} (1b)$$

Applying Snell's law of refraction at the air – core interface:

$$\mu_1^0 = \frac{\sin\theta_0}{\sin r}\text{------------------------}(2)$$

Critical angle (i_c) and angle of refraction (r) are complementary angles (i_c + r =90), hence

$$\mu_1^0 = \frac{\sin\theta_0}{\sin r} = \frac{\sin\theta_0}{\sin(90-i_c)} = \frac{\sin\theta_0}{\cos i_c}\text{-------}(3)$$

Using (1a) , (3) and $\mu_0 = 1$ for air we get:

$$\frac{\sin\theta_0}{\cos i_c} = \frac{\mu_1}{\mu_0} = \mu_1\text{-------}(4)$$

Applying Snell's law of refraction at the core-cladding interface:

$$\mu_2^1 = \frac{\sin i_c}{\sin 90}\text{------}(5)$$

Using (1b) , (5) and $\sin\sin 90 = 1$ we get:

$$\frac{\mu_2}{\mu_1} = \sin i_c\text{-------}(6)$$

Using $(\sin x)^2 + (\cos x)^2 = 1$, (4) and (6) we get:

$$(\sin i_c)^2 + (\cos i_c)^2 = \frac{{\mu_2}^2}{{\mu_1}^2} + \frac{(\sin\theta_0)^2}{{\mu_1}^2} = 1$$

$$\frac{(\sin\theta_0)^2}{{\mu_1}^2} = 1 - \frac{{\mu_2}^2}{{\mu_1}^2}$$

$$(\sin\theta_0)^2 = {\mu_1}^2 - {\mu_2}^2$$

Numerical Aperture: Sine of acceptance angle is called Numerical Aperture.

$$\sin\theta_0 = \sqrt{{\mu_1}^2 - {\mu_2}^2}$$

Thus the expression for acceptance angle is :

$$\theta_0 = sin^{-1}(\sqrt{\mu_1{}^2 - \mu_2{}^2})$$

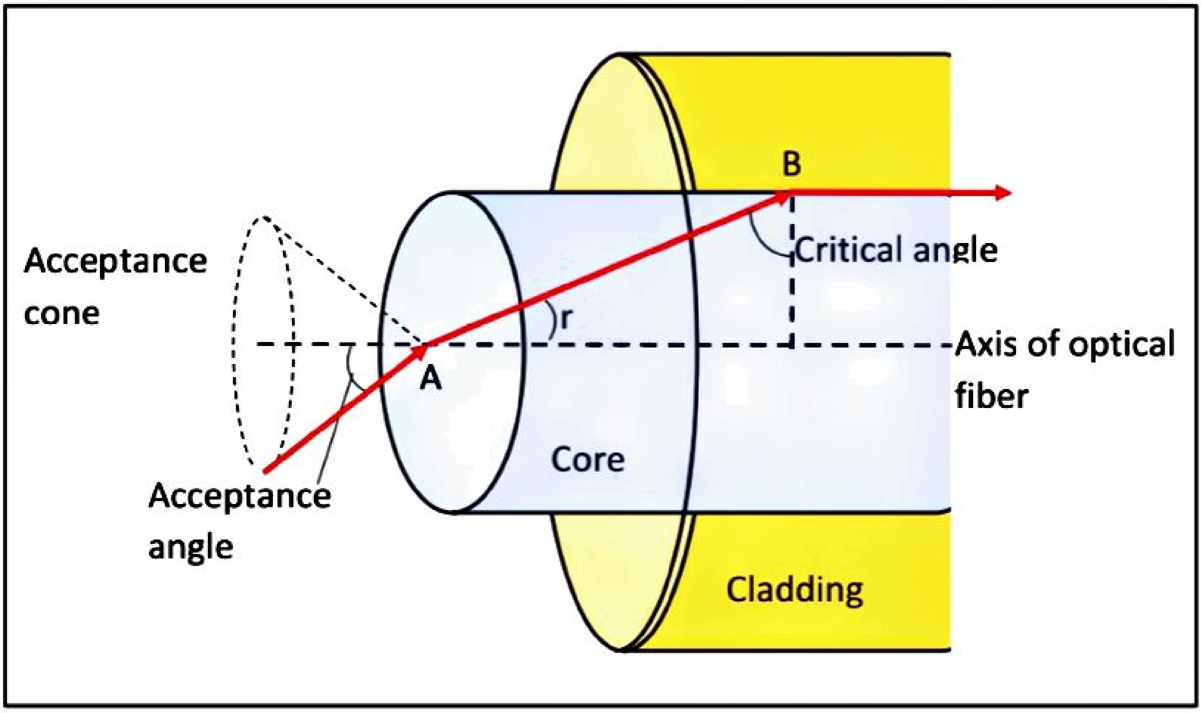


Figure 2b.4.1: Acceptance angle for fibre optic cable

Acceptance Cone: The solid angle made by the acceptance angle in all directions is called acceptance cone all light incident in this cone propagates through the fibre successfully.

Q2b.5. Distinguish between single mode and multimode fibres.

(M.U. Dec 2009; May 2013) (3 m)

Single / Mono mode fibre (SMF)	Multi-mode fibre (MMF)
Support only one mode of propagation	Support more than one mode
Core diameter is small	Core diameter is large

Usually step index type	Further divided as step index and graded index

Q2b.6. Differentiate between SI fibre and GRIN fibre.

(M.U. Dec 2003, 05, 10, 16; May 2013, 15) (5m)

Step index optical fibre	Graded Index optical fibre
Discontinuity of index profile at core cladding junction.	R.I. of core decreases gradually to attain R.I. of cladding at core-cladding.
R.I. of core is constant.	R.I. of core decreases nearly in parabolic manner.
High attenuation.	Low attenuation.
For a given diameter the Numerical Aperture (N.A.) is greater.	For a given diameter the Numerical Aperture (N.A.) is lesser compared to SI.

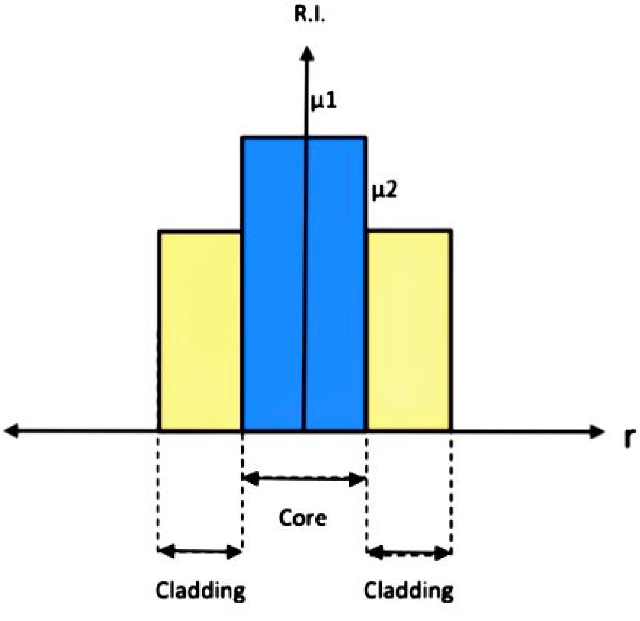


Figure 2b.6.1: Step Index fibre

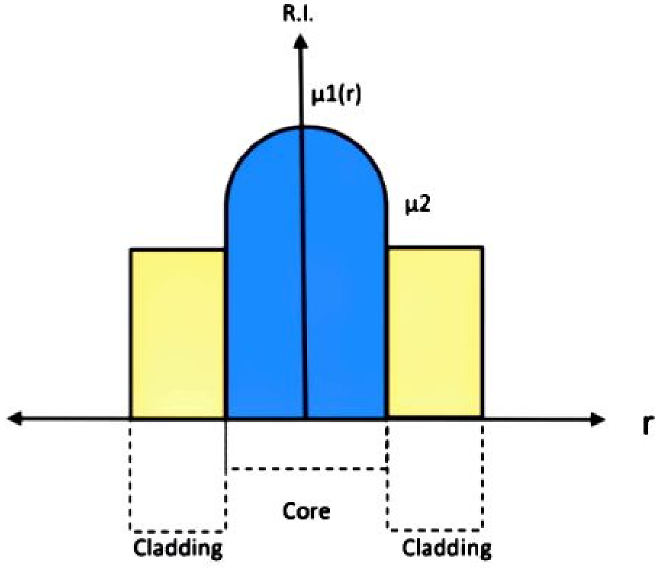


Figure 2b.6.2: Graded Index fibre

Q2b.7. Describe fibre optic communication system.

(M.U. Nov. 2018) (5m)

Transfer of information from one place to another is called communication. For communication to occur a system should consist following three main parts as shown in *Figure 2b.7.1*:

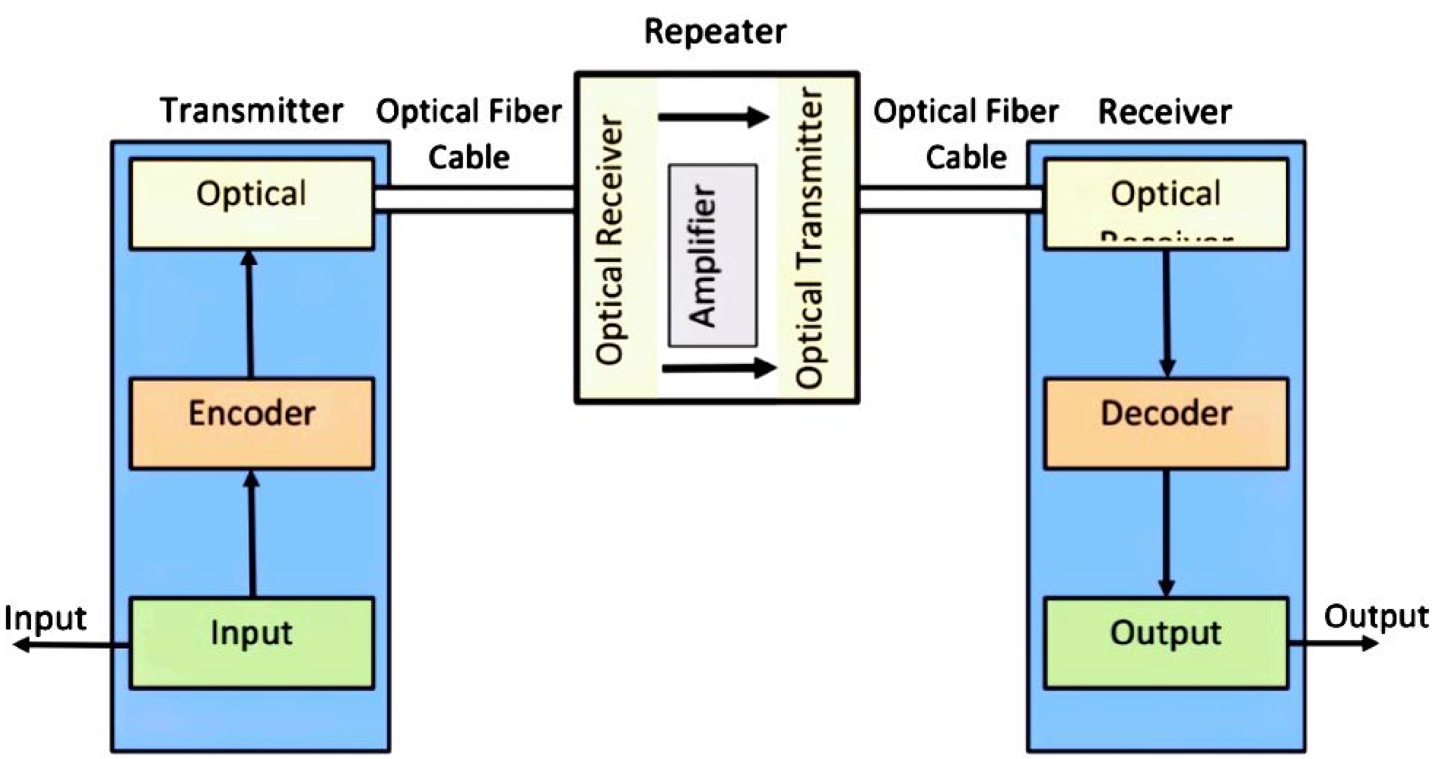
A. Transmitter (T)
B. Receiver (R)
C. Channel for propagation of information from (T) to (R)

Fibre optic communication system is specialized in a sense that the information propagates in the form of light and hence the transmitter and receiver in this type of system has additional components to support this modification. A block diagram for a typical optical fibre communication system is shown in the figure below.

Principle Elements of typical Fibre optical communication system are:

1. **Input Device:** A typical input device would be a Telephone or mike in case of a voice input to be transmitted across the channel to the receiver
2. **Encoder:** This block collects the input signal from the Input device, mixes it with a high frequency carrier as and when required and converts it to an electrical signal which is sent to the optical transmitter.
3. **Optical transmitter:** This is the special device in the transmitter that converts electrical signal to light and launches it appropriately to propagate through the optical fiber. Eg. LED.
4. **Optical fibre:** This is the three layered fibre consisting of a core-cladding interface covered in a sheath. The information propagates through the core in the form of light due to the phenomenon of total internal reflection in a zig – zag manner.

5. **Optical receiver:** This is the special device in the Receiver that converts light from the optical fiber to electrical signal. Eg. Photodiode.
6. **Decoder:** This block collects the signal from the Optical receiver, removes the high frequency carrier as and when required and converts electrical signal from the optical receiver into a user understandable output (Audio).
7. **Output Device:** A typical output device would be a Telephone or speaker in case a voice output is expected at the receiver end.
8. **Repeater:** A typical amplification device suitable for long distance communication through optical fibres.



Q2b.8. What are the advantages of using fibre optic communication systems?

(M.U. May 2008, 16, 17; Dec. 2012) (3m)

The advantages of using a fiber optic communication system are:

1. **Greater Bandwidth:** Fiber optic cables provide more bandwidth than copper cables of the same diameter, in carrying more data.
2. **Faster Speeds:** Fiber optic cables have a center that carries light to transmit data. This allows fiber optic cables to carry signals at a faster pace.
3. **Better Reliability:** Fiber optic cable is immune to temperature changes, severe weather and moisture, all of which can hamper the connectivity of copper cable. Plus, fiber doesn't carry current , so it's not bothered by electromagnetic interference (EMI) which will interrupt data transmission.
4. **Thinner and Sturdier:** In comparison to copper cables, fibre optic cables are lighter and thinner. They are also less susceptible to breakage and can also withstand a higher amount of pull and pressure.
5. **Lower Total Cost of Ownership:** Although some fiber optic cables may have a better initial cost than copper, the sturdiness and reliability of fiber can make the entire cost of ownership (TCO) lower. Day by day costs continue to decrease for fiber optic cables as technology advances.

Formula List for Fiber Optics

1. Numerical Aperture (N.A.)

$$N.A. = \sin\theta_0 = \sqrt{\mu_1^2 - \mu_2^2}$$

Where;

- θ_0 = Acceptance angle
- μ_1 = Refractive index of core
- μ_2 = Refractive index of cladding

2. Numerical Aperture (N.A.) of fiber in other medium

$$N.A. = \sin\theta_0 = \frac{\sqrt{\mu_1^2 - \mu_2^2}}{\mu_o}$$

Where;

- θ_0 = Acceptance angle
- μ_1 = Refractive index of core
- μ_2 = Refractive index of cladding
- μ_o = Refractive index of other outer medium

3. Fractional Refractive Index (Δ)

$$\Delta = \frac{\mu_1 - \mu_2}{\mu_1}$$

Where;

- μ_1 = Refractive index of core
- μ_2 = Refractive index of cladding

4. Numerical Aperture in terms of Δ

$$\mathrm{N.A.} = \mu_1\sqrt{2\Delta}$$

Where;

- μ_1 = Refractive index of core
- Δ = Fractional refractive index of the fibre

5. Critical angle at core-cladding interface (φ_c)

$$\varphi_c = \sin^{-1}\frac{\mu_2}{\mu_1}$$

Where;

- μ_1 = Refractive index of core
- μ_2 = Refractive index of cladding

6. Normalized Frequency (V)

$$V = \frac{2\pi a}{\lambda}\sqrt{{\mu_1}^2 - {\mu_2}^2}$$

Where;

- μ_1 = Refractive index of core
- μ_2 = Refractive index of cladding
- λ = Wavelength travelling through the fiber
- a = Radius of the core

7. Number of modes (N_m)

$$N_m = \frac{V^2}{2}$$

Where;

- V = Normalized Frequency

FIBRE OPTICS PROBLEMS

Q1. Calculate the numerical aperture and hence the acceptance angle for an optical fiber. Given that the refractive indices of the core and the cladding are 1.45 and 1.40 respectively.

Given:- μ_1 =1.45; μ_2=1.40

Formula :- $N.A. = \sin\theta_0 = \sqrt{\mu_1^2 - \mu_2^2}$

<u>**Solution**</u>:- N.A.= $\sqrt{1.45^2 - 1.40^2}$

= 0.3775

θ_0 = sin^{-1}(N.A.) = sin^{-1}(0.1425)

= 8.192°

<u>**Ans:- N.A. of fiber is 0.3775 and its acceptance angle is 8.192°**</u>

Q2. The refractive index of core and cladding of a SI fiber are 1.52 and 1.41 respectively. Calculate (i) critical angle (ii) NA and (iii) maximum incidence angle.

Given:-$\mu_1 = 1.52$; $\mu_2 = 1.41$

Formula:- $\varphi_c = \sin^{-1}\frac{\mu_2}{\mu_1}$; $N.A. = \sin\theta_0 = \sqrt{\mu_1^2 - \mu_2^2}$

<u>**Solution**</u>:- φ_c = sin^{-1}($\frac{1.41}{1.52}$)= 68.06°

N.A.= $\sqrt{1.52^2 - 1.41^2}$ = 0.5677

θ_0= sin^{-1}(0.5677)

= 34.59°

<u>**Ans:- The critical angle is 68.06° and N.A. is 0.5677 and θ_0 is 34.59°**</u>

Q3. An optical fiber has a NA of 0.20 and refractive index of cladding is 1.59. Determine the core refractive index and the acceptance angle for the fiber in water which has a refractive index of 1.33.

Given:- NA=0.20; μ_2=1.59; μ_o=1.33

Formula:- $N.A. = \sin\theta_0 = \frac{\sqrt{\mu_1^2-\mu_2^2}}{\mu_o}$

<u>Solution:-</u> N.A.=$\sqrt{\mu_1{}^2 - \mu_2{}^2}$

μ_1=$\sqrt{N.A.^2 + \mu_2{}^2}$ =$\sqrt{0.2^2 + 1.59^2}$ =1.6025

$\theta_0 = \sin^{-1}\frac{N.A.}{\mu_o}$ =8.64°

<u>Ans:- The R.I. of core is 1.6025 and acceptance angle 8.64°</u>

Q4. A typical relative refractive index difference for an optical fiber is 1%. Estimated the numerical aperture and the critical angle at the core cladding interface if the core refractive index is 1.46.

Given:- Δ=0.01; μ_1=1.46

Formula:- $\Delta=\frac{\mu_1-\mu_2}{\mu_1}$; N.A.= $\mu_1\sqrt{2\Delta}$; $\varphi_c = \sin^{-1}\frac{\mu_2}{\mu_1}$

<u>Solution:-</u> N.A.=1.46(2 x 0.01)$^{1/2}$ = 0.2064

Δ= 1- $\frac{\mu_2}{\mu_1}$

$\frac{\mu_2}{\mu_1}$ = 1- Δ = 0.99

φ_c=sin^{-1}0.99 = 81.89°

<u>Ans:- The N.A. is 0.2064 and the critical angle is 81.89°.</u>

Q5. A glass clad fiber is made with core glass of refractive index 1.5 and the cladding is doped to get a frictional index difference of

0.0005. Find [a] the refractive index of the cladding. [b] the critical internal reflection angle [c] Acceptance angle.

Given :- μ_1=1.5; $\Delta = 0.0005$

Formula:-Δ= 1- $\frac{\mu_2}{\mu_1}$; N.A.= $\sqrt{\mu_1{}^2 - \mu_2{}^2}$

Solution:- μ_2= μ_1(1-Δ)=1.5(1-0.0005) =1.49925

φ_c=$\sin^{-1}\frac{\mu_2}{\mu_1}$= $\sin^{-1}\frac{1.49925}{1.5}$=88.18°

$\theta_0 = \sin^{-1}\sqrt{\mu_1{}^2 - \mu_2{}^2}$

=$\sin^{-1}\sqrt{1.5^2 - 1.49925^2}$ = 2.718°

Ans:- The R.I of cladding is 1.49925 and the critical angle is 88.18° and the acceptance angle is 2.718°.

Q6. A step index fiber has core diameter 29x10^{-6}m . The refractive indices of the core and the cladding are 1.52 and 1.5189 resp. If the light of wavelength 1.3μm is transmitted through the fiber, determine [a]Normalized frequency of the fiber. [b] The number of modes fiber will support.

Given :- d=29x10^{-6}m; $\lambda = 1.3 \times 10^{-6} m$; μ_1= 1.52; μ_2=1.5189.

Formula :- V=$\frac{2\pi r\sqrt{\mu_1^2-\mu_2^2}}{\lambda}$; N_m=$\frac{V^2}{2}$

Solution:- V=$\frac{3.14\times 29\times 10^{-6}}{1.3\times 10^{-6}}$ x $\sqrt{(1.5)^2 - (1.5189)^2}$= 4.049

N_m=$\frac{V^2}{2}$ = $\frac{(4.049)^2}{2}$= 8.197

Ans:- Normalized frequency is 4.049 and the number of modes is 8.

Unit 4: RELATIVITY

Q4.1. What do you mean by an inertial and non-inertial frame of reference?

A Reference frame: -

Three space coordinates x, y, z and time t define a reference frame. All physical phenomenon's need a reference frame with respect to which they can be studied.

Inertial frame: -

If a reference frame is either at rest or moving with a uniform velocity then it is called an inertial frame of reference. Example: A person standing on a railway station and a person standing in a train that is moving with constant velocity. Both persons are in inertial frames.

Non inertial frame: -

An accelerating frame of reference is called non inertial frame of reference. Example: If the train in the previous example does not move with constant velocity, then the train would have been an example of non-inertial frame of reference.

Q4.2. State the fundamental postulates of Special theory of relativity

1. The Principle of Relativity
   All physical laws are the same in all inertial frames that are moving relative to each other with constant velocity.
2. Principal of independence of velocity of light
   The speed of light in free space has the same value 'c' in all inertial reference frames.

Q4.3. Discuss the Galilean transformations for space and time.

Different inertial frames may have different values of physical quantities, but laws of physics do not depend on the choice of the frame. Galilean transformations are used to relate the physical quantities in one inertial frame to another inertial frame.

Let S and S' be Representations of two inertial frames with coordinate systems (x, y, z) and (x' y' z') respectively. Let S' be moving with uniform velocity v with respect to S reference frame. Let an event take place at point P. The coordinates of point P with respect to individual frame is S (x, y, z, t) and S' (x', y', z', t') for simplicity let us assume that the x, y and z axes of coordinate systems are parallel to each other. We start counting from the time when origin of S i.e. O and origin of S' i.e., O' coincide as shown in *Figure 4.3.1*

After lapse of time 't', 'S' moving with velocity 'v' would have covered a distance 'd = v t'. Let coordinates system move along x axis with respect to each other.

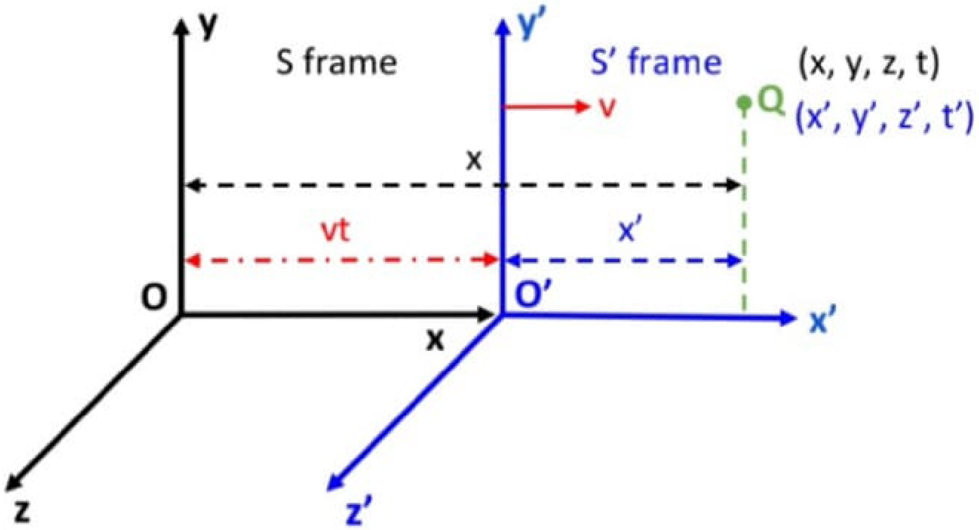


Figure 4.3.1: Galilean Transformation

Hence, the relation between coordinates of P in one frame is related to those in the other frame by the equations called the Galilean transformation equations.

The transformations for space and time are given as:

$$x' = x - vt \quad \text{\_\_\_\_\_\_} (1)$$

As the object does not undergo any relative motion in the y and z direction:

$$y' = y \quad \_\_\_\_\_\_ (2)$$

$$z' = z \quad \_\_\_\_\_\_ (3)$$

Also,

$$t' = t \quad \_\_\_\_\_\_ (4)$$

The Galilean Transformations for velocity are as follows:

Assuming the object is moving with a velocity u, let u_x, u_y, and u_z be the velocities in the x, y and z directions respectively, with respect to frame S and u_x', u_y' and u_z' with respect to frame S'.

Consider equation 1, upon differentiating with respect to time, we get the following result:

$$(dx'/dt) = (dx/dt) - v_x \quad \_\_\_\_\_\_ (5) \text{ or,}$$

$$u_x' = u_x - v_x \quad \_\_\_\_\_\_ (6)$$

Similarly, we get,

$$u_y' = u_y - v_y \quad \_\_\_\_\_\_ (7)$$

$$u_z' = u_z - v_z \quad \_\_\_\_\_\_ (8)$$

However, Galilean Transformations do not satisfy the postulates of the Special Theory of Relativity. Galilean Transformations give very different results while converting some physical quantities in different frames which violates the first postulate. Also, according to Galilean Transformations, the speed of light would be different in different inertial frames, which violates the second postulate.

Q4.4. Derive the Lorentz transformation equations for space and time

The Galilean transformations relate space time coordinates in one inertial frame to those in the other frame. But these transformations are not valid for the case where velocity of motion of one frame with respect to other frame approaches 'c' i.e., velocity of light.

The transformations that apply to all speeds and are valid up to 'c' i.e., velocity of light is known as Lorentz transformation.

Consider two inertial frames S and S' where standards of measuring distance and time are the same. Let S be stationary and S' be moving with a constant velocity 'v' with respect to frame S along x' and x axes of frames S' and S respectively. Let us assume x and x' are the same line while y and y' also z and z' are parallel. Let at initial time t=t'=0 and this is when Origin O and O' of frames S and S' coincide as shown in *Figure 4.4.1*.

As space and time are regarded as homogeneous. The relation between their coordinate and time in different frames is linear

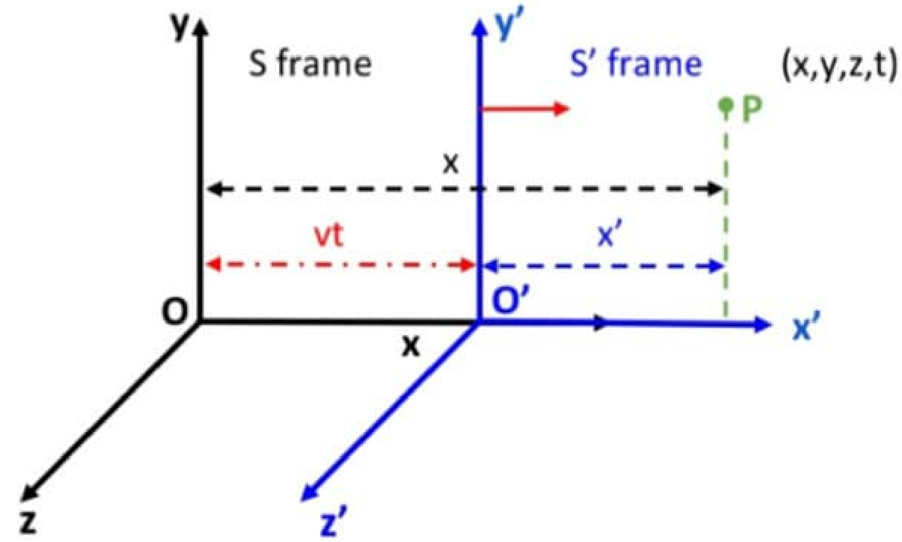


Figure 4.4.1: Lorentz transformation

$x' = ax + bt$ ....(1)

$t' = fx + gt$ ....(2)

Equation (1) can be written as:

$x' = a\left(x + \frac{b}{a}t\right)$ ....(3)

After time t, origin O' of frame S' is at position x=vt with respect to frame S

And x' for origin O' corresponds to x'=0, putting these values in Equation (3)

$0 = a\left(vt + \frac{b}{a}t\right)$ this gives,

$v = -\frac{b}{a}$ ....(4)

Substituting (4) in Equation (3) we get,

$x' = a(x - vt)$ ....(5)

Similarly, we can write, $x = a(x' + vt)$ ....(6)

Since S' moves along x axis only, the y and z coordinates do not change, hence:

$y = y'$ and $z' = z$ ....(7)

We know that for light velocity 'c',

$x' = c\,t'$, whereas $x = ct$ putting these two in Equation (5) we have:

$c\,t' = a\,(c\,t - v\,t)$

$\therefore c\,t' = a\,c\,t\,(1 - \frac{v}{c})$

$t' = a\,t\,(1 - \frac{v}{c})$ ....(8)

Hence for t we can write, $t = a\,t'\,(1 + \frac{v}{c})$ ....(9)

Using Equation (9) in Equation (8);

$t' = a\,a\,t'\,(1 + \frac{v}{c})\,(1 - \frac{v}{c})$ this becomes

$a^2 = \frac{1}{1 - \frac{v^2}{c^2}}$

More often the constant 'a' is denoted by 'γ'

$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ ....(10)

Using Equation (10) in Equation (8) and Equation (5),

$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$ ....(11)

$t' = \frac{t\left(1 - \frac{v}{c}\right)}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t - \frac{vtc}{c\,c}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$ (using ct=x) ....(12)

Thus, the Lorentz transformation of coordinate from system S' to frame S are

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$y' = y; \quad z' = z$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots(13)$$

Similarly, the Inverse Lorentz transformation equations can be written as:

$$x = \frac{x' + vt}{\sqrt{1 - \frac{v^2}{c^2}}};\ y = y';\ z = z';\ t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots(14)$$

Q4.5. Explain length contraction and time dilation

Relativistic effects appear to conflict common sense. This is due to the fact that the velocities that we have observed in our everyday experience are very small as compared to velocity of light 'c'. Special theory of relativity predicts observers will measure different time and length in different inertial frames only when the frames are moving with a velocity comparable to 'c' with respect to each other. Length contraction and time dilation are two relativistic effects explained below:

1. Length Contraction:
   Consider two inertial frames S and S', where S' is moving with a constant velocity 'v' with respect to S. Let x_1' and x_2' be two ends of a rod at rest as seen by an observer in S' frame. Let L be the length of the rod measured by the observer in the S frame as shown in *Figure 4.5.1*.

   $$L = x_2 - x_1$$
   $$L' = x_2' - x_1'$$

   Both measurements are done simultaneously: $t_2 = t_1 = t$

   Using Lorentz transformation equations, we get:

   $$L' = x_2' - x_1' = \gamma\ [\ (x_2 - vt) - (x_1 - vt)]$$

$$L' = \gamma\,(x_2 - x_1)$$
$$L' = \gamma\, L$$

The Rod is at rest in S': L' =Lo true length of the rod and the rod appears to be in motion to an observer in S frame so L= apparent length

$$L = \frac{L_0}{\gamma} = L_0\sqrt{1-\frac{v^2}{c^2}}$$

So, length contraction is explained by the equation $L = L_0\sqrt{1-\frac{v^2}{c^2}}$

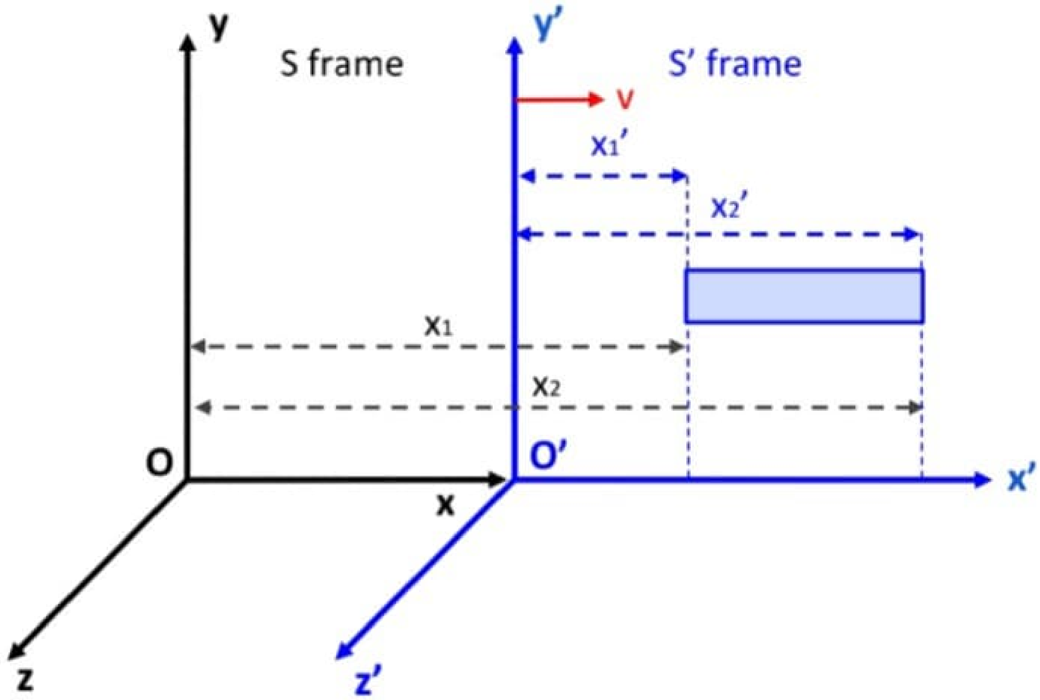


Figure 4.5.1: Length contraction

2. Time dilation:

   Let a clock be at rest in S frame at point x. Suppose it produces two ticks at times t_1' and t_2' in frame S'. The time interval between these two ticks may be given by,

   $$\Delta t' = t_2' - t_1'$$

   Using Lorentz transformation equations,

$$\Delta t' = t_2' - t_1' = \gamma\ [\ (t_2 - \frac{v}{c^2}x) - (t_1 - \frac{v}{c^2}x)\}$$
$$\Delta t' = \gamma\ (t_2 - t_1) = \gamma\ \Delta t$$

Let $\Delta t = t_o$ as the clock was at rest in the S frame

$\Delta t' = t$ as the apparent time interval measured in the S' frame

$$\therefore t = \gamma t_o$$

So, the time dilation is explained by equation $t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}}$

Q4.6. Derive Einstein's Mass energy relation.

Relativistic mass:

Mass of a body is supposed to be independent of its velocity. Due to momentum conservation, we require that momentum of an isolated system be conserved.

Relativistic to an isolated system to conserve momentum it is observed that mass must depend on velocity and the relation that govern this dependence is,

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \text{..........................} (1)$$

Where,

m= moving mass; m_o= rest mass; v= velocity of motion; c = velocity of light

Relativistic momentum:

$$P = mv = \frac{m_o \times v}{\sqrt{1-\frac{v^2}{c^2}}} \text{.......................} (2)$$

This is the resultant relativistic momentum of the particle after substituting the relativistic mass that is dependent on velocity of the particle 'v'.

Kinetic energy:

Newton's second law states that force is equal to the rate of change of momentum.

$$F=\frac{d\,(mv)}{dt}=m\frac{d\,(v)}{dt}+v\frac{d\,(m)}{dt}\text{.....................}(3)$$

Kinetic energy of a moving body is force into displacement,

$$dE_k = F.\,dx=\left(m\frac{d\,(v)}{dt}+v\frac{d\,(m)}{dt}\right).dx$$

$$dE_k = mv\,dv+ v^2\,dm \text{ .. ..}(4).\text{ using } \frac{dx}{dt}=v$$

We know that, $m=\frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$ and hence

$$dm = m_o\left(\frac{-1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{\frac{-3}{2}}\left(\frac{-2v}{c^2}dv\right)$$

$$dm=\frac{m_0\,vdv}{c^2\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}$$

m_o can be replaced by $m\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}$......using (1)

$$\therefore dm=\frac{mvdv}{c^2\left(1-\frac{v^2}{c^2}\right)}=\frac{mvdv}{(c^2-v^2)}\text{ ....}(5)$$

Rearranging (5) we can write,

$$c^2\,dm - v^2dm = m\,vdv \text{ ....}(6)$$

Comparing Equations (6) and (4) we get

$$dE_k= c^2dm$$

$$E_k=\int_0^{E_k} dE_k = c^2\int_{m_0}^{m} dm$$

$$E_k = c^2\,(m-m_o)$$

Thus, relativistic kinetic energy of a body is equal to the gain in mass multiplied by square of speed of light.

$\therefore m_oc^2$ is the energy of the body at rest.

Total energy of a body: $E=E_k+$ rest energy

$E = c^2 (m - m_o) + m_o c^2$

This is the Energy mass relation of Einstein i.e., $\mathbf{E = m c^2}$

Formula List for Relativity

1. Length contraction

$$l = l_0\sqrt{1-\frac{v^2}{c^2}}$$

Where;

- l = length of the object while its moving
- l_0= length of the object when at rest
- v= Velocity with which the object is moving
- c= velocity of light

2. Time dilation

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where;

- t = time measured by clock which is moving
- t_0= time measured by clock at rest
- v= Velocity with which the object is moving
- c= velocity of light

RELATIVITY PROBLEMS

Q1. What will be the length of a meter rod appear to a person travelling parallel to the length of the rod at a speed of 0.8c relative to rod?

Given:- l_0=1.0m ; v=0.8c

Formula :- $l = l_0\sqrt{1-\frac{v^2}{c^2}}$

Solution :- $l = 1.0\sqrt{1-0.8^2}$ =0.6m

Ans:- The apparent length of the rod while in motion is 0.6m.

Q2. A meter ruler moves past an observer on the earth with a velocity of 2.5×10^{10} cm/ sec, along the direction of its length. What is its apparent length with respect to the observer?

Given:- l_0= 1.0m ; v=2.5×10^8m/s

Formula:- $l = l_0\sqrt{1-\frac{v^2}{c^2}}$

Solution :- l =1.0 $\sqrt{1-\left(\frac{2.5\times 10^8}{3\times 10^8}\right)^2}$ =55.27cm

Ans:- The apparent length of the moving ruler is 55.27cm.

Q3. A spaceship 50m long passes the earth at a speed of 2.8×10^8 m/s. what will be its apparent length?

Given:- l_0= 50m ; v=2.8×10^8m/s

Formula:- $l = l_0\sqrt{1-\frac{v^2}{c^2}}$

Solution :- $l = 50\sqrt{1-\left(\frac{2.8\times 10^8}{3\times 10^8}\right)^2}$ =18m

Ans:- The apparent length of the moving spaceship is 18m.

Q4. A rocketship is 50m long. When it is on flight its length appears to be 49.5 to an observer on ground. Find the speed of the rocket?

Given:- l =49.5m; l_0=50m

Formula:- $l = l_0\sqrt{1-\frac{v^2}{c^2}}$

Solution:- $49.5 = 50\sqrt{1-\frac{v^2}{c^2}}$

$1-\frac{v^2}{c^2}$=0.9801

v=0.141c

Ans:- The speed of the rocket is 0.141c .

Q5. A certain particle called meson has a life time 2 ×10^{-6}sec; a] What is the mean life time when the particle is travelling with a speed of 2.9994× 10^8 m/sec?; b] How far does it go during one mean life?

Given:- t_0 =2 ×10^{-6}sec ; v= 2.9994× 10^8 m/sec

Formula:- $t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$; distance (d) = Speed (v) x time (t)

Solution:- $t = \frac{2\times 10^{-6}}{\sqrt{1-\left(\frac{2.9994\times 10^8}{3\times 10^8}\right)^2}}$ =31.63 x 10^{-6} sec

Distance travelled by meson during mean life time,

d = (2.994x10^8)(31.63x10^{-6})

=9470m

Ans:- The mean life time is 31.63x10^{-6}sec and the distance is 9470m.

Q6. The mean life of meson is $2x10^{-8}$ sec. calculate the mean life of a meson moving with a velocity 0.8c?

Given :- $t_0 = 2x10^{-8}$; v=0.8c

Formula:- $t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$

<u>**Solution:-**</u> $t = \frac{2\times 10^{-8}}{\sqrt{1-(0.8)^2}}$ =3.3 x 10^{-8} sec

<u>**Ans:- The mean life is 3.3x10^{-8}sec .**</u>

Q7. A wrist watch keeps correct time on earth. If it is worn by pilot in spaceship, leaving earth with constant velocity of 10^9 cm/sec. How many seconds does it appear to lose in one day with respect to the observer on the Earth.

Given:- v=10^9 cm/sec =10^7 m/sec; t = 24 hrs in a day

Formula:- $t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$

<u>**Solution:-**</u> 24= $\frac{t_o}{\sqrt{1-\frac{1}{900}}}$

$$t_o = 24[1-\frac{1}{900}]^{-1/2}$$

Here, 1/900 is very small w.r.t 1 hence we can use the binomial expansion and neglect higher order terms

$$t_o = 24\ [1-\frac{1}{1800}] = 24-\frac{1}{75}\text{ hrs}$$

hence loss in 24 hours= $\frac{1}{75}$ hrs =48 sec

<u>**Ans:- wrist watch looses 48 sec in a day with respect to observer on earth.**</u>

Unit 3: ELECTRODYNAMICS

Q3.1. Describe three different types of Coordinate Systems

In electromagnetics, most of the quantities are either functions of space or time. In order to describe the spatial variations of these quantities, all the points in space must be defined uniquely using an appropriate coordinate system.

The most useful three coordinate systems for this purpose are:-

1. Cartesian, or rectangular, coordinates
2. Cylindrical, or circular, coordinates
3. Spherical, or polar, coordinates.

1. Cartesian or Rectangular Coordinates (x, y, z)

A point P in Cartesian coordinates is represented as P(x, y, z) as shown in *Figure 3.1.1*

The ranges of coordinate variables are

$-\infty < x < \infty$

$-\infty < y < \infty$

$-\infty < z < \infty$

A vector $\vec{A}$ in the Cartesian coordinate system is written as,

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

Where, $\hat{i}, \hat{j}$ and $\hat{k}$ are the unit vectors along the x, y and z directions respectively.

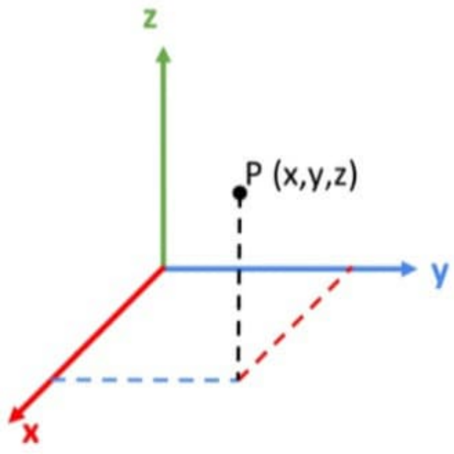


Figure 3.1.1: Cartesian coordinates

2. Cylindrical Coordinates P (r, ϕ, z)

A point P in cylindrical coordinates is represented as P(r, ϕ, z) as shown in *Figure 3.1.2* ,

The Ranges of Coordinates are,

$$0 \le r < \infty$$

$$0 \leq \emptyset < 2\pi$$

$$-\infty < z < \infty$$

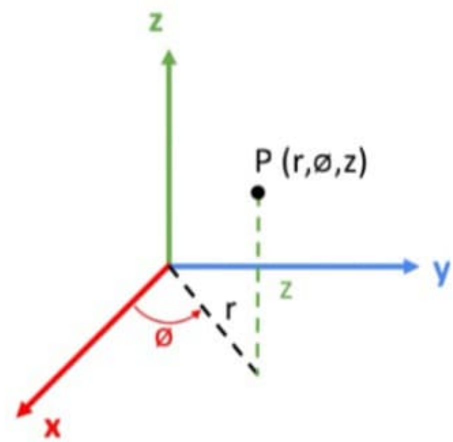


Figure 3.1.2: Cylindrical coordinates

Where,

r=Radius of the cylinder passing through P

ϕ=Angle from the X-axis in the xy-plane, known as azimuthal angle

z= same as in Cartesian coordinates

A vector ' $\vec{A}$ ' in cylindrical coordinate system is written as,

$$\overrightarrow{A} = A_r\hat{r} + A_\emptyset\hat{\emptyset} + +A_z\hat{z}$$

Where $\hat{r}, \hat{\emptyset}$ and $\hat{z}$ are the unit vectors along the r, ø and z directions respectively.

3.Spherical or Polar Coordinates (r,θ,ø)

A point P in spherical coordinates is represented as P(r,θ,ϕ) as shown in *Figure 3.1.3* . The ranges of coordinate variables are,

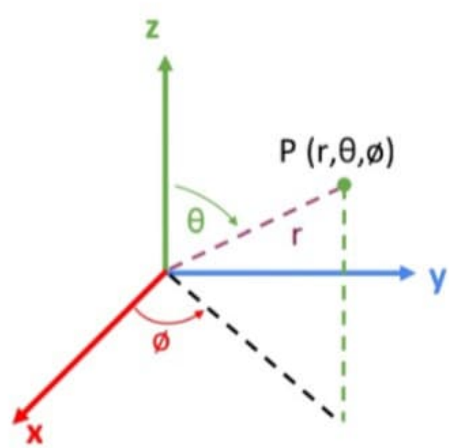


Figure 3.1.3: Spherical coordinates

$$0 \leq r < \infty$$

$$0 < \theta < \pi$$

$$0 \leq \emptyset < 2\pi$$

Where,

r= Distance of the point from the origin

θ = Angle between the z-axis and P

ϕ =Angle from the X-axis in the xy-plane, known as azimuthal angle

A vector ' $\vec{A}$ ' inspherical coordinate system is written as,

$$\vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_\phi\hat{\phi}$$

Where, $\hat{r}, \hat{\theta} and \hat{\phi}$ are the unit vectors along the r,Θ,and ø directions respectively.

Q3.2. Obtain relation between Cartesian and Spherical coordinates and also between Cartesian and Cylindrical coordinates

Relation between Cartesian(x,y,z) and Spherical (r,Θ,ϕ) Coordinates

r=$\sqrt{x^2+y^2+z^2}$ Θ=$\tan^{-1}(\frac{\sqrt{x^2+y^2}}{z})$ ϕ=$\tan^{-1}\frac{y}{x}$

And

x=r sinΘ cosϕ y= r sinΘ sinϕ z=r cosΘ

Relations between Cartesian (x, y, z) and Cylindrical (r, ø, z) Coordinates

r=$\sqrt{x^2+y^2}$ ϕ=$\tan^{-1}\frac{y}{x}$ z = z

And

x=r cosϕ y= r sinϕ z = z

Q3.3. Define a field. What are scalar and vector fields

Field: Behaviour of a physical quantity in a given region is described by its value at each point in the region, the function describing this value is called a field.

Scalar field: A scalar field is something that has a particular value at every point in space. An example of a scalar field is temperature.

Vector field: A vector field is something that has a particularvalue and direction at every point in space.An example of a vector field is Electric field.

Q3.4. What is ∇ (DEL) Operator

The collection of partial derivative operators is called DEL operator. Hence DEL can be viewed as the derivative in multi-dimensional space.DEL operator is defined as a vector differential operator. A DEL operator is not a vector in itself, but when acts on a scalar function, it becomes a vector.DEL is not simply a vector; it is a vector operator. Whereas a vector is a quantity with both a magnitude and direction, DEL does not have a precise until it is allowed to operate on something.In Cartesian coordinate system Del operator is given as:

$$\vec{\nabla}= \hat{\imath}\frac{\partial}{\partial x}+\hat{\jmath}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}$$

This operator is useful or significant in defining

- Gradient of a scalar V: $(\vec{\nabla}V)$
- Divergence of a vector A:$(\vec{\nabla}.\vec{A})$
- Curl of a vector A: $(\vec{\nabla}\times\vec{A})$

Q3.5. What is gradient of a scalar field? Express in Cartesian form

The gradient of a scalar field provides a vector field that states how the scalar value is changing throughout space – a change that has both a magnitude and direction.

The physical meaning of the gradient of a scalar is that it represents the steepness of the slope or line. For example, let T be the scalar function of temperature then the first term of the gradient defines rate of change of temperature along x axis, second term defines change along y axis and third term along z axis. It is mathematically represented as:

$$\vec{\nabla}T = \hat{\imath}\frac{\partial T}{\partial x}+\hat{\jmath}\frac{\partial T}{\partial y}+\hat{k}\frac{\partial T}{\partial z}$$

Thus, the Gradient is also called the directional derivative of a scalar function.

Q3.6. What is Curl of a Vector field ? Express in Cartesian form

Curl of a Vector Field (Curl A) in <u>Cartesian Coordinate system</u> is given as:

$$\vec{\nabla}\times\vec{A} = \text{determinant}\left|\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}\right|$$

The direction of the curl is the axis of rotation, as determined by the right hand thumb rule and the magnitude of the curl is the magnitude of rotation. If the curl of a vector field $\vec{A}$ is zero, then the vector field $\vec{A}$ is said to be irrotational. In such cases, the circulation of $\vec{A}$ around a closed path is zero; it implies that the line integral of $\vec{A}$ is independent of the chosen path. Hence an irrotational field is also known as a conservative field.

Q3.7. Explain the physical significance of Divergence.

(M.U. Dec 2019) (5 m)

Divergence of a vector 'A' $(\nabla . A)$ is the measure of the extent to which the vector 'A' spreads. In other words, $\nabla . A$ indicates how much vector A diverges (spreads out) from a given point.

$(\nabla . A)$indicates how much vector A diverges (spreads out) from a given point P.

$\int_0^S (\nabla . A)\, ds$ indicates how much vector A diverges (spreads out) from a given surface 'S'.

$\int_0^V (\nabla . A)\, dV$ indicates how much vector A diverges (spreads out) from a given Volume 'V'.

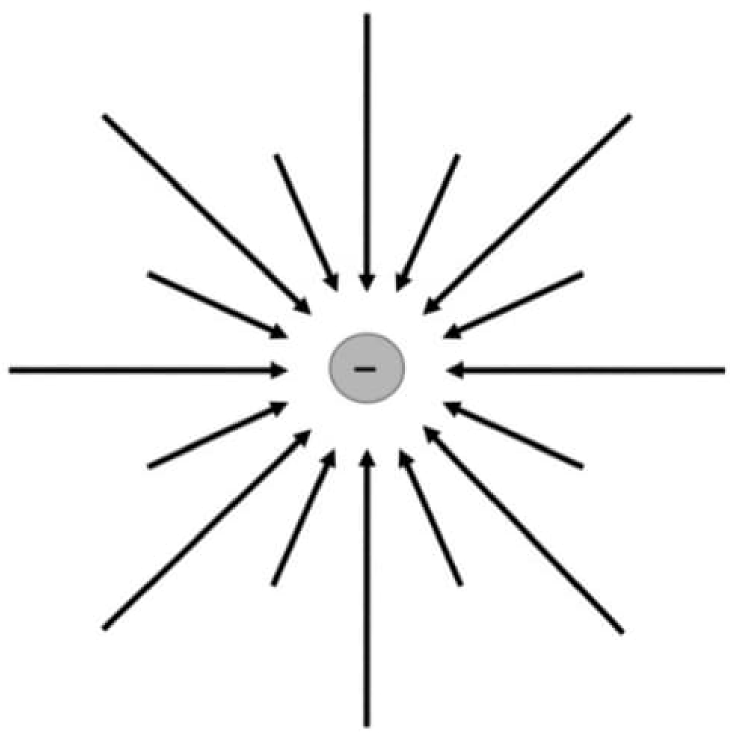


Figure 3.7.1: Negative divergence

The value of the divergence could be positive, negative or zero. Depending on its value following conclusions can be derived.

1. $(\nabla . A)$= Negative indicates that the vector A is closing in on point P. Then point P where divergence is found negative is called the sink for vector field A as shown in Figure 3.7.1.

2. $(\nabla . A)$= Positive indicates that the vector A is spreading out from point P. Then point P where divergence is found positive is called the source for vector field A as shown in Figure 3.7.2.

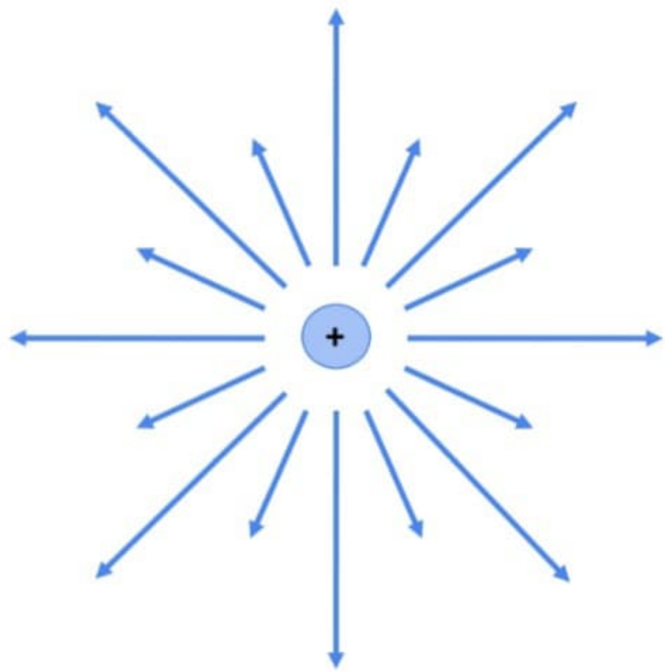


Figure 3.7.2: Positive divergence

3. $(\nabla . A)$=0 then A vector field is called solenoidal, incompressible vector field or divergence less vector field. This means that point 'P' is neither a source nor a sink. It also indicates the amount of field flux entering is equal to amount of the field flux leaving the point 'P' as shown in Figure 3.7.3.

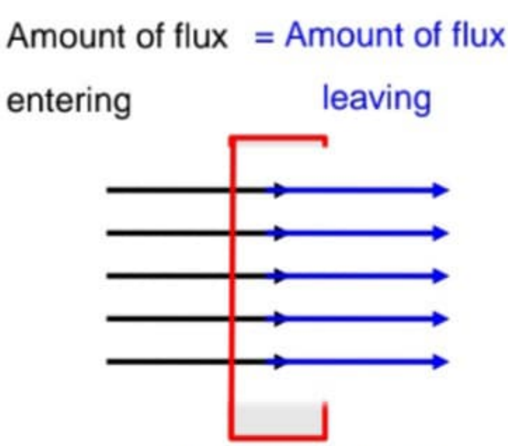


Figure 3.7.3: Zero divergence

Q3.8. What is Divergence of a Vector field? Express in Cartesian form

(M.U. May 2017) (3 m)

Divergence of a vector field $\vec{A}$in Cartesian coordinate system is given as:

$$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

When divergence is applied to a vector function it yields a scalar. Divergence of a vector field $\vec{A}$ is a measure of how much a vector field converges to or diverges from a given point. In simple terms it is a measure of the outgoingness of a vector field.A vector field with constant zero divergence is called solenoid or divergence less or incompressible ($\vec{\nabla} . \vec{A} = 0$). In such cases no net flow can occur across any closedsurface.

Q3.9. Show that divergence of the curl of a vector is zero.

(M.U. Dec. 2017, 19) (3 m)

Ans. Let $A = A_x i + A_y j + A_z k$

$$A = \nabla \times A = \text{determinant} \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) i - \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) j + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) k$$

Now, $\text{div}(\text{curl } A) = \nabla . (\nabla \times A)$

$$= \frac{\partial}{\partial x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$= 0$$

Hence divergence of a curl vanishes.

Q3.10. State two important Theorem for electrodynamics.

1. Divergence Theorem.
   This theorem states that volume integral of divergence of Vector $\vec{A}$ taken over volume 'V' equals surface integral of Vector $\vec{A}$ taken over surface 'S' enclosing the volume.

$$\int_0^V \vec{\nabla}.\vec{A}\,dv = \oint_0^S \vec{A}.\overrightarrow{dS}$$

2. Stokes Theorem

   The Surface integral of curl of a vector $\vec{A}$ over an open surface 'S' equals line integral of a vector $\vec{A}$ over the enclosed curve 'l' surrounding surface area 'S'.

$$\int_0^s (\overrightarrow{\vec{\nabla}x\overline{A})}.\ dS = \oint_0^l \vec{A}.\overrightarrow{dl}$$

Q3.11. Derive point as well as integral form of Maxwell's equations

(M.U. Dec. 2017) (5 m)

1. Maxwell's First Equation (Gauss Law for Electric Field).

   Statement: Electric flux passing through any closed surface 'S' is equal to the total charge enclosed by the surface. Mathematically

$$ø = Q_{enclosed}/\mathcal{E}_0 \text{-----------}(1)$$

Electric field E is electric flux ø per unit area, thus flux can be written as:

$$ø\ =\ \oint_0^S \vec{E}.\overrightarrow{dS} \text{-------------}(2)$$

And we know that change enclosed inside a closed volume V is

$$Q_{enclosed} = \int_0^V \rho\ dV;\ \text{-----------}(3)$$

Where, $\rho = \frac{\text{Charge}}{\text{Volume}}$ change density

Using (1) ,(2) and (3) we get ;

$$\oint_0^S \vec{E}.\overrightarrow{dS} = \int_0^V \rho\ dV/\mathcal{E}_0 \text{------------}(4)$$

By Divergence Theorem, $\oint_0^S \vec{E}.\overrightarrow{dS} = \int_0^V \vec{\nabla}.\vec{E}\ dV$ ------------(5)

Therefore, $\int_0^V \vec{\nabla}.\vec{E}\ dV = \int_0^V \frac{\rho}{\mathcal{E}_0} dV$

Hence, $\overline{\vec{\nabla}.\vec{E}} = \rho/\mathcal{E}_0$. This is Maxwell's first equation.

2. Maxwell's Second Equation (Gauss Law for Magnetic Field).

Statement: In a magnetic field, the magnetic lines are closed on themselves. Hence, total magnetic flux is zero. This in other words means that there is no magnetic monopole. Mathematically :

$$\phi_B = 0$$

Magnetic field B is magnetic flux ø per unit area, hence magnetic flux is:

$$\phi_B = \oint_0^S \vec{B}.\overrightarrow{dS}$$

Where, B is magnetic flux density

By Divergence Theorem, $\oint_0^S \vec{B}.\overrightarrow{dS} = \int_0^V \vec{\nabla}.\vec{B}\ dV$

Using above three equations we get $\int_0^V \vec{\nabla}.\vec{B}\ dV = 0$

If any volume integral is zero than the integrand must be zero

Hence $\vec{\nabla}.\vec{B} = 0.$ This is Maxwell's Second equation.

3. Faraday's Law (Maxwell's third equation for steady fields).

(M.U. Nov. 2018) (5 m)

Statement: In a static electric field the work done in moving the test charge once around the closed path is equal to zero. Mathematically;

$$\oint_0^l \vec{E}.\vec{dl} = 0 \text{------------}(1)$$

{Because ; Work done = 0; Force x displacement =0
Force = charge x E (i.e. E is the force on unit charge)}

By Stoke's Law, $\oint_0^l \vec{E}.\vec{dl} = \oint_0^S (\vec{\nabla} \times \vec{E}).\overrightarrow{dS}$

Therefore, $\vec{\nabla} \times \vec{E} = 0.$ This is Maxwell's Third equation.

4. Ampere's law (Maxwell's fourth equation for steady fields).

Statement: The line integral of magnetic field $\vec{H}$ around a closed path is exactly equal to the direct current enclosed by that path.

$$\oint \vec{H}.\vec{dl} = I$$

But we know that, $\vec{B} = \mu_0 \vec{H}$ for free space, hence

$$\oint_0^c \vec{B}.\vec{dl} = \mu_0 I$$

Let J be the current density i.e. current per unit cross sectional area and therefore current enclosed inside a surface S is written in terms of J as:

$$I = \int_0^s \vec{J}.\overrightarrow{ds}$$

Hence from above equations we get;

$$\oint_0^c \vec{B}.\overrightarrow{dl} = \mu_0 \int_0^s \vec{J}.\overrightarrow{ds}$$

Using strokes theorem $\oint_0^c \vec{B}.\overrightarrow{dl} = \int_0^s (\vec{\nabla} \times \vec{B}).\overrightarrow{ds}$

Therefore, $\vec{\nabla} \times \vec{B} = \mu_0 J$. This is Maxwell's Fourth equation.

Maxwell's Equations for Static fields:

S.NO.	DIFFERENTIAL FORM	INTEGRAL FORM	SIGNIFICANCE
1.	$\vec{\nabla}.\vec{E} = \rho/\varepsilon_0$	$\oint_0^S \vec{E}.\overrightarrow{dS} = Q/\varepsilon_0$	Gauss Law of Electrostatics
2.	$\vec{\nabla}.\vec{B} = 0$	$\oint_0^S \vec{B}.\overrightarrow{dS} = 0$	Gauss Law of Magneto statics
3.	$\vec{\nabla} \times \vec{E} = 0$	$\oint_0^l \vec{E}.\overrightarrow{dl} = 0$	Faraday's Law
4.	$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$	$\oint_0^l \vec{B}.\overrightarrow{dl} = \mu_0 \vec{I}$	Ampere's Circuital Law

Q3.12. What is continuity equation.

Consider a small volume element ΔV located inside a conducting medium the current density (J) has the direction of current flow.

If there is no source or sink of charge inside ΔV, the current is steady and continuous so divergence of the current in that volume is zero;

$$\int_0^V \vec{\nabla}.\vec{J} dv = 0 \text{---------} (1)$$

Using the Divergence theorem, $\int_0^V \vec{\nabla}.\vec{J}dv = \oint_0^s \vec{J}.\overrightarrow{ds} = 0$

On the contrary the case where the current is not steady, difference between the current flowing into the volume and that flowing out of the volume mustbe equal to rate of change of electric charge inside the volume. Mathematically this is ;

$$\int_0^V \vec{\nabla}.\vec{J}dv = -\int \frac{\partial\rho}{\partial t}dv$$

When the net flow of current is outward$\nabla.J$is positive; but at the same time the charge inside the volume decreases giving a negative rate ;

Rate of decrease of charge $(-\frac{\partial\rho}{\partial t})$ this is expressed by continuity equation

$$\vec{\nabla}.\vec{J} = -\frac{\partial\rho_V}{\partial t}$$

This equation is the famous continuity equation that relates the current density to the charge density.

Q3.13. Obtain Maxwell's third and fourth equation for time varying fields in differential and integral forms

Maxwell's Third Equation with time varying field:

Statement: A time varying magnetic field produces an electromotive force which may establish a current in a suitable closed circuit.

Electromotive force induced in closed loop is negative rate of change of magnetic flux ø given as the lens law:

$$e = -\frac{d\phi}{dt}$$

$$\phi = \oint_0^S \vec{B}.ds$$

Therefore, $e = -\oint_0^S \left(\frac{d\vec{B}}{dt}\right).dS$--------------------(1)

The electromotive force is the work done in carrying unit charge around a closed loop.

Therefore, e = $\oint_0^l \vec{E}.\vec{dl}$ ---------------(2)

Using (1) and (2) we get;

$$\oint_0^l \vec{E}.\vec{dl} = -\oint_0^S \left(\frac{d\vec{B}}{dt}\right).dS \text{ --------------(3)}$$

By Stoke's Theorem,

$$\oint_0^l \vec{E}.\vec{dl} = \int_0^S (\vec{\nabla} \times \vec{E}).\vec{dS} \text{ --------------(4)}$$

Using (3) and (4) we get ,

$$\int_0^S (\vec{\nabla} \times \vec{E}).\vec{dS} = -\oint_0^S \left(\frac{d\vec{B}}{dt}\right).dS$$

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

This is the Maxwell's third equation for time varying fields.

Maxwell's Fourth Equation with time varying field:

Statement for Ampere's law: The line integral of magnetic field $\overrightarrow{H}$ around a closed path is exactly equal to the direct current enclosed by that path.

$$\oint \vec{H}.\vec{dl} = I$$

This leads to Maxwell's fourth Equation $\vec{\nabla} \times \vec{B} = \mu_0 J$. -------(5)

Taking the divergence on both sides of above equation (5) gives;

$$\vec{\nabla}.(\vec{\nabla} \times \vec{B}) = \mu_0 \vec{\nabla}.\vec{J}$$

As the divergence of a curl is always zero; hence the above equation implies that $\vec{\nabla}.\vec{J} = 0$; which is not true for non steady fields as it contradicts the continuity equation.

Thus for the case of non steady field, let us assume ,

$$\vec{\nabla} \times \vec{B} = \mu_0(\vec{J} + \overrightarrow{G})\text{----(6)}$$

Where, G is some unknown function. Taking the divergence of eq.(6) again

$$\vec{\nabla}.(\vec{\nabla} \times \vec{B}) = \mu_0 \overline{(\vec{\nabla}.\vec{J} + \vec{\nabla}.\vec{G})} = 0$$

We get;

$$\nabla . J = -\nabla . G \text{--------(7)}$$

From the continuity equation(refer Q10.) and Eq. (7) we have;

$$\nabla . G = \frac{\partial \rho}{\partial t} \text{-----------(8)}$$

From Maxwell's first Equation;

Electrostatics Gauss law: $\rho = \varepsilon_0 (\nabla . E)$---------(9)

Putting (9) in Eq.(8) we get$\nabla . G = \varepsilon_0 \nabla . \frac{\partial E}{\partial t}$-------------(10)

Thus $G = \varepsilon_0 \frac{\partial E}{\partial t}$-------------(11)

Putting Eq.(11) in Eq. (6) We get ;

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

This is the Maxwell's Fourth equation for time varying fields.

Maxwell's Equations for Time-Varying Fields:

S.NO.	DIFFERENTIAL FORM
1	$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$
2	$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

Q3.14. State the significance of Maxwells Equations.

(M.U. May 2019) (5 m)

1. Maxwell's equations are set of four complicated equations that describe the world of electromagnetism in a concise way.
2. Maxwell's equations describe how electric and magnetic field propagate, interact and how are they influenced by objects.
3. Maxwell's equations unite electromagnetism and optics. Since Maxwell's equations electricity, magnetism and light are understood as aspect of single objective the electromagnetic field.

4. In short Maxwell's equations for the first time summarized the fundamentals of electricity and magnetism in the most elegant way, forming a theory of electrodynamics.
5. Maxwell's equations are critical in understanding working of antennas, waveguide and satellite communication

Beauty of these equations makes these one of the greatest intellectual achievements of mankind.

Formula List for Electrodynamics

1. Divergence of a vector

$$(\vec{\nabla}.\vec{A})= \hat{\imath}\frac{\partial}{\partial x}A_x + \hat{\jmath}\frac{\partial}{\partial y}A_y + \hat{k}\frac{\partial}{\partial z}A_z$$

Where;

- $\vec{\nabla}= \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ is the del operator
- $\vec{A}$= Any arbitrary vector
- $\vec{\nabla}.\vec{A}$= divergence of vector A

2. Curl of a vector

$$(\vec{\nabla} \times \vec{A})=\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Where;

- $\vec{\nabla}= \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ is the del operator
- $\vec{A}$= Any arbitrary vector
- $(\vec{\nabla} \times \vec{A})$= curl of vector A

3. Gradient of a Scalar function

$$\vec{\nabla}f(x,y,z)= \hat{\imath}\frac{\partial}{\partial x}f(x,y,z) + \hat{\jmath}\frac{\partial}{\partial y}f(x,y,z) + \hat{k}\frac{\partial}{\partial z}f(x,y,z)$$

Where;

- $\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ is the del operator
- $f(x, y, z)$= Any arbitrary function of x, y and z.
- $\vec{\nabla} f(x, y, z)$= Gradient of a function f(x,y,z)

4. Conversion of Cartesian co-ordinates to cylindrical

$$r = \sqrt{x^2 + y^2}$$
$$\emptyset = tan^{-1}\left(\frac{y}{x}\right)$$
$$z = z$$

Where;

- $r, \emptyset, z$ = The Cylindrical co-ordinates
- x, y, z = The Cartesian co-ordinates

5. Conversion of Cartesian co-ordinates to Spherical

$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right)$$
$$\emptyset = \tan^{-1}\left(\frac{y}{x}\right)$$

Where;

- $r, \theta, \emptyset$ = The Spherical co-ordinates
- x, y, z = The Cartesian co-ordinates

ELECTRODYNAMICS PROBLEMS

Q1. Convert the point P(1,3,5) from Cartesian to Cylindrical and Spherical polar coordinates

Given:- X= 1; Y= 3; Z=5

Formula:- Cylindrical- Cartesian $r = \sqrt{x^2 + y^2}$; $\emptyset = tan^{-1}\left(\frac{y}{x}\right)$; $z = z$

Spherical-Cartesian;

$r = \sqrt{x^2 + y^2 + z^2}$; $\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$; $\emptyset = tan^{-1}\left(\frac{y}{x}\right)$

Solution:-

In cylindrical polar coordinates, (r, $\emptyset, z$)

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 3^2} = \sqrt{10} = 3.162$$

$$\emptyset = tan^{-1}\left(\frac{y}{x}\right) = tan^{-1}(3) = 71.57$$

Z = 5

In Spherical polar coordinates (r,$\theta, \emptyset$)

$$r=\sqrt{x^2 + y^2 + z^2} = \sqrt{1^2 + 3^2 + 5^2} = \sqrt{35} = 5.916$$

$$\emptyset = tan^{-1}\left(\frac{y}{x}\right) = tan^{-1}(3) = 71.57$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) = \tan^{-1}\left(\frac{\sqrt{1^2+3^2}}{5}\right) = 32.3$$

Ans:- Coordinates of P in cylindrical coordinates are (3.162, 71.57, 5) and in spherical polar coordinates are (5.916, 71.57, 32.3)

Q2. Given vector $\vec{A}(x, y, z) = y\hat{\imath} + (x + z)\hat{\jmath}$ in Cartesian coordinate system at point P(-2,6,3). Convert the vector $\vec{A}$ into cylindrical and spherical coordinates.

Given:-$\vec{A} = y\hat{\imath} + (x + z)\hat{\jmath}$, x=-2, y=6, z=3

Formula:- Cylindrical- Cartesian $r = \sqrt{x^2 + y^2}$; $\emptyset = tan^{-1}\left(\frac{y}{x}\right)$; $z = z$

Spherical-Cartesian;

$r = \sqrt{x^2 + y^2 + z^2}$; $\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$; $\emptyset = tan^{-1}\left(\frac{y}{x}\right)$

__**Solution:-**__

Cylindrical coordinates:

$$r = \sqrt{(-2)^2 + 6^2} = 6.32$$

$$\emptyset = \frac{6}{-2} = 108.43^{\circ}$$

Z=3

Spherical coordinates:

$$r = \sqrt{(-2)^2 + 6^2 + 3^2} = 7$$

$$\theta = \frac{\sqrt{(-2)^2 + 6^2}}{3} = 64.62^{\circ}$$

$$\emptyset = \frac{6}{-2} = 108.43^{\circ}$$

__**Ans:-**$\vec{A} = (6.32\,\hat{r},\ 108.43\,\hat{\emptyset},\ 3\,\hat{z})$ **is cylindrical coordinates**__

__$\vec{A} = (7\,\hat{r},\ 64.62\,\hat{\theta},\ 108.43\,\hat{\emptyset})$ **is spherical coordinates.**__

Q3. Find the divergence and curl of the field $F = 30\hat{\imath} + 2xy\hat{\jmath} + 5xz^2\hat{k}$ in Cartesian coordinates.

Given:- $F = 30\hat{\imath} + 2xy\hat{\jmath} + 5xz^2\hat{k}$

Formula:-Divergence $(\vec{\nabla}.\vec{A})$= $\hat{\imath}\frac{\partial}{\partial x}A_x + \hat{\jmath}\frac{\partial}{\partial y}A_y + \hat{k}\frac{\partial}{\partial z}A_z$

Curl: $(\vec{\nabla} \times \vec{A})$=$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

Solution:-

Divergence, $\vec{\nabla}.\vec{F} = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).(30\hat{\imath} + 2xy\hat{\jmath} + 5xz^2\hat{k})$

$=\frac{\partial}{\partial x}(30) + \frac{\partial}{\partial y}(2xy) + \frac{\partial}{\partial z}(2xz^2) = 2x + 10xz$

$\vec{\nabla}.\vec{F} = 2x(1 + 5z)$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 30 & 2xy & 5xz^2 \end{vmatrix} = -5z^2\,\hat{\jmath} + 2y\,\hat{k}$$

Ans:- Divergence of field is $2x(1+5z)$ and its curl $= -5z^2\,\hat{\jmath} + 2y\,\hat{k}$

Q4. If $\emptyset(x,y,z) = 3(x^2y - y^2x)$, Calculate gradient.

Given:-$\emptyset(x,y,z) = 3(x^2y - y^2x)$

Formula:-

Solution:-

Grad $\emptyset = \nabla\emptyset = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\{3(x^2y - y^2x)\}$

$= \hat{\imath}(6xy - 3y^2) + \hat{\jmath}(3x^2 - 2yx) + \hat{k}(0)$

Therefore, $\nabla\emptyset = \hat{\imath}(6xy - 3y^2) + \hat{\jmath}(3x^2 - 2yx)$

Ans:- The gradient is , $\nabla\emptyset = \hat{\imath}(6xy - 3y^2) + \hat{\jmath}(3x^2 - 2yx)$.

Q5. A region is specified by the potential function given by $\emptyset = 4x^2 + 3y^2 - 9z^2$. Calculate electric field strength.

Given:- $\emptyset = 4x^2 + 3y^2 - 9z^2$.

Formula:- $\vec{E} = -grad(potential\ function)$

<u>**Solution:-**</u>

$$\begin{aligned}\vec{E} &= -grad(potential\ function)\\ &= -grad\ \emptyset = -\nabla[4x^2 + 3y^2 - 9z^2]\\ &= -\left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(4x^2 + 3y^2 - 9z^2)\end{aligned}$$

Therefore, $\vec{E} = -8x\hat{\imath} - 6y\hat{\jmath} + 18z\hat{k}$

<u>**Ans:- The electric field strength is $\vec{E} = -8x\hat{\imath} - 6y\hat{\jmath} + 18z\hat{k}$.**</u>

Unit 5: <u>NANOTECHNOLOGY</u>

Q5.1. What are nano materials and what is nanotechnology?

We know all materials are composed of atoms with different sizes. Any material having very closely packed atoms within the size range of 1 to 100 nanometres are called nanomaterials.

The technology emerged from this is called Nanotechnology. Using these highly latest technology nano materials can be formed from metals, ceramics polymers and even from liquids.

Q5.2. What are two approaches in nanotechnology?

(M.U. Dec. 2015, 16, 17; May 2015, 19) [5 marks]

There are two methods for the production of nanomaterials: as shown in *Figure 5.2.1*.

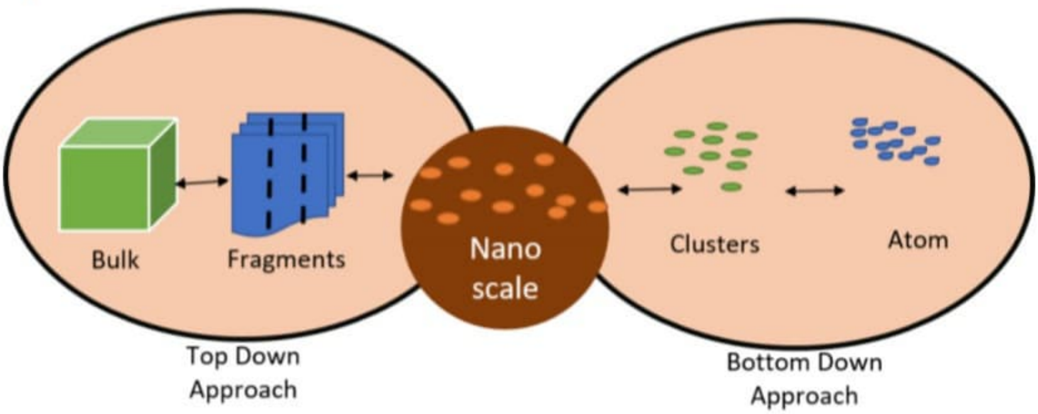


Figure 5.2.1: Techniques of Nano synthesis

(I) Bottom-up approach:
In these nanomaterials are made by building atom by atom or molecule by molecule.
These approaches include:
1)chemical synthesis
2)self-assembly
3)positional assembly

(II) Top-down approach:
In this a bulk material is broken or reduced in size or pattern. The techniques developed under this are modified or improved one what we have in use to fabricate micro-processors, micro-electro-mechanical system.

Q5.3. Write a short note on Scanning Electron Microscope (SEM)?

(M.U. May 2013, 14, 17; Nov 2018; Dec 2013, 14, 16, 19) [5marks]

- SEM is used to obtain images of surface of thickness. Also, a thin specimen can be studied. Construction of TEM includes an arrangement that makes it possible for an electron beam to scan the specimen similar to that we have in TV picture tubes.
- Here as shown in *Figure 5.3.1* electron beam is obtained from electron gun and it is made to pass through a condenser lens. Next stage is the scanning coil which is used to focus the electron beam on a small spot-on specimen surface and also to scan the surface like electron beam scans in TV picture tubes.

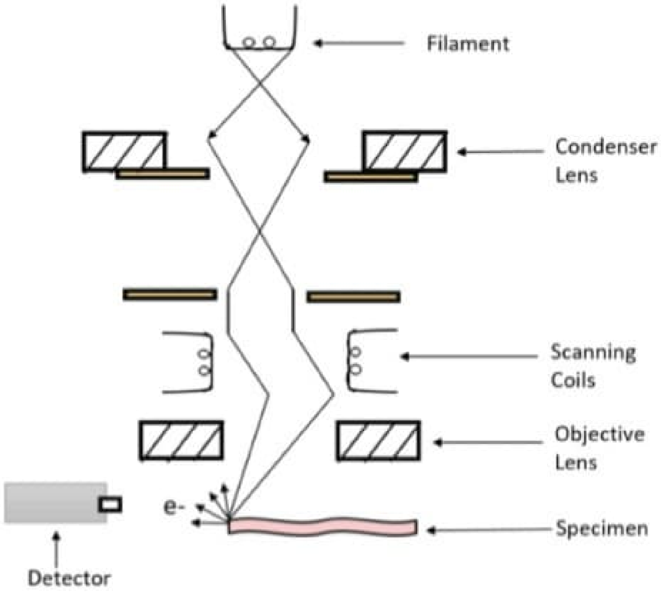


Figure 5.3.1: Scanning Electron Microscope

- Image formation in SEM is due to two main combining aspects
    a. Scattering of electron beams is because of atoms on the surface of the specimen and these atoms have different scattering power.
    b. Topographical variations of the surface.
- Actually, the aspects mentioned above are also responsible for the contrast which is essential for image formation.
- During the scanning of atoms by electron beam, the scattered electrons intensities are measured by detector and then displayed on the screen. If the scattering is high at a particular point during the scanning, the corresponding point on the viewing screen will be bright and for low scattering, the corresponding point on the screen will be dark. This develops required contrast for a clear image of the specimen.
- Specimen as small as 50A° size may be clearly resolved by SEM.

Q5.4. Write a short note on Atomic Force Microscope?

(M.U. Dec. 2012, 16, 17; May 2015, 16, 19) [5 marks]

Atomic Force Microscope or the AFM is a scanning microscope used as an imaging device. It works on the basic principle of recording the scattering of electromagnetic radiation i.e. when the laser beams are scattered off of the surface of the sample material, the readings are recorded by the microscope. These readings are detailed to the atomic level and the three dimensional images produced have atomic resolution.

- The AFM consists of a sharp tip probe which is attached to a cantilever on the top. The tip of the probe is about 1 nm in radius and the length of the cantilever is about 10 nm . The surface of the cantilever is highly reflective.
- A laser beam is passed through an optical fibre to be made incident on the surface of the cantilever. As the probe moves around the surface of the sample it experiences different kinds of forces.
- This in turn makes the cantilever undergo deflection. These deflections are recorded with the help of photo detectors.
    - Depending upon the material the forces could be electrostatic, mechanical, magnetic or even Van Der Waals forces.
- The readings from the photo detectors are then processed and used to create three dimensional images of the topography of the sample.
- The resolution of the image is in nanometers.

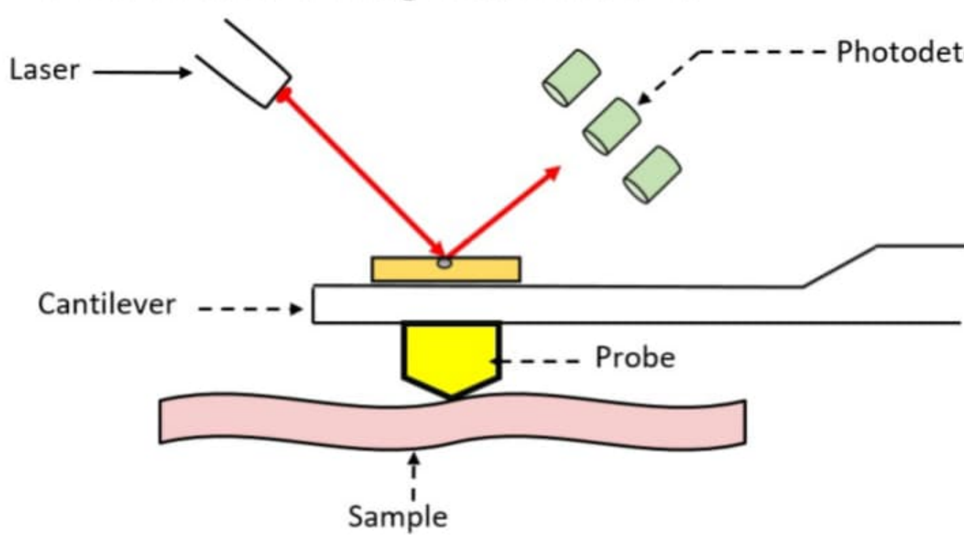


Figure 5.4.1: Atomic Force Microscope

Q5.5. Explain the Ball Milling method of nanoparticle synthesis?
(M.U. May 2013, 14, 17, 18; Nov. 2018; Dec. 2013, 14, 16, 19) [5 marks]

- Ball Milling method is a physical/mechanical process of creating nanoparticles.
- In this method small hard steel balls are used to make nanoparticles.
- These balls are kept in a container along with the powdered form of the material.
- These containers are made to rotate about their own axis while revolving around in a circular motion.
- This creates centrifugal and centripetal forces which cause the steel balls to collide with the powdered material continuously and create nanoparticles.
- The size of the steel balls are inversely proportional to the size of the nanoparticles they create.
- This method is generally used to create nanoparticles of metals and alloys.

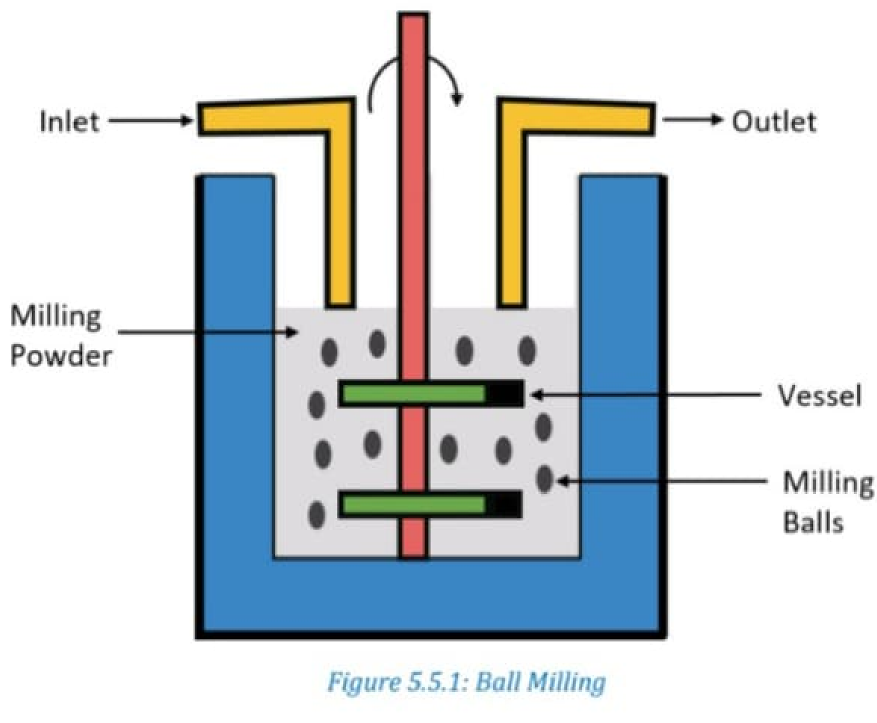


Figure 5.5.1: Ball Milling

Q5.6. Explain the Sputtering method of nanoparticle synthesis?

(M.U. May 2019) [5 mark]

Sputtering is a process in which the target material is subjected to high energy particles. This causes the material to vaporise and the nanoparticles of this material is collected in the collecting rod.

A typical arrangement of thin film deposition using sputtering is shown in the given figure. A sufficiently large potential difference is maintained between the substrate and the target. Argon gas is introduced into the enclosure at low pressure which can be varied.

The argon atoms get ionized due to the large potential difference. The positive argon ions hit the target with a large velocity and dislodge its atoms. The atoms move towards the substrate and get deposited on it.

The thickness of the film can be controlled by varying the argon gas pressure and the time for which the sputtering process is carried out. Thickness as small as a fraction of a nanometre, i.e., atomic monolayers, have been successfully deposited using this method.

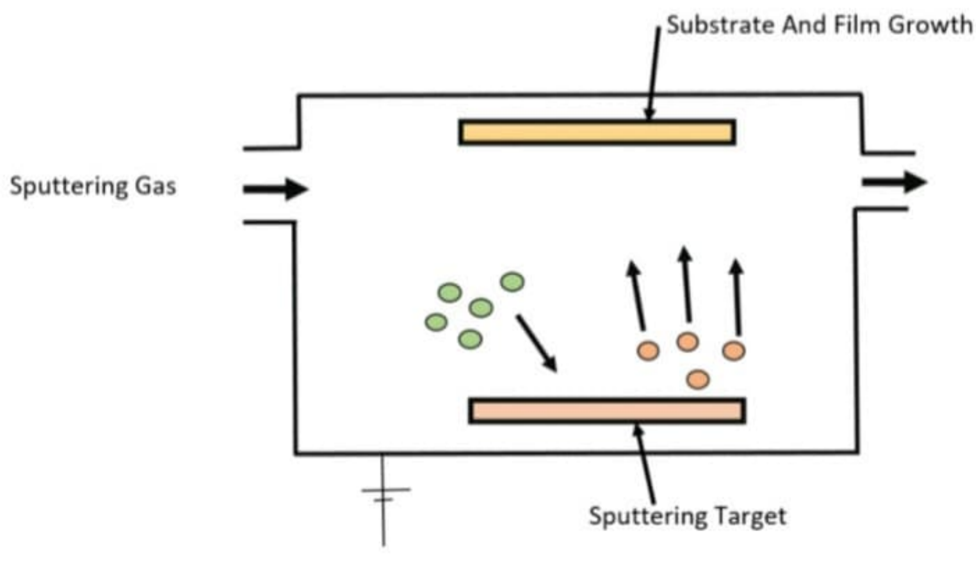


Figure 5.6.1: Sputtering

Q5.7. Explain the Vapour Deposition method of nanoparticle synthesis?

(M.U. May 2013, 14, 17, 18; Nov. 2018; Dec. 2013, 14, 16, 19) [5 marks]

The bulk material kept in a crucible is evaporated and the particles formed are blown away by using an inert gas towards the liquid nitrogen cooled cylinder called cold finger.

This assembly is placed in an evacuated chamber.

The evaporated particles get condensed and are collected on the cold finger, which are scraped off and fall into collection tray from the funnel.

The size of particles is controlled by changing the distance between the crucibles and the cold finger and by changing the inert gas pressure.

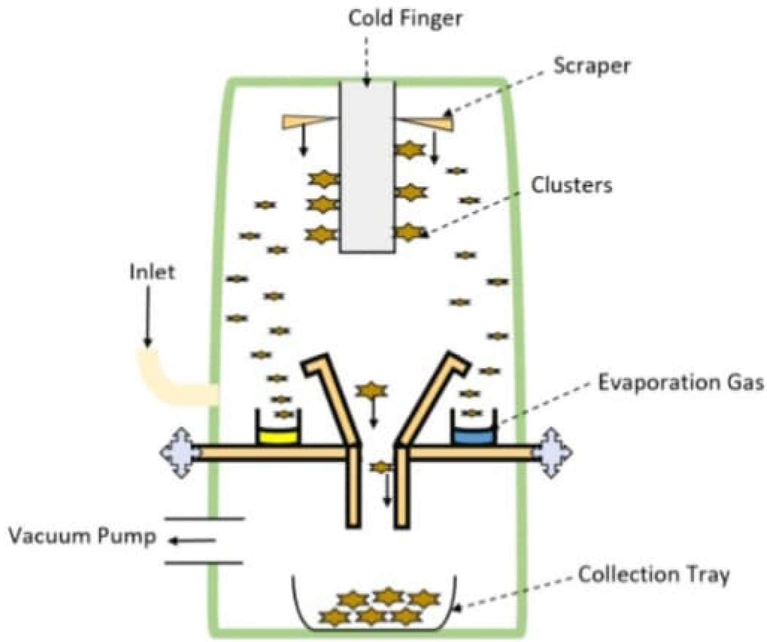


Figure 5.7.1: Vapour Deposition

Q5.8. Explain the SOL- Gel Technique method of nanoparticle synthesis?

(M.U. May 2013, 14, 17, 18; Nov. 2018; Dec. 2013, 14, 16, 19) [5 marks]

- Sol-gel method is a chemical process used to synthesize nanoparticles. It is one of the best and efficient methods to synthesize nanoparticles.
- In this method a solution is used with particles suspended in it.
- A powdered form of the material to be synthesized is mixed with chemicals to form the solution or sol. As time passes, long chained polymers are formed in this solution.
- These long chained polymers result in the formation of gel. Only a part of the sol gets converted to gel.
- The sol-gel method with the cavitation effect produces nanoparticles.
  - The sol-gel method is a bottom up approach and it can be used to produce nanoparticles of almost any material.

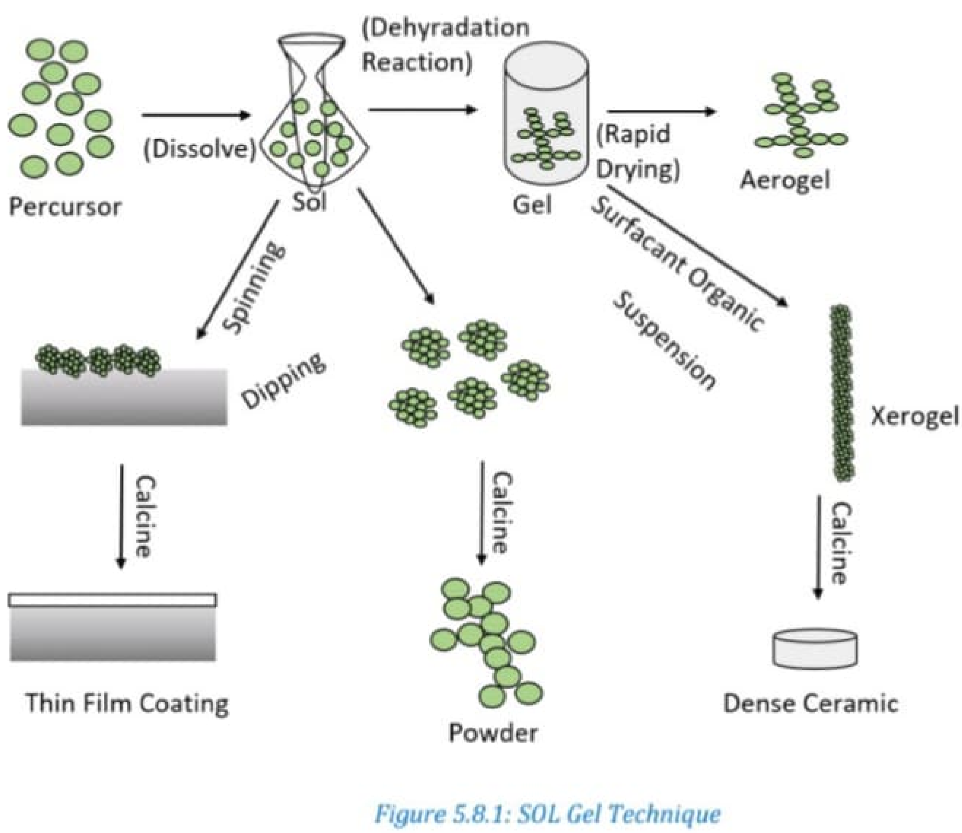


Figure 5.8.1: SOL Gel Technique